8

The pn Junction Diode

I n the last chapter, we discussed the electrostatics of the pn junction in thermal equilibrium and under reverse bias. We determined the built-in potential barrier at thermal equilibrium and calculated the electric field in the space charge region. We also considered the junction capacitance.

In this chapter, we consider the pn junction with a forward-bias voltage applied and determine the current–voltage characteristics. The potential barrier of the pn junction is lowered when a forward-bias voltage is applied, allowing electrons and holes to flow across the space charge region. When holes flow from the p region across the space charge region into the n region, they become excess minority carrier holes and are subject to the excess minority carrier diffusion, drift, and recombination processes discussed in Chapter 6. Likewise, when electrons from the n region flow across the space charge region into the p region, they become excess minority carrier electrons and are subject to these same processes. ■

8.0 | PREVIEW

In this chapter, we will:

■ Consider the process by which the potential barrier of a pn junction is lowered when a forward-bias voltage is applied, so holes and electrons can flow across the junction generating a diode current.

■ Derive the boundary conditions for excess holes in the n region and excess-electrons in the p region, and analyze the behavior of these excess carriers under a forward bias.

■ Derive the ideal current–voltage relation of the forward-biased pn junction diode.

■ Describe and analyze nonideal effects in the pn junction diode such as high-level injection, and generation and recombination currents.

■ Develop a small-signal equivalent circuit of the pn junction diode. This equivalent circuit is used to relate small time-varying currents and voltages in the pn junction.

■ Discuss large signal diode switching characteristics.
■ Describe a specialized pn junction called a tunnel diode.

8.1 | pn JUNCTION CURRENT

When a forward-bias voltage is applied to a pn junction, a current will be induced in the device. We initially consider a qualitative discussion of how charges flow in the pn junction and then consider the mathematical derivation of the current–voltage relationship.

8.1.1   Qualitative Description of Charge Flow in a pn Junction

We can qualitatively understand the mechanism of the current in a pn junction by again considering the energy-band diagrams. Figure 8.1a shows the energy-band diagram of a pn junction in thermal equilibrium that was developed in the last chapter. We argued that the potential barrier seen by the electrons, for example, holds back the large concentration of electrons in the n region and keeps them from flowing into the p region. Similarly, the potential barrier seen by the holes holds back the large concentration of holes in the p region and keeps them from flowing into the n region. The potential barrier, then, maintains thermal equilibrium.

Figure 8.1b shows the energy-band diagram of a reverse-biased pn junction. The potential of the n region is positive with respect to the p region so the Fermi energy
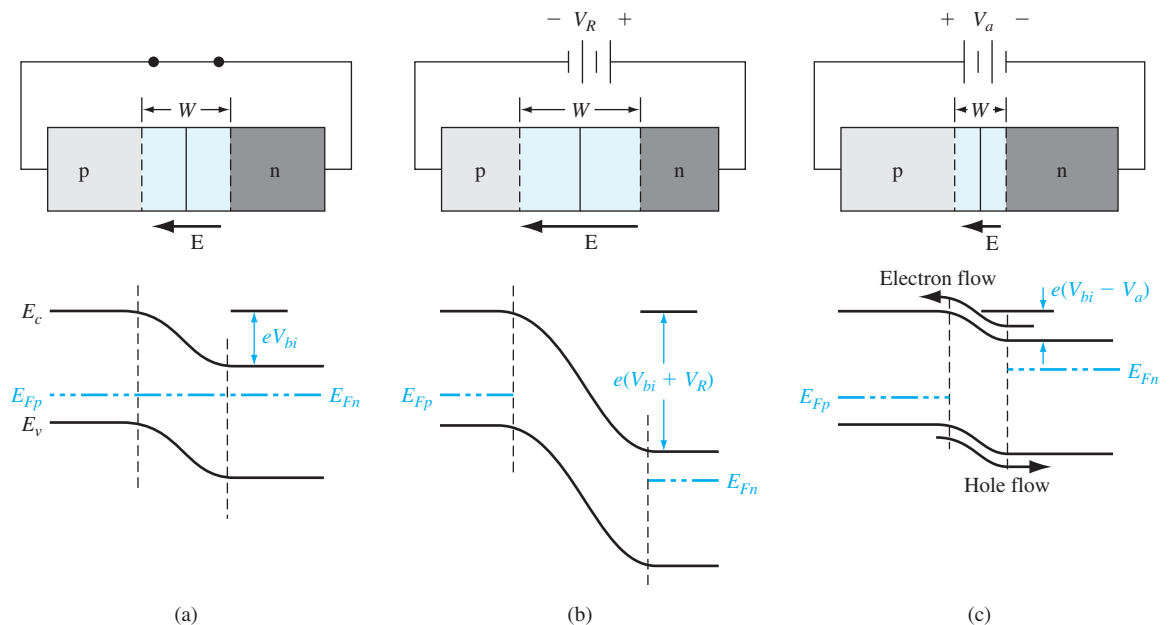

