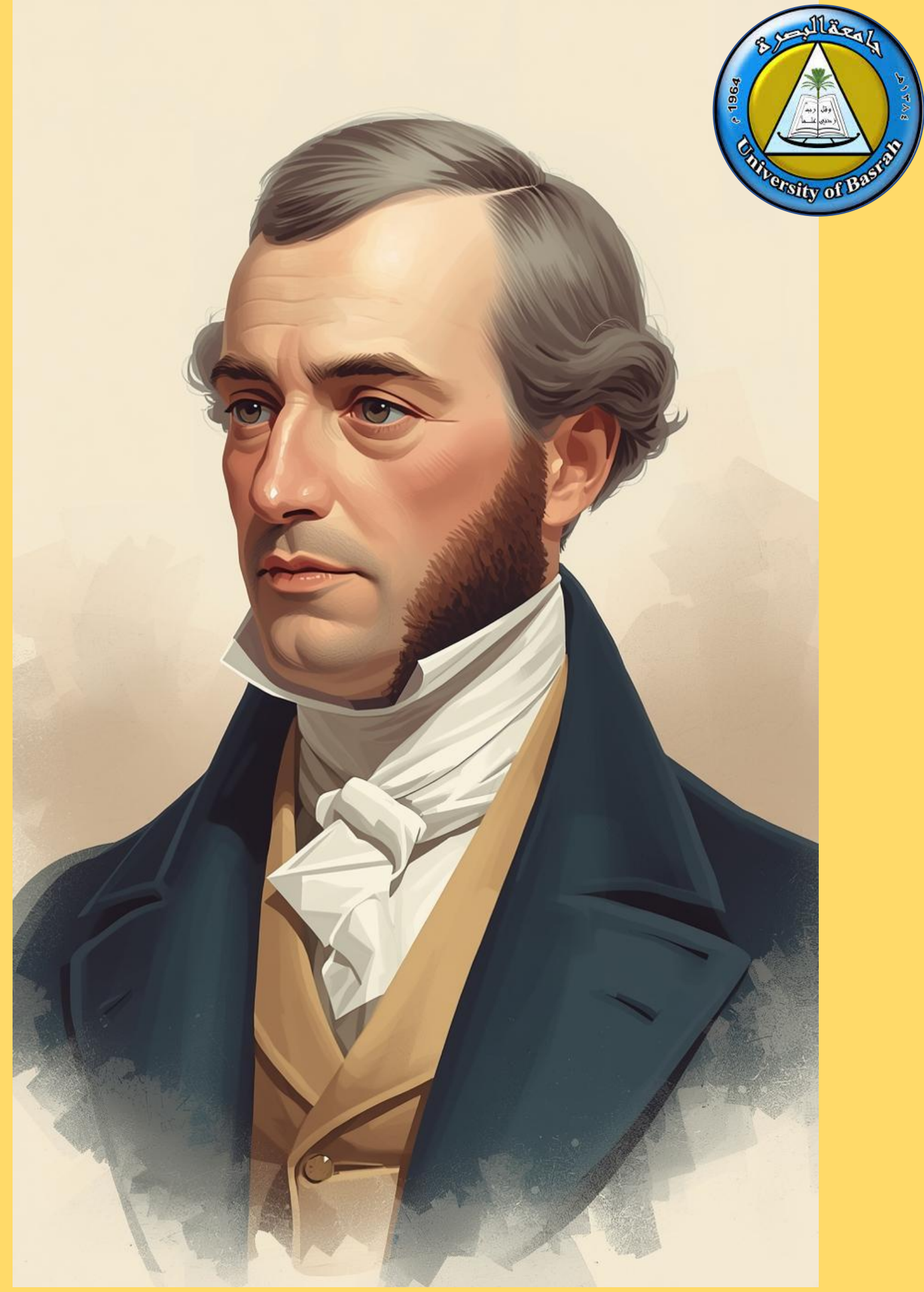# Poisson Distribution

Asst. Lecture : Shereen M. Ibrahim
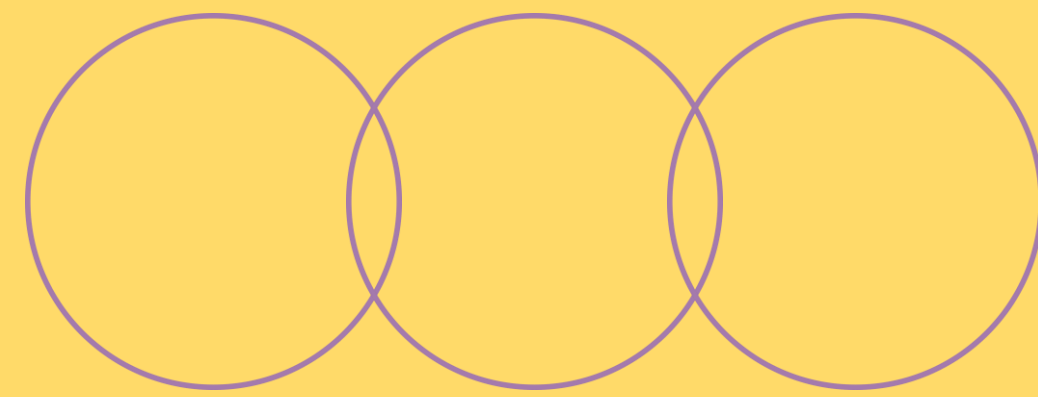
Overview of Poisson Distribution

The Poisson distribution is named after Siméon Denis Poisson

it's a mathematical formula **named after a famous French mathematician** who studied random events and probabilities.
