Composite Structures:

Syllabus:

1. Introduction
2. Simply-Supported Composite Beams
3. Continuous Beams
4. The Shear Connection
5. Composite Columns
6. Composite Slabs

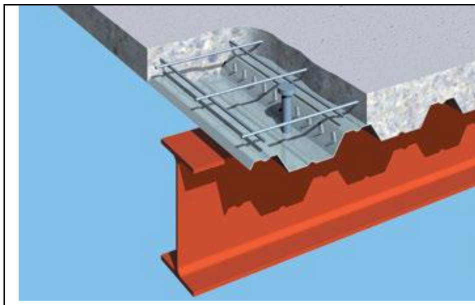
References:

1. William T. Segui "Steel Design" 5th edition, 2013.
2. Mc CORMAC JACK C. STEPHEN F. CSERNAK "STRUCTURAL STEEL DESIGN" 5TH Edition, 2012.
3. Roger P. Johnson *"Composite* **Structures of Steel and Concrete** Beams, Slabs, Columns and Frames for Buildings" Fourth edition, 2019.
4. Chen Wai-Fah "Structural Engineering Handbook", CRC Press LLC, 1999.
5. ANSI/AISC 360-22 "Specification for Structural Steel Buildings" August 1, 2022.

Introduction

1. Definition:

- A composite structure consists of a flexural stiff steel member – for example an I or H-section – with a longitudinal shear connection connected to the concrete.



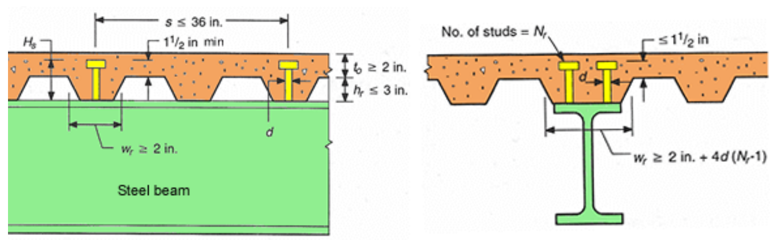
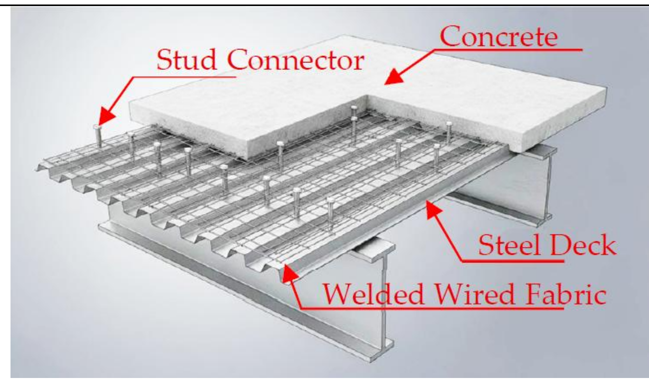


Figure 1: Composite deck



- From this structural action, a composite structure obtains its load-bearing resistance.
- The choice between steel, concrete and composite construction for a particular structure depends on many factors.
- Composite construction is particularly competitive for medium- or long-span structures where a concrete slab or deck is needed for other reasons, where there is a premium for rapid construction, and where a low or medium level of fire protection to steelwork is sufficient.
- Although steel and concrete materials behave very differently, they complement each other perfectly: -

i. Concrete excels to resist compressive forces; steel is better to resist tensile forces.

ii. Steel elements are slender and therefore sensitive to flexural and lateral buckling, and local buckling. Due to composite action, these types of instability are prevented.
iii. Concrete protects steel against corrosion.
iv. Concrete protects steel in a fire situation, because steel heats up less quickly due to the large mass of concrete. In a fire situation, additionally redistribution of forces from the (warmer) steel to the (cooler) concrete takes place.
v. Steel ensures that the structure has sufficient plastic deformation capacity. This prevents the structure from sudden collapse without 'warning mechanism' like large deformations. Deformation capacity is also required to allow for a redistribution of internal forces.

2. Historical background:

- Use of composite or hybrid material solutions is of particular interest, due to the significant potential in overall performance improvement obtained in manufacturing and constructional technologies.
- Successful combinations of materials may even generate a new material, as in the case of reinforced concrete or, more recently, fibre-reinforced plastics.
- However, most often the synergy between structural components made of different materials has shown to be a fairly efficient choice.
- The most important example in this field is represented by the steel-concrete composite construction, the enormous potential of which is not yet fully exploited after more than one century since its first appearance.
- Composite bridges" and "composite buildings" appeared in the U.S. in the same year, 1894:

1. **The Rock Rapids Bridge in Rock Rapids, Iowa, made use of curved steel I-beams embedded in concrete.**
2. **The Methodist Building in Pittsburgh had concrete-encased floor beams.**

- The composite action in these cases relied on interfacial bond between concrete and steel.
- Efficiency and reliability of bond being rather limited, attempts to improve concrete-to-steel joining systems were made since the very beginning of the last century, as shown by the shearing tabs system patented by Julius Kahn in 1903 (Figure 2).

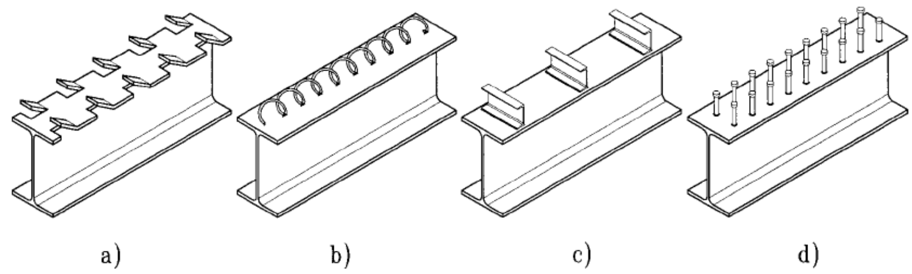


Figure 2: Historical development of shear connectors.
(a) Shearing tabs system (Julius Kahn 1903).
(b) Spiral connectors.
(c) Channels.
(d)Welded studs.

- Development of efficient mechanical shear connectors progressed quite slowly, despite the remarkable efforts both in Europe (spiral connectors and rigid connectors) and North America (flexible channel connectors).
- The use of welded headed studs (in 1956) was hence a substantial breakthrough.
- Since then, the metal studs have been by far the most popular shear transferring device in steel-concrete composite systems for both building and bridge structures.
- The significant interest raised by this "new material (steel-concrete composite)" prompted a number of studies on composite members (columns and beams) and connecting devices.
- The increasing level of knowledge then enabled development of Code provisions, which first appeared for buildings (the New York City Building Code in 1930) and subsequently for bridges (the AASHO specifications in 1944).

3. <u>Advantages of Composite Construction:</u>

a) high stiffness and strength (beams, girders, columns, and moment connections)
b) high ductility and toughness, and satisfactory damping properties (e.g., encased columns, beam-to-column connections)
c) satisfactory performance under fire conditions (all members and the whole system)
d) high constructability (e.g., floor decks, tubular infilled columns, moment connections)

4. <u>Design Codes:</u>

- The complexity of the response of composite steel-concrete systems, and the number of possible different situations in practice led to the use of design methods developed by empirical processes.

- They are based on, and calibrated against, a set of test data.
- Therefore, their applicability is limited to the range of parameters covered by the specific experimental background.
- This feature makes the reference to codes, and in particular to their application rules, of substantial importance for any text dealing with design of composite structures.
- Example of extensively used codes are: AISC-LRFD Specifications and Eurocode.

5. Materials:

Information on the properties of structural steel, profiled sheeting, concrete and reinforcement is readily available. Only that which has particular relevance to composite structures is given here.

- Concrete reaches its maximum compressive stress at a strain of between 0.002 and 0.003, and at higher strains it crushes, losing almost all of its compressive strength.
- It is very brittle in tension, having a strain capacity of only about 0.0001 (i.e. 0.1mm per metre) before it cracks.
- Steel yields at a strain similar to that given for the maximum stress in concrete, but on further straining the stress continues to increase slowly, until (for a typical structural steel) the total strain is **at least 30 times the yield strain**.

a) Concrete:

- Composite action implies that forces are transferred between steel and concrete components. The transfer mechanisms are fairly complex.
- Design methods are supported mainly by experience and test data, and their use **should be restricted to the range of concrete grades** and strength classes sufficiently investigated.
- It should be noted that concrete strength significantly affects the local and overall performance of the shear connection, **due to the inverse relation between the resistance and the strain capacity of this material**.
- Therefore, the capability of redistribution of forces within the shear connection is limited by the use of high strength concretes, and consequently the applicability of plastic

analysis and of design methods based on full redistribution of the shear forces supported by the connectors.

- The LRFD specifications prescribe for composite flexural elements that concrete meet quality requirements of ACI, made with ASTM C33 with concrete unit weight not less than 14.4 kN/m^3.
- This allows for the development of the full flexural capacity.
- A restriction is also imposed on the concrete strength in composite compressed members to ensure consistency of the specifications with available experimental data: the strength upper limit is 55 N/mm^2 and the lower limit is 20 N/mm^2 for normal weight concrete, and 27 N/mm^2 for lightweight concrete.
- The recommendations of Eurocode 4 are applicable for concrete strength classes up the C50/60, i.e., to concretes with cylinder characteristic strength up to 50 N/mm^2.
- The use of higher classes should be justified by test data.
- The strength classes for concrete defined in EN 1992-1-1 extend to class C90/105.
- Class C80/95 concrete was used in columns for the Shard building in London in 2012, and concrete-filled steel tube (CFST) columns with f'c > 100MPa have been used in buildings in the US and Japan.
- Lightweight concretes with unit weight not less than 16 kN/m^3 can be used.

Note: In British codes for concrete, f_{ck} and f_{cu} are respectively characteristic cylinder and cube strengths.

b) Reinforcing Steel:

- Rebars with yield strength up to 500N/mm^2 are acceptable in most instances.
- The reinforcing steel should have **adequate ductility when plastic analysis is adopted** for continuous beams.
- This factor should be carefully considered in the selection of the steel grade, in particular when high strength steels are used.
- A different requirement is implied by the limitation of 380 N/mm^2 (55 ksi) specified by AISC for the yield strength of the reinforcement in encased composite columns; this is

aimed at ensuring that **buckling of the reinforcement does not occur before complete yielding** of the steel components.

- The modulus of elasticity for reinforcement, Es, is normally taken as 200kN/mm^2; but in a composite section it may be assumed to have the value for structural steel, Ea = 210 kN/mm^2, as the error is negligible.

c) Structural Steel

- Structural steel alloys with yield strength **up to 355 N/mm^2** (50 ksi for American grades) can be used in composite members, without any particular restriction.
- Studies of the performance of composite members and joints made of high strength steel are available covering a yield strength range up to 780 N/mm^2 (113 ksi). However, significant further research is needed to extend the range of structural steels up to such levels of strength.
- Rules applicable to steel grades Fe420 and Fe460 (with fy D 420 and 460 N/mm^2, respectively) have been recently included in the Eurocode 4.
- Account is taken of the influence of the higher strain at yielding on the possibility to develop the full plastic sagging moment of the cross-section, and of the greater importance of buckling of the steel components.
- The AISC specification applies the same limitation to the yield strength of structural steel as for the reinforcement.

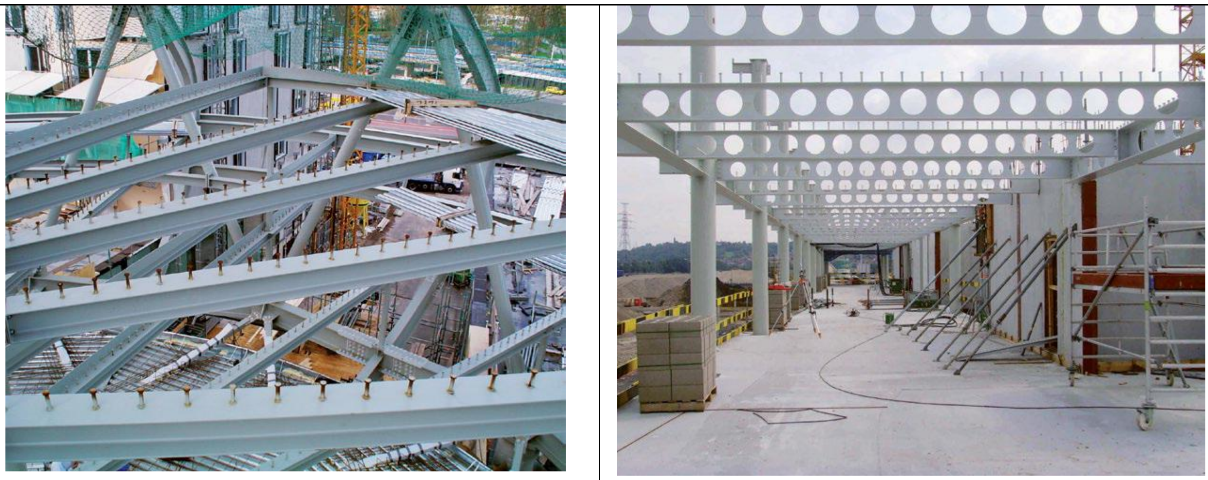


Figure 3: Steel sections used in composite construction

d) Steel Decking (Profiled steel sheeting):

- A wide range of shapes, depths (from 38 to 200 mm), thicknesses (from 0.76 to 1.52 mm), and steel grades (with yield strength from 235 to 460 MPa) may be adopted.
- Mild steels are commonly used, which ensure satisfactory ductility.
- Elastic properties of the material may be assumed to be as for structural steel.
- The minimum thickness of the sheeting is dictated by protection requirements against corrosion.
- Zinc coating about 0.02mm thick on each face should be selected, the total mass of which should depend on the level of aggressiveness of the environment.
- A coating of total mass 275 g/m^2 may be considered adequate for internal floors in a non-aggressive environment.

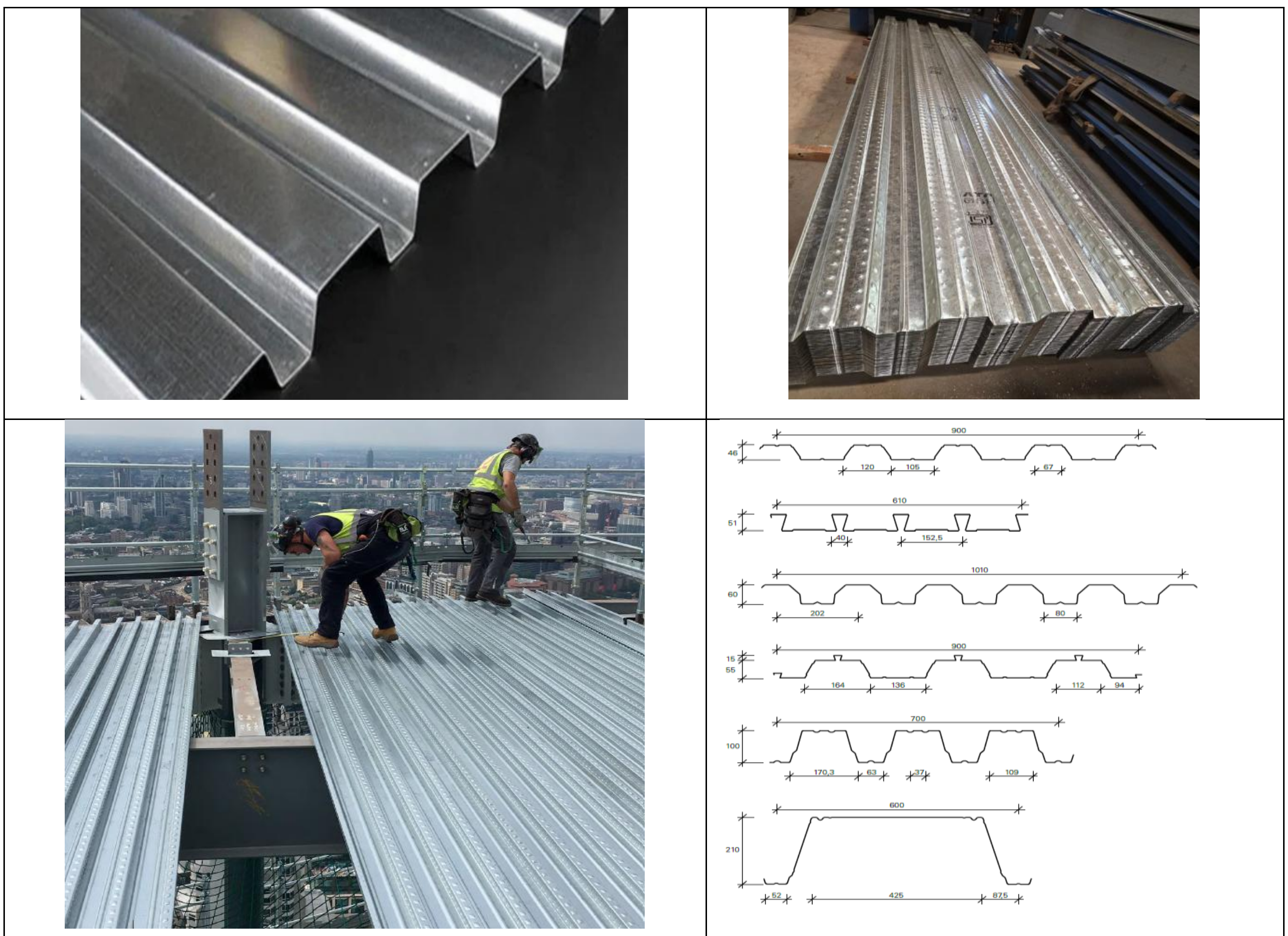


Figure 4: Steel decking sheets

e) Shear connectors:

- In the early years of composite construction, many types of connectors were in use.
- This market is now dominated by automatically-welded headed studs.
- The steel quality of the connectors should be selected according to the method of fixing (usually welding or screwing).
- The welding techniques also should be considered for welded connectors (studs, anchors, hoops, etc.).
- Design methods implying redistribution of shear forces among connectors impose that the connectors do possess adequate deformation capacity.
- A problem arises concerning the mechanical properties to be required to the stud connectors.
- Standards for material testing of welded studs are not available.
- These connectors are obtained by cold working the bar material, which is then subject to localized plastic straining during the heading process.
- The Eurocode hence specifies requirements for the ultimate-to-yield strength ratio (fu/fy ≥1.2) and to the elongation at failure (not less than 12% on a gauge length of $5.65\sqrt{A_o}$ where A_o is the cross-sectional area of the tensile specimen) to be fulfilled by the finished (cold drawn) product.
- Such a difficulty in setting an appropriate definition of requirements in terms of material properties leads many codes to prescribe, for studs, cold bending tests after welding as a means to check "ductility".

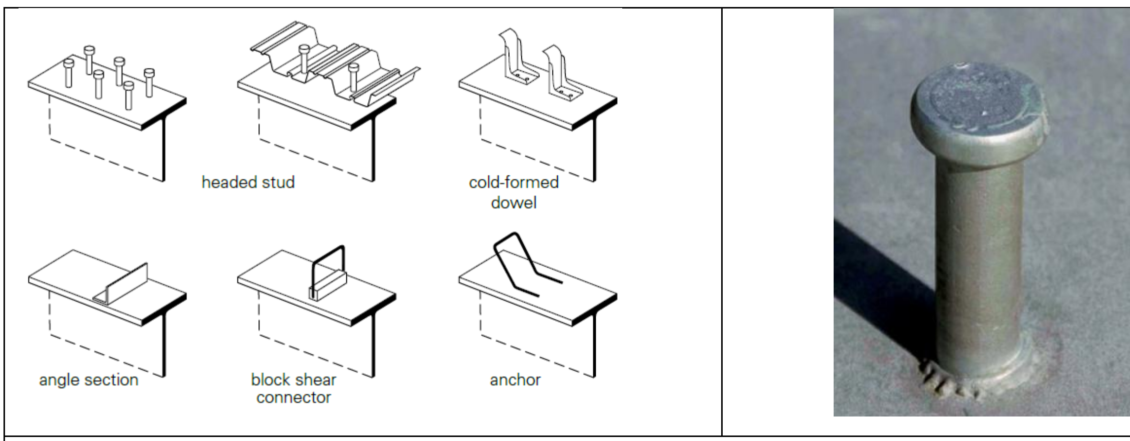


Figure 5: Types of shear connectors

Nominal Strength of Composite Sections According to AISC 360 -22:

- The nominal strength of composite sections shall be determined in accordance with either:
  a) The plastic stress distribution method,
  b) the strain compatibility method,
  c) the elastic stress distribution method, or
  d) the effective stress-strain method.
- The tensile strength of the concrete shall be neglected in the determination of the nominal strength of composite members.
- Local buckling effects shall be evaluated for filled composite members.
- Local buckling effects need not be evaluated for encased composite members or composite plate shear walls meeting the requirements of AISC 360.

a) Plastic Stress Distribution Method

- For the plastic stress distribution method, the nominal strength shall be computed assuming that steel components have reached a stress of fy in either tension or compression, and concrete components in compression due to axial force and/or flexure have reached a stress of 0.85 $f'c$, where $f'c$ is the specified compressive strength of concrete, (MPa).
- For round HSS (Hollow Structural Sections) filled with concrete, a stress of 0.95$f'c$ is permitted to be used for concrete components in compression due to axial force and/or flexure to account for the effects of concrete confinement.

b) Strain Compatibility Method

- For the strain compatibility method, a linear distribution of strains across the section shall be assumed, with the maximum concrete compressive strain equal to 0.003 (mm/mm).
- The stress-strain relationships for steel and concrete shall be obtained from tests or from published results.

