University of Basrah/College of Pharmacy

Mathematics(Calculus 1 – Fundamentals of Calculus)

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1. Introduction to Calculus

Calculus is a branch of mathematics that focuses on change and accumulation. It is used to understand motion, growth, and variation in natural and social sciences. Calculus is divided into two main branches:

- **Differential Calculus: studies rates of change (derivatives).**
- **Integral Calculus: studies accumulation of quantities (integrals). Applications appear in physics (motion, force), economics (cost and revenue), biology (population growth), and engineering (optimization and modeling).**

2. Functions

A function assigns each input exactly one output. Examples:
1.$f(x) = x^2$(quadratic function)
2. $f(x) = e^x$ (exponential function)
3. $f(x) = \ln(x)$ (logarithmic function)
4. $f(x) = \sin(x)$ (trigonometric function)

3. Limits

A limit describes the value a function approaches as the input approaches a point.

Example 1:

$$\lim_{x\to 2} f(x) = x^2 + x - 2 = 2^2 + 2 - 2 = 4.$$

Example 2: $\lim_{x\to 2} \frac{3x+2}{x-1} = \frac{3.2+2}{2-1} = 8.$

Example 3: *Let* $f(x) = \frac{1}{x^2}$ *Find* $\lim_{x\to 0} f(x)$ *if it exists?*

Solution: *The values of f (x) can be made arbitrarily large by taking x close enough to 0. Thus the values of f (x) do not approach 0 number, so* $\lim_{x\to 0}\frac{1}{x^2}$ *does not exist.*

4. Derivatives

The derivative measures the instantaneous rate of change of a function. It can be viewed as the slope of the tangent line to a curve at a given point.

Derivative definition *:the function* $f' = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ *is called the derivative with respect to* x *of the function* f*. The domain of f0consists of all the points for which the limit exists. The domain of* f' *consists of all the points for which the limit exists.*

Basic Rules:

1. If $y = k$, where k is constant , $y' = 0$.
2. If $y = k.\, f(x)$, *then* $y' = k.\, f'(x)$.
3. If $f(x) = x^n$, then for every real value of x, $f'(x) = nx^{n-1}$.
4. If $y = f(x) \mp g(x)$, *then* $y' = f'(x) \mp g'(x)$.
5. If $y = f(x)\, .\, g(x)$, *then* $y' = f(x) * g'(x) + g(x) * f'(x)$.
6. If $f(x) = \frac{f(x)}{g(x)}$, where $g(x) \neq 0$ *for every* x , *then* $y' = \frac{g(x).f'(x)-f(x).g'(x)}{(g(x))^2}$.
7. If $y = (f(x))^n$, (*Any real number* n) then $y' = n(f(x))^{n-1}.f'(x)$.
8. If $y = \sin(u)$, *then* $y' = \cos(u).u'$
9. If $y = \cos(u)$, *then* $y' = -\sin(u).u'$
10. If $y = e^u$, *then* $y' = e^u.u'$
11. If $y = ln(u)$, *then* $y' = \frac{u'}{u}$

Example 4: If $y = (x^2 + x + 1)^3$ *then* $y' = 3(x^2 + x + 1)^2 * (2x + 1)$

Example 5: If $y = \frac{x^2}{2x-1}$, find y'?

$$y' = \frac{(2x-1)*(2x)-x^2*2}{(2x-1)2} = \frac{4x^2-2x-2x^2}{(2x-1)2} = \frac{2x^2-2x}{(2x-1)2} = \frac{2(x^2-x)}{(2x-1)2}$$

Example 6: **(Application – Velocity)**

If $s(t) = 5t^2 + 2t$ is the position (in meters), find velocity?

v(t) = s'(t) = 10t + 2 is velocity.

Example 7: **If the concentration of a drug in the plasma is given by the equation:**

$C(X)5X^3$,What is the instantaneous rate of increase in concentration at any time t ?

$C'(X) = 15X^2$ mg·L^{-1}, t in hours.

Example 8 : **If Drug plasma concentration after bolus: $C(t) = C_0e^{-kt}$, find $\frac{dC}{dt}$?**

Solution: $\frac{dC}{dt} = C_0(-k)e^{-kt} = -kC_0e^{-kt}$.

Interpretation: The instantaneous rate of change is proportional to (−k)times the current concentration (exponential decay). Negative sign = concentration decreasing.

Example 9: **If $PH = -log_{10}[H^+]$, So how does pH change when the concentration of hydrogen ions [H$^+$] changes?**

Solution: **Use natural log relation $log_{10}x = \frac{lnx}{ln10}$,so $log_{10}[H^+], = \frac{ln[H^+]}{ln10}$**

Differentiate:$\frac{dPH}{d[H^+]} = \frac{1}{ln10} \cdot \frac{1}{[H^+]}$.