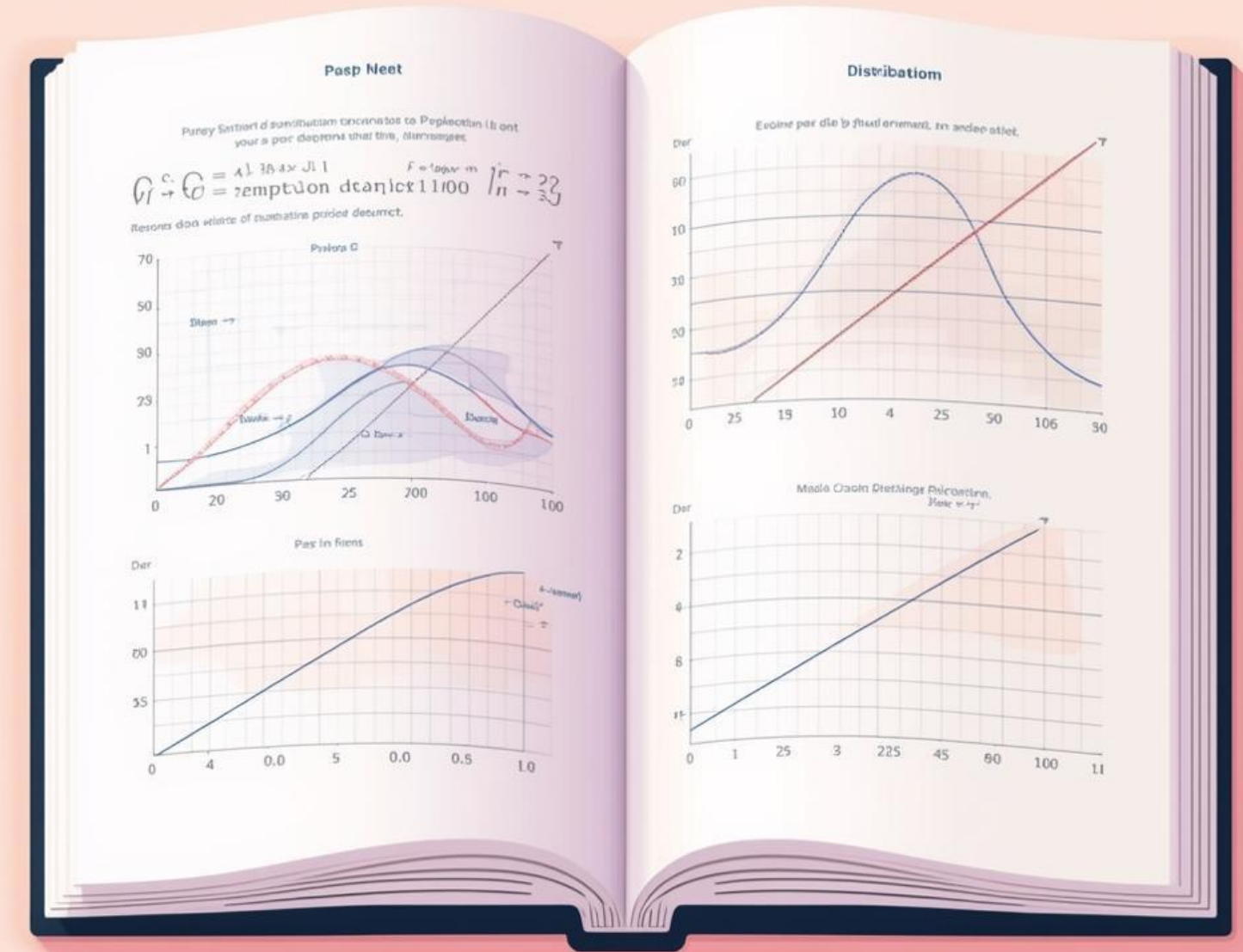
The Poisson distribution models the **number of events** occurring within a fixed interval, distinguishing itself from the commonly misunderstood term "poison." Key topics include its formula, properties, and real-world applications.