Note: The strain compatibility method can be used to determine nominal strength for irregular sections and for cases where the steel does not exhibit elastoplastic behavior.

c) Elastic Stress Distribution Method

- For the elastic stress distribution method, the nominal strength shall be determined from the superposition of elastic stresses for the limit state of yielding or concrete crushing.

a) Effective Stress-Strain Method

For the effective stress-strain method, the nominal strength shall be computed assuming strain compatibility and effective stress-strain relationships for structural steel, reinforcing steel, and concrete components accounting for the effects of local buckling, yielding, interaction, and concrete confinement.

II. Material Limitations

For concrete, structural steel, and reinforcing steel in composite systems, the following limitations shall be met unless the design is based on the requirements of Appendix 2:

(a) For the determination of the available strength, concrete shall have a specified compressive strength, *f'c*, of not less than 21 MPa nor more than 69 MPa for normal weight concrete and not less than 21 MPa nor more than 41 MPa for lightweight concrete.

(b) The specified minimum yield stress of structural steel used in calculating the strength of composite members shall not exceed 525 MPa.

(c) The specified minimum yield stress of reinforcing bars used in calculating the strength of composite members shall not exceed 550 MPa.

- The design of filled composite members constructed from materials with strengths above the limits noted in this section shall be in accordance with Appendix 2.

Note: Appendix 2 of AISC 360-22 includes equations for determining the available strength of rectangular filled composite members with either the specified minimum yield stress of structural steel exceeding 525 MPa but less than 690 MPa or specified compressive strength, *f'c* exceeding 69 MPa) but less than 100 MPa.

Table 1: Limitation on characteristic strength (N/mm2) in modern design codes.

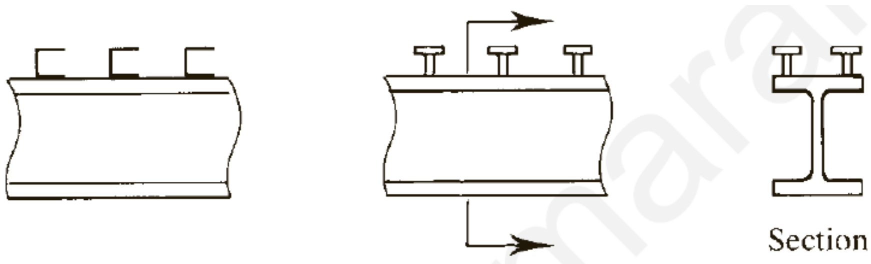
Codes	Steel yield strength (N/mm^2)	Concrete cylinder strength, (N/mm^2)
EN 1994–1–1	235 ~ 460	20 ~ 50
AISC 360–16	≤525	21 ~ 69
GB 50936	235 ~ 420	25 ~ 67
Architectural Institute of Japan (AIJ)	235 ~ 440	18 ~ 90
EN 1992–1–1	–	12 ~ 90
EN 1993–1–1, EN 1993–1–12	235 ~ 700	–

Table 2: Limitations on ductility, elongation at failure and ultimate strain for mild steel and high tensile steel.

Steel	Ratio f_u/f_y	Elongation at failure	Ultimate strain
Mild steel	≥1.10	15%	$\varepsilon_u \geq 15\varepsilon_y$ ($\varepsilon_y = f_y/E$)
High tensile steel	≥1.05	10%	$\varepsilon_u \geq 15\varepsilon_y$

<u>Full and Partial Connection:</u>

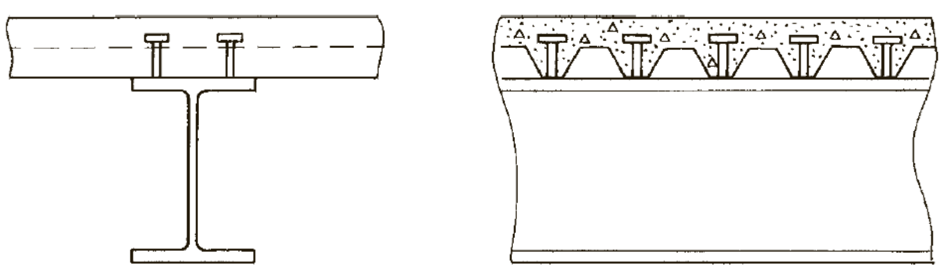
The unified behavior of the section, composed of steel shape and reinforced concrete, is possible only if horizontal slippage between the two components is prevented. That can be accomplished if the horizontal shear at the interface is resisted by connecting devices known as anchors or shear connectors. These devices which can be steel headed studs or short lengths of small steel channel shapes are welded to the top flange of the steel beam at prescribed intervals and provide the connection mechanically through anchorage in the hardened concrete.



Studs are the most commonly used type of anchors, and more than one can be used at each location if the flange is wide enough to accommodate them.
One reason for the popularity of steel headed stud anchors is their ease of installation. It is essentially a one-worker job made possible by an automatic tool that allows the operator to position the stud and weld it to the beam in one operation.
A certain number of anchors will be required to make a beam fully composite. Any fewer than this number will permit some slippage to occur between the steel and concrete; such a beam is said to be partially composite. Partially composite beams actually are more efficient than fully composite beams.

Deck floors:

Most composite construction in buildings utilizes formed steel deck, which serves as formwork for the concrete slab and is left in place after the concrete cures. This metal deck also contributes to the strength of the slab.
The deck can be used with its ribs oriented either perpendicular or parallel to the beams. In the usual floor system, the ribs will be perpendicular to the floor beams and parallel to the supporting girders.
The studs are welded to the beams from above, through the deck. Since the studs can be placed only in the ribs, their spacing along the length of the beam is limited to multiples of the rib spacing. The figure below shows a slab with formed steel deck and the ribs perpendicular to the beam axis.



- Almost all highway bridges that use steel beams are of composite construction, and composite beams are frequently the most economical alternative in buildings.
- Although smaller, lighter rolled steel beams can be used with composite construction, this advantage will sometimes be offset by the additional cost of the studs. Even so, other

advantages may make composite construction attractive. Shallower beams can be used, and deflections will be smaller than with conventional non-composite construction.

EFFECTIVE FLANGE WIDTH:

- AISC I3.1a requires that the effective width of floor slab on each side of the beam centerline be taken as the smallest of:

1. one eighth of the span length,
2. one half of the beam center-to-center spacing, or
3. the distance from the beam centerline to the edge of the slab.

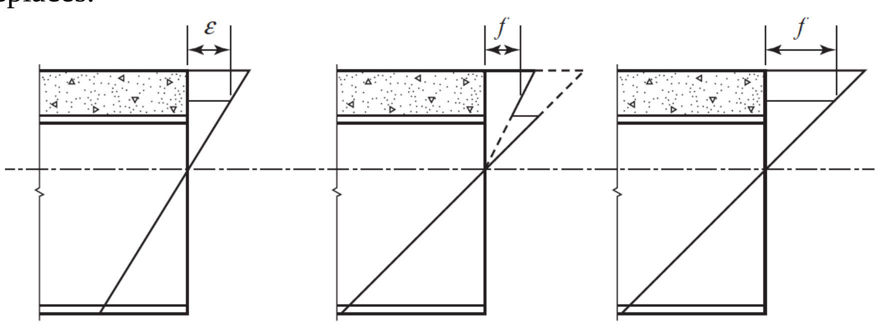
- The third criterion will apply to edge beams only, so for *interior* beams, the full effective width will be the smaller of one fourth the span length or the center-tocenter spacing of the beams (assuming that the beams are evenly spaced).

Elastic Analysis:

- When the interface slip can be neglected, a similar procedure for the analysis of reinforced concrete sections can be adopted for composite members subject to bending. In fact, the cross-sections remain plane and then the strains vary linearly along the section depth.
- Flexural and shearing stresses in beams of homogeneous materials can be computed from the formulas:

  $f_b = \frac{M\,c}{I}$ and $f_v = \frac{V\,Q}{I\,t}$

- The composite beams are not homogeneous, however, and these formulas are not valid. To be able to use them, an artifice known as the transformed section is employed to "convert" the concrete into an amount of steel that has the same effect as the concrete.
- This procedure requires the strains in the fictitious steel to be the same as those in the concrete it replaces.



(a) Strain (ε) (b) Actual Stress ($f = E\varepsilon$) (c) Stress in Transformed Section

- If the slab is properly attached to the rolled steel shape, the strains will be as shown, with cross sections that are plane before bending remaining plane after bending.
- However, a continuous linear stress distribution as shown in Part c of the figure is valid only if the beam is assumed to be homogeneous.

- It is assumed that the strain in the concrete at any point be equal to the strain in any
- replacement steel at that point:

$$\varepsilon_c = \varepsilon_s \quad \text{or} \quad \frac{f_c}{E_c} = \frac{f_s}{E_s}$$

and

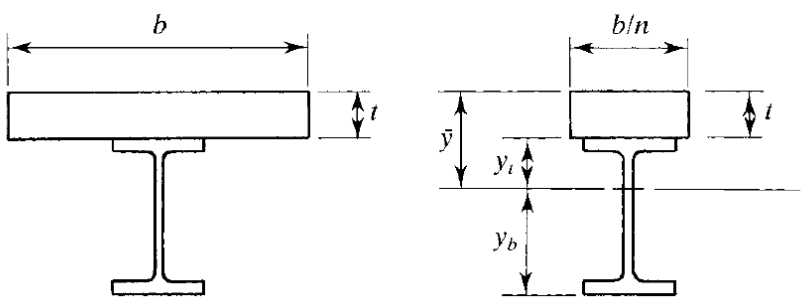
$$f_s = \frac{E_s}{E_c} f_c = n f_c$$

Ec = modulus of elasticity of concrete = $0.043 w_c^{1.5} \sqrt{f_c'}$ MPa
wc = weight of concrete per unit volume (1 500 ≤ *wc* ≤ 2 500 kg/m3)
Ec can be taken as $(4700\sqrt{f_c'})$ *for normal concrete*
Es = modulus of elasticity of steel = 200 000 MPa

- The stress diagram is also linear if the concrete stress is multiplied by the modular ratio:

$$n = \frac{E_s}{E_c}$$

The theory of the transformed sections can be used, i.e., the composite section is replaced by an equivalent all-steel section. The flange of which has a breadth equal to (b_{eff} /n).

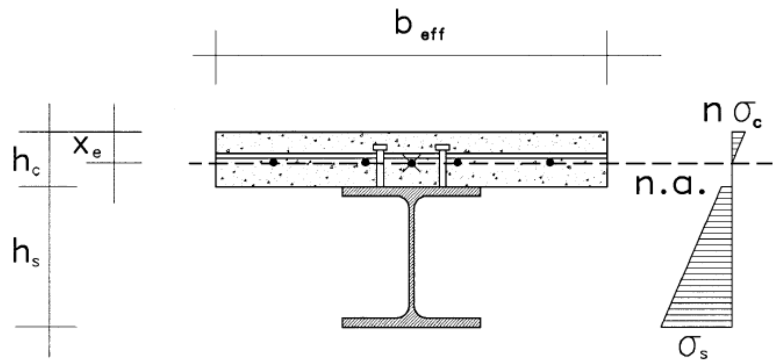


(a) **(b)**

To compute stresses, the neutral axis of this composite shape if first located and compute the corresponding moment of inertia. We can then compute bending stresses with the flexure formula. At the top of the steel.
The position of the neutral axis can be determined by imposing that the first moment of the effective area of the cross-section is equal to zero.
In the case of a solid concrete slab, and if the elastic neutral axis lies in the slab, this condition leads to the equation:

$$S = \frac{1}{n}\frac{b_{\text{eff}} \cdot x_e^2}{2} - A_s \cdot \left(\frac{h_s}{2} + h_c - x_e\right) = 0$$

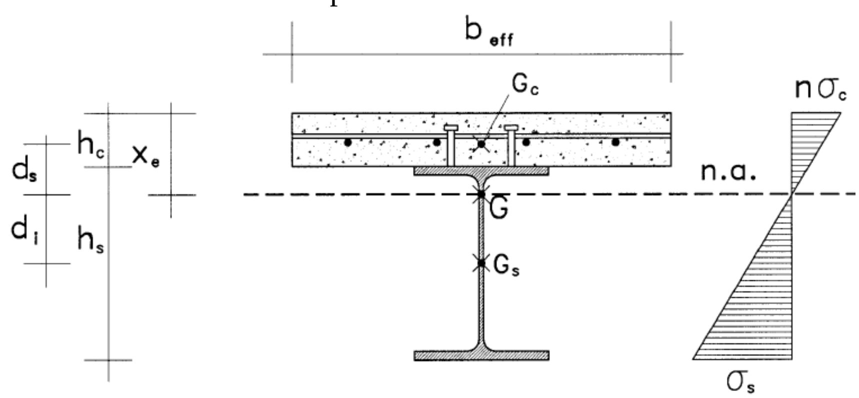
x_e: is the distance of elastic neutral axis to the top fiber of the concrete slab.



Once the value of x_e is calculated, the second moment of area of the transformed cross-section can be evaluated by the following expression:

$$I = \frac{1}{n}\frac{b_{\text{eff}} \cdot x_e^3}{3} + I_s + A_s \cdot \left(\frac{h_s}{2} + h_c - x_e\right)^2$$

If the elastic neutral axis lies in the steel profile the whole cross-section is effective.



In this case it results:

$$x_e = d_s + \frac{h_c}{2} \tag{6.7}$$

where

$$d_s = \frac{A_s}{A_s + b_{\text{eff}} \cdot h_c / n} \cdot \frac{h_c + h_s}{2}$$

where d_s is the distance between the centroid of the slab and the centroid of the transformed section;

$$I = I_s + \frac{1}{n}\frac{b_{\text{eff}} \cdot h_c^3}{12} + A^* \cdot \frac{(h_s + h_c)^2}{4} \tag{6.8}$$

where

$$A^* = \frac{A_s \cdot b_{\text{eff}} \cdot h_c / n}{A_s + b_{\text{eff}} \cdot h_c / n}$$

When the neutral axis depth and the second moment of area of the composite section are known, the maximum stress of concrete in compression and of structural steel in tension associated with a bending moment M are evaluated by the following expressions:

$$\sigma_c = \frac{1}{n}\frac{M}{I}\cdot x_e$$
$$\sigma_s = \frac{M}{I}\cdot(h_s + h_c - x_e)$$

These stresses must be lower than the relevant maximum design stresses allowed at the elastic limit condition.
NOTE: In the case of unshored construction, determination of the elastic stress distribution should take into account that the steel section alone resists all the permanent loads acting on the steelwork before composite action can develop.
In many instances, it is useful to define an "elastic moment of resistance" as the moment at which the strength of either structural steel or concrete is achieved.
This elastic limit moment can be determined as the lowest of the moments associated with the attainment of the elastic limit of steel and concrete, i.e.

$$M_{el} = \min\left\{ f_{c.d}\cdot\frac{n\cdot I}{x_e},\ f_{y.s.d}\cdot\frac{I}{h_s + h_c - x_e}\right\}$$

The stress check is then indirectly satisfied if (and only if) it results:

$$M \le M_{el}$$

where M is the maximum value of the bending moment for the load combination considered.

Example: A composite beam consists of a W410×53 (410mm×53kg/m) of A992 steel with a 125mm-thick and 2210mm wide reinforced concrete slab at the top. The strength of the concrete is $f'c = 28$MPa. Determine the maximum stresses in the steel and concrete resulting from a positive bending moment of 217 kN.m.
Note that A992 steel yield stress is (345 to 450)MPa, ultimate strength is (450 to 550)MPa, and E=200GPa.
Solution:
$E_c = 4700\sqrt{f_c'} = 24.87\ GPa$
n=Es/Ec= 8.04 , use n=8.0
Effective width= L/8= 2210/8.0 = 276.25mm
The location of the neutral axis can be found by applying the principle of moments with the axis of moments at the top of the slab.
1) try x ≤ hc

$$\frac{1}{n}\frac{b_{\text{eff}}\cdot x_e^2}{2} - A_s\cdot\left(\frac{h_s}{2} + h_c - x_e\right) = 0$$

Can be reduced to: $A.x^2 + B.x + C = 0$

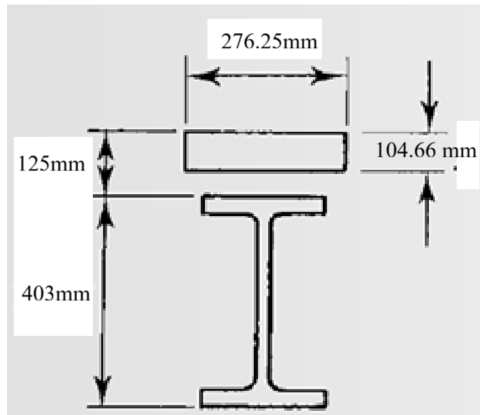
$$A = \frac{b_e}{2n} \qquad B = A_s \qquad C = -A_s\left(\frac{h_s}{2} + h_c\right)$$

A= 138.125 B= 6820 C= - -2.23E+06

X= 104.66mm < hc=125 mm   O.K.

$$I = \frac{1}{n}\frac{b_{\text{eff}} \cdot x_e^3}{3} + I_s + A_s \cdot \left(\frac{h_s}{2} + h_c - x_e\right)^2$$

I= 6.27E+08 mm^4



Maximum stresses:

$$\sigma_c = \frac{1}{n}\frac{M}{I} \cdot x_e$$
$$\sigma_s = \frac{M}{I} \cdot (h_s + h_c - x_e)$$

Compression stress in concrete is

$\sigma_c = 4.53\ MPa$

$\sigma_s = 146\ MPa$

Wide-Flange Sections or W Shapes SI Units

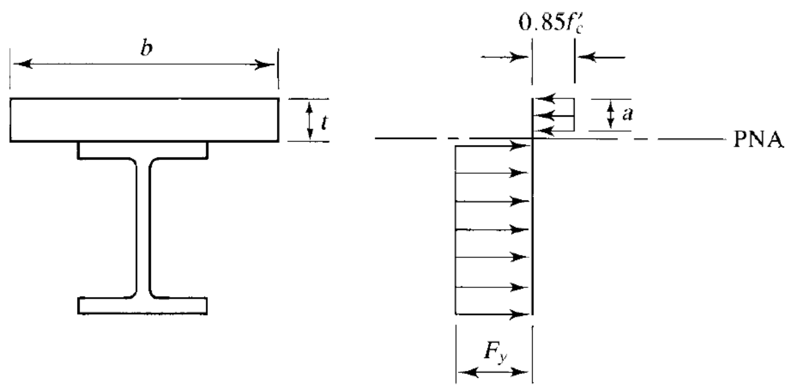
Designation	Area A	Depth d	Web thickness t_w	Flange width b_f	Flange thickness t_f	x–x axis I	x–x axis S	x–x axis r	y–y axis I	y–y axis S	y–y axis r
mm × kg/m	mm^2	mm	mm	mm	mm	$10^6\ \text{mm}^4$	$10^3\ \text{mm}^3$	mm	$10^6\ \text{mm}^4$	$10^3\ \text{mm}^3$	mm
W610 × 155	19 800	611	12.70	324.0	19.0	1 290	4 220	255	108	667	73.9
W610 × 140	17 900	617	13.10	230.0	22.2	1 120	3 630	250	45.1	392	50.2
W610 × 125	15 900	612	11.90	229.0	19.6	985	3 220	249	39.3	343	49.7
W610 × 113	14 400	608	11.20	228.0	17.3	875	2 880	247	34.3	301	48.8
W610 × 101	12 900	603	10.50	228.0	14.9	764	2 530	243	29.5	259	47.8
W610 × 92	11 800	603	10.90	179.0	15.0	646	2 140	234	14.4	161	34.9
W610 × 82	10 500	599	10.00	178.0	12.8	560	1 870	231	12.1	136	33.9
W460 × 97	12 300	466	11.40	193.0	19.0	445	1 910	190	22.8	236	43.1
W460 × 89	11 400	463	10.50	192.0	17.7	410	1 770	190	20.9	218	42.8
W460 × 82	10 400	460	9.91	191.0	16.0	370	1 610	189	18.6	195	42.3
W460 × 74	9 460	457	9.02	190.0	14.5	333	1 460	188	16.6	175	41.9
W460 × 68	8 730	459	9.14	154.0	15.4	297	1 290	184	9.41	122	32.8
W460 × 60	7 590	455	8.00	153.0	13.3	255	1 120	183	7.96	104	32.4
W460 × 52	6 640	450	7.62	152.0	10.8	212	942	179	6.34	83.4	30.9
W410 × 85	10 800	417	10.90	181.0	18.2	315	1 510	171	18.0	199	40.8
W410 × 74	9 510	413	9.65	180.0	16.0	275	1 330	170	15.6	173	40.5
W410 × 67	8 560	410	8.76	179.0	14.4	245	1 200	169	13.8	154	40.2
W410 × 53	6 820	403	7.49	177.0	10.9	186	923	165	10.1	114	38.5
W410 × 46	5 890	403	6.99	140.0	11.2	156	774	163	5.14	73.4	29.5
W410 × 39	4 960	399	6.35	140.0	8.8	126	632	159	4.02	57.4	28.5
W360 × 79	10 100	354	9.40	205.0	16.8	227	1 280	150	24.2	236	48.9
W360 × 64	8 150	347	7.75	203.0	13.5	179	1 030	148	18.8	185	48.0
W360 × 57	7 200	358	7.87	172.0	13.1	160	894	149	11.1	129	39.3
W360 × 51	6 450	355	7.24	171.0	11.6	141	794	148	9.68	113	38.7
W360 × 45	5 710	352	6.86	171.0	9.8	121	688	146	8.16	95.4	37.8
W360 × 39	4 960	353	6.48	128.0	10.7	102	578	143	3.75	58.6	27.5
W360 × 33	4 190	349	5.84	127.0	8.5	82.9	475	141	2.91	45.8	26.4

Ultimate Flexural Strength:

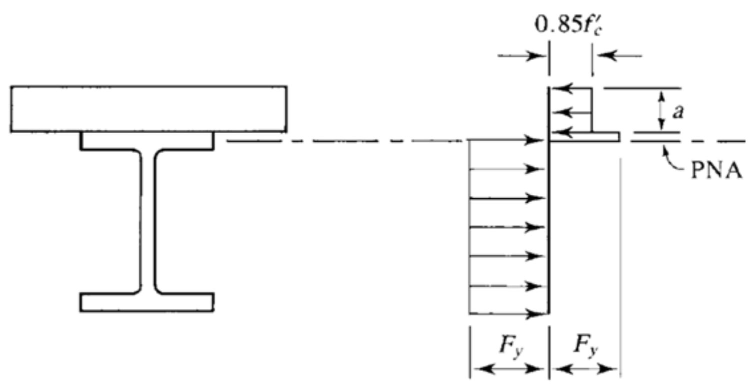
- The nonlinear stress state of composite beams under sagging moments usually permits the plastic moment of the composite section to be achieved.
- In most instances the plastic neutral axis lies in the slab and the whole of the steel section is in tension, which results in:
  1. local buckling not being a critical phenomenon
  2. concrete strains being limited, even when the full yielding condition of the steel beam is achieved.
- Therefore, the plastic method of analysis is applicable to most simply supported composite beams.
- This approach is based on equilibrium equations at ultimate, and does not depend on the constitutive relations of the materials and on the construction method. The plastic moment can be computed by application of the rectangular stress block theory. Moreover, the concrete may be assumed, in composite beams, to be stressed uniformly over the full depth $(a = x_{pl})$ of the compression side of the plastic neutral axis, while for the reinforced concrete sections usually the stress block depth is limited to $(\beta_1 x_{pl})$.
- In most cases, the nominal flexural strength will be reached when the entire steel cross section yields and the concrete crushes in compression (for positive bending moment).
- The corresponding stress distribution on the composite section is called a *plastic stress distribution.*
- The AISC Specification provisions for flexural strength are as follows:
  - For shapes with compact webs—that is, $h/t_w \leq 3.76\sqrt{E/F_y}$—the nominal strength M_n is obtained from the plastic stress distribution.
  - For shapes with $h/t_w > 3.76\sqrt{E/F_y}$, M_n is obtained from the elastic stress distribution corresponding to first yielding of the steel.
  - For LRFD, the design strength is $\phi_b M_n$, where $\phi_b = 0.90$.
  - For ASD, the allowable strength is M_n/Ω_b, where $\Omega_b = 1.67$.
- All W, M, and S shapes tabulated in the AISC *Manual* have compact webs (for flexure) for $Fy \leq 385$ MPa, so the first condition will govern for all composite beams except those with built-up steel shapes.
- The concrete stress is a uniform compressive stress of 0.85*f'c*, extending from the top of the slab to a depth that may be equal to or less than the total slab thickness.
- This distribution is the *Whitney equivalent stress distribution*, which has a resultant that matches that of the actual stress distribution (ACI-318).
- The nominal moment capacity can be found by computing the moment of the couple formed by the compressive and tensile resultants.
- This can be accomplished by summing the moments of the resultants about any convenient point.

