University of Basrah/College of Pharmacy

Mathematics(Application of the area under the curve)

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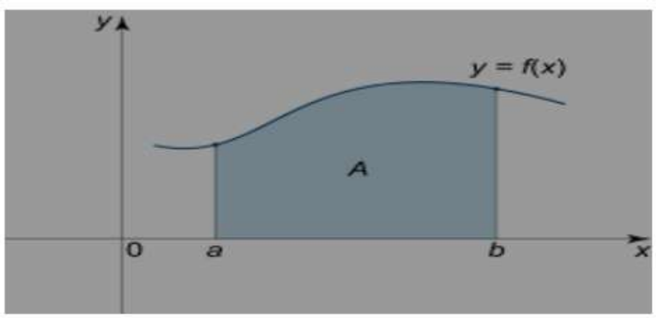
Application of the area under the curve

Definition : Let $f(x)$ be continuing function over the closed value [a, b], then the area under the curve define: -



$$A = \int_a^b f(x)dx \ \ \boldsymbol{with\ x - axis}$$

Or

$$A = \int_a^b f(y)dy \ \ \boldsymbol{with}\ y - \boldsymbol{axis}$$

Remark :

- In pharmacokinetics, AUC (Area Under the Curve) represents the total drug exposure over a time period. Mathematically, it is expressed as:

$$AUC = \int_{t_1}^{t_2} C(t)dt$$

where:

- $C(t)$: drug concentration at time t.
- t_1, t_2 : time boundaries.

Properties of the Definite Integral:

1. $\int_a^a f(x)dx = 0$

2. $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$.

3. $\int_a^b k.f(x)dx = k\int_a^b f(x)dx$,for any constant k.

4. $\int_a^b (f(x) + g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx$.

5. $\int_a^b f(x)dx = -\int_b^a f(x)dx$

Example 1: Oral dose yields $C(t) = 4e^{-0.3t}$ (mg/L). Find the AUC from $0\ to\ 10\ h$?

$(e^{-3} = 0.049\,, e^0 = 1)$

Solution: $AUC = \int_0^{10} 4e^{-0.3t}dt = 4\frac{1}{-0.3}\int_0^{10}(-0.3).e^{-0.3t}dt$

$$= \frac{-4}{0.3}e^{-0.3t}|_0^{10} = \frac{-4}{0.3}\left[e^{-0.3(10)} - e^{-0.3(0)}\right] = \frac{-4}{0.3}[0.049 - 1\ ] = 12.68$$

Example 2: concentration given by $C(t) = 3t + 5$. Find AUC between $t = 0$ and $t = 6$??

Solution: $AUC = \int_0^6 (3t + 5)dt = 3\frac{t^2}{2} + 5t|_0^6$

$$= [\tfrac{3}{2}(6)^2 + 5(6)] - [\tfrac{3}{2}(0)^2 + 5(0)] = 84.$$

Example 3: Concentration follows $C(t) = \sqrt{t}$. Compute the AUC from $0\ to\ 9\ h$?

Solution: $AUC = \int_0^9 \sqrt{t}\,dt = \frac{2}{3}t^{\frac{3}{2}}\Big|_0^9 = \frac{2}{3}.(9)^{\frac{3}{2}} = 18$

Example 4: Find the area of the region bounded by $y = 4x - x^2$ and $x - axis$??

Solution: We find the intersection point with $x - axis\ \rightarrow y = 0 \rightarrow 4x - x^2 = 0$

$\Longrightarrow x(4 - x) = 0 \Longrightarrow x = 0\ or\ x = 4$

$A = \int_0^4 4x - x^2 dt = 2x^2 - \frac{x^3}{3}\Big|_0^4 = (2(4^2) - \frac{4^3}{3}) = \frac{32}{3}$ unit area.