Figure 8.1 | A pn junction and its associated energy-band diagram for (a) zero bias, (b) reverse bias, and (c) forward bias.

in the n region is lower than that in the p region. The total potential barrier is now larger than that for the zero-bias case. We argued in the last chapter that the increased potential barrier continues to hold back the electrons and holes so that there is still essentially no charge flow and hence essentially no current.

Figure 8.1c shows the energy-band diagram for the case when a positive voltage is applied to the p region with respect to the n region. The Fermi level in the p region is now lower than that in the n region. The total potential barrier is now reduced. The smaller potential barrier means that the electric field in the depletion region is also reduced. The smaller electric field means that the electrons and holes are no longer held back in the n and p regions, respectively. There will be a diffusion of holes from the p region across the space charge region where they will flow into the n region. Similarly, there will be a diffusion of electrons from the n region across the space charge region where they will flow into the p region. The flow of charge generates a current through the pn junction.

The injection of holes into the n region means that these holes are minority carriers. Likewise, the injection of electrons into the p region means that these electrons are minority carriers. The behavior of these minority carriers is described by the ambipolar transport equations that were discussed in Chapter 6. There will be diffusion as well as recombination of excess carriers in these regions. The diffusion of carriers implies that there will be diffusion currents. The mathematical derivation of the pn junction current–voltage relationship is considered in the next section.

8.1.2   Ideal Current–Voltage Relationship

The ideal current–voltage relationship of a pn junction is derived on the basis of four assumptions. (The last assumption has three parts, but each part deals with current.) They are:

 1. The abrupt depletion layer approximation applies. The space charge regions have abrupt boundaries, and the semiconductor is neutral outside of the depletion region.
 2. The Maxwell–Boltzmann approximation applies to carrier statistics.
 3. The concepts of low injection and complete ionization apply.
 4a. The total current is a constant throughout the entire pn structure.
 4b. The individual electron and hole currents are continuous functions through the pn structure.
 4c. The individual electron and hole currents are constant throughout the depletion region.

Notation can sometimes appear to be overwhelming in the equations in this chapter. Table 8.1 lists some of the various electron and hole concentration terms that appear. Many terms have already been used in previous chapters but are repeated here for convenience.

Table 8.1 | Commonly used terms and notation for this chapter

Term	Meaning
N_a	Acceptor concentration in the p region of the pn junction
N_d	Donor concentration in the n region of the pn junction
$n_{n0} = N_d$	Thermal-equilibrium majority carrier electron concentration in the n region
$p_{p0} = N_a$	Thermal-equilibrium majority carrier hole concentration in the p region
$n_{p0} = n_i^2/N_a$	Thermal-equilibrium minority carrier electron concentration in the p region
$p_{n0} = n_i^2/N_d$	Thermal-equilibrium minority carrier hole concentration in the n region
n_p	Total minority carrier electron concentration in the p region
p_n	Total minority carrier hole concentration in the n region
$n_p(-x_p)$	Minority carrier electron concentration in the p region at the space charge edge
$p_n(x_n)$	Minority carrier hole concentration in the n region at the space charge edge
$\delta n_p = n_p - n_{p0}$	Excess minority carrier electron concentration in the p region
$\delta p_n = p_n - p_{n0}$	Excess minority carrier hole concentration in the n region

8.1.3 Boundary Conditions

Figure 8.2 shows the conduction-band energy through the pn junction in thermal equilibrium. The n region contains many more electrons in the conduction band than the p region; the built-in potential barrier prevents this large density of electrons from flowing into the p region. The built-in potential barrier maintains equilibrium between the carrier distributions on either side of the junction.

An expression for the built-in potential barrier was derived in the last chapter and was given by Equation (7.10) as
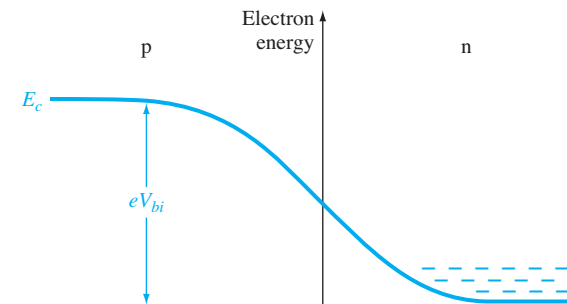
$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

Figure 8.2 | Conduction-band energy through a pn junction.