What is Poisson distribution

The **Poisson distribution** is a probability distribution that models the number of times an event occurs **within a fixed interval** of:
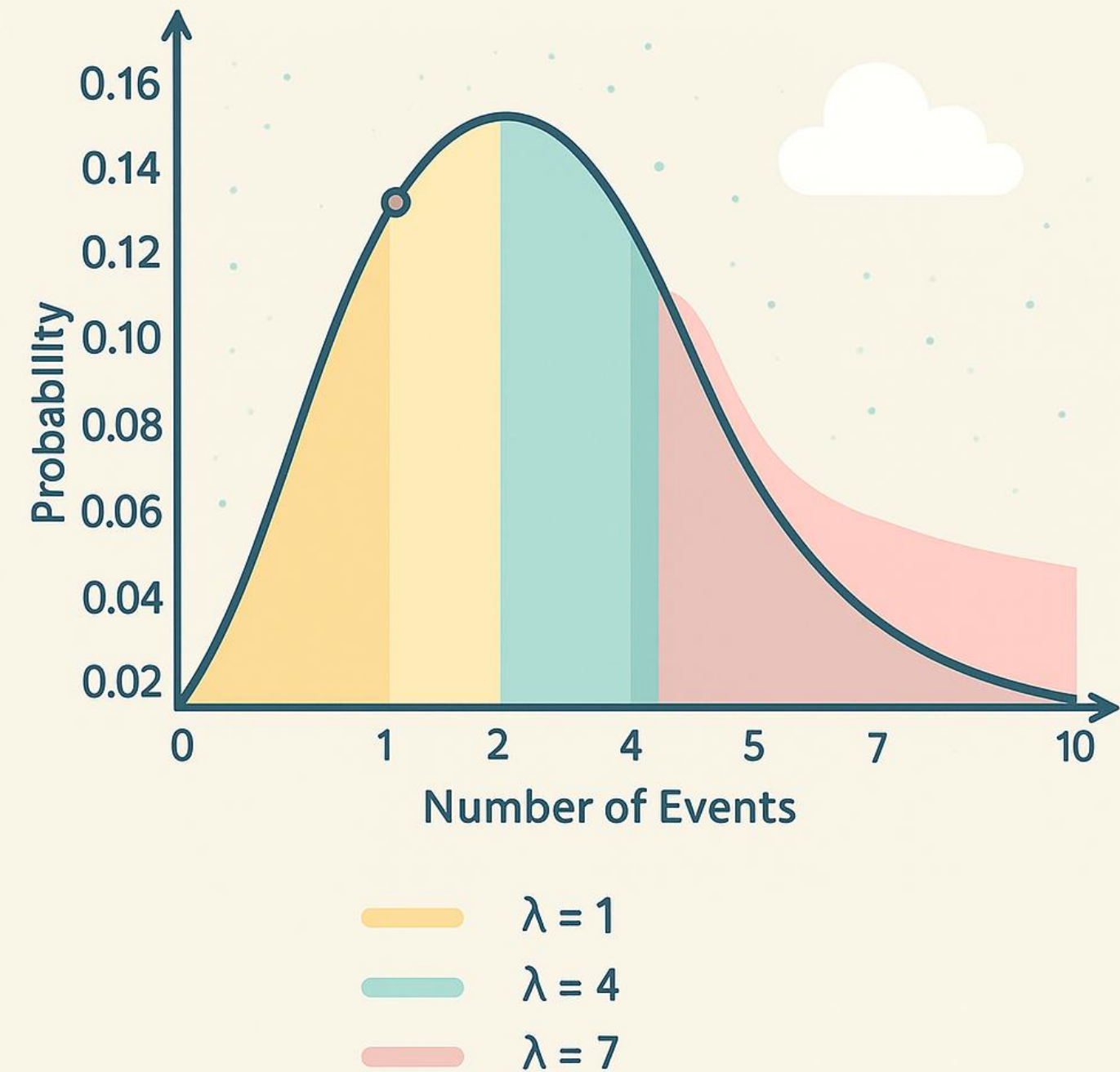
- **Time**
- **Distance**
- **Area**
- **Volume**