- Because of the connection of the steel shape to the concrete slab, lateral-torsional buckling is no problem once the concrete has cured and composite action has been achieved.
- When a composite beam has reached the plastic limit state, the stresses will be distributed in one of the three ways shown in Figure:

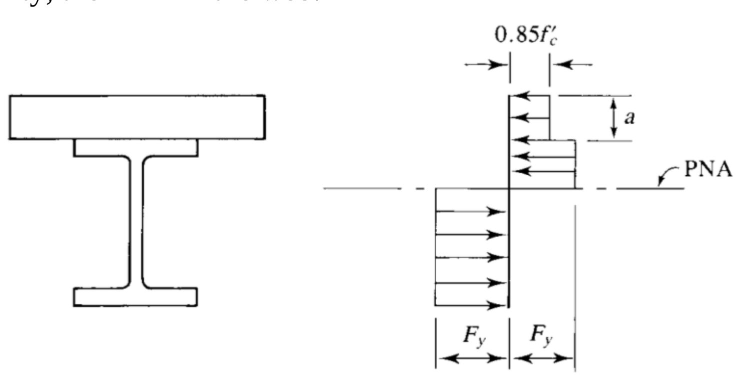
1) The plastic neutral axis (PNA) is in the slab and full tensile yielding stress section. The tensile strength of concrete is small and is discounted, so no stress is shown where tension is applied to the concrete. This condition will usually prevail when there are enough stud anchors provided to prevent slip completely, that is, to ensure full composite behavior.



2) The concrete stress block extends the full depth of the slab, and the PNA is in the flange of the steel shape. Part of the flange will therefore be in compression to augment the compressive force in the slab.



3) The third possibility, the PNA in the web.



- To determine which of the three cases governs, compute the compressive resultant as the smallest of

1. $F_{s,max} = A_s F_y$
2. $F_{c,max} = 0.85 f_c' A_c$
3. $\sum Q_n$

Where:
A_s: cross-sectional area of steel shape
A_c: area of concrete $= h_c \times b_e$
$\sum Q_n$: total shear strength of the stud anchors.

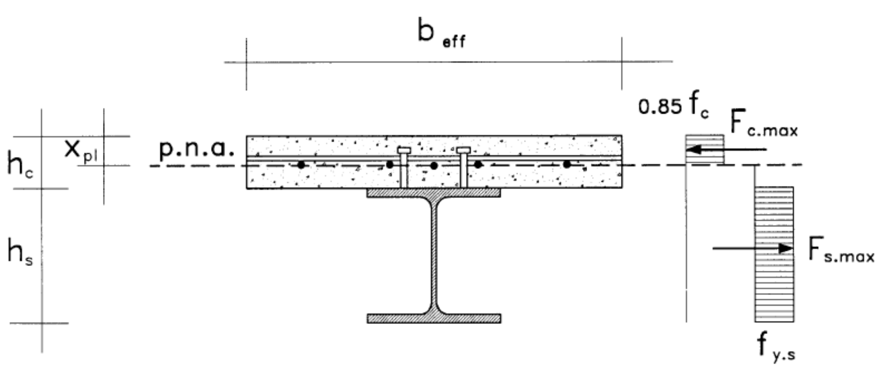
- Each possibility represents a horizontal shear force at the interface between the steel and the concrete.
- When the first possibility controls, the steel is being fully utilized, and the PNA is in the slab.
- The second possibility corresponds to the concrete controlling, and the PNA will be in the steel, either in the flange or in the web.
- The third case governs only when there are fewer studs than required for full composite behavior, resulting in partial composite action.

<u>I.</u> If $F_{c,max}$ is greater than $F_{s,max}$ the plastic neutral axis lies in the slab; in this case the interaction force between slab and steel profile is $F_{s,max}$ and the plastic neutral axis depth is defined by a simple first order equation:

$$F_{c.\max} > F_{s.\max} \Rightarrow F_c = F_s = A_s \cdot f_{y.s}$$

$$0.85 b_{\text{eff}} \cdot x_{pl} \cdot f_c = A_s \cdot f_{y.s}$$

$$x_{pl} = \frac{A_s \cdot f_{y.s}}{0.85 b_{\text{eff}} \cdot f_c}$$



2. If $F_{s,max}$ is greater than $F_{c,max}$, the neutral axis lies in the steel profile; in this case it results:

$$F_{c.\max} < F_{s.\max} \Rightarrow F_c = F_s = 0.85 b_{\text{eff}} \cdot h_c \cdot f_c$$

Two different cases can take place; in the first case:

$$F_c > F_w = d \cdot t_w \cdot f_{y.s}$$

where
t_w = the web thickness
d = the clear distance between the flanges
and:

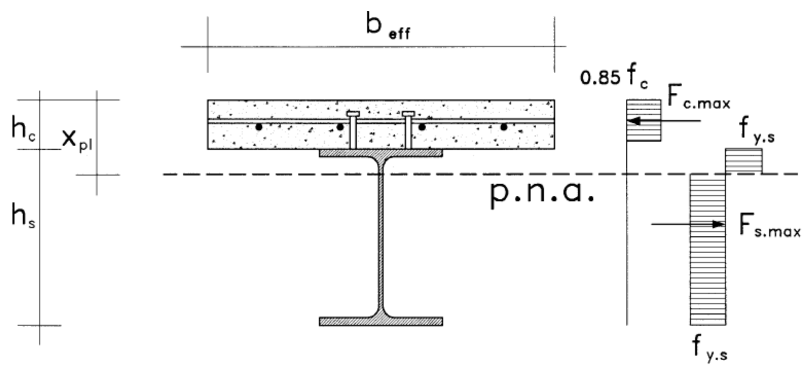
$$M_{pl} \simeq F_{s.\max} \cdot \frac{h_s}{2} + F_c \cdot \frac{h_c}{2}$$

In the second case:

$$F_c < F_w = d \cdot t_w \cdot f_{y.s}$$

and:

$$M_n = M_{pl} = M_{pl.s} + F_c \cdot \frac{h_s + h_c}{2} - \frac{F_c^2}{4 \cdot t_w \cdot f_{y \cdot s}}$$



Example: Compute the available strength of the composite beam of the previous example. Assume that sufficient stud anchors are provided for full composite behavior. The steel section is W410×53 (410mm×53kg/m), the slab is 125mm-thick and 2210mm wide. $f'c$ = 28MPa fy= 345MPa.

Solution:

The tensile force in steel section, T is

$T = A_s F_y = 345 \times 6820 =$ 2352900 N= 2352.9 kN

The compression force in concrete, C is

$C = 0.85 f_c' A_c = 0.85 \times 28 \times 125 \times 2210 =$6574750 N= 6574.75 kN

- The steel controls. This means that the PNA lies in RC slab and full depth of the slab is not needed to develop the required compression force.
- The resultant compressive force can also be expressed as:

$C = 0.85f_c' \times a \times b_e = T = 2352900\ N$

from which we obtain

$$a = x_{pl} = \frac{2352900}{0.85f_c' \times b_e} = 44.73\ mm$$

- The force C will be located at the centroid of the compressive area at a depth of a/2 from the top of the slab.
- The resultant tensile force T (equal to C) will be located at the centroid of the steel area.
- The moment arm of the couple formed by C and T is

$$d = \frac{h_s}{2} + h_c - \frac{a}{2} = \frac{403}{2} + 125 - \frac{44.73}{2} = 304.1mm$$

The nominal strength is the moment of the couple,

$$M_n = T \times d = C \times d = 2352.9 \times 0.3041 = 715.5\ kN.m$$

The design (ultimate) strength according to LRFD, is $\emptyset M_n = 0.9 \times 715.5 = 644.0\ kN.m$

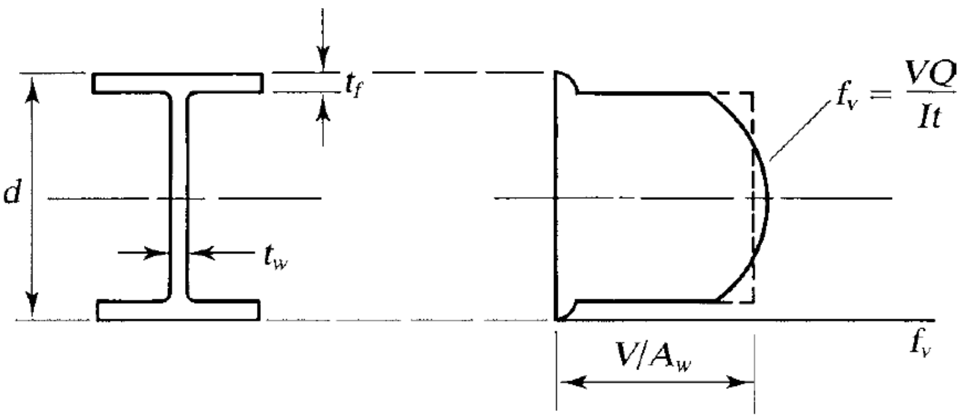
The design (allowable) strength according to ASD, is $M_n/\Omega = 715.5/1.67 = 428.5\ kN.m$

Vertical Shear:

In composite elements shear is carried mostly by the web of the steel profile; the contributions of concrete slab and steel flanges can be neglected in the design due to their width. Taking the shear yield stress as 60% of the tensile yield stress, it can write the equation for the stress in the web at failure as:

$$f_v = \frac{V_n}{A_w} = 0.6F_y$$

where Aw = area of the web.



AISC Specification Requirements for Shear:

For LRFD, the relationship between required and available strength is

$$V_u \leq \phi_v V_n$$

where

V_u = maximum shear based on the controlling combination of factored loads
ϕ_v = resistance factor for shear

For ASD, the relationship is

$$V_a \leq \frac{V_n}{\Omega_v}$$

where

V_a = maximum shear based on the controlling combination of service loads
Ω_v = safety factor for shear

Section G2.1 of the AISC Specification covers both beams with stiffened webs and beams with unstiffened webs. In most cases, hot-rolled beams will not have stiffeners. The basic strength equation is

$$V_n = 0.6F_yA_wC_v \qquad \text{(AISC Equation G2-1)}$$

where

A_w = area of the web ≈ dt_w

d = overall depth of the beam

C_v = ratio of critical web stress to shear yield stress

The value of C_v depends on whether the limit state is web yielding, web inelastic buckling, or web elastic buckling.

- For hot-rolled I shapes with

$$\frac{h}{t_w} \leq 2.24\sqrt{\frac{E}{F_y}}$$

The limit state is shear yielding, and

$$C_v = 1.0 \qquad \text{(AISC Equation G2-2)}$$

$$\phi_v = 1.00$$

$$\Omega_v = 1.50$$

Most W shapes with $F_y \leq 385$MPa fall into this category (see User Note in AISC G2.1).

This means that the nominal shear strength is therefore

$$V_n = 0.6F_yA_w$$

provided that there is no shear buckling of the web.

- The shear-moment interaction is not important in simply supported beams (in fact for usual loading conditions where the moment is maximum the shear is zero; where the shear is maximum the moment is zero). The situation in continuous beams is different.

Serviceability Limit States:

The adequacy of the performance under service loads requires that:

a) The use, efficiency, or appearance of the structure are not impaired.
b) Besides, the stress state in concrete also needs to be limited due to the possible associated durability problems.
c) Micro-cracking of concrete (when stressed over 0.5 *fc*) may allow development of rebars corrosion in aggressive environments.

The stiffness of a composite beams in sagging bending is far higher than in the case of steel members of equal depth, due to the significant contribution of the concrete flange. Therefore, deflection limitation is less critical than in steel systems.
However, the effect of concrete creep and shrinkage has to be evaluated, which may significantly increase the beam deformation as computed for short-term loads.

- Because of the large moment of inertia of the transformed section, deflections in composite beams are smaller than in non-composite beams.
- For un-shored construction, this larger moment of inertia is available only after the concrete slab has cured.
- Deflections caused by loads applied before the concrete cures must be computed with the moment of inertia of the steel shape only.
- A complication arises if the beam is subject to sustained loading, such as the weight of partitions, after the concrete cures.
- In positive moment regions, the concrete will be in compression continuously and is subject to a phenomenon known as creep.
- Creep is a deformation that takes place under sustained compressive load. After the initial deformation, additional deformation will take place at a very slow rate over a long period of time.
- The effect on a composite beam is to increase the curvature and hence the vertical deflection.
- Long-term deflection can only be estimated; one technique is to use a reduced area of concrete in the transformed section so as to obtain a smaller moment of inertia and a larger computed deflection.
- The reduced area is computed by using 2n or 3n instead of the actual modular ratio n.
- For ordinary buildings, the additional creep deflection caused by sustained dead load is very small.
- If a significant portion of the live load is considered to be sustained, then creep should be accounted for.
- A method for estimating the shrinkage deflection can be found in the Commentary to the AISC Specification.
- Use of the transformed moment of inertia to compute deflections in composite beams tends to underestimate the actual deflection (Viest, et al., 1997).

- To compensate for this, the Commentary to the Specification recommends that the computed transformed moment of inertia be reduced by one of two methods.

1. Reduce the computed moment of inertia by 25%.
2. Use a lower-bound moment of inertia (I_{LB}) which is a conservative underestimation of the elastic moment of inertia.

- The lower-bound moment of inertia approach is recommended by Viest, et al. (1997).
- The chief simplification made in the computation of (I_{LB}) is that only part of the concrete is used in computing the transformed area. Only that part of the concrete used in resisting the bending moment is considered to be effective.

- Under the assumption of full interaction, the usual formulae for beam deflection calculation can be used. As an example, the deflection under a uniformly distributed load, p, is obtained as:

$$\delta = \frac{5}{384}\frac{p \cdot l^4}{E_s \cdot I}$$

The value of the moment of inertia I of the transformed section, and hence the value of δ, depends on the modular ratio, n. Therefore, the effective modulus (EM) theory enables the effect of concrete creep to be incorporated in design calculations without any additional complexity. Determination of the deflection under sustained loads simply requires that an effective modular ration $n_{ef} = E_s/E_{c.ef}$ is used when computing I via Equation 6.6 or 6.8.

The effect of the shrinkage strain ε_{sh} could be evaluated considering that the compatibility of the composite beam requires a tension force N_{sh} to develop in the slab equal to:

$$N_{sh} = \varepsilon_{sh} \cdot E_{c.ef} \cdot b \cdot h_c \tag{6.29}$$

This force is applied in the centroid of the slab and, due to equilibrium, produces a positive moment M_{sh} equal to:

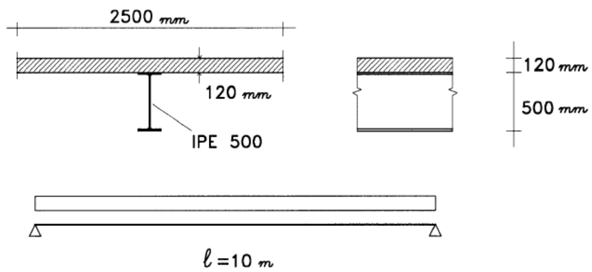
$$M_{sh} = N_{sh} \cdot d_s \tag{6.30}$$

The additional deflection can be determined as:

$$\delta_{sh} = \frac{M_{sh} \cdot l^2}{8 \cdot E_s \cdot I} = 0.125 \cdot \varepsilon_{sh} \cdot \frac{b \cdot h_c \cdot d_s}{n_{ef} \cdot I} \cdot l^2 \tag{6.31}$$

- The influence of shrinkage on the deflection is usually important in a dry environment and for span-to-beam depth ratios greater than 20.In partially composite beams, the deflection associated with the interface slip has also to be accounted for.
- The total deflection should be lower than limit values compatible with the serviceability requirements specific to the building system considered.

EXAMPLE on Composite beam with solid concrete slab: The cross-section of a simply supported beam with a span length *l* equals 10 m; the steel profile (IPE 500) is characterized by *As*= 11600 mm^2 and *Is*= 4.82 ×10^{8}mm^4. The solid slab has a thickness of 120 mm; the spacing with adjacent beams is 5000 mm. The beam is subjected to a uniform load p=40 kN/m (16 kN/m of dead load, 24 kN/m of live load) at the serviceability condition. A shored construction is considered.



Solution:

1. Determination of design moment:
   The design moment for service conditions is

$$M = \frac{40 \cdot 10^2}{8} = 500 \text{ kNm}$$

For the ultimate limit state, considering a factor of 1.35 for the dead load and 1.5 for the live load, it is

[1.2(16)+1.6(24)](10.10)/8= 720 kN.m

$$M = \frac{(1.35 \cdot 16 + 1.5 \cdot 24) \cdot 10^2}{8} = 720 \text{ kNm}$$

2. Evaluation of the effective width (Section 6.3.2):
   Applying Equation 6.4 it results:

$$b_{\text{eff}} = 2 \cdot \frac{l}{8} = 2 \cdot \frac{10000}{8} = 2500 \text{ mm}$$

Thus, only 2500 mm of 5000 mm are considered as effective.

3. Elastic analysis of the cross-section (Section 6.3.3):
   The following assumptions are made:

$$E_c = 30500 \text{ N / mm}^2$$
$$E_s = 210000 \text{ N / mm}^2$$

Therefore, at the short time the modular ratio results:

$$n = \frac{210000}{30500} = 6.89$$

Equation 6.5 becomes:

$$\frac{1}{6.89}\frac{2500}{2} \cdot x_e^2 - 11600 \cdot \left(\frac{500}{2} + 120 - x_e\right) = 0$$

then:

$$181.4x_e^2 + 11600 \cdot x_e - 4292000 = 0$$
$$x_e = 125.1 \text{ mm } > 120 \text{ mm}$$

i.e., the entire slab is under compression and Equation 6.7 shall be considered:

$$x_e = \frac{120}{2} + \frac{11600}{11600 + 120 \cdot 2500/6.89} \frac{120 + 500}{2} = 125.2 \text{ mm}$$

From Equation 6.8 it results:

$$\begin{aligned} A^* &= \frac{11600 \cdot 2500 \cdot 120/6.89}{11600 + 2500 \cdot 120/6.89} = 9160 \text{ mm}^2 \\ I &= \frac{1}{6.89} \frac{2500 \cdot 120^3}{12} + 4.82 \cdot 10^8 + 9160 \frac{(500 + 120)^2}{4} = 1.41 \cdot 10^9 \text{ mm}^4 \end{aligned}$$

The maximum stress in concrete and steel are the following (see Equations 6.9 and 6.10):

$$\begin{aligned} \sigma_c &= \frac{500 \cdot 10^6}{6.89 \cdot 1.41 \cdot 10^9} \cdot 125.2 = 6.4 \text{ N / mm}^2 \\ \sigma_s &= \frac{500 \cdot 10^6}{1.41 \cdot 10^9} \cdot (500 + 120 - 125.2) = 175.5 \text{ N / mm}^2 \end{aligned}$$

For the long term calculation, a creep coefficient $\phi = 2$ is assumed obtaining the following modular ratio:

$$n_{ef} = 6.89 \cdot 3 = 20.67$$

Equation 6.6 gives:

$$\begin{gathered} \tfrac{2500}{20.67 \cdot 2} \cdot x_e^2 - 11600 \cdot \left(\tfrac{500}{2} + 120 - x_e \right) = 0 \\ 60.47 \cdot x_e^2 + 11600 \cdot x_e - 4292000 = 0 \\ x_e = 187.2 \text{ mm } > 120 \text{ mm} \end{gathered}$$

The elastic neutral axis lies in the steel profile web and the slab is entirely compressed. Therefore, it is (Equations 6.7, and 6.8):

$$\begin{aligned} x_e &= \frac{120}{2} + \frac{11600}{11600 + 120 \cdot 2500/20.67} \frac{120 + 500}{2} = 197.7 \text{ mm} \\ A^* &= \frac{11600 \cdot 2500 \cdot 120/20.67}{11600 + 2500 \cdot 120/20.67} = 6447 \text{ mm}^2 \\ I &= \frac{2500 \cdot 120^3}{20.67 \cdot 12} + 4.82 \cdot 10^8 + 6447 \cdot \frac{(500 + 120)^2}{4} = 1.12 \cdot 10^9 \text{ mm}^4 \end{aligned}$$

The stresses result:

$$\begin{aligned} \sigma_c &= \frac{500 \cdot 10^6}{20.67 \cdot 1.12 \cdot 10^9} \cdot 197.7 = 4.3 \text{ N / mm}^2 \\ \sigma_s &= \frac{500 \cdot 10^6}{1.12 \cdot 10^9} \cdot (620 - 197.7) = 188.5 \text{ N / mm}^2 \end{aligned}$$

The concrete stress decreases 33% while the steel stress increases 7%.