If we divide the equation by $V_t = kT/e$, take the exponential of both sides, and then take the reciprocal, we obtain

$$\frac{n_i^2}{N_a N_d} = \exp\left(\frac{-eV_{bi}}{kT}\right) \tag{8.1}$$

If we assume complete ionization, we can write

$$n_{n0} \approx N_d \tag{8.2}$$

where n_{n0} is the thermal-equilibrium concentration of majority carrier electrons in the n region. In the p region, we can write

$$n_{p0} \approx \frac{n_i^2}{N_a} \tag{8.3}$$

where n_{p0} is the thermal-equilibrium concentration of minority carrier electrons. Substituting Equations (8.2) and (8.3) into Equation (8.1), we obtain

$$n_{p0} = n_{n0} \exp\left(\frac{-eV_{bi}}{kT}\right) \tag{8.4}$$

This equation relates the minority carrier electron concentration on the p side of the junction to the majority carrier electron concentration on the n side of the junction in thermal equilibrium.

If a positive voltage is applied to the p region with respect to the n region, the potential barrier is reduced. Figure 8.3a shows a pn junction with an applied voltage V_a. The electric field in the bulk p and n regions is normally very small. Essentially all of the applied voltage is across the junction region. The electric field E_{app} induced by the applied voltage is in the opposite direction to the thermal-equilibrium space charge electric field, so the net electric field in the space charge region is reduced below the equilibrium value. The delicate balance between diffusion and the E-field
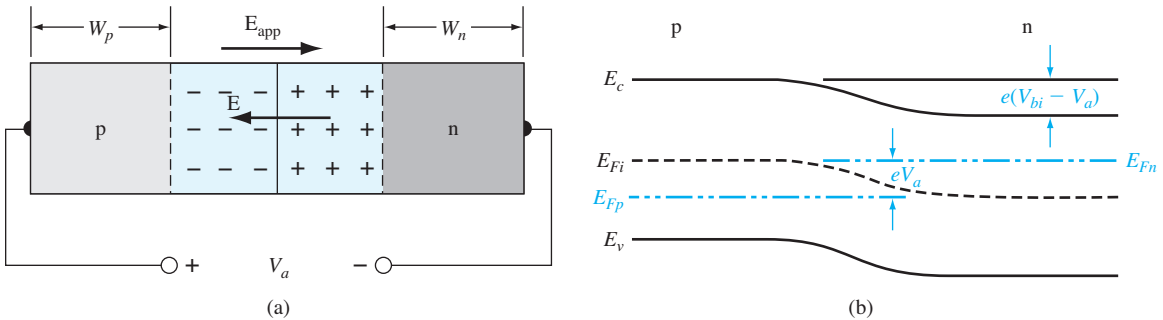

Figure 8.3 | (a) A pn junction with an applied forward-bias voltage showing the directions of the electric field induced by V_a and the space charge electric field. (b) Energy-band diagram of the forward-biased pn junction.

force achieved at thermal equilibrium is upset. The electric field force that prevented majority carriers from crossing the space charge region is reduced; majority carrier electrons from the n side are now injected across the depletion region into the p material, and majority carrier holes from the p side are injected across the depletion region into the n material. As long as the bias V_a is applied, the injection of carriers across the space charge region continues and a current is created in the pn junction. This bias condition is known as forward bias; the energy-band diagram of the forward-biased pn junction is shown in Figure 8.3b.

The potential barrier V_{bi} in Equation (8.4) can be replaced by $(V_{bi} - V_a)$ when the junction is forward biased. Equation (8.4) becomes

$$n_p = n_{n0} \exp\left(\frac{-e\,(V_{bi} - V_a)}{kT}\right) = n_{n0} \exp\left(\frac{-eV_{bi}}{kT}\right) \exp\left(\frac{+eV_a}{kT}\right) \qquad (8.5)$$

If we assume low injection, the majority carrier electron concentration n_{n0}, for example, does not change significantly. However, the minority carrier concentration, n_p, can deviate from its thermal-equilibrium value n_{p0} by orders of magnitude. Using Equation (8.4), we can write Equation (8.5) as

$$n_p = n_{p0} \exp\left(\frac{eV_a}{kT}\right) \qquad (8.6)$$

When a forward-bias voltage is applied to the pn junction, the junction is no longer in thermal equilibrium. The left side of Equation (8.6) is the total minority carrier electron concentration in the p region, which is now greater than the thermal equilibrium value. The forward-bias voltage lowers the potential barrier so that majority carrier electrons from the n region are injected across the junction into the p region, thereby increasing the minority carrier electron concentration. We have produced excess minority carrier electrons in the p region.