The Poisson distribution is widely used today in engineering, physics, insurance, and finance

It is used when events:

✔ Occur **independently**

✔ Occur **randomly**

✔ Occur at a **constant average rate (λ)**

✔ Are **rare** relative to the interval

POISSON DISTRIBUTION

Probability

0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02

0   1   2   4   5   7   10

Number of Events

— λ = 1
— λ = 4
— λ = 7

Poisson PMF Formula

The Poisson probability mass function (PMF) is expressed as **The Poisson Probability Formula:**

$$P(X = k) = \frac{e^{-\lambda} \cdot \lambda^k}{k!}$$

- $e \approx 2.71828 \rightarrow$ **the mathematical constant (base of natural logarithms).**

- k=0,1,2,… $\rightarrow$ the number of events observed.

- $\lambda \rightarrow$ the average (mean) number of events per interval.

- k! $\rightarrow$ factorial of k, meaning k·(k−1)·(k−2)····.

Properties

- **Mean:** $\mathbb{E}[X] = \lambda$

- **Variance:** $\text{Var}(X) = \lambda$

Applications in Oil and Gas:

The Poisson distribution is widely applied in

Risk assessment, reliability engineering, and safety analysis in the petroleum industry

Engineering Example

Consider A 100 Km Section Of Pipeline Where The Average Number Of Leaks Per Year Is

$\lambda = 0.5$. **What Is The Probability Of Observing At Least One Leak In A Year?**

Step-by-step solution

1. Write the Poisson PMF:

$$P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}, k = 0, 1, 2, \dots$$

2. Use the complement rule:

$$P(X \geq 1) = 1 - P(X = 0)$$

3. Compute $P(X = 0)$ with $\lambda = 0.5$:

$$P(X = 0) = \frac{e^{-0.5} \cdot 0.5^0}{0!} = e^{-0.5}$$

4.Evaluate $e^{-0.5}$:

$$e^{-0.5} \approx 0.60653$$

5.Find the final probability:

