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Statistical(Measures of Dispersion)

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Dispersion: is the distance(deviation) away from the center (mean). The distance away from the center is expressed as:

$$(x_i - \bar{x})$$

❖ If we have the following values, $x_1, x_2, x_3, \dots, x_n$, then the mean is:

$$\bar{x} = \frac{\sum x_i}{n}$$

Where:

- x_i= each value
- n = total number of values

Remark: The sum of the deviations of the values from their mean is zero.

$$\therefore \sum(x_i - \bar{x}) = 0$$

Prove $\sum(x_i - \bar{x}) = 0$**??**

Proof: $\sum_{i=1}^{n}(x_i - \bar{x}) = \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} \bar{x} = \sum_{i=1}^{n} x_i - n\bar{x}$

$$= \sum_{i=1}^{n} x_i - n\frac{\sum_{i=1}^{n} x_i}{n} = \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} x_i = 0$$

Example 1: These data show duration of hospital stay for 5 patients.

NO.	Duration of hospital stay	$\overline{x}$	$x_i - \overline{x}$
1	0	3	-3
2	1	3	-2
3	1	3	-2
4	2	3	-1
5	11	3	8
total	15		0

The mean for the data $\overline{x} = \frac{15}{5} = \mathbf{3}$

❖ Measures of dispersion tell us " how dispersed" the values are from their center . measures of dispersion are numerous, e.g.

1)Range ($\boldsymbol{R}$).

2) Variance ($\boldsymbol{s^2}$).

3) Standard deviation ($\boldsymbol{s}$).

4) Coefficient of variation ($\boldsymbol{cv}$).

Range: The simplest measure of variability for a set of data is the range and is defined as the difference between the largest and smallest values in the set.

$$\text{Range} = \text{maximum value} - \text{minimum value}$$

$$\boldsymbol{R} = \boldsymbol{max} - \boldsymbol{min}$$

Example 2: Find the range for the sample observations:

13, 23, 11, 17, 25, 18, 14, 24

Solution: We see that the largest observation is 25 and the smallest observation is 11.

∴ Range is 25-11=14.

Example 3: A pharmacist measures the weights (mg) of 5 tablets from a batch to check uniformity. Tablet weights (mg):

498, 502, 505, 500, 497 , Calculate the Range??

Solution: $\boldsymbol{Range} = \boldsymbol{max} - \boldsymbol{min}$

$\boldsymbol{max}$ = 505
$\boldsymbol{min}$= 497

$\boldsymbol{Range}$ =505−497=8 mg.

Variance (s^2): is the mean sum of the squares of the deviations

of the data from the arithmetic mean of the data. Variance (s^2) can be calculated as follows:

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2 \quad \textbf{Variance of sample}$$

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \overline{x})^2 \quad \textbf{Variance of population}$$

Where :

$\boldsymbol{n}$: is the number of data.

$\boldsymbol{x_i}$: is the ith data point in the data set x.

$\overline{\boldsymbol{x}}$: is the mean of the data set x.

Example 4: These data show the weight ($\boldsymbol{x}$) of 5 children (in kg) are (5,6,6,3,5) Calculate the variance??

Solution: Mean weight for these children $\overline{(\boldsymbol{x})} = \frac{25}{5} = \mathbf{5}$

x	$\overline{x}$	$x - \overline{x}$	$(x - \overline{x})^2$
5	5	0	0
6	5	1	1
6	5	1	1
3	5	-2	4
5	5	0	0
			Total=6

$$\sum(x-\bar{x})^2 = 6$$

$$n = 5 \rightarrow n-1 = 4$$

$$s^2 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1} = \frac{6}{4} = 1.5\ kg^2$$

Example 5: A quality-control lab measures the dissolution percentage of a drug from 6 tablets, Dissolution %: 87, 90, 92, 88, 91, 89 , Calculate the variance??

Solution:

$$\bar{x} = \frac{87+90+92+88+91+89}{6} = \frac{537}{6} = 89.5$$

x	$\bar{x}$	$x-\bar{x}$	$(x-\bar{x})^2$
87	89.5	-2.5	6.25
90	89.5	0.5	0.25
92	89.5	2.5	6.25
88	89.5	-1.5	2.25
91	89.5	1.5	2.25
89	89.5	-0.5	0.25
			Total=17.5

Variance of sample: $s^2 = \frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1} = \frac{17.5}{5} = 3.5$

The standard Deviation (s)

Standard deviation (s) is the square root of the variance. The standard deviation can be calculated as follows:

$$s = \sqrt{\frac{\sum_{i=1}^{n}(x_i-\bar{x})^2}{n-1}}$$

Example 6: In example number 4, we found the variance to be equal to Variance $s^2 = 1.5\ kg$, find Standard deviation (s)??

Solution: Standard deviation $s = \sqrt{s^2} = \sqrt{1.5 =}\ 1.22\ kg$

Example 7: Measuring the amount of active ingredient in eye-drop samples (mg/mL): 9.8, 10.2, 10.0, 9.7, 10.3 ,find **Range , Variance (sample), Standard deviation?**

Solution:

- **Range** = $10.3 - 9.7 = 0.6$
- **Variance (sample):**

  Mean ($\bar{x}$) $= 10$

$$s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$$

$$= \frac{(9.8-10)^2 + (10.2-10)^2 + (10.0-10)^2 + (9.7-10)^2 + (10.3-10)^2}{4}$$

$$s^2 = 0.06$$

- **Standard deviation:**

$$s = \sqrt{s^2} = \sqrt{0.06} = 0.245$$

Coefficient of Variation (cv):

coefficient of variation (cv) is the percentage of the standard deviation to the mean, as follows:

$$cv = \frac{standard\ deviation}{mean} \times 100\%$$

$$cv = \frac{s}{\bar{x}} \times 100\%$$

Example 8: these data show the weight (x) of 5 children (in kg). as previously calculated: mean($\bar{x}$) = 5 kg , standard deviation(s) =1.22 kg , find coefficient of variation (cv)?

Solution: $cv = \frac{s}{\bar{x}} \times 100\% = \frac{1.22 \times 100}{5} = 24.4\ \%$

Example 9: We have a medication containing the following active ingredient (mg) in 5 capsules ,Values: 98, 102, 100, 101, 99

Where $\bar{x} = 100$,and $s = 1.58$, find coefficient of variation (cv)?

Solution: $\boldsymbol{cv} = \frac{\boldsymbol{s}}{\boldsymbol{\bar{x}}} \times 100\% = \frac{1.58 \times 100}{\mathbf{100}} = 1.58\ \%$**.**

Example 10: A company wants to evaluate the accuracy of a device for measuring drug concentration, We have 3 readings: 102 ,100,98 ,where Where $\bar{x} = 100$,and $s = 2$, find coefficient of variation (cv)?

Solution: $\boldsymbol{cv} = \frac{\boldsymbol{s}}{\boldsymbol{\bar{x}}} \times 100\% = \frac{2 \times 100}{\mathbf{100}} = 2\ \%$

H.W

1) If the deviations of 7 values from their mean are:

2.2 , -2.1, 0.1 , -1.2 , - 0.7 , k ,1.3 Find the value of k?

2) We have the following values $8,10,12, b, 14$, and their mean $\boldsymbol{\bar{x} = 11,}$ Find the value of b?

3) If the standard deviation = 2 and the mean = 7 for the following values:

$m, 9,7,5$, Find the value of m?

4) We have data containing mean = 50 ,and coefficient of variation (cv)=10% , find standard deviation?