4. Plastic analysis of the cross-section (Section 6.3.4):
The design strength of materials are assumed as $f_{c.d} = 14.2$ N/mm^2 (including the coefficient 0.85) for concrete and $f_{y.s.d} = 213.6$ N/mm^2 for steel. By means Equations 6.13, and 6.14 it is:

$$F_{c.\max} = 2500 \cdot 120 \cdot 14.2 = 4260000\text{N} = 4260 \text{ kN}$$
$$F_{s.\max} = 11600 \cdot 213.6 = 2477760\text{N} = 2478 \text{ kN}$$

Consequently, it is assumed:

$$F_c = F_s = 2478 \text{ kN}$$

and, considering the design strength in Equation 6.17:

$$x_{pl} = \frac{11600 \cdot 213.6}{2500 \cdot 14.2} = 69.8 \text{ mm}$$

The internal arm is (Equation 6.18):

$$h^* = \frac{500}{2} + 120 - \frac{69.8}{2} = 335 \text{ mm}$$

and the plastic moment results (Equation 6.19):

$$M_{pl} = 2478 \cdot 0.335 = 830 \text{ kNm}$$

Thus, the value of the plastic resistance is greater than the design moment at the ultimate conditions ($M = 720$ kNm).

5. Ultimate shear of the section:

$$V_n = 0.6 F_v A_v C_v$$

For all rolled I-sections the ratio

$$\frac{h}{t_w} \le 2.24\sqrt{\frac{E}{F_y}}$$

The limit state is shear yielding, and

$C_v = 1.0$ (AISC Equation G2-2)

$\phi_v = 1.00$

$\Omega_v = 1.50$

$$\frac{h}{t_w} = \frac{500}{10.2} = 49.02$$

$$2.24\sqrt{\frac{E}{F_y}} = 2.24\sqrt{\frac{200{,}000}{213.6}} = 68.54$$

Thus, $\frac{h}{t_w} < 2.24\sqrt{\frac{E}{F_y}}$ O.k.

$F_v = 213.6\ MPa$

$A_v = h_s \times t_w = 500 \times 10.2 = 5100$ mm^2

$V_n = 0.6(213.6)(5100)(1.0) = 653{,}616\ N = 653.6\ kN$

Design shear strength $= \Phi_v V_n$

$\Phi_v V_n$ =1.0×653.6=653.6 kN

The applied shear force is
$V_u = (1.2DL + 1.6LL) \times L = (1.2(16) + 1.6(24)) \times 10$ = 576 kN

$\Phi_v V_n > V_u$ O.K.

6. Control of deflection (Section 6.3.6):
The deflection at short term is

$$\delta(t = 0) = \frac{5}{384} \cdot \frac{40 \cdot 10000^4}{210000 \cdot 1.41 \cdot 10^9} = 17.6 \text{ mm} = \frac{1}{568} \cdot l$$

The deflection at long term is increased by the creep and shrinkage effects. The 50% of the live load is considered as long term load; thus, in this verification the load is $16 + 0.5 \cdot 24 = 28$ kN/m. If only the creep effect is taken into account, the following result is obtained:

$$\delta(t = \infty) = \frac{5}{384} \cdot \frac{28 \cdot 10000^4}{210000 \cdot 1.12 \cdot 10^9} = 15.5 \text{ mm}$$

As regards the shrinkage effect, a final value of the strain is assumed equal to

$$\varepsilon_{sh} = 200 \cdot 10^{-6}$$

and the increment of deflection due to shrinkage results (Equation 6.32):

$$\delta_{sh} = 0.125 \cdot 200 \cdot 10^{-6} \cdot \frac{120 \cdot 5000 \cdot (197.7 - 60)}{20.67 \cdot 1.12 \cdot 10^9} \cdot 10000^2 = 8.9 \text{ mm}$$

The final value of the deflection is

$$\delta(t = \infty) = 15.5 + 8.9 = 24.4 \text{ mm} = \frac{l}{410}$$

SHORED VERSUS UNSHORED CONSTRUCTION:

- Before the concrete has cured and attained its design strength (at least 75% of its 28-day compressive strength, *fc′*), there can be no composite behavior.
- This means that the weight of the slab in addition to other loads must be supported by the steel section only.
- Once the concrete has cured, composite action is possible, and all subsequently applied loads will be resisted by the composite beam.
- If the steel shape is supported at a sufficient number of points along its length before the slab is placed, the weight of the wet concrete will be supported by these temporary *shores* rather than by the steel.
- Once the concrete has cured, the temporary shoring can be removed, and the weight of the slab, as well as any additional loads, will be carried by the composite beam.
- If shoring is not used, however, the rolled steel shape must support not only its own weight, but also the weight of the slab and its formwork during the curing period.
- Once composite behavior is attained, additional loads, both dead and live, will be supported by the composite beam.

I. Unshored: Before Concrete Cures:

- AISC I3.lb requires that when temporary shoring is not provided, the steel shape alone must have sufficient strength to resist all loads applied before the concrete attains 75% of its strength, *fc′*.
- The flexural strength is computed in the usual way, based on Chapter (F) of the Specification (DESIGN OF MEMBERS FOR FLEXURE).
- Depending on its design, the formwork for the concrete slab may or may not provide lateral support for the steel beam.
- If not, the unbraced length *Lb* must be taken into account, and lateral-torsional buckling may control the flexural strength.
- If temporary shoring is not used, the steel beam may also be called on to resist incidental construction loads. To account for these loads, an additional 1 kN/m^2 is recommended (Hansell et al., 1978).

II. Unshored: After Concrete Cures:

- After composite behavior is achieved, all loads subsequently applied will be supported by the composite beam.
- At failure, however, all loads will be resisted by the internal couple corresponding to the stress distribution at failure.
- Thus, the composite section must have adequate strength to resist all loads, including those applied to the steel beam before the concrete cures (except for construction loads, which will no longer be present).

III. Shored Construction:

- In shored construction, only the composite beam need be considered, because the steel shape will not be required to support anything other than its own weight.

Example: A W300×74 (W-shape) acts compositely with a 100mm-thick concrete slab. The effective slab width is 1830 mm. Shoring is not used, and the applied bending moments are as follows: from the beam weight, $M_{beam} = 18$ kN.m; from the slab weight, $M_{slab} = 100$ kN.m; and from the live load, $M_L = 200$ kN.m. (Do not consider any additional construction loads in this example.) Steel is A992, and f'c= 28 MPa. Determine whether the flexural strength of this beam is adequate. Assume full composite action and assume that the formwork provides lateral support of the steel section before curing of the concrete.

Wide-Flange Sections or W Shapes SI Units

Designation	Area A	Depth d	Web thickness t_w	Flange width b_f	Flange thickness t_f	x–x axis I	x–x axis S	x–x axis r	y–y axis I	y–y axis S	y–y axis r
mm × kg/m	mm^2	mm	mm	mm	mm	10^6 mm^4	10^3 mm^3	mm	10^6 mm^4	10^3 mm^3	mm
W310 × 129	16 500	318	13.10	308.0	20.6	308	1940	137	100	649	77.8
W310 × 74	9 480	310	9.40	205.0	16.3	165	1060	132	23.4	228	49.7
W310 × 67	8 530	306	8.51	204.0	14.6	145	948	130	20.7	203	49.3
W310 × 39	4 930	310	5.84	165.0	9.7	84.8	547	131	7.23	87.6	38.3
W310 × 33	4 180	313	6.60	102.0	10.8	65.0	415	125	1.92	37.6	21.4
W310 × 24	3 040	305	5.59	101.0	6.7	42.8	281	119	1.16	23.0	19.5
W310 × 21	2 680	303	5.08	101.0	5.7	37.0	244	117	0.986	19.5	19.2

Solution:

Compute the nominal strength of the composite section. The compressive force C is the smaller of

The tensile force in steel section, T is

$$T = A_s F_y = 345 \times 9480 =$$

The compression force in concrete, C is

$$C = 0.85 f_c' A_c =$$

The PNA is in the concrete slab

a= 75.1 mm

The moment arm Z is

$$d = \frac{h_s}{2} + h_c - \frac{a}{2} = 217\ mm$$

The moment strength of the composite section is

$$M_n = T \times d = C \times d = 710\ kN.m$$

A. <u>The LRFD solution:</u>

1. Loads and strength of steel section only:

Before the concrete cures, there is only dead load (no construction load in this example). Hence, load combination A4-1 controls, and the factored load moment is

$M_u = 1.4(MD) = 1.4(18 + 100) = 165.2$ kN.m

$(M_{n,steel}) = Z_p \times f_y = 1162.23 \times 10^3 \times 345 = 401$ kN.m

$\Phi M_{n,steel} = 0.9 \times 401 = 361\ kN.m > M_u = 165.2\ kN.m$ (O.K.)

2. Loads and strength of composite section:

After the concrete has cured, the factored load moment that must be resisted by the composite beam is

Mu = 1.2MD + 1.6ML= *461.6 kN.m*
$\Phi M_n = 0.9 \times 710 = 639\ kN.m > M_u = 461.6\ kN.m$ (O.K.)

B. <u>The ASD solution:</u>

1. Before the concrete cures, there is only dead load (no constructional loads):
M=18+100= 118 kN.m
$(M_{n,steel}) = Z_p \times f_y$ =401 kN.m
Allowable moment capacity= $\frac{M_{n,steel}}{\Omega} = \frac{401}{1.67} = 240.1\ kN.m$ (which is greater than 118 kN.m)
2. After the concrete has cured, the required moment strength is:
$M_a = M_{DL} + M_{LL}$= 118+ 200= 318 kN.m
The nominal strength of composite section is:
$M_n = 710\ kN.m$
Allowable moment capacity= $\frac{M_n}{\Omega} = \frac{710}{1.67} = 425.1\ kN.m$ (which is greater than 218 kN.m)

The beam has sufficient flexural strength

<u>Note:</u>

- Obviously, shored construction is more efficient than unshored construction because the steel section is not called on to support anything other than its own weight.
- In some situations, the use of shoring will enable a smaller steel shape to be used.
- Most composite construction is unshored, however, because the additional cost of the shores, especially the labor cost, outweighs the small savings in steel weight that may result.

Example: A composite floor system consists of (W460 · 52) steel beams spaced at 2.75m and supporting a 120 mm-thick reinforced concrete slab. The span length is 9.1m. In addition to the weight of the slab, there is a 1.0 kN/m^2 moveable partition load and a live load of 6 kN.m^2. The steel is A992 (fy= 345MPa), and the concrete strength is $f'c$ = 28MPa. Investigate a typical interior beam for compliance with the AISC Specification if no temporary shores are used. Assume full lateral support during construction and an additional construction load of 1.0 kN/m^2. Sufficient steel anchors are provided for full composite action.

Designation	Area A	Depth d	Web thickness t_w	Flange width b_f	Flange thickness t_f	x–x axis I	x–x axis S	x–x axis r	y–y axis I	y–y axis S	y–y axis r
mm × kg/m	mm^2	mm	mm	mm	mm	10^6 mm^4	10^3 mm^3	mm	10^6 mm^4	10^3 mm^3	mm
W460 × 97	12 300	466	11.40	193.0	19.0	445	1 910	190	22.8	236	43.1
W460 × 89	11 400	463	10.50	192.0	17.7	410	1 770	190	20.9	218	42.8
W460 × 82	10 400	460	9.91	191.0	16.0	370	1 610	189	18.6	195	42.3
W460 × 74	9 460	457	9.02	190.0	14.5	333	1 460	188	16.6	175	41.9
W460 × 68	8 730	459	9.14	154.0	15.4	297	1 290	184	9.41	122	32.8
W460 × 60	7 590	455	8.00	153.0	13.3	255	1 120	183	7.96	104	32.4
W460 × 52	6 640	450	7.62	152.0	10.8	212	942	179	6.34	83.4	30.9

Solution:
The loads and the strength of the composite section are common to both the LRFD and ASD solutions.
Loads applied before the concrete cures:
Slab weight= 24*0.12= 2.88 kN/m^2
Slab weight on beam= 2.88*2.75=7.92 kN/m
Beam self-weight= 52*9.81= 0.510 kN/m
Total dead load on beam is D.L.= 8.43 kN/m
The construction load is treated as live load= 1.0 *2.75 = 2.75 kN/m

Loads applied after the concrete cures: After the concrete cures, the construction loads do not act, but the partition load does, and it will be treated as live load.
D.L.= 8.43 kN/m
The live load is, L.L.= (1+6.0)*2.75= 19.25 kN/m
Strength of the composite section: To obtain the effective flange width, use either:
Span/4=2.275m or Beam spacing=2.75m
Since this member is an interior beam, the third criterion is not applicable.
Use b_e= 2.275m as the effective flange width.
Then, the compressive force is the smaller of
Fs=As fy or Fc= 0.85 f'c*be*h
Fs is the smaller
The depth of compression stress block is: a= 42.7 mm
The moment lever arm Z or d= 317.8m
Mn= 728 kN.m

1. LRFD
a) Before the concrete cures, the factored load and moment are:
w_u= 1.2 D.L. +1.6 L.L. =1.2(8.43)+1.6(2.75)=14.516 kN/m
M_u= 150 kN.m
$\Phi M_n =$337 kN.m > 150 kN.m
b) After the concrete cures, the factored load and moment are:
w_u= 1.2DL+ 1.6LL= 1.2*8.43+1.6*19.25 = 40.916 kN/m

M_u= 423.5 kN.m

Mn= 728 kN.m
The design strength of the composite section is:

$\Phi M_n = 655.2$ kN.m

Check shear:

V_u= w_u L/2 = 40.916*9.1/2=

$V_n = 0.6F_v A_v C_v$

Design shear strength is $\Phi V_n =$ > V_u O.K.

1. ASD solution:

1. Before the concrete cures, the load and moment are:

W=11.18 kN/m M= 115.7 kN.m

Design allowable moment strength is:

$\frac{M_{n,steel}}{\Omega} = \frac{375}{1.67} =$ > 115.7 kN.m

2. After the concrete cures, the load and moment are:

W=8.43+19.25= 27.68 kN/m M= 286.5 kN.m

Design allowable moment strength is:

$\frac{M_{n,steel}}{\Omega} = \frac{728}{1.67} =$436 kN.m > 286.5 kN.m O.K.

Check shear:

Va= wL/2= 286.5*9.1/2=

Design allowable shear strength is:

$\frac{V_n}{\Omega} = \frac{}{1.67} =$ > Va O.K.

SHEAR TRANSFER:

- The concrete slabs may rest directly on top of the steel beams, or the beams may be completely encased in concrete for fireproofing purposes. This latter case, however, is very expensive and thus is rarely used.
- The longitudinal shear can be transferred between the two elements by bond and shear (and possibly some type of shear reinforcing), if needed, when the beams are encased. When not encased, mechanical connectors must transfer the load.
- Fireproofing is not necessary for bridges, and the slab is placed on top of the steel beams. Bridges are subject to heavy impactive loads, and the bond between the beams and the deck, which is easily broken, is considered negligible. For this reason, steel anchors are designed to resist all of the shear between bridge slabs and beams.
- Various types of steel anchors have been tried, including spiral bars, channels, zees, angles, and studs.
- Economic considerations have usually led to the use of round studs welded to the top flanges of the beams.

- The studs actually consist of rounded steel bars welded on one end to the steel beams.The other end is upset or headed to prevent vertical separation of the slab from the beam. These studs can be quickly attached to the steel beams through the steel decks with stud-welding guns by semiskilled workers.
- The AISC Commentary (I3.2d) describes special procedures needed for 16-gage and thicker decks and for decks with heavy galvanized coatings (> 0.38 kg/m^2).
- A rather interesting practical method is used by many engineers in the field to check the adequacy of the welds used to connect the studs to the steel beams.They take a hammer (about 2.5kg) and hit occasional studs a sufficient number of times to cause them to bend over roughly 25 or 30 degrees. If the studs don't break loose during this hammering, the welds are considered to be satisfactory and the studs are left in their bent positions, which is OK because they will later be encased in the concrete. Should the welds be poor, as where they were made during wet conditions, they may break loose and have to be replaced.

The Shear Connection:

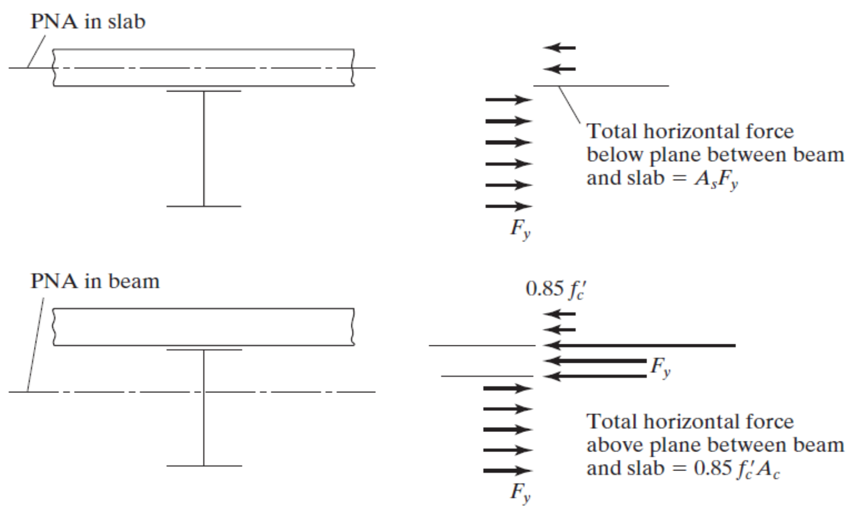
- In design, the main attention is focused on the forces (or stresses) parallel to interface, i.e., on the longitudinal shear forces (and stresses).
- The components of the interface forces perpendicular to the interface, which may play a significant role in the transfer mechanism, are principally considered through the selection of suitable detailing.
- The key behavioral parameters of shear connectors are:

1) Stiffness: a shear connection realizes either:
    i) full interaction (the connection is "rigid" and no slip occurs under stress at the steel-concrete interface), or
    ii) partial interaction (the connection is flexible and interface slip occurs).
2) Resistance: when the overall resistance of the connection can be conveniently considered as in plastic design, a full connection has the shear strength sufficient to make the composite structural element (beam or slab) to develop its ultimate flexural resistance before collapse is achieved. If this condition is not fulfilled, the connection is a partial connection. A structural element with full shear connection is a fully composite structural element. A structural element with partial shear connection is a partially composite structural element. The ratio $F_c/F_{c,f}$ between the resistance of the shear connection and the minimum resistance required by the full connection condition defines the degree of shear connection.
3) Ductility: a connection is ductile if its deformation (slip) capacity is adequate for a complete redistribution of the forces acting on the individual connectors. The ductility demand depends on the span and the degree of shear connection.

- Shop installation of steel anchors initially is more economical, but there is a growing tendency to use field installation.
- There are two major reasons for this trend:

1) The anchors may easily be damaged during transportation and setting of the beams, and
2) they serve as a hindrance to the workers walking along the top flanges during the early phases of construction.

- If the plastic neutral axis (PNA) fall in the slab, the maximum horizontal shear (or horizontal force on the plane between the concrete and the steel) is said to be ($A_s fy$) and if the plastic neutral axis is in the steel section, the maximum horizontal shear is considered to equal ($0.85 f_c' A_c$) where A_c is the effective area of the concrete slab.



The Shear Transfer Mechanisms:

Various forms of shear transfer can be identified, namely:

a) adhesion and chemical bond
b) interface friction
c) mechanical interlock
d) dowel action

Shear transfer via adhesion and bond has the non-negligible advantage of being associated with no steel-concrete slip. However, tests show a rather low maximum shear resistance, which decreases rapidly and, remarkably, in the post-ultimate range of response. Moreover, this form of shear strength is highly dependent on factors, such as the quality of the steel surface and the concrete shrinkage, the control and quantification of which is difficult, if at all possible. Nevertheless, bond might be sufficient when the demand of interface shear capacity is limited as in composite columns or in fully encased beams at least in the elastic range (see also the AISC specifications at clause I1 and I3).