Example 10: If $y = sin^4(x)\ ,find\ y\,'$?

$y\,' = 4\ sin^3(x).cos(x).1$

Example 11: If $y = (7sin(5x) - cos(\sqrt{x}))^3\ ,find\ y\,'$?

$y\,' = 3\ (7sin(5x) - cos(\sqrt{x}))^2.(35cos(5x) + \frac{1}{2\sqrt{x}}\ sin(\sqrt{x}).$

Example 12: If $f(x) = cos(e^{2x})\ ,,find\ f\,'(x)$?

$f\,'(x) = -sin(e^{2x}).e^{2x}.2 = -2e^{2x}.sin(e^{2x})$.

Indefinite Integral

Def **:A function $F(x)$ is called an anti-derivative of a function $f(x)$ if $F'(x) = f(x)$. If $F(x)$a is any anti-derivative of $f(x)$,then the most general anti-derivative of $f(x)$ is called an intdefinite integral and denoted $\int f(x)dx = F(x) + c$, c is any constant.**

- **In this definition the $\int$ *is* called the integral symbol, $f(x)$is called the integrand , x is called the integration variable and the c is called the constant of integration.**

Basic Integration formulas:

1. $\int x^n dx = \frac{x^{n+1}}{n+1} + C$

2. $\int \left(f(x)\right)^n . f'(x) dx = \frac{\left(f(x)\right)^{n+1}}{n+1} + C$

Example 1: **If the rate of drug accumulation is: $\frac{dA}{dt} = 5t^2$, Find the general expression for the amount of drug $A(t)$?**

Solution: $A(t) = \int 5t^2 dt = 5\frac{t^3}{3} + C$

Example 2: **If infusion-rate (mg·hr⁻¹) modeled as: $\frac{dA}{dt} = 3t^2 + 2t + 1$, Find A(t)?**

Solution: $A(t) = \int 3t^2 + 2t + 1dt = 3\frac{t^3}{3} + 2\frac{t^2}{2} + t + C = t^3 + t^2 + t + C.$

Example 3: **If rate of concentration change is: $\frac{dC}{dt} = 0.5t^2 - t + 0.2$ $(mg.L^{-1}.hr^{-1})$.**

Find $C(t)$??

Solution:$C(t) = \int (0.5t^2 - t + 0.2)dt = 0.5\frac{t^3}{3} - \frac{t^2}{2} + 0.2t + C.$

Example 4: **Suppose rate: $\frac{dA}{dt} = (t+1)(t^2 + 2t)$, Find $A(t)$??**

Solution: $A(t) = \int (t+1)(t^2+2t)dt = \int (t^3+3t^2+2t)dt$

$$= \frac{t^4}{4} + 3\frac{t^3}{3} + 2\,\frac{t^2}{2} + C = \frac{t^4}{4} + t^3 + t^2 + C.$$

Example 5: A topical gels potency index changes in time as: $(3t+2)^4$, Find the antiderivative $P(t)$??

Solution: $P(t) = \int (3t+2)^4 dt = \frac{1}{3}\int 3\,(3t+2)^4 dt = \frac{1}{3}\frac{(3t+2)^5}{5} + C = \frac{(3t+2)^5}{15} + C.$

Example 6: In an advanced pharmaceutical formulation study, the rate of change of an excipient's activity inside a liquid preparation is modeled as: $R(t) = \frac{t}{(t^2+1)^2}$, where:

- **$R(t)$: rate of change of excipient activity**
- **t: time (hours)**

Find the expression for the cumulative activity of the excipient??

Solution: **Let** $A(t) = \int \frac{t}{(t^2+1)^2} dt = \int t\,(t^2+1)^{-2} dt = \frac{1}{2}\int 2t\,(t^2+1)^{-2} dt$

$$= \frac{1}{2}\frac{(t^2+1)^{-1}}{-1} + C = -\frac{1}{2(t^2+1)} + C.$$

Example 7: In a drug dissolution test, the rate of drug dissolution in a simulated medium is given by: $D(t) = 3t^2(2t+1)$

where:

- **$D(t)$ = dissolution rate (mg/hr)**
- **t= time (hr)**

Find the total amount of drug dissolved up to time t??

Solution: **Let** $Q(t) = \int 3t^2(2t+1)dt = \int (6t^3+3t^2)dt$

$$= 6\,\frac{t^4}{4} + 3\frac{t^3}{3} + C = \frac{3}{2}\,t^4 + t^3 + C.$$