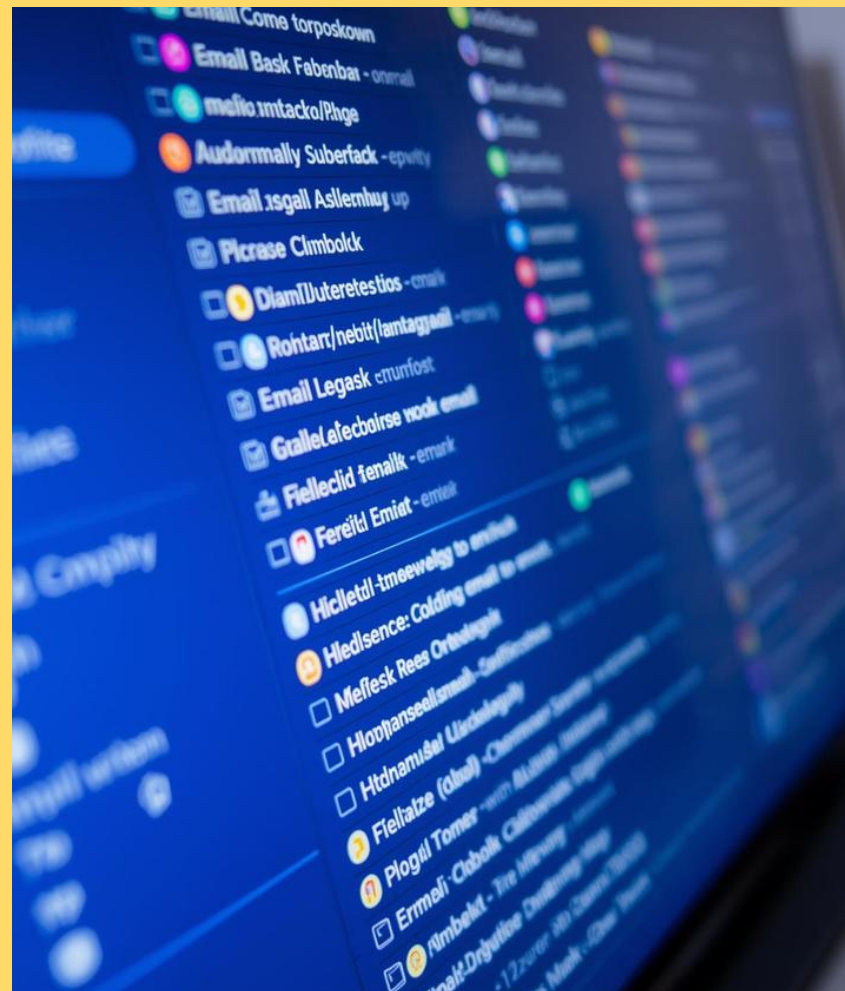$$P(X \geq 1) = 1 - 0.60653 \approx 0.39347$$

There is about a 39.35% chance of observing one or more leaks in a year on this 100 km pipeline section.