From this information, expressions for the shear to be taken by the anchors can be determined: The AISC (I3.2d) says that, for composite action, the total horizontal shear between the points of maximum positive moment and zero moment is to be taken as the least of the following, where ΣQ_n is the total nominal strength of the steel anchors provided,

a. For concrete crushing

$$V' = 0.85\, f'_c A_c \qquad \text{(AISC Equation I3-1a)}$$

b. For tensile yielding of the steel section (for hybrid beams, this yield force must be calculated separately for each of the components of the cross section)

$$V' = F_y A_s \qquad \text{(AISC Equation I3-1b)}$$

c. For strength of steel anchors

$$V' = \Sigma Q_n \qquad \text{(AISC Equation I3-1c)}$$

Strength of Steel Headed Stud Anchors:

As we have shown, the horizontal shear force to be transferred between the concrete and the steel is equal to the compressive force in the concrete, C. We denote this horizontal shear force V'. Thus V' is given by the smallest of A_sF_y, $0.85f_c'A_c$, or ΣQ_n. If A_sF_y or $0.85f_c'A_c$ controls, full composite action will exist and the number of anchors required between the points of zero moment and maximum moment is

$$N_1 = \frac{V'}{Q_n} \tag{9.2}$$

where Q_n is the nominal shear strength of one anchor. The N_1 anchors should be uniformly spaced within the length where they are required. The AISC Specification gives equations for the strength of both stud and channel anchors. As indicated at the beginning of this chapter, stud anchors are the most common, and we consider only this type. For one stud,

$$Q_n = 0.5A_{sa}\sqrt{f_c'E_c} \le R_gR_pA_{sa}F_u \qquad \text{(AISC Equation I8-1)}$$

where

A_{sa} = cross-sectional area of steel headed stud anchor, in.^{2} (mm^2)
E_c = modulus of elasticity of concrete
= $w_c^{1.5}\sqrt{f_c'}$, ksi ($0.043w_c^{1.5}\sqrt{f_c'}$, MPa)
F_u = specified minimum tensile strength of a steel headed stud anchor, ksi (MPa)

R_g = 1.0 for the following:

(a) One steel headed stud anchor welded in a steel deck rib with the deck oriented perpendicular to the steel shape

(b) Any number of steel headed stud anchors welded in a row directly to the steel shape

(c) Any number of steel headed stud anchors welded in a row through steel deck with the deck oriented parallel to the steel shape and the ratio of the average rib width to rib depth ≥ 1.5

= 0.85 for the following:

(a) Two steel headed stud anchors welded in a steel deck rib with the deck oriented perpendicular to the steel shape

(b) One steel headed stud anchor welded through steel deck with the deck oriented parallel to the steel shape and the ratio of the average rib width to rib depth < 1.5

= 0.7 for three or more steel headed stud anchors welded in a steel deck rib with the deck oriented perpendicular to the steel shape

R_p = 0.75 for the following:

(a) Steel headed stud anchors welded directly to the steel shape

(b) Steel headed stud anchors welded in a composite slab with the deck oriented perpendicular to the beam and $e_{mid\text{-}ht} \geq 2$ in. (50 mm)

(c) Steel headed stud anchors welded through steel deck, or steel sheet used as girder filler material and embedded in a composite slab with the deck oriented parallel to the beam

= 0.6 for steel headed stud anchors welded in a composite slab with deck oriented perpendicular to the beam and $e_{mid\text{-}ht} < 2$ in. (50 mm)

$e_{mid\text{-}ht}$ = distance from the edge of steel headed stud anchor shank to the steel deck web, measured at mid-height of the deck rib, and in the load bearing direction of the steel headed stud anchor (in other words, in the direction of maximum moment for a simply supported beam), in. (mm)

Condition	R_g	R_p
No decking	1.0	0.75
Decking oriented parallel to the steel shape		
$\frac{w_r}{h_r} \geq 1.5$	1.0	0.75
$\frac{w_r}{h_r} < 1.5$	0.85[a]	0.75
Decking oriented perpendicular to the steel shape Number of steel headed stud anchors occupying the same decking rib:		
1	1.0	0.6[b]
2	0.85	0.6[b]
3 or more	0.7	0.6[b]

h_r = nominal rib height, in. (mm)
w_r = average width of concrete rib or haunch (as defined in Section I3.2c), in. (mm)
[a]For a single steel headed stud anchor
[b]This value may be increased to 0.75 when $e_{mid\text{-}ht} \geq 2$ in. (50 mm).

Strength of Steel Channel Anchors

The nominal shear strength of one hot-rolled channel anchor embedded in a solid concrete slab shall be determined as

$$Q_n = 0.3(t_f + 0.5t_w)l_a\sqrt{f_c' E_c} \tag{I8-2}$$

where

l_a = length of channel anchor, in. (mm)
t_f = thickness of channel anchor flange, in. (mm)
t_w = thickness of channel anchor web, in. (mm)

The strength of the channel anchor shall be developed by welding the channel to the beam flange for a force equal to Q_n, considering eccentricity on the anchor.

Required Number of Steel Anchors:

The number of anchors required between the section of maximum bending moment, positive or negative, and the adjacent section of zero moment shall be equal to the horizontal shear (V') as described before divided by the nominal shear strength of one steel anchor as determined from Eqs. I8.a or I8.2. The number of steel anchors required between any concentrated load and the nearest point of zero moment shall be sufficient to develop the maximum moment required at the concentrated load point.
The required number of anchors is determined from the equation:

$$N_1 = \frac{V'}{Q_n}$$

Where: $Q_n$is the nominal shear strength of one anchor, and
V' is the horizontal shear force determined from:

(a) Concrete crushing

$$V' = 0.85 f_c' A_c \tag{I3-1a}$$

(b) Tensile yielding of the steel section

$$V' = F_y A_s \tag{I3-1b}$$

(c) Shear strength of steel headed stud or steel channel anchors

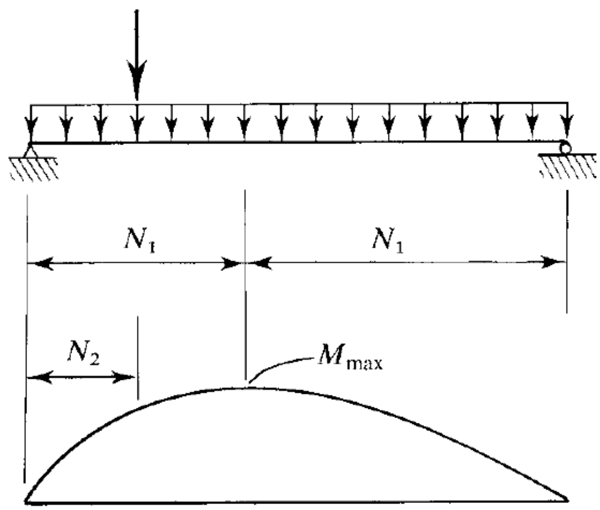
$$V' = \Sigma Q_n \tag{I3-1c}$$

where

A_c = area of concrete slab within effective width, in.^{2} (mm^2)
A_s = cross-sectional area of structural steel section, in.^{2} (mm^2)
ΣQ_n = sum of nominal shear strengths of steel headed stud or steel channel anchors between the point of maximum positive moment and the point of zero moment, kips (N)

Equation of (N_1) gives the number of anchors required between the point of zero moment and the point of maximum moment. Consequently, for a simply supported, uniformly loaded beam, $2N_1$ anchors will be required, and they should be equally spaced.

When concentrated loads are present, AISC I8.2c requires that enough of the N_1 anchors be placed between the concentrated load and the adjacent point of zero moment to develop the moment required at the load.



Additional Requirements for Steel Headed Stud Anchors:

The following requirements are from AISC I8.1, I8.2, and I8.2d:

• Maximum diameter= 2.5 × flange thickness of steel shape (unless placed directly over the web)

• Minimum length = 4 × stud diameter

• Minimum longitudinal spacing (center-to-center) = 6 × stud diameter

• Maximum longitudinal spacing (center-to-center) = 8 × slab thickness ≤ 900 mm.

• Minimum transverse spacing (center-to-center) = 4 × stud diameter

• Minimum concrete cover in the direction perpendicular to the shear force, *V′*, = 25mm.

• Minimum distance from the center of a stud to a free edge in the direction of the shear force = 200mm for normal-weight concrete and 250mm for lightweight concrete.

Example: Design steel headed stud anchors for the floor system of the previous example. Steel section (W460 · 52) , spaced at 2.75m , slab thickness=120 mm, span length is 9.1m.
Partition load= 1.0 kN/m^2 , L.L.= 6 kN.m^2, fy= 345MPa), $f'c$ = 28MPa, construction load=1.0 kN/m^2.

Solution:

The force= Fs= 6640*345= 2290.8kN
Try 12.5mm diameter, 50mm long studs.
The maximum permissible diameter is:
2.5 (flange thick)= 2.5*10.8=27mm O.K.
Cross sectional area, Asa= 122 mm^2

$E_c = 4700\sqrt{f_c'} = 24870\ MPa$

$$Q_n = 0.5 A_{sa}\sqrt{f_c' E_c} \leq R_g R_p A_{sa} F_u$$

$Q_n = 0.5 \times 122 \times \sqrt{28 * 24870} = 50{,}903\ N = 50.9\ kN$

$R_a R_p A_{sa} F_u = 1.0 \times 0.75 \times 122 \times 450 = 41175\ N = 41.175\ kN < 50.9$ kN

Use $Q_n = 41.175\ kN$

and

Minimum longitudinal spacing is 6d = 75mm
Minimum transverse spacing is 4d = 50 mm
Maximum longitudinal spacing is 8hs = 960mm (upper limit is 900 mm)
The number of studs required between the end of the beam and midspan is

$N_1 = \frac{V'}{Q_n} = \frac{2290.8}{41.175} = 57.02$

Use 58 studs for each half of the beam, and a total of 116 anchors.
The spacing is:

$s = \frac{9100}{116} = 78.4\ mm$ (for one stud per section)

Or

$s = \frac{9100}{58} = 156.9\ mm$ (for two stud per section)

Either arrangement is satisfactory; either spacing will be between the lower and upper limits. The stud layout is not necessarily the final form the arrangement will take. Its purpose is to show that the number of studs is compatible with the spacing requirements. A simple statement, such as "Use 108 studs, placed in equally spaced pairs," would suffice.

Note: Note that there is no distinction between LRFD and ASD when designing steel anchors for full composite behavior. This is because the required number of studs is determined by dividing a nominal strength V' by a nominal strength Q_n, and no applied loads are involved.

Design of composite sections:

- Composite construction is an economic choice when loads are heavy, spans are long, and beams are spaced at fairly large intervals.
- For steel building frames, composite construction is economical for spans varying roughly from 7.5 to 15m, with particular advantage in the longer spans.
- For bridges, simple spans have been economically constructed up to approximately 35m and continuous spans of 55m.
- Composite bridges are generally economical for simple spans greater than about 12m and for continuous spans greater than about 18m.
- Occasionally, cover plates are welded to the bottom flanges of steel beams, with improved economy. One can see that with the slab acting as part of the beam, there is a very large compressive area available and that by adding cover plates to the tensile flange, a slightly better balance is obtained.
- In tall buildings where headroom is a problem, it is desirable to use the minimum possible overall floor thicknesses. For buildings, minimum depth-span ratios of approximately 1/24 are recommended if the loads are fairly static and 1/20 if the loads are of such a nature as to cause appreciable vibration.
- The thicknesses of the floor slabs are known (from the concrete design), and the depths of the steel beams can be fairly well estimated from these ratios.
- Before we attempt some composite designs, several additional points relating to lateral bracing, shoring, estimated steel beam weights, and lower bound moments of inertia are discussed in the paragraphs to follow.

a) <u>Lateral Bracing:</u>

- After the concrete slab hardens, it will provide sufficient lateral bracing for the compression flange of the steel beam.
- However, during the construction phase before the concrete hardens, lateral bracing may be insufficient and its design strength may have to be reduced, depending on the estimated unbraced length.
- When steel-formed decking or concrete forms are attached to the beam’s compression flange, they usually will provide sufficient lateral bracing.
- The designer must very carefully consider lateral bracing for fully encased beams.

b) <u>Beams with Shoring:</u>

- If beams are shored during construction, wit will assume that all loads are resisted by the composite section after the shoring is removed.

c) <u>Beams without Shoring:</u>

- If temporary shoring is not used during construction, the steel beam alone must be able to support all the loads before the concrete is sufficiently hardened to provide composite action.
- Without shoring, the wet concrete loads tend to cause large beam deflections, which may lead us to build thicker slabs where the beam deflections are larger.
- This situation can be counteracted by cambering the beams.
- The AISC Specification does not provide any extra margin against yield stresses occurring in beams during construction of unshored composite floors.

d) Extra Reinforcing:

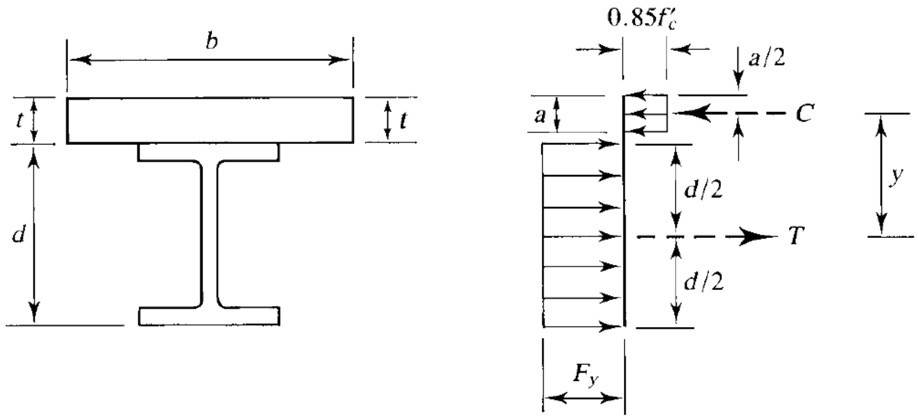
- For building design calculations, the spans often are considered to be simply supported, but the steel beams generally do not have perfectly simple ends. The result is that some negative moment may occur at the beam ends, with possible cracking of the slab above.
- To prevent or minimize cracking, some extra steel can be placed in the top of the slab, extending 0.5 to 1.0m out into the slab.
- The amount of steel added is in addition to that needed to meet the temperature and shrinkage requirements specified by the American Concrete Institute.

Procedure of Design of Composite Floors:

- The first step in the design of a floor system is to select the thickness of the floor slab, whether it is solid or ribbed (formed with steel deck). The thickness will be a function of the beam spacing, and several combinations of slab thickness and beam spacing may need to be investigated so that the most economical system can be found.
- The design of the slab and beam spacing are assumed to be known.
- Having made this assumption, we can take the following steps to complete the design of an unshored floor system:

1) Compute the moments acting before and after the concrete cures.
2) Estimate the depth and weight of the steel section.
3) Select a steel shape for trial.
4) Compare the available strength of the steel shape to the required moment strength acting before the concrete cures. Account for the unbraced length if the formwork does not provide adequate lateral support. If this shape is not satisfactory, try a larger one.
5) Compute the available strength of the composite section and compare it to the *total* required moment strength. If the composite section is inadequate, select another steel shape for trial.
6) Check the shear strength of the steel shape.
7) Design the steel anchors:
    a) Compute V', the horizontal shear force at the interface between the concrete and the steel.
    b) Divide this force by Qn, the shear capacity of a single stud, to obtain Nl, which, in most cases, is half the total number of studs required. Using this number of studs will provide full composite behavior. If partial composite action is desired, the number of studs can be reduced.
8) Check deflections.

- The major task in the trial-and-error procedure just outlined is the selection of a trial steel shape.
- A formula that will give the required area (or, alternatively, the required weight per meter of length) can be developed if a beam depth is assumed.
- Assuming full composite action and the PNA in the slab (i.e., steel controlling, the most common case for full composite action), we can write the nominal strength as:



$$M_n = Ty = A_s F_y y$$

LRFD Procedure. Equate the design strength to the factored load moment and solve for A_s:

$$\phi_b M_n = \phi_b A_s F_y y = M_u$$

$$A_s = \frac{M_u}{\phi_b F_y y}$$

or

$$A_s = \frac{M_u}{\phi_b F_y \left(\frac{d}{2} + t - \frac{a}{2}\right)}$$

The above equation can also be written in terms of weight rather than area.
Mass per meter length = 7850*As.

$$w = \frac{7850\ M_u}{\emptyset F_y \left(\frac{d}{2} + t - \frac{a}{2}\right)}$$

ASD Procedure:
Equate the allowable strength to the service load moment and solve for As:

$$\frac{M_n}{\Omega_b} = \frac{A_s F_y y}{\Omega_b} = M_a$$

$$A_s = \frac{\Omega_b M_a}{F_y y}$$

or

$$A_s = \frac{\Omega_b M_a}{F_y\left(\frac{d}{2} + t - \frac{a}{2}\right)}$$

And the estimated weight per unit length is:

$$w = \frac{7850\ \Omega_b\ M_a}{F_y\left(\frac{d}{2} + t - \frac{a}{2}\right)}$$

Where t: is the distance from top of steel section to top of concrete flange.

- Both Equations of (LRFD) and (ASD) require an assumed depth and an estimate of $d/2$.

Assumed values are as following:

d: can be assumed as d= L/20 to L/25.

a: can be assumed as a= 25 to 50mm.

Example: The span length of a certain floor system is 9.1m, and the beam spacing is 3.0 m center-to-center. Select a rolled steel shape and the steel anchors needed to achieve full composite behavior with a 90mm-thick reinforced concrete floor slab. Superimposed loading consists of a 1.0 kN/m^2 partition load and a 4.8kN/m^2 live load. Concrete strength is fc′ = 28 MPa, and A992 steel is to be used. Assume that the beam has full lateral support during construction and that there is a 1 kN/m^2 construction load.

Solution:
a) Loads to be supported before the concrete cures are:
Slab: 0.09*24 = 2.16 kN/m^2
Weight per linear meter: 2.16(3) = 6.48 kN/m
Construction load: 1.0(3) = 3 kN/m
b) Loads to be supported after the concrete cures are:
$w_D = w_{slab}$ = 6.48 kN/m
wL = (4.8 + 1.0)(3) = 17.4 kN/m (where the 1.0 kN/m^2 partition load is treated as a live load).

I) LRFD solution:
The composite section must resist a factored load and moment of
w_u =1.2DL+1.6LL= 1.2*6.48+1.6*17.4= 35.62 kN/m
Mu= wu*L^2/8 = 35.62*9.1^2/8= 368.7 kN.m
Try from d= 9.1/25 = 0.364m to 9.1/20= 0.455m
Try d= 0.4m
t= 90mm = 0.09m
try a= 50mm= 0.05m

$$w = \frac{7850\ M_u}{\emptyset F_y\left(\frac{d}{2}+t-\frac{a}{2}\right)} = = \frac{7850 \times (368.7 \times 10^3)}{0.9 \times 345 \times 10^6 \times \left(\frac{0.4}{2}+0.09-\frac{0.05}{2}\right)}$$

w= 35.2 kg/m

Try W16×26lb/ft = W410×39kg/m

The complete solution of the example in US units is as follows:

The span length of a certain floor system is 30 feet, and the beam spacing is 10 feet center-to-center. Select a rolled steel shape and the steel anchors needed to achieve full composite behavior with a 3.5-inch-thick reinforced concrete floor slab. Superimposed loading consists of a 20 psf partition load and a 100 psf live load. Concrete strength is $f_c' = 4$ ksi, and A992 steel is to be used. Assume that the beam has full lateral support during construction and that there is a 20 psf construction load.

SOLUTION Loads to be supported before the concrete cures are

Slab: (3.5/12)(150) = 43.75 psf
Weight per linear foot: 43.75(10) = 437.5 lb/ft
Construction load: 20(10) = 200 lb/ft

(The beam weight will be accounted for later.)

Loads to be supported after the concrete cures are

$$w_D = w_{slab} = 437.5 \text{ lb/ft}$$
$$w_L = (100 + 20)(10) = 1200 \text{ lb/ft}$$

where the 20 psf partition load is treated as a live load.

LRFD SOLUTION The composite section must resist a factored load and moment of

$$w_u = 1.2w_D + 1.6w_L = 1.2(437.5) + 1.6(1200) = 2445 \text{ lb/ft}$$

$$M_u = \frac{1}{8}w_uL^2 = \frac{1}{8}(2.445)(30)^2 = 275 \text{ ft-kips}$$

Try a nominal depth of d = 16 inches. From Equation 9.4, the estimated beam weight is

$$w = \frac{3.4M_u}{\phi_b F_y\left(\frac{d}{2}+t-\frac{a}{2}\right)} = \frac{3.4(275 \times 12)}{0.90(50)\left(\frac{16}{2}+3.5-0.5\right)} = 22.7 \text{ lb/ft}$$

Try a W16 × 26. Check the unshored steel shape for loads applied before the concrete cures (the weight of the slab, the weight of the beam, and the construction load).

$$w_u = 1.2(0.4375 + 0.026) + 1.6(0.200) = 0.8762 \text{ kips/ft}$$

$$M_u = \frac{1}{8}(0.8762)(30)^2 = 98.6 \text{ ft-kips}$$

From the Z_x table,

$$\phi_b M_n = \phi_b M_p = 166 \text{ ft-kips} > 98.6 \text{ ft-kips} \qquad \text{(OK)}$$

After the concrete cures and composite behavior has been achieved,

$$w_D = w_{\text{slab}} + w_{\text{beam}} = 0.4375 + 0.026 = 0.4635 \text{ kips/ft}$$

$$w_u = 1.2w_D + 1.6w_L = 1.2(0.4635) + 1.6(1.200) = 2.476 \text{ kips/ft}$$

$$M_u = \frac{1}{8}w_u L^2 = \frac{1}{8}(2.476)(30)^2 = 279 \text{ ft-kips}$$

Before computing the design strength of the composite section, we must first determine the effective slab width. For an interior beam, the effective width is the smaller of

$$\frac{\text{Span}}{4} = \frac{30(12)}{4} = 90 \text{ in.} \quad \text{or} \quad \text{Beam spacing} = 10(12) = 120 \text{ in.}$$

Use $b = 90$ in. For full composite behavior, the compressive force in the concrete at ultimate (equal to the horizontal shear at the interface between the concrete and steel) will be the smaller of

$$A_s F_y = 7.68(50) = 384 \text{ kips}$$

or

$$0.85 f_c' A_c = 0.85(4)(90)(3.5) = 1071 \text{ kips}$$

Use $C = V' = 384$ kips. The depth of the compressive stress block in the slab is

$$a = \frac{C}{0.85 f_c' b} = \frac{384}{0.85(4)(90)} = 1.255 \text{ in.}$$

and the moment arm of the internal resisting couple is

$$y = \frac{d}{2} + t - \frac{a}{2} = \frac{15.7}{2} + 3.5 - \frac{1.255}{2} = 10.72 \text{ in.}$$

The design flexural strength is

$$\phi_b M_n = \phi_b(Cy) = 0.90(384)(10.72)$$
$$= 3705 \text{ in.-kips} = 309 \text{ ft-kips} > 279 \text{ ft-kips} \qquad \text{(OK)}$$

Check the shear:

$$V_u = \frac{w_u L}{2} = \frac{2.476(30)}{2} = 37.1 \text{ kips}$$

From the Z_x table,

$$\phi_v V_n = 106 \text{ kips} > 37.1 \text{ kips} \qquad \text{(OK)}$$

ANSWER Use a W16 × 26.