Example 5: Calculate the area bounded by the x-axis and the curve $y = 6 - x - x^2$?
Solution: We find the intersection point with $x - axis \quad \rightarrow y = 0$
$\rightarrow 6 - x - x^2 = 0 \rightarrow (3 + x)(2 - x) = 0$
Which gives $x = -3 \; and \; x = 2$
The area is:

$$A = \int_{-3}^{2} (6 - x - x^2)dx = 6x - \frac{x^2}{2} - \frac{x^3}{3}\Big|_{-3}^{2} = \left(12 - 2 - \frac{8}{3}\right) - \left(-18 - \frac{9}{2} + 9\right) = \frac{125}{6}$$

Example 6: A drug's plasma concentration (mg/L) follows: $C(t) = mt - t^2$,

$0 \leq t \leq 4$ hours. If the total drug exposure $AUC = \frac{32}{3}$ mg · hr/L , find the value of m??

Solution: $AUC = \int_0^4 (mt - t^2)dt = m\,\frac{t^2}{2} - \frac{t^3}{3}\Big|_0^4 = m8 - \frac{64}{3}$

$\therefore AUC = \frac{32}{3} \rightarrow m8 - \frac{64}{3} = \frac{32}{3} \rightarrow m8 = \frac{32}{3} + \frac{64}{3} = \frac{96}{3} = 32$

$\rightarrow m = \frac{32}{8} \rightarrow m = 4.$

Example 7: A tablet dissolves at a rate: $R(t) = 4t + 2, 0 \leq t \leq b$ hours , if the total drug dissolved equal 12 mg find the value of b??

Solution: $AUC = \int_0^b (4t + 2)dt = 2\,t^2 + 2t\,|_0^b = 2b^2 + 2b$

$\therefore AUC = 12 \rightarrow 2b^2 + 2b = 12 \rightarrow b^2 + b - 6 = 0 \rightarrow (b + 3)(b - 2) = 0$

$\rightarrow b = -3 \; or \; b = 2$.

Since b is a time (hours) in the dissolution problem, we need a real, nonnegative solution. The only real solution is $b = 2$ hours .

Example 8: If the area between the curve $y = x(x - 3)$, and $x = 0$ & $x = 5$ is A ,find $6A$??

Solution: $A = \int_0^5 \big(x(x - 3)\big)dx = \int_0^5 (x^2 - 3x)dx = \frac{x^3}{3} - 3\frac{x^2}{2}\,|_0^5$

$$= \frac{5^3}{3} - 3\frac{5^2}{2} = \frac{125}{3} - \frac{75}{2}$$

$$= \frac{250 - 225}{6} = \frac{25}{6}$$

$$\rightarrow 6A = 6\left(\frac{25}{6}\right) = 25.$$

Example 9: If the area between the curve $y = 2x + k,$ and $x = 1$ to $x = 3$ equal $14\ units^2$, find the value of k??

Solution: $A = \int_1^3 (2x + k,)dx = x^2 + kx \,|_1^3$

$$A = (9 + 3k) - (1 + k) = 8 + 2k$$

$\therefore A = 14 \rightarrow 8 + 2k = 14$

$$\rightarrow 2k = 14 - 8 \rightarrow k = 3$$

H.W

1)Let $A_1 = \int_a^b k.x\,dx$ and $A_2 = \int_a^b k\,dx$,find $Z = \frac{A_1}{A_2}$?

2)Let$\int_0^2 f(x)dx = 3$ and $\int_2^6 f(x)dx = -2$,find:

- $2\int_0^2 f(x)dx - 5\int_2^6 f(x)dx??$
- $\int_0^6 f(x)\,dx$??
- $\int_6^2 f(x)\,dx$??

3) Calculate $\int_0^{10} \frac{3}{\sqrt{5x-1}} dx$??

4) A drug produces the concentration profile: $C(t) = 0.2t^2 + 2t$, after administering a 50 mg dose. If the dose is doubled to 100 mg, the expected concentration profile becomes: $C(t) = 0.4t^2 + 4t$

Calculate:

a) The AUC from 0 to 12 hours for the 50 mg dose?
b) The AUC for the 100 mg dose ?
c) The ratio $\frac{AUC_2}{AUC_1}$?