When the electrons are injected into the p region, these excess carriers are subject to the diffusion and recombination processes we discussed in Chapter 6. Equation (8.6), then, is the expression for the minority carrier electron concentration at the edge of the space charge region in the p region.

Exactly the same process occurs for majority carrier holes in the p region, which are injected across the space charge region into the n region under a forward-bias voltage. We can write that

$$p_n = p_{n0} \exp\left(\frac{eV_a}{kT}\right) \qquad (8.7)$$

where p_n is the concentration of minority carrier holes at the edge of the space charge region in the n region. Figure 8.4 shows these results. By applying a forward-bias voltage, we create excess minority carriers in each region of the pn junction.
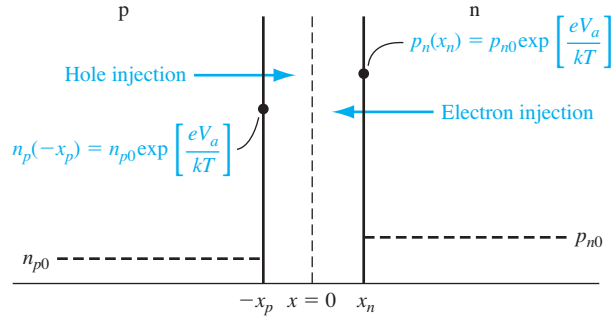
Figure 8.4 | Excess minority carrier concentrations at the space charge edges generated by the forward-bias voltage.

EXAMPLE 8.1

Objective: Calculate the minority carrier concentrations at the edge of the space charge regions in a forward-biased pn junction.

Consider a silicon pn junction at $T = 300$ K. Assume the doping concentration in the n region is $N_d = 10^{16}$ cm^{-3} and the doping concentration in the p region is $N_a = 6 \times 10^{15}$ cm^{-3}, and assume that a forward bias of 0.60 V is applied to the pn junction.

■ **Solution**

From Equations (8.6) and (8.7) and from Figure 8.4, we have

$$n_p\left(-x_p\right) = n_{po} \exp\left(\frac{eV_a}{kT}\right) \qquad \text{and} \qquad p_n(x_n) = p_{no} \exp\left(\frac{eV_a}{kT}\right)$$

The thermal-equilibrium minority carrier concentrations are

$$n_{po} = \frac{n_i^2}{N_a} = \frac{(1.5 \times 10^{10})^2}{6 \times 10^{15}} = 3.75 \times 10^4 \text{ cm}^{-3}$$

and

$$p_{no} = \frac{n_i^2}{N_d} = \frac{(1.5 \times 10^{10})^2}{10^{16}} = 2.25 \times 10^4 \text{ cm}^{-3}$$

We then have

$$n_p\left(-x_p\right) = 3.75 \times 10^4 \exp\left(\frac{0.60}{0.0259}\right) = 4.31 \times 10^{14} \text{ cm}^{-3}$$

and

$$p_n(x_n) = 2.25 \times 10^4 \exp\left(\frac{0.60}{0.0259}\right) = 2.59 \times 10^{14} \text{ cm}^{-3}$$

■ **Comment**

The minority carrier concentrations can increase by many orders of magnitude when a relatively small forward-bias voltage is applied. Low injection still applies, however, since the excess minority carrier concentrations at the space-charge edges are much less than the thermal-equilibrium majority carrier concentrations.

■ **EXERCISE PROBLEM**

Ex 8.1 A silicon pn junction at $T = 300$ K is doped with impurity concentrations of $N_d = 2 \times 10^{16}$ cm^{-3} and $N_a = 5 \times 10^{16}$ cm^{-3}. The junction is forward biased at $V_a = 0.650$ V. Determine the minority carrier concentrations at the space charge edges. Does low injection still apply?