Design the steel anchors.

Maximum diameter $= 2.5t_f = 2.5(0.345) = 0.863$ in.

Try ½ in. × 2 in. studs.

$$d = 1/2 \text{ in.} < 0.863 \text{ in.} \qquad \text{(OK)}$$

$$A_{sa} = \frac{\pi d^2}{4} = \frac{\pi (0.5)^2}{4} = 0.1963 \text{ in.}^2$$

For normal weight concrete,

$$E_c = w^{1.5}\sqrt{f_c'} = (145)^{1.5}\sqrt{4} = 3492 \text{ ksi}$$

From AISC Equation I3-3,

$$Q_n = 0.5A_{sa}\sqrt{f_c' E_c} \le R_g R_p A_{sa} F_u$$
$$= 0.5(0.1963)\sqrt{4(3492)} = 11.60 \text{ kips}$$
$$R_g R_p A_{sa} F_u = 1.0(0.75)(0.1963)(65)$$
$$= 9.570 \text{ kips} < 11.60 \text{ kips} \quad \therefore \text{ Use } Q_n = 9.570 \text{ kips.}$$

The number of studs required between the end of the beam and midspan is

$$N_1 = \frac{V'}{Q_n} = \frac{384}{9.570} = 40.1 \quad \therefore \text{ Use 41 for half the beam, or 82 total}$$

and

Minimum longitudinal spacing is $6d = 6(0.5) = 3$ in.
Minimum transverse spacing is $4d = 4(0.5) = 2$ in.
Maximum longitudinal spacing is $8t = 8(3.5) = 28$ in. (< 36 in. upper limit)

If one stud is used at each section, the approximate spacing will be

$$s = \frac{30(12)}{82} = 4.4 \text{ in.}$$

This spacing is between the upper and lower limits and is therefore satisfactory.

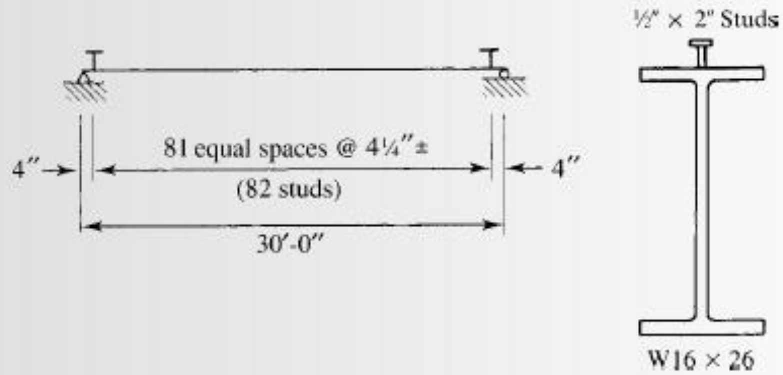
ANSWER Use the design shown in Figure 9.13.

ASD SOLUTION The composite section must resist a service load and moment of

$$w_a = w_D + w_L = 437.5 + 1200 = 1638 \text{ lb/ft}$$

$$M_u = \frac{1}{8}w_a L^2 = \frac{1}{8}(1.638)(30)^2 = 184.3 \text{ ft-kips}$$

FIGURE 9.13



Try a nominal depth of $d = 16$ inches. From Equation 9.6, the estimated beam weight is

$$w = \frac{3.4\Omega_b M_a}{F_y\left(\frac{d}{2}+t-\frac{a}{2}\right)} = \frac{3.4(1.67)(184.3\times 12)}{50\left(\frac{16}{2}+3.5-0.5\right)} = 22.8 \text{ lb/ft}$$

Try a W16 × 26. Check the unshored steel shape for loads applied before the concrete cures (the weight of the slab, the weight of the beam, and the construction load).

$$w_a = w_{\text{slab}} + w_{\text{beam}} + w_{\text{const}} = 0.4375 + 0.026 + 0.200 = 0.6635 \text{ lb/ft}$$

$$M_a = \frac{1}{8}w_a L^2 = \frac{1}{8}(0.6635)(30)^2 = 74.6 \text{ ft-kips}$$

From the Z_x table,

$$\frac{M_n}{\Omega_b} = \frac{M_p}{\Omega_b} = 110 \text{ ft-kips} > 74.6 \text{ ft-kips} \qquad \text{(OK)}$$

After the concrete cures and composite behavior has been achieved, the load and moment are

$$w_a = w_{\text{slab}} + w_{\text{beam}} + w_{\text{part}} + w_L = 0.4375 + 0.026 + 0.200 + 1.000 = 1.664 \text{ kips/ft}$$

$$M_a = \frac{1}{8}(1.664)(30)^2 = 187 \text{ ft-kips}$$

The effective slab width for a typical interior beam is the smaller of

$$\frac{\text{Span}}{4} = \frac{30\times 12}{4} = 90 \text{ in.} \qquad \text{or} \qquad \text{Beam spacing} = 10\times 12 = 120 \text{ in.}$$

Use $b = 90$ in. For full composite behavior, the compressive force in the concrete at ultimate (equal to the horizontal shear at the interface between the concrete and steel) will be the smaller of

$$A_s F_y = 7.68(50) = 384 \text{ kips}$$

or

$$0.85 f'_c A_c = 0.85(4)\,(3.5 \times 90) = 1071 \text{ kips}$$

Use $C = 384$ kips.

$$a = \frac{C}{0.85 f'_c b} = \frac{384}{0.85(4)(90)} = 1.255 \text{ in.}$$

$$y = \frac{d}{2} + t - \frac{a}{2} = \frac{15.7}{2} + 3.5 - \frac{1.255}{2} = 10.72 \text{ in.}$$

The allowable flexural strength is

$$\frac{M_n}{\Omega_b} = \frac{Cy}{\Omega_b} = \frac{384(10.72)}{1.67} = 2465 \text{ in.-kips} = 205 \text{ ft-kips} > 187 \text{ ft-kips} \qquad \text{(OK)}$$

Check shear:

$$V_a = \frac{w_a L}{2} = \frac{1.664(30)}{2} = 25.0 \text{ kips}$$

From the Z_x table,

$$\frac{V_n}{\Omega_v} = 70.5 \text{ kips} > 25.0 \text{ kips} \qquad \text{(OK)}$$

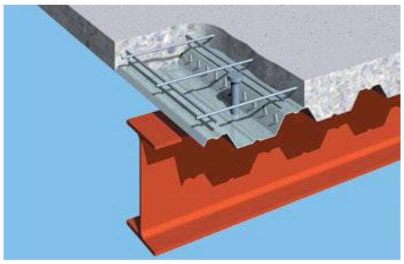
ANSWER Use a W16 × 26.

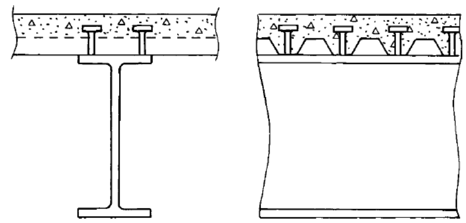
The ASD design of the steel anchors is identical to what was done for LRFD and will not be repeated here.

ANSWER Use the design shown in Figure 9.13.

COMPOSITE BEAMS WITH FORMED STEEL DECK:

- The floor slab in many steel-framed buildings is formed on ribbed steel deck, which is left in place to become an integral part of the structure. Although there are exceptions, the ribs of the deck are usually oriented perpendicular to floor beams and parallel to supporting girders.





- The installation of steel headed stud anchors is done in the same way as without the deck; the studs are welded to the beam flange directly through the deck.
- The attachment of the deck to the beam can be considered to provide lateral support for the beam before the concrete has cured.
- The design or analysis of composite beams with formed steel deck is essentially the same as with slabs of uniform thickness, with the following exceptions:

1) The concrete in the ribs—that is, below the top of the deck—is neglected in determining section properties when the ribs are perpendicular to the beam [AISC I3.2c(2)]. When the ribs are parallel to the beam, the concrete may be included in determining section properties, and it *must* be included in computing *Ac* [AISC I3.2c(3)].
2) The shear strength of the studs could be reduced when a deck is used. This will depend on the placement of the studs in the ribs.
3) Full composite behavior will not usually be possible. The reason is that the spacing of the studs is limited by the spacing of the ribs, and the exact number of required studs cannot always be used. Although partial composite design can be used without formed steel deck, it is covered here because it is almost a necessity with formed steel deck. This is not a disadvantage; in fact, it will be the most economical alternative.

- Most composite beams with formed steel deck are floor beams with the deck ribs oriented perpendicular to the beam. Special requirements that apply when the ribs are parallel to the beam are presented in AISC I3.2c(3) and I8.2a.

Shear Strength of Anchors Used with Steel Deck:

The shear strength of one stud depends on the values of R_p and R_g in

$$Q_n = 0.5A_{sa}\sqrt{f_c' E_c} \leq R_g R_p A_{sa} F_u \qquad \text{(AISC Equation I8-1)}$$

If the studs are welded directly to the top flange of the steel beam (no deck), $R_p = 1.0$ and $R_g = 0.75$. With a formed steel deck, these constants can take on different values. For deck ribs oriented perpendicular to the beam, the values are

$R_p = 1.0$ for one stud per rib.

= 0.85 for two studs per rib.

= 0.7 for three or more studs per rib.

$R_p = 0.75\ for\ e_{mid-ht} \geq 50mm$

$= 0.6\ for\ e_{mid-ht} < 50mm$

e_{mid-ht}= distance from mid-height of the rib to the stud, measured in the loadbearing direction (toward the point of maximum moment in a simply supported beam).

- Most steel deck is manufactured with a longitudinal stiffener in the middle of the rib, so the stud must be placed on one side or the other of the stiffener.
- Tests have shown that placement on the side farthest from the point of maximum moment results in a higher strength. Since it is difficult to know in advance where the stud will actually be placed, it is conservative to use a value of ***Rp* = 0.6.**

Example: Determine the shear strength of 12mm ×62mm stud, where there are two studs per rib.

The 28-day compressive strength of the concrete is *f'c* = 28MPa, Fy=345MPa, Fu= 450MPa.

Solution:

$A_{sa} = 113\ mm^2$

$Q_n = 0.5A_{sa}\sqrt{f_c'E_c} \leq R_g R_p A_{sa} F_u$

$E_c = 4700\sqrt{f_c'}$ 24870 MPa

$Q_n = 0.5A_{sa}\sqrt{f_c'E_c}$ = 46,958 N

Upper limit = $R_g R_p A_{sa} F_u = 0.85(0.6)(113)(450)$ = 25,933 N < 46,958 N

The upper limit controls, thus

Q_n = 25,933 N = 25.9 kN

Partial Composite Action:

- Partial composite action exists when there are not enough steel anchors to completely prevent slip between the concrete and steel.
- Neither the full strength of the concrete nor that of the steel can be developed, and the compressive force is limited to the maximum force that can be transferred across the interface between the steel and the concrete, that is, the strength of the studs, $\sum Q_n$.
- Recall that the force C is the smallest of $A_s f_y$, $0.85 f_c' A_c$, and $\sum Q_n$.
- With partial composite action, the plastic neutral axis (PNA) will usually fall within the steel cross section. This location will make the strength analysis somewhat more difficult than if the PNA were in the slab, but the basic principles are the same.
- The steel strength will not be fully developed in a partially composite beam, so a larger shape will be required than with full composite behavior.
- Assume that it was needed to select a steel section that will have an LRFD design strength of 200 kN.m when made composite with the concrete slab. Assume also that the trial section selected from the Manual, has a moment capacity equal to 250 kN.m (when made composite with the slab).
- If the steel anchors were installed for full composite action, the section will have a design strength of 250 kN.m. But the required capacity is only 200 kN.m.
- It seems logical to assume that we may decide to provide only a sufficient number of anchors to develop a design strength of 200 kN.m. In this way we can reduce the number of anchors and reduce costs (perhaps substantially if we repeat this section many times in the structure). The resulting section is a partially composite section, one that does not have a sufficient number of anchors to develop the full flexural strength of the composite beam.

Miscellaneous Requirements:

The following requirements are from AISC Section I3.2c. Only those provisions not already discussed are listed.

• Maximum rib height h_r = 75mm.
• Minimum average width of rib w_r = 50mm, but the value of w_r used in the calculations shall not exceed the clear width at the top of the deck.
• Maximum stud diameter = 20mm. This requirement for formed steel deck is in addition to the usual maximum diameter of $2.5T_f$.
• Minimum height of stud above the top of the deck = 38mm.
• Minimum cover above studs = 13mm.
• Minimum slab thickness above the top of the deck = 50mm.
• The deck must be attached to the beam flange at intervals of no more than 450mm, either by the studs or spot welds. This is for the purpose of resisting uplift.

Slab and Deck Weight:

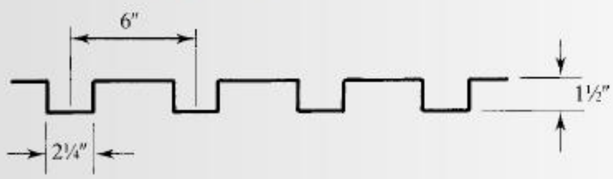
- To simplify computation of the slab weight, we use the full depth of the slab, from bottom of deck to top of slab. Although this approach overestimates the volume of concrete, it is conservative.
- For the unit weight of reinforced concrete, we use the weight of plain concrete plus 0.8 kN/m^3.
- Because slabs on formed metal deck are usually lightly reinforced (sometimes welded wire mesh, rather than reinforcing bars, is used), adding 0.8kN/m^3 for reinforcement may seem excessive, but the deck itself can weigh between 0.3 and 0.5 kN/m^3.
- An alternative approach is to use the thickness of the slab above the deck plus half the height of the rib as the thickness of concrete in computing the weight of the slab.
- In practice, the combined weight of the slab and deck can usually be found in tables furnished by the deck manufacturer.

EXAMPLE 9.10

Floor beams are to be used with the formed steel deck shown in Figure 9.17 and a reinforced concrete slab whose total thickness is 4.75 inches. The deck ribs are perpendicular to the beams. The span length is 30 feet, and the beams are spaced at 10 feet center-to-center. The structural steel is A992, and the concrete strength is $f_c' = 4$ ksi. The slab and deck combination weighs 50 psf. The live load is 120 psf, and there is a partition load of 10 psf. No shoring is used, and there is a construction load of 20 psf.

a. Select a W-shape.
b. Design the steel anchors.
c. Compute deflections. The maximum permissible live-load deflection is $1/360$ of the span length.

FIGURE 9.17



SOLUTION Compute the loads (other than the weight of the steel shape). Before the concrete cures,

Slab wt. = 50(10) = 500 lb/ft

Construction load = 20(10) = 200 lb/ft

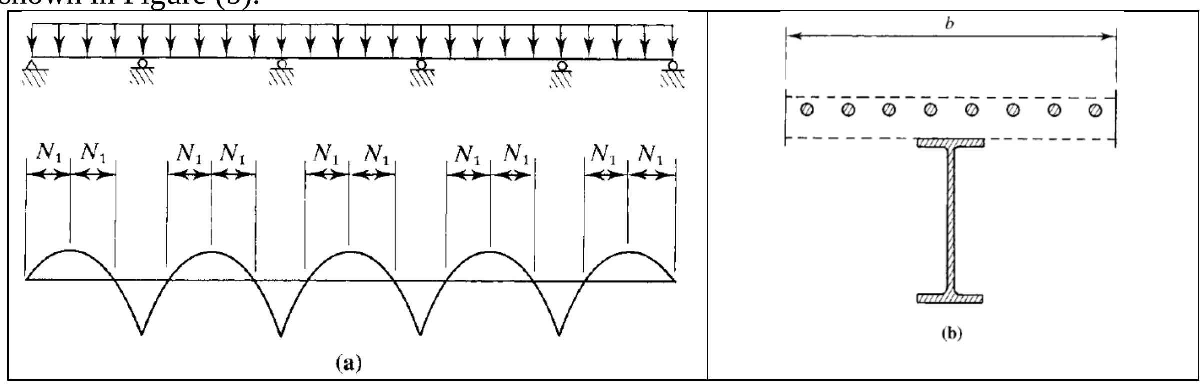
After the concrete cures,

Partition load = 10(10) = 100 lb/ft

Live load = 120(10) = 1200 lb/ft

CONTINUOUS BEAMS:

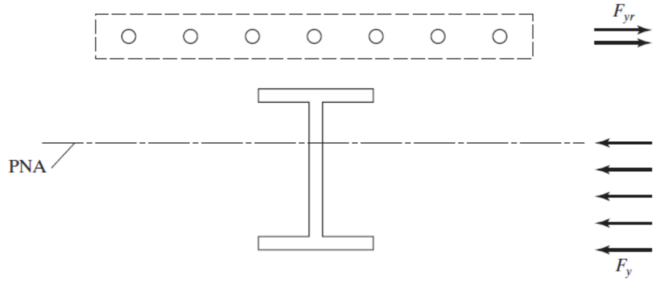
- In a simply supported beam, the point of zero moment is at the support. The number of steel anchors required between each support and the point of maximum positive moment will be half the total number required.
- In a continuous beam, the points of inflection are also points of zero moment and, in general, $2N_1$ anchors will be required for each span.
- Figure (a) below shows a typical continuous beam and the regions in which anchors would be required.
- In the negative moment zones, the concrete slab will be in tension and therefore ineffective. In these regions there will be no composite behavior in the sense that we have considered thus far.
- The only type of composite behavior possible is that between the structural steel beam and the longitudinal reinforcing steel in the slab. The corresponding composite cross section is shown in Figure (b).



- If this concept is used, a sufficient number of steel anchors must be provided to achieve a degree of continuity between the steel shape and the reinforcing.
- The AISC Specification (I3.2b) permits the use of continuous composite sections.
- The flexural strength of a composite section in a negative moment region may be considered to equal ($\emptyset_b M_n$) for the steel section alone, or it may be based upon the plastic strength of a composite section considered to be made up of the steel beam and the longitudinal reinforcement in the slab. For this latter method to be used, the following conditions must be met:

1. The steel section must be compact and adequately braced.
2. The slab must be connected to the steel beams in the negative moment region with shear connectors.
3. The longitudinal reinforcing in the slab parallel to the steel beam and within the effective width of the slab must have adequate development lengths.

- For a particular beam, the total horizontal shear force between the point of zero moment and the point of maximum negative moments is to be taken as the smaller of ($A_{sr}F_{ysr}$) and ($\sum Q_n$). where $A_{sr}$is the cross-sectional area of the properly developed reinforcing and $F_{ysr}$is the yield stress of the bars.



Example: Determine the depth of the plastic neutral axis and the value of the nominal negative moment capacity of the composite section shown. The steel section is W310×39 kg/m, A992 (Fy=345MPa), and the yield strength of steel bars is 414MPa. Effective width=1.22m, slab thick= 100mm, 10 bars of 16mm diameter within be, at slab mid depth.

Solution:

Determine the PNA: the slab is in tension then the concrete is not effective.

The reinforcing bars provide the nominal tension force:

$T_{sr}= A_{sr}. F_{sr}= 10(201)(414)= 832.14$ kN

The nominal compression force in steel section is

$C=As.F_y= 4930(345)= 1700.85$ kN

C is greater than Tsr, then the PNA is within the steel section.

Check the location of PNA:

C-Tsr= 1700.85-832.14= 868.71 kN

(C-Tsr)/2= 434.36 kN

Try the PNA is in the flange of steel section:

Y*bf*Fy=434,360

Y*165.0*345=434,360

Y= 7.63mm < thickness of flange=9.7mm

The PNA is within the top flange

Xp= 107.63 mm (from top)

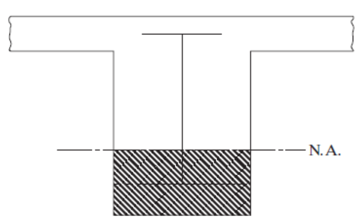
Mn=

CONCRETE-ENCASED SECTIONS:

- For fireproofing purposes, it is possible to completely encase in concrete the steel beams used for building floors.
- This practice definitely is not economical, because lightweight spray-on fire protection is so much cheaper. Furthermore, encased beams may increase the floor system dead load by as much as 15 percent.
- For the rare situation in which encased beams are used, steel anchors shall be provided.
- The nominal flexural strength, Mn, shall be determined using one of several methods of AISC Specification I3.3.
  1. With one method, the design strength of the encased section may be based on the plastic moment capacity or of the steel section alone.
  2. By another method, the design strength is based on the first yield of the tension flange, assuming composite action between the concrete, which is in compression, and the steel section.
- Again, and If the second method is used and we have unshored construction, the stresses in the steel section caused by the wet concrete and other construction loads are calculated. Then the stresses in the composite section caused by loads applied after the concrete hardens are computed. These stresses are superimposed on the first set of stresses.
- If we have shored construction, all of the loads may be assumed to be supported by the composite section and the stresses computed accordingly.
- For stress calculations, the properties of a composite section are computed by the transformed area method.
- In this method, the cross-sectional area of one of the two materials is replaced or transformed into an equivalent area of the other.
- For composite design, it is customary to replace the concrete with an equivalent area of steel, whereas the reverse procedure is used in the working stress design method for reinforced-concrete design.
- There are no slenderness limitations required by the AISC Specification for either of the two methods, because the encasement is effective in preventing both local and lateral buckling.