Practical Applications of Poisson Distribution

Emails

Counting emails received per hour

Accidents

Number of accidents per day

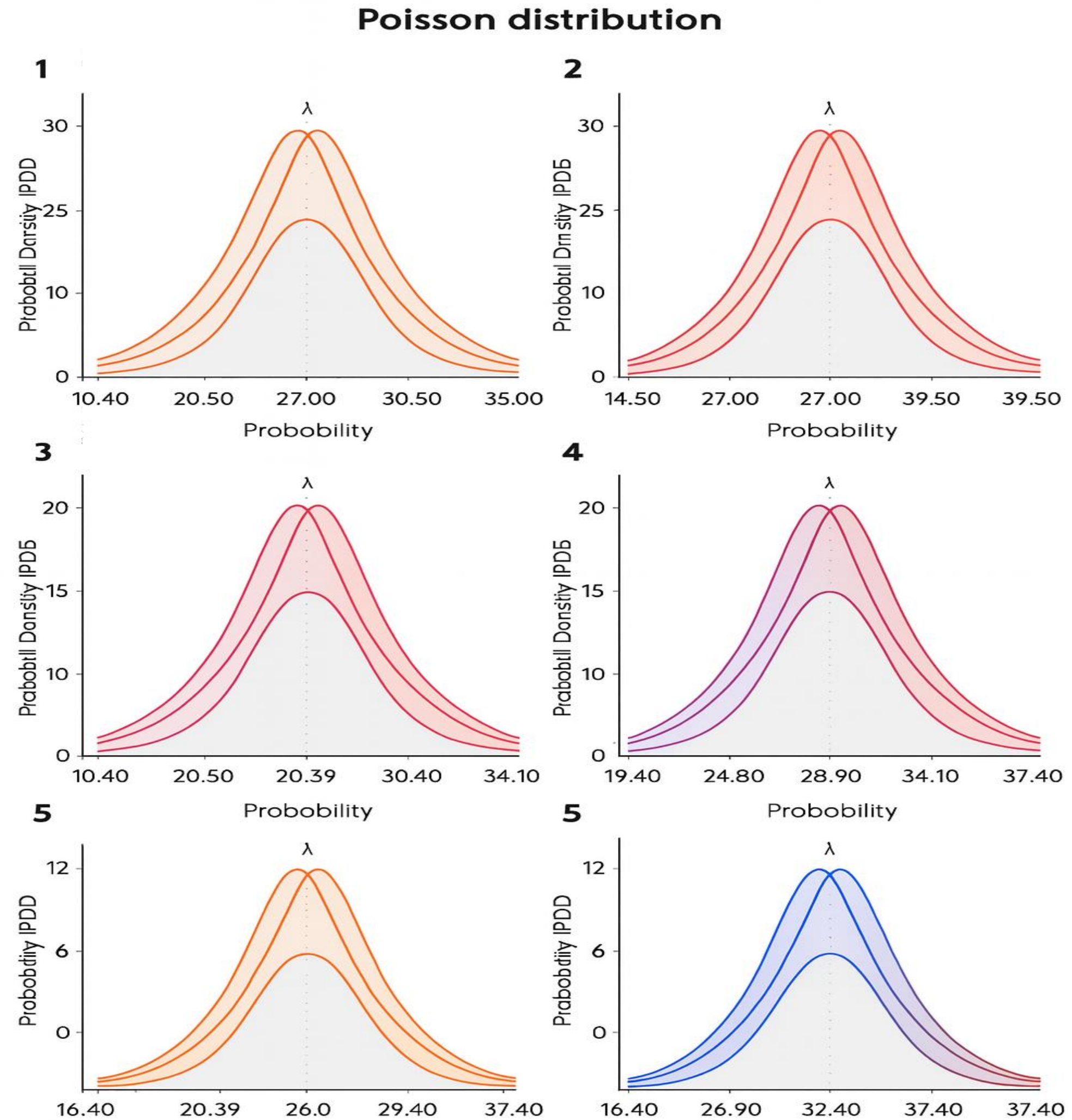**Defects**

Counting defects in a batch

Rare Events

Modeling rare occurrences effectively
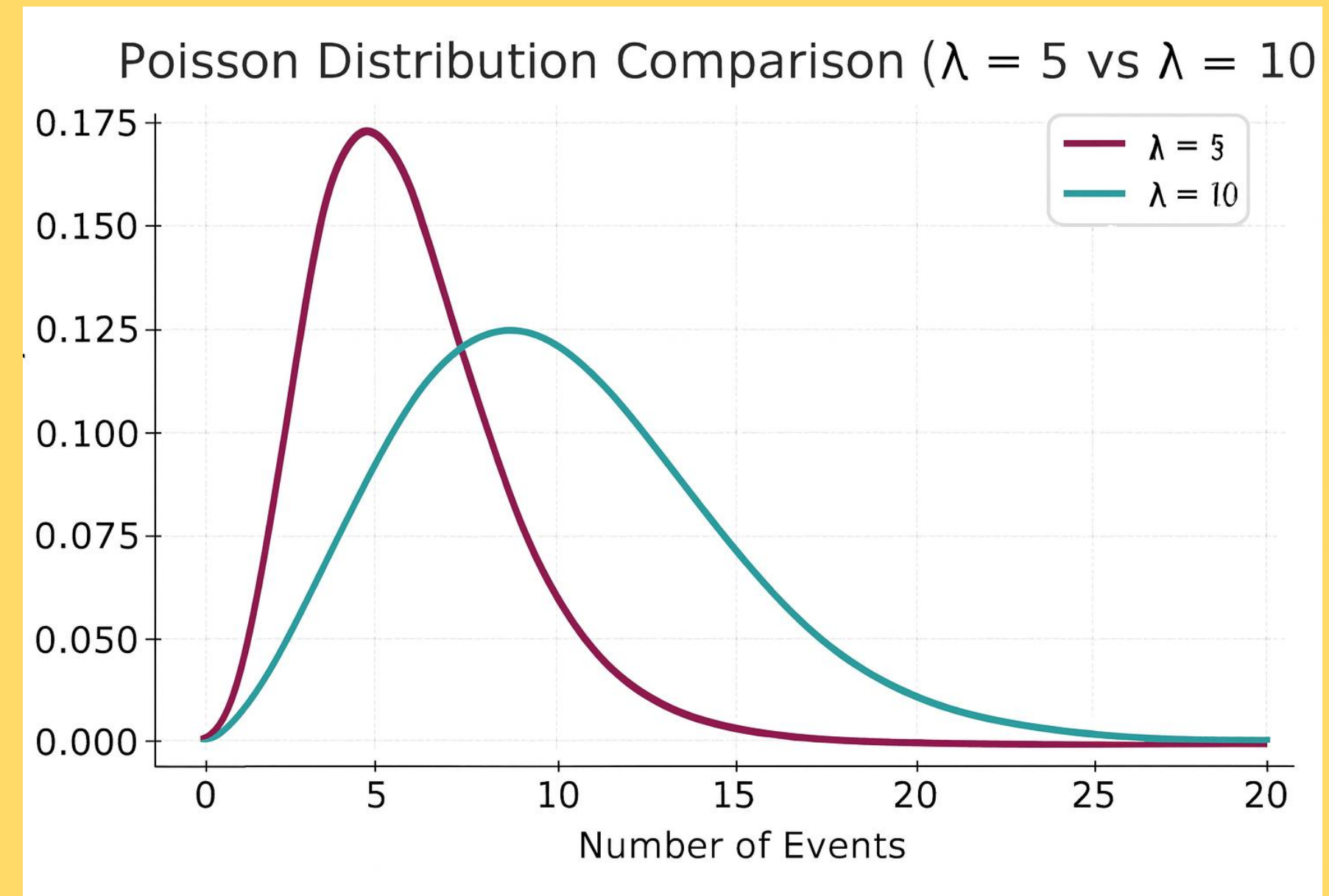
Distribution Shape

The shape of the Poisson distribution varies significantly with different λ values, illustrating how probabilities of event counts change from skewed to symmetric distributions.

Real-World Applications

A petroleum reservoir in which we monitor the number of high-pressure events occurring in a single well over the course of one year. Assume that: The **average number of high-pressure events per year** is **5**.

Therefore, the Poisson parameter is: $\lambda = 5$. Now suppose operating conditions change and the **average increases to 10 events per year**.

Poisson Distribution Comparison ($\lambda = 5$ vs $\lambda = 10$)

- This example demonstrates how the Poisson distribution shifts and changes shape in real engineering applications when the event rate (λ) increases.

H.W 16

Application : Poisson Distribution in Petroleum Engineering using (MATLAB)

A petroleum engineer is analyzing the **number of high-pressure events** occurring in a production well per year. Historical data shows that: $\lambda = 5 \; events \; per \; year$, We want to calculate the probability of observing **0 to 12 events**