[Ans. $n_p(-x_p) = 3.57 \times 10^{14}$ cm^{-3}, $p_n(x_n) = 8.92 \times 10^{14}$ cm^{-3}, yes]

The minority carrier concentrations at the space charge edges, given by Equations (8.6) and (8.7), were derived assuming that a forward-bias voltage ($V_a > 0$) was applied across the pn junction. However, nothing in the derivation prevents V_a from being negative (reverse bias). If a reverse-biased voltage greater than a few tenths of a volt is applied to the pn junction, then we see from Equations (8.6) and (8.7) that the minority carrier concentrations at the space charge edge are essentially zero. The minority carrier concentrations for the reverse-biased condition drop below the thermal-equilibrium values.

8.1.4 Minority Carrier Distribution

We developed, in Chapter 6, the ambipolar transport equation for excess minority carrier holes in an n region. This equation, in one dimension, is

$$D_p \frac{\partial^2 (\delta p_n)}{\partial x^2} - \mu_p E \frac{\partial (\delta p_n)}{\partial x} + g' - \frac{\delta p_n}{\tau_{p0}} = \frac{\partial (\delta p_n)}{\partial t} \tag{8.8}$$

where $\delta p_n = p_n - p_{n0}$ is the excess minority carrier hole concentration and is the difference between the total and thermal equilibrium minority carrier concentrations. The ambipolar transport equation describes the behavior of excess carriers as a function of time and spatial coordinates.

In Chapter 5, we calculated drift current densities in a semiconductor. We determined that relatively large currents could be created with fairly small electric fields. As a first approximation, we assume that the electric field is zero in both the neutral p and n regions. In the n region for $x > x_n$, we have that $E = 0$ and $g' = 0$. If we also assume steady state so $\partial (\delta p_n)/\partial t = 0$, then Equation (8.8) reduces to

$$\frac{d^2 (\delta p_n)}{dx^2} - \frac{\delta p_n}{L_p^2} = 0 \qquad (x > x_n) \tag{8.9}$$

where $L_p^2 = D_p \tau_{p0}$. For the same set of conditions, the excess minority carrier electron concentration in the p region is determined from

$$\frac{d^2 (\delta n_p)}{dx^2} - \frac{\delta n_p}{L_n^2} = 0 \qquad (x < x_p) \tag{8.10}$$

where $L_n^2 = D_n \tau_{n0}$.

The boundary conditions for the total minority carrier concentrations are

$$p_n(x_n) = p_{n0} \exp\left(\frac{eV_a}{kT}\right) \tag{8.11a}$$

$$n_p(-x_p) = n_{p0} \exp\left(\frac{eV_a}{kT}\right) \tag{8.11b}$$

$$p_n(x \to +\infty) = p_{n0} \tag{8.11c}$$

$$n_p(x \to -\infty) = n_{p0} \tag{8.11d}$$

As minority carriers diffuse from the space charge edge into the neutral semiconductor regions, they recombine with majority carriers. We assume that the lengths W_n and W_p shown in Figure 8.3a are very long, meaning in particular that $W_n \gg L_p$ and $W_p \gg L_n$. The excess minority carrier concentrations must approach zero at distances far from the space charge region. The structure is referred to as a long pn junction.

The general solution to Equation (8.9) is

$$\delta p_n(x) = p_n(x) - p_{n0} = Ae^{x/L_p} + Be^{-x/L_p} \qquad (x \geq x_n) \tag{8.12}$$

and the general solution to Equation (8.10) is

$$\delta n_p(x) = n_p(x) - n_{p0} = Ce^{x/L_n} + De^{-x/L_n} \qquad (x \leq -x_p) \tag{8.13}$$

Applying the boundary conditions from Equations (8.11c) and (8.11d), the coefficients A and D must be zero. The coefficients B and C may be determined from the boundary conditions given by Equations (8.11a) and (8.11b). The excess carrier concentrations are then found to be, for $(x \geq x_n)$,

$$\delta p_n(x) = p_n(x) - p_{n0} = p_{n0}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\exp\left(\frac{x_n - x}{L_p}\right) \tag{8.14}$$

and, for $(x \leq -x_p)$,

$$\delta n_p(x) = n_p(x) - n_{p0} = n_{p0}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\exp\left(\frac{x_p + x}{L_n}\right) \tag{8.15}$$

The minority carrier concentrations decay exponentially with distance away from the junction to their thermal-equilibrium values. Figure 8.5 shows these results. Again, we have assumed that both the n-region and the p-region lengths are long compared to the minority carrier diffusion lengths.