Continuous encased beams:

- For buildings, continuous composite construction with encased sections is permissible.
- For continuous construction, the positive moments are handled exactly as has been illustrated.
- For negative moments, however, the transformed section is taken as shown in figure.



- The crosshatched area represents the concrete in compression, and all concrete on the tensile side of the neutral axis (that is, above the neutral axis) is neglected.

Example:

Review the encased beam section shown in Fig. 16.14 if no shoring is used and the following data are assumed:

Simple span = 36 ft
Service dead load = 0.50 k/ft before concrete hardens plus an additional 0.25 k/ft after concrete hardens
Construction live loads = 0.2 k/ft
Service live load = 1.0 k/ft after concrete hardens
Effective flange width b_e = 60 in and $n = 9$
$F_y = 50$ ksi

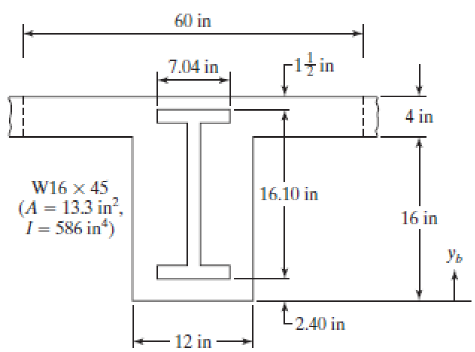


FIGURE 16.14

Solution. Calculated Properties of Composite Section: Neglecting concrete area below the slab.

$$A = 13.3\text{ in}^2 + \frac{(4\text{ in})(60\text{ in})}{9} = 39.96\text{ in}^2$$

$$y_b = \frac{(13.3\text{ in}^2)(10.45\text{ in}) + (26.66\text{ in}^2)(18\text{ in})}{39.96} = 15.50\text{ in}$$

$$I = 586\text{ in}^4 + (13.3\text{ in}^2)(5.05\text{ in})^2 + \left(\tfrac{1}{12}\right)\left(\tfrac{60}{9}\text{ in}\right)(4\text{ in})^3 + (26.66\text{ in}^2)(2.5\text{ in})^2$$
$$= 1127\text{ in}^4$$

Stresses before concrete hardens

Assume that wet concrete is a live load

$$w_u = (1.6)(0.5\text{ k/ft} + 0.2\text{ k/ft}) = 1.12\text{ k/ft}$$

$$M_u = \frac{(1.12\text{ k/ft})(36\text{ ft})^2}{8} = 181.4\text{ ft-k}$$

Assume beam only properties

$$f_t = \frac{(12\text{ in/ft})(181.4\text{ ft-k})(8.05\text{ in})}{586\text{ in}^4} = 29.90\text{ ksi}$$

$$< \phi_b F_y = (0.9)(50) = 45\text{ ksi} \qquad \text{(OK)}$$

Stresses after concrete hardens

$$w_u = (1.2)(0.25\text{ k/ft}) + (1.6)(1.0\text{ k/ft}) = 1.9\text{ k/ft}$$

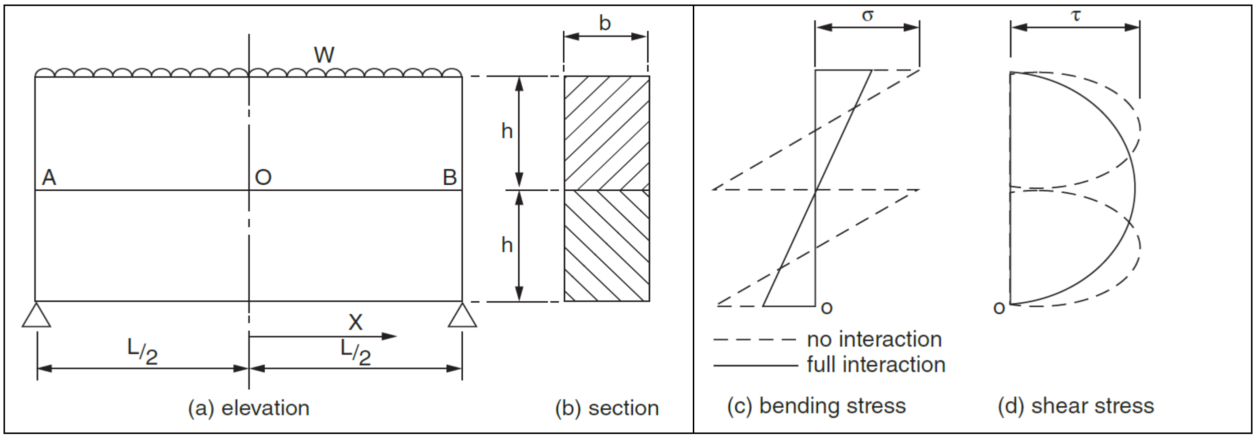
$$M_u = \frac{(1.9\text{ k/ft})(36\text{ ft})^2}{8} = 307.8\text{ ft-k}$$

$$f_t = \frac{(12\text{ in/ft})(307.8\text{ ft-k})(15.50\text{ in} - 2.40\text{ in})}{1127\text{ in}^4} = 42.93\text{ ksi}$$

Total $f_t = 29.90 + 42.93 = 72.83\text{ ksi} > 0.9F_y = 45\text{ ksi}$ (NG)

Effect of Shear Connection on the flexural behaviour of beams:

To simplify the description this effect, a simply supported beam is considered. The beam cross section is made from two members of the same cross-section, each one is (b×h). The span/depth ratio is assumed to be about 20, so that shear deformation is negligible compared with flexural deformation and is subjected to a lateral loading of *w* per unit length. The two components are made of an elastic material with Young's modulus *E*.



(a) elevation (b) section (c) bending stress (d) shear stress

I. **Case of No shear connection:**

- Assume first that there is no shear connection or friction on the interface AB.
- The upper beam cannot deflect more than the lower one, that is the two members has the same deflection along the beam span.
- Since the two members have the same section and material properties, then each one carries a load of $w/2$ per unit length.
- The mid-span bending moment in each beam is $wL^2/16$.
- By elementary beam theory, the longitudinal stress distribution at mid-span is given by the dashed line shown in figure (c), and the maximum bending stress in each component, σ, is given by:

$$\sigma = \frac{My_{\max}}{I} = \frac{wL^2}{16}\frac{12}{bh^3}\frac{h}{2} = \frac{3wL^2}{8bh^2}$$

- The maximum longitudinal and vertical shear stress, τ, occurs near a support. The two parabolic distributions given by simple elastic and at the horizontal centre-line of each beam:

$$\tau = \frac{3}{2}\frac{wL}{4}\frac{1}{bh} = \frac{3wL}{8bh}$$

The maximum deflection, δ, is given by the usual formula

$$\delta = \frac{5(w/2)L^4}{384EI} = \frac{5wL^4}{64Ebh^3}$$

The bending moment in each beam at a section distant x from mid-span is $M_x = w(L^2 - 4x^2)/16$, so that the longitudinal strain ε_x at the bottom fibre of the upper beam is

$$\varepsilon_x = \frac{My_{max}}{EI} = \frac{3w}{8Ebh^2}(L^2 - 4x^2) \tag{2.4}$$

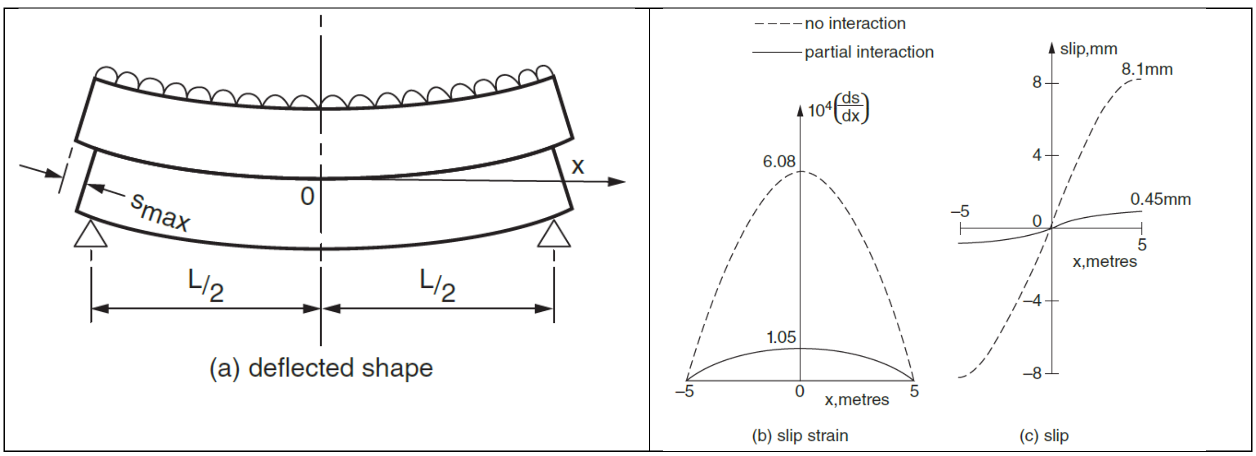
There is an equal and opposite strain in the top fibre of the lower beam, so that the difference between the strains in these adjacent fibres, known as the *slip strain*, is $2\varepsilon_x$.

It is easy to show by experiment with two or more flexible wooden laths or rulers that, under load, the end faces of the two-component beam have the shape shown in Figure 2.3a. The slip at the interface, s, is zero at $x = 0$ (from symmetry) and a maximum at $x = \pm L/2$. The cross-section at $x = 0$ is the only one where a plane cross-section through both members remains plane. The slip strain, defined above, is not the same as slip. In the same way that strain is rate of change of displacement, slip strain is the rate of change of slip along the beam. Thus from Equation 2.4:

$$\frac{ds}{dx} = 2\varepsilon_x = \frac{3w}{4Ebh^2}(L^2 - 4x^2) \tag{2.5}$$

Integration gives:

$$s = \frac{w}{4Ebh^2}(3L^2x - 4x^3) \tag{2.6}$$



- The constant of integration is zero, since s = 0 when x = 0, so that Equation 2.6 gives the distribution of slip along the beam.
- Results (2.5) and (2.6) for an example beam are plotted in Figure 2.3. The curves 'no interaction' show that, at mid-span, slip strain is a maximum and slip is zero and, at the ends of the beam, slip is a maximum and slip strain is zero.
- From Equation 2.6, the maximum slip (when x=L/2) is ($wL^3/4Ebh^2$).
- Some idea of the magnitude of this slip is given by relating it to the maximum deflection of the two beams.
- From Equation 2.3, the ratio of slip to deflection is 3.2h/L. For a beam with a typical span/depth ratio (L/2h here) of 20, the end slip is less than a tenth of the deflection. This shows that shear connection must be very stiff if it is to be effective.

II. Case of Full interaction:

- It is now assumed that the two halves of the beam are joined together by an infinitely stiff shear connection. The two members then behave as one.
- Slip and slip strain are everywhere zero, and it can be assumed that plane sections remain plane.
- This situation is known as full interaction. Except in design with partial shear connection, all design of composite beams and columns in practice is based on the assumption that full interaction is achieved.

For the composite beam of breadth b and depth $2h$, $I = 2bh^3/3$, and elementary theory gives the mid-span bending moment as $wL^2/8$. The extreme fibre bending stress is

$$\sigma = \frac{My_{\max}}{I} = \frac{wL^2}{8}\frac{3}{2bh^3}h = \frac{3wL^2}{16bh^2} \quad (2.7)$$

The vertical shear at section x is:

$$V_x = wx \quad (2.8)$$

so the shear stress at the neutral axis is

$$\tau_x = \frac{3}{2}wx\frac{1}{2bh} = \frac{3wx}{4bh} \quad (2.9)$$

and the maximum shear stress is

$$\tau = \frac{3wL}{8bh} \quad (2.10)$$

The stresses are compared in Figure 2.2 (c) and (d) with those for the non-composite beam. The provision of the shear connection does not change the maximum shear stress, but the maximum bending stress is halved.

The mid-span deflection is

$$\delta = \frac{5wL^4}{384EI} = \frac{5wL^4}{256Ebh^3} \quad (2.11)$$

which is a quarter of the previous deflection (Equation 2.3). Thus the provision of shear connection increases both the bending strength and stiffness of a beam of given size, and in practice leads to a reduction in the size of the beam required for a given loading, and usually to a reduction in its cost.

In this example – but not always – the interface AOB coincides with the neutral axis of the composite member, so that the maximum longitudinal shear stress at the interface is equal to the maximum vertical shear stress, which occurs at $x = \pm L/2$ and is $3wL/8bh$, from Equation 2.10.

The shear connection must be designed for the longitudinal shear per unit length, v_L, which is known as the *shear flow*. In this example it is given by

$$v_{L,x} = \tau_x b = \frac{3wx}{4h} \quad (2.12)$$

The total shear flow in a half span is found, by integration of Equation 2.12, to be $3wL^2/(32h)$. Typically, $L/2h = 20$, so the shear connection in the whole span has to resist a total shear force

$$2 \times \frac{3}{32}\frac{wL^2}{h} = 7.5wL$$

Thus, this shear force is almost eight times the total load carried by the beam. A useful rule of thumb is that the resistance of the shear connection for a beam should be an order of magnitude greater than the load to be carried; it shows that *shear connection must be very strong*.

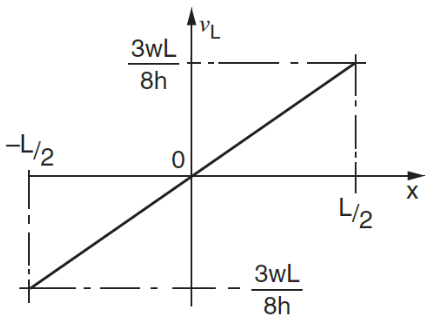


Figure 2.4 Shear flow for 'triangular' spacing of connectors

In design based on elastic behaviour, the shear connectors are spaced in accordance with the shear flow. Thus, if the design shear resistance of a connector is P_{Rd}, the pitch or spacing at which single connectors should be provided, p, is given by $pv_{L,x} \leq P_{Rd}$. From Equation 2.12 this is

$$p \leq \frac{4P_{Rd}h}{3wx} \tag{2.13}$$

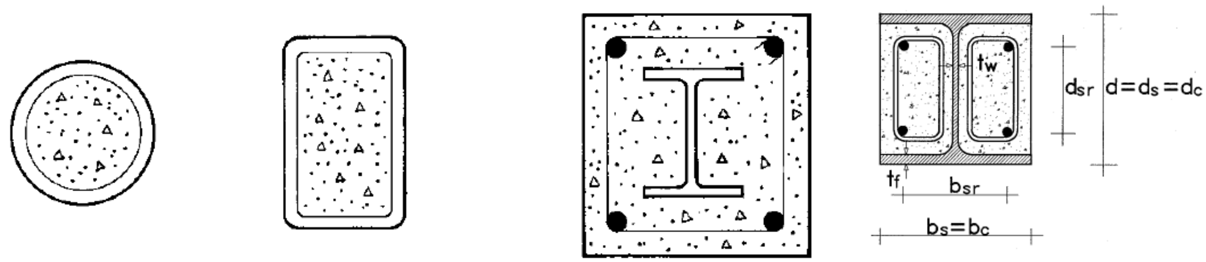
This is known as 'triangular' spacing, from the shape of the graph of v_L against x.

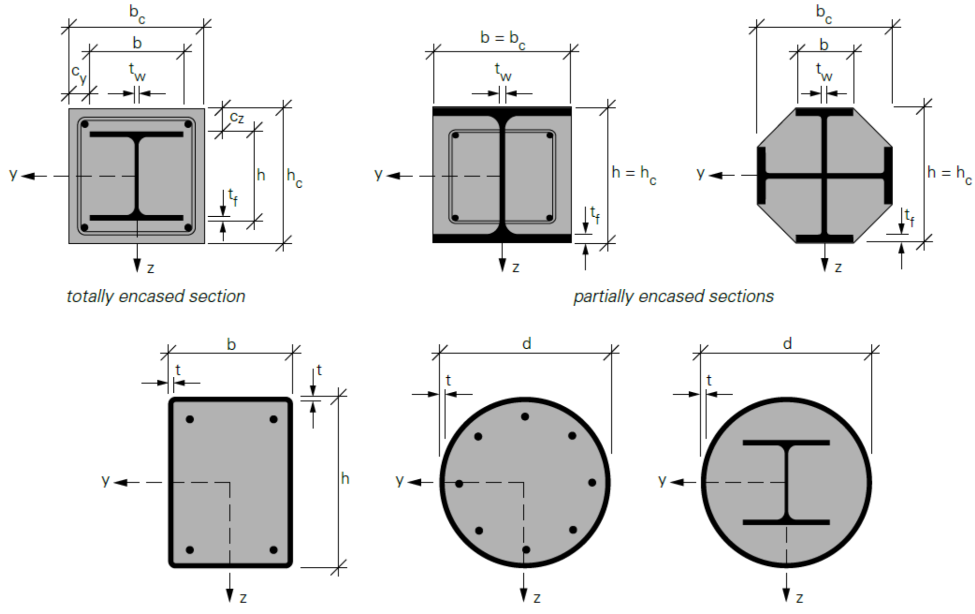
Uplift:

- In the preceding example, the stress normal to the interface AOB was everywhere compressive, and equal to $w/2b$ except above the supports.
- The stress would have been tensile if the load w had been applied to the lower member. Such loading is unlikely, except when travelling cranes are suspended from the steelwork of a composite floor above.
- But there are other situations in which stresses tending to cause vertical separation of the two members (uplift) can occur at the interface. These arise from complex effects such as the torsional stiffness of reinforced concrete slabs forming flanges of composite beams, the triaxial stresses in the vicinity of shear connectors and, in box-girder bridges, the torsional stiffness of the steel box.
- Tension across the interface can also occur in beams of non-uniform section or with partially completed flanges.

COMPOSITE COLUMNS:

Composite columns can take one of two forms: a rolled steel shape encased in concrete with both longitudinal reinforcing bars and transverse reinforcement in the form of ties or spirals (as in a reinforced concrete column) or a hollow shape filled with plain concrete. The encased columns can be fully or partially encased.





Strength of Encased Composite Columns

- The AISC Specification covers encased composite columns in Section I2.1.
- If buckling were not an issue, the member strength could be taken as the summation of the axial compressive strengths of the component materials:

$$P_{no} = F_y A_s + F_{ysr} A_{sr} + 0.85 f'_c A_c \qquad \text{(AISC Equation I2-4)}$$

where

F_y = yield stress of the steel shape

A_s = cross-sectional area of the steel shape

F_{ysr} = yield stress of the longitudinal reinforcing steel bars

A_{sr} = cross-sectional area of the reinforcing steel

- The strength *Pno* is sometimes called the "squash" load; it is the nominal strength when length effects (slenderness effects) are not taken into account.

Effect of slenderness ration:

- Because of slenderness effects, the strength predicted by AISC Equation I2-4 cannot be achieved.
- To account for slenderness, the relationship between *Pno* and *Pe* is used, where *Pe* is the Euler buckling load and is defined as:

1b. Compressive Strength

The design compressive strength, $\phi_c P_n$, and allowable compressive strength, P_n/Ω_c, of doubly symmetric axially loaded encased composite members shall be determined for the limit state of flexural buckling based on member slenderness as follows:

$$\phi_c = 0.75 \text{ (LRFD)} \qquad \Omega_c = 2.00 \text{ (ASD)}$$

(a) When $\dfrac{P_{no}}{P_e} \le 2.25$

$$P_n = P_{no}\left(0.658^{\frac{P_{no}}{P_e}}\right) \tag{I2-2}$$

(b) When $\dfrac{P_{no}}{P_e} > 2.25$

$$P_n = 0.877 P_e \tag{I2-3}$$

where

P_e = elastic critical buckling load determined in accordance with Chapter C or Appendix 7, kips (N)

$$= \pi^2 (EI)_{eff} / L_c^2 \tag{I2-4}$$

$(EI)_{eff}$ = effective stiffness of composite section, kip-in.^{2} (N-mm^2)

$$= E_s I_s + E_s I_{sr} + C_1 E_c I_c \tag{I2-5}$$

C_1 = coefficient for calculation of effective rigidity of an encased composite compression member

$$= 0.25 + 3\left(\frac{A_s + A_{sr}}{A_g}\right) \le 0.7 \tag{I2-6}$$

E_c = modulus of elasticity of concrete

$= w_c^{1.5}\sqrt{f_c'}$, ksi $(0.043 w_c^{1.5}\sqrt{f_c'}$, MPa)

$$P_e = \frac{\pi^2 (EI)_{eff}}{(KL)^2} \qquad \text{(AISC Equation I2-5)}$$

where $(EI)_{eff}$ is the effective flexural rigidity of the composite section and is given by

$$(EI)_{eff} = E_s I_s + 0.5 E_s I_{sr} + C_1 E_c I_c \qquad \text{(AISC Equation I2-6)}$$

where

I_s = moment of inertia of the steel shape about the axis of buckling
I_{sr} = moment of inertia of the longitudinal reinforcing bars about the axis of buckling

The nominal strength equations are similar to those for noncomposite members.
When $\frac{P_{no}}{P_e} \leq 2.25$,

$$P_n = P_{no}\left[0.658^{\left(\frac{P_{no}}{P_e}\right)}\right] \qquad \text{(AISC Equation I2-2)}$$

When $\frac{P_{no}}{P_e} > 2.25$,

$$P_n = 0.877 P_e \qquad \text{(AISC Equation I2-3)}$$

Note that if the reinforced concrete terms in AISC Equations I2-4 and I2-6 are omitted,

$$P_{no} = A_s F_y$$

$$(EI)_{eff} = E_s I_s$$

$$P_e = \frac{\pi^2 E_s I_s}{(KL)^2}$$

and

$$\frac{P_{no}}{P_e} = \frac{A_s F_y}{\pi^2 E_s I_s/(KL)^2} = \frac{F_y}{\pi^2 E_s A_s r^2/A_s (KL)^2} = \frac{F_y}{\pi^2 E_s/(KL/r)^2} = \frac{F_y}{F_e}$$

where F_e is defined in AISC Chapter E as the elastic buckling stress.