Use the code

```
% Poisson Distribution Example in Petroleum Engineering
% λ = 5 high-pressure events per year

lambda = 5;
x = 0:12;

% Compute Poisson PMF
P = poisspdf(x, lambda);

% Display values
disp('X values:');
disp(x);
disp('Probability P(X):');
disp(P);

% Plot
figure;
stem(x, P, 'LineWidth', 2);
title('Poisson Distribution of High-Pressure Events (λ = 5)');
xlabel('Number of Events per Year');
ylabel('Probability');
grid on;
```
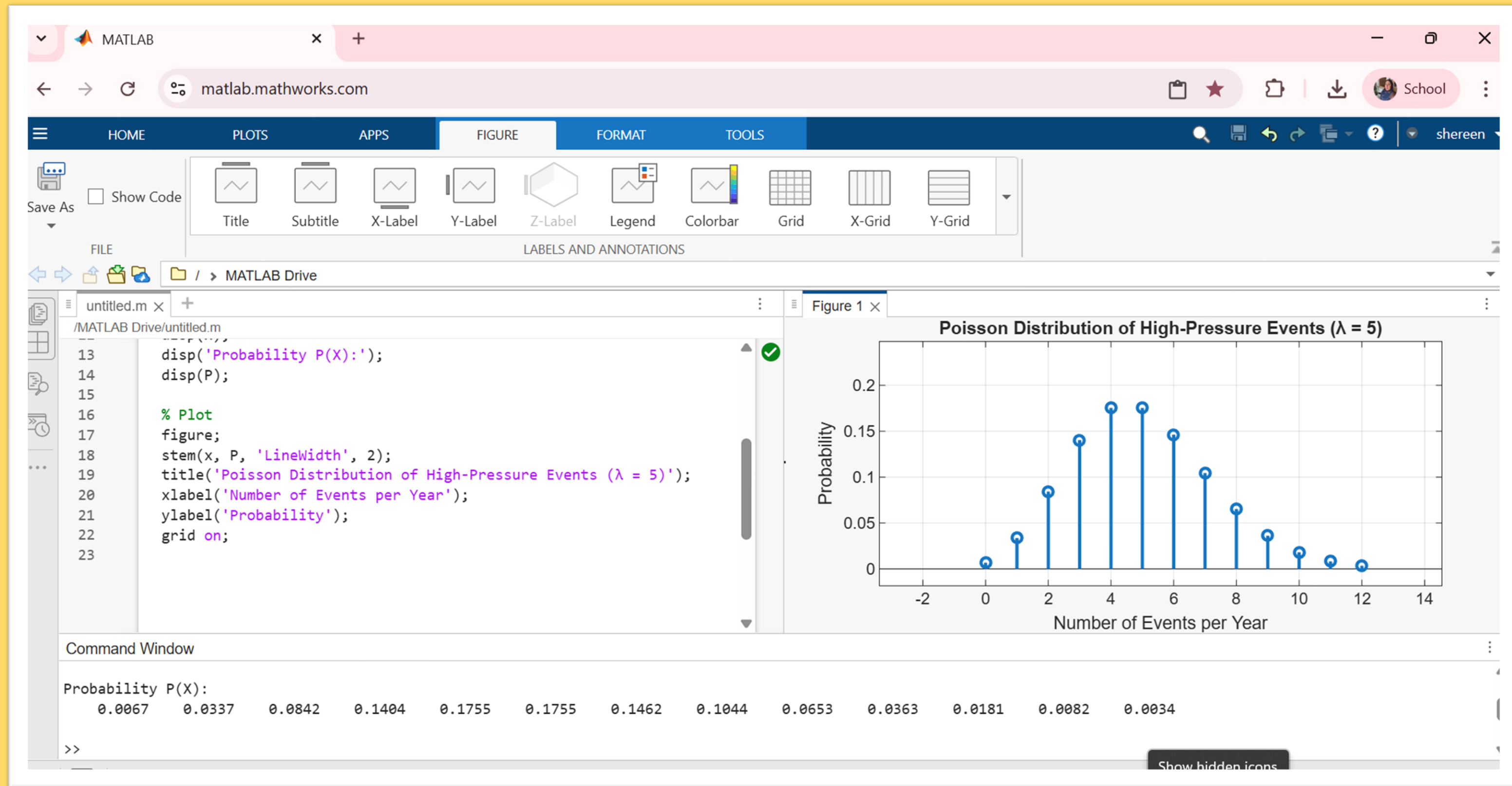
Screen shot your page like below , then upload it on google classroom

https://classroom.google.com/c/ODE1MTc5MzkxMzQ2/a/ODMyMzMyMjk2MDA3/details

Thank You for
Your Attention!