AISC Equation I2-2 then becomes

$$P_n = A_s\left[0.658^{\left(\frac{F_y}{F_e}\right)}\right] F_y \qquad (9.7)$$

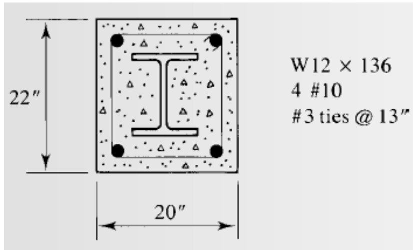
and AISC Equation I2-3 becomes

$$P_n = A_s (0.877) F_e \qquad (9.8)$$

Equations 9.7 and 9.8 give the same strength as the equations in AISC Chapter E for noncomposite compression members.

For LRFD, the design strength is $\phi_c P_n$, where $\phi_c = 0.75$. For ASD, the allowable strength is P_n/Ω_c, where $\Omega_c = 2.00$.

Example: A composite compression member consists of a W12 · 136 (in SI units W310 . 202 kg/m) encased in a 20-in. 22-in. (500mm . 550mm) concrete column as shown in figure. Four #10 (32mm) bars are used for longitudinal reinforcement, and #3 (10mm) ties spaced 13 inches (330mm) center-to-center provide the lateral reinforcement. Assume a concrete cover of 2.5 inches ((63.5mm) to the center of the longitudinal reinforcement. The steel yield stress is $Fy = 50$ ksi (345 MPa) , and Grade 60 (414 MPa) reinforcing bars are used. The concrete strength is $f'c = 5$ ksi (35MPa). Compute the available strength for an effective length of 16 feet (4.9m) with respect to both axes.



SOLUTION Values needed for the AISC strength equations are as follows. For the W12 × 136, $A_s = 39.9$ in.^{2} and $I_s = I_y = 398$ in.^{4} For the longitudinal reinforcement,

$$A_{sr} = 4(1.27) = 5.08 \text{ in.}^2$$

$$I_{sr} = \sum Ad^2 = 4 \times 1.27\left(\frac{20 - 2 \times 2.5}{2}\right)^2 = 285.8 \text{ in.}^2 \text{ (about the weak axis)}$$

For the concrete,

$$A_c = \text{net area of concrete} = 20(22) - A_s - A_{sr} = 440 - 39.9 - 5.08 = 395.0 \text{ in.}^2$$

$$E_c = w^{1.5}\sqrt{f_c'} = (145)^{1.5}\sqrt{5} = 3904 \text{ ksi}$$

$$I_c = \frac{22(20)^3}{12} = 14{,}670 \text{ in.}^4 \text{ (about the weak axis)}$$

From AISC Equation I2-4,

$$P_{no} = F_y A_s + F_{ysr} A_{sr} + 0.85 f_c' A_c = 50(39.9) + 60(5.08) + 0.85(5)(395.0)$$
$$= 3979 \text{ kips}$$

From AISC Equation I2-7,

$$C_1 = 0.25 + 3\left(\frac{A_s + A_{sr}}{A_g}\right) \leq 0.7$$

$$= 0.25 + 3\left(\frac{39.9 + 5.08}{440}\right) = 0.5567 < 0.7$$

From AISC Equation I2-6,

$$(EI)_{\text{eff}} = E_s I_s + 0.5 E_s I_{sr} + C_1 E_c I_c$$
$$= 29{,}000(398) + 0.5(29{,}000)(285.8) + 0.5567(3904)(14{,}670)$$
$$= 4.757 \times 10^7 \text{ kip-in.}^2$$

From AISC Equation I2-5,

$$P_e = \frac{\pi^2 (EI)_{\text{eff}}}{(L_c)^2} = \frac{\pi^2(4.757 \times 10^7)}{(16 \times 12)^2} = 12{,}740 \text{ kips}$$

Then

$$\frac{P_{no}}{P_e} = \frac{3979}{12{,}740} = 0.3123 < 2.25 \qquad \therefore \text{ Use AISC Equation I2-2.}$$

$$P_n = P_{no}\left[0.658^{\left(\frac{P_{no}}{P_e}\right)}\right] = 3979(0.658)^{0.3123} = 3491 \text{ kips}$$

Answer For LRFD, the design strength is $\phi_c P_n = 0.75(3491) = 2620$ kips.
For ASD, the allowable strength is $P_n / \Omega_c = 3491/2.00 = 1750$ kips.

Example:

Compute the values of $\phi_c P_n$ and P_n/Ω_c for the axially loaded encased composite column shown in Fig. 17.3 if KL = 12.0 ft, F_y = 50 ksi, and f_c' = 3.5 ksi. The concrete weighs 145 lb/ft^3.

Solution

Using a W12 × 72(A_s = 21.1 in^2, I_{sx} = 597 in^4, I_{sy} = 195 in^4)

$$A_c = (20 \text{ in})(20 \text{ in}) - 21.1 \text{ in}^2 - (4)(1.0 \text{ in}^2) = 374.9 \text{ in}^2$$

$$P_{no} = A_s F_y + A_{sr} F_{ysr} + 0.85 A_c f_c' \qquad \text{(AISC Equation I2-4)}$$
$$= (21.1 \text{ in}^2)(50 \text{ ksi}) + (4.0 \text{ in}^2)(60 \text{ ksi}) + (0.85)(374.9 \text{ in})(3.5 \text{ ksi}) = 2410 \text{ k}$$

$$C_1 = 0.1 + 2\left(\frac{A_s}{A_c + A_s}\right) \leq 0.3 \qquad \text{(AISC Equation I2-7)}$$

This equation to be modified

$$= 0.1 + 2\left(\frac{21.1}{374.9 + 21.1}\right) = 0.2066 < 0.3 \qquad \textbf{OK}$$

$$E_c = w_c^{1.5}\sqrt{f_c'} = 145^{1.5}\sqrt{3.5} = 3.267 \times 10^3 \text{ ksi}$$

$$I_c = \left(\frac{1}{12}\right)(20)(20)^3 - 195 = 13{,}138 \text{ in}^4$$

$$EI_{eff} = E_s I_s + 0.5 E_s I_{sr} + C_1 E_c I_c \qquad \text{(AISC Equation I2-6)}$$
$$= (29 \times 10^3)(195) + (0.5)(29 \times 10^3)(4 \times 1.0 \times 7.5^2)$$
$$+ (0.2066)(3.267 \times 10^3)(13{,}138) = 17.785 \times 10^6 \text{ k-in}^2$$

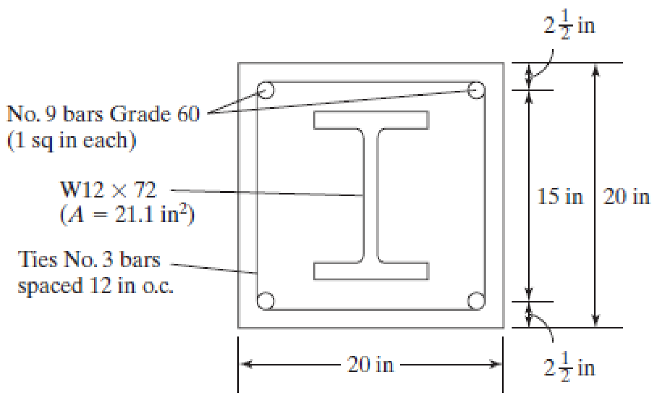


FIGURE 17.3

$$P_e = \frac{(\pi^2)(EI_{eff})}{(KL)^2} \qquad \text{(AISC Equation I2-5)}$$

$$= \frac{(\pi)^2(17.785 \times 10^6)}{(12 \times 12)^2} = 8465 \text{ k}$$

$$\frac{P_{no}}{P_e} = \frac{2410}{8465} = 0.28 \leq 2.25$$

$\therefore$ Must use AISC Equation I2-2 for P_n.

$$P_n = P_{no}\left[0.658^{\frac{P_{no}}{P_e}}\right] = 2410\left[0.658^{\frac{2410}{8465}}\right] = 2139 \text{ k}$$

LRFD $\phi_c = 0.75$	ASD $\Omega_c = 2.00$
$\phi_c P_n = (0.75)(2139) = 1604$ k	$\frac{P_n}{\Omega_c} = \frac{2139}{2.00} = 1070$ k

Note: The W12 × 72 alone has $\phi_c P_n = 807$ k and $P_n/\Omega_c = 537$ k.

Limitations and Detailing regarding Encased Composite Members (AISC):

a) Limitations:

For encased composite members, the following limitations shall be met:

a) The cross-sectional area of the steel core shall comprise at least 1% of the total composite cross section.
b) Concrete encasement of the steel core shall be reinforced with continuous longitudinal bars and transverse reinforcement consisting of ties, hoops, and/or spirals. Detailing and placement of longitudinal reinforcement, including bar spacing and concrete cover requirements, shall conform to ACI 318.
c) Transverse reinforcement where specified as ties or hoops shall consist of a minimum of either a No. 3 (10 mm) bar spaced at a maximum of 12 in. (300 mm) on center, or a No. 4 (13 mm) bar or larger spaced at a maximum of 16 in. (400 mm) on center. Deformed wire or welded wire reinforcement of equivalent area is permitted.
d) Maximum spacing of ties or hoops shall not exceed 0.5 times the smaller column dimension.
e) The minimum reinforcement ratio for continuous longitudinal reinforcement, ρ_{sr}, shall be 0.004, where ρ_{sr} is given by

$$\rho_{sr} = \frac{A_{sr}}{A_g} \qquad \text{(I2-1)}$$

where
A_g = gross area of composite member, in.^{2} (mm^2)
A_{sr} = area of continuous longitudinal reinforcing bars, in.^{2} (mm^2)

f) The maximum reinforcement ratio for continuous longitudinal reinforcement, ρ_{sr}, shall meet ACI 318 with the gross area of concrete, A_g, assumed in the calculations.

b) Detailing:

For encased composite members, the following detailing requirements shall be met:

(a) Clear spacing between the steel core and longitudinal reinforcing bars shall be a minimum of 1.5 longitudinal reinforcing bar diameters, but not less than 1.5 in. (38 mm).

(b) If the composite cross section is built up from two or more encased steel shapes, the shapes shall be interconnected with lacing, tie plates, or comparable components to prevent buckling of individual shapes due to loads applied prior to hardening of the concrete.

2. Filled Composite Members

2a. Limitations

For filled composite members, the following limitations shall be met:

(a) The cross-sectional area of the structural steel section shall comprise at least 1% of the total composite cross section.

(b) Filled composite members shall be classified for local buckling according to Section I1.4.

(c) Minimum longitudinal reinforcement is not required.

- If longitudinal reinforcement is provided, internal transverse reinforcement is not required for strength; however, minimum internal transverse reinforcement shall be provided.
- Transverse reinforcement where specified as ties or hoops shall consist of a minimum of either a No. 3 (10 mm) bar spaced at a maximum of 12 in. (300 mm) on center, or a No. 4 (13 mm) bar or larger spaced at a maximum of 16 in. (400 mm) on center.
- Deformed wire or welded wire reinforcement of equivalent area is permitted.

(d) If longitudinal reinforcing steel is provided for strength, the maximum reinforcement ratio shall be based on ACI 318 requirements for the gross area of concrete.

Strength of Filled Composite Columns:

Compressive Strength

The available compressive strength of axially loaded doubly symmetric filled composite members shall be determined for the limit state of flexural buckling in accordance with Section I2.1b with the following modifications:

(a) For compact composite sections

$$P_{no} = P_p \tag{I2-9a}$$

where

P_p = plastic axial compressive strength, kips (N)

$$= F_y A_s + C_2 f_c' \left(A_c + A_{sr} \frac{E_s}{E_c} \right) \tag{I2-9b}$$

C_2 = 0.85 for rectangular sections and 0.95 for round sections

(b) For noncompact composite sections

$$P_{no} = P_p - \frac{P_p - P_y}{\left(\lambda_r - \lambda_p\right)^2} \left(\lambda - \lambda_p\right)^2 \tag{I2-9c}$$

where

λ_p and λ_r are width-to-thickness ratios determined from Table I1.1a.

P_p is determined from Equation I2-9b.

$$P_y = F_y A_s + 0.7 f_c' \left(A_c + A_{sr} \frac{E_s}{E_c} \right) \tag{I2-9d}$$

(c) For slender composite sections

$$P_{no} = F_n A_s + 0.7 f_c' \left(A_c + A_{sr} \frac{E_s}{E_c} \right) \tag{I2-9e}$$

where

the critical buckling stress for the structural steel section of filled composite members, F_n, is determined as follows:

(1) For rectangular filled sections

$$F_n = \frac{9E_s}{\lambda^2} \tag{I2-10}$$

(2) For round filled sections

$$F_n = \frac{0.72F_y}{\left[\left(\frac{D}{t}\right)\frac{F_y}{E_s}\right]^{0.2}} \tag{I2-11}$$

See Section I1.4 for definitions of maximum width-to-thickness ratio, λ; width, D; and thickness, t, for rectangular and round HSS and box sections of uniform thickness.

The effective stiffness of the composite section, $(EI)_{eff}$, for all sections shall be

$$(EI)_{eff} = E_s I_s + E_s I_{sr} + C_3 E_c I_c \tag{I2-12}$$

where

C_3 = coefficient for calculation of effective rigidity of a filled composite compression member

$$= 0.45 + 3\left(\frac{A_s + A_{sr}}{A_g}\right) \le 0.9 \tag{I2-13}$$

The available compressive strength need not be less than that determined for the bare steel member in accordance with Chapter E.

TABLE I1.1a
Limiting Width-to-Thickness Ratios for Compression Steel Elements in Composite Members Subjected to Axial Compression for Use with Section I2.2

Description of Element	Width-to-Thickness Ratio	λ_p Compact Composite/ Noncompact Composite	λ_r Noncompact Composite/ Slender-Element Composite	Maximum Permitted
Walls of rectangular HSS and box sections of uniform thickness	b/t	$2.26\sqrt{\frac{E}{F_y}}$	$3.00\sqrt{\frac{E}{F_y}}$	$5.00\sqrt{\frac{E}{F_y}}$
Round HSS	D/t	$\frac{0.15E}{F_y}$	$\frac{0.19E}{F_y}$	$\frac{0.31E}{F_y}$

EXAMPLE 9.14

An HSS 7 × 0.125 with an effective length of 13 feet is filled with normal-weight concrete and used as a column. No longitudinal reinforcement is used. The concrete strength is $f_c' = 5$ ksi. Compute the available compressive strength.

SOLUTION The following dimensions and properties from Part 1 of the *Manual* will be needed: design wall thickness = 0.116 in., $A_s = 2.51$ in.^{2}, $D/t = 60.3$ in.^{4}, and $I_s = 14.9$ in.^{4}. The following values will also be required:

$$A_c = \frac{\pi d_{\text{inside}}^2}{4} = \frac{\pi(7.000 - 2 \times 0.116)^2}{4} = 35.98 \text{ in.}^2$$

$$E_c = w^{1.5}\sqrt{f_c'} = (145)^{1.5}\sqrt{5} = 3904 \text{ ksi}$$

$$I_c = \frac{\pi d_{\text{inside}}^4}{64} = \frac{\pi(7.000 - 2 \times 0.116)^4}{64} = 103.0 \text{ in.}^4$$

Determine the classification of the section:

$$\lambda = \frac{D}{t} = 60.3$$

$$\lambda_p = \frac{0.15E}{F_y} = \frac{0.15(29,000)}{42} = 103.6$$

Since $\lambda < \lambda_p$, the section is compact, and from AISC Equation I2-9b,

$$P_{no} = P_p = F_y A_s + C_2 f_c' \left(A_c + A_{sr}\frac{E_s}{E_c} \right)$$

$$= 42(2.51) + 0.95(5)(35.98 + 0) = 276.3 \text{ kips}$$

Next, compute $(EI)_{eff}$ and P_e. From AISC Equation I2-13,

$$C_3 = 0.6 + 2\left(\frac{A_s}{A_c + A_s}\right) \le 0.9$$

$$= 0.6 + 2\left(\frac{2.51}{35.98 + 2.51}\right) = 0.7304 < 0.9$$

and from AISC Equation I2-12,

$$(EI)_{eff} = E_s I_s + E_s I_{sr} + C_3 E_c I_c$$

$$= 29,000(14.9) + 0 + 0.7304(3904)(103.0) = 7.258 \times 10^5 \text{ kip-in.}^2$$

From AISC Equation I2-5,

$$P_e = \frac{\pi^2 (EI)_{eff}}{(KL)^2} = \frac{\pi^2(7.258 \times 10^5)}{(13 \times 12)^2} = 294.4 \text{ kips}$$

Determine P_n:

$$\frac{P_{no}}{P_e} = \frac{276.3}{294.4} = 0.9385 < 2.25 \qquad \therefore \text{ Use AISC Equation I2-2.}$$

$$P_n = P_{no}\left[0.658^{\frac{P_{no}}{P_e}}\right] = 276.3\left[0.658^{\frac{276.3}{294.4}}\right] = 186.5 \text{ kips}$$

ANSWER For LRFD, the design strength is $\phi_c P_n = 0.75(186.5) = 140$ kips.

For ASD, the allowable strength is $P_n/\Omega_c = 186.5/2.00 = 93.3$ kips.

Example 17-2

Determine the LRFD design strength $\phi_c P_n$ and the ASD allowable strength P_n/Ω_c of a concrete-filled 46 ksi HSS 12 × 12 × 1/2 section filled with 4 ksi concrete weighing 145 lb/ft^3. $(KL)_x = (KL)_y = 16$ ft.

Solution

Using an HSS $12 \times 12 \times \frac{1}{2}$ ($A_s = 20.9$ in^2, $I_x = I_y = 457$ in^4, $t = 0.465$ in)

$$\frac{b}{t} = \frac{12}{0.465} = 25.81 < 2.26\sqrt{\frac{E}{F_y}} = 2.26\sqrt{\frac{29 \times 10^3}{46}} = 56.7 \therefore \text{Compact section}$$

$$P_{no} = P_p \qquad \text{(AISC Equation I2-9a)}$$

C_2 given in specification = 0.85 (for rectangular section)

$$A_c = (12)(12) - (20.9) = 123.1 \text{ in}^2$$

$$P_p = A_s F_y + C_2 f'_c \left[A_c + A_{sr}\left(\frac{E_s}{E_c}\right)\right] \qquad \text{(AISC Equation I2-9b)}$$

$$= (20.9)(46) + 0.85(4)\,[123.1 + 0] = 1380 \text{ k} = P_p = P_{no}$$

$$C_3 = 0.6 + 2\left(\frac{A_s}{A_c + A_s}\right) \leq 0.9 \qquad \text{(AISC Equation I2-13)}$$

$$= 0.6 + 2\left(\frac{20.9}{(12 \times 12) + 20.9}\right) = 0.85 < 0.9 \qquad \textbf{OK}$$

$$E_c = w_c^{1.5}\sqrt{f'_c} = (145)^{1.5}\sqrt{4} = 3.492 \times 10^3 \text{ ksi}$$

$$I_c = \left(\frac{1}{12}\right)(12)(12)^3 - 457 = 1271 \text{ in}^4$$

$$EI_{eff} = E_s I_s + E_s I_{sr} + C_3 E_c I_c \qquad \text{(AISC Equation I2-12)}$$

$$= (29 \times 10^3)(457) + 0 + (0.85)(3.492 \times 10^3)(1271)$$

$$= 17.026 \times 10^6 \text{ k-in}^2$$

$$P_e = \frac{\pi^2 EI_{eff}}{(KL)^2} \qquad \text{(AISC Equation I2-5)}$$

$$= \frac{(\pi^2)(17.026 \times 10^6)}{(12 \times 16)^2} = 4558 \text{ k}$$

$$\frac{P_{no}}{P_e} = \frac{1380}{4558} = 0.30 \leq 2.25$$

$\therefore$ Use AISC Equation I2-2.

$$P_n = P_{no}\left[0.658^{\frac{P_{no}}{P_e}}\right] = 1380\left[0.658^{\frac{1380}{4558}}\right] = 1216 \text{ k}$$

LRFD $\phi_c = 0.75$	ASD $\Omega_c = 2.00$
$\phi_c P_n = (0.75)(1216) = 912$ k	$\frac{P_n}{\Omega_c} = \frac{1216}{2.00} = 608$ k

From AISC Table 4-15, the values are $\phi_c P_n = 911$ k and $P_n/\Omega_c = 607$ k. In computing the properties of the column, the author did not account for the fillets on the inside corners of the HSS. For this reason, his results vary a little from those given in the AISC tables.