In Chapter 6, we discussed the concept of quasi-Fermi levels, which apply to excess carriers in a nonequilibium condition. Since excess electrons exist in the neutral p region and excess holes exist in the neutral n region, we can apply quasi-Fermi levels to these regions. We had defined quasi-Fermi levels in terms of carrier concentrations as

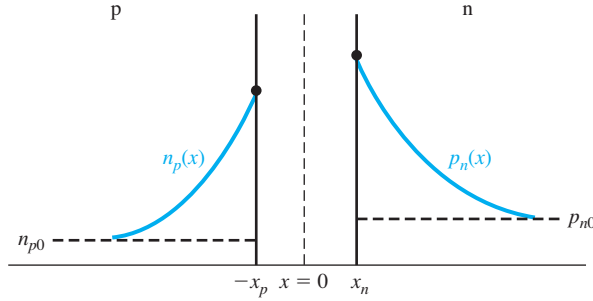$$p = p_o + \delta p = n_i \exp\left(\frac{E_{Fi} - E_{Fp}}{kT}\right) \tag{8.16}$$

Figure 8.5 | Steady-state minority carrier concentrations in a pn junction under forward bias.
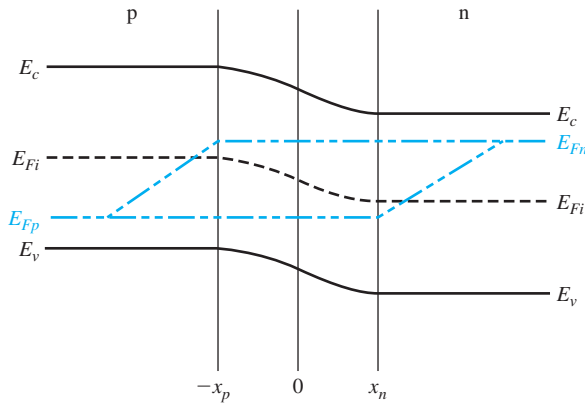

Figure 8.6 | Quasi-Fermi levels through a forward-biased pn junction.

and

$$n = n_o + \delta n = n_i \exp\left(\frac{E_{Fn} - E_{Fi}}{kT}\right) \tag{8.17}$$

Figure 8.6 shows the quasi-Fermi levels through the pn junction. From Equations (8.14) and (8.15), the carrier concentrations are exponential functions of distance, and from Equations (8.16) and (8.17), the carrier concentrations are exponential functions of the quasi-Fermi levels. The quasi-Fermi levels are then linear functions of distance in the neutral p and n regions as shown in Figure 8.6.

We may note that close to the space charge edge in the p region, $E_{Fn} - E_{Fi} > 0$ which means that $\delta n > n_i$. Further from the space charge edge, $E_{Fn} - E_{Fi} < 0$ which means that $\delta n < n_i$ and the excess electron concentration is approaching zero. The same discussion applies to the excess hole concentration in the n region.

At the space charge edge at $x = x_n$, we can write, for low injection

$$n_o p_n(x_n) = n_o p_{no} \exp\left(\frac{V_a}{V_t}\right) = n_i^2 \exp\left(\frac{V_a}{V_t}\right) \tag{8.18}$$

Combining Equations (8.16) and (8.17), we can write

$$np = n_i^2 \exp\left(\frac{E_{Fn} - E_{Fp}}{kT}\right) \tag{8.19}$$

Comparing Equations (8.18) and (8.19), the difference in quasi-Fermi levels is related to the applied bias V_a and represents the deviation from thermal equilibrium. The difference between E_{Fn} and E_{Fp} is nearly constant through the depletion region.

To review, a forward-bias voltage lowers the built-in potential barrier of a pn junction so that electrons from the n region are injected across the space charge region, creating excess minority carriers in the p region. These excess electrons begin diffusing into the bulk p region where they can recombine with majority carrier holes. The excess minority carrier electron concentration then decreases with distance from the junction. The same discussion applies to holes injected across the space charge region into the n region.

8.1.5 Ideal pn Junction Current

The approach we use to determine the current in a pn junction is based on the three parts of the fourth assumption stated earlier in this section. The total current in the junction is the sum of the individual electron and hole currents that are constant through the depletion region. Since the electron and hole currents are continuous functions through the pn junction, the total pn junction current will be the minority carrier hole diffusion current at $x = x_n$ plus the minority carrier electron diffusion current at $x = -x_p$. The gradients in the minority carrier concentrations, as shown in Figure 8.5, produce diffusion currents, and since we are assuming the electric field to be zero at the space charge edges, we can neglect any minority carrier drift current component. This approach in determining the pn junction current is shown in Figure 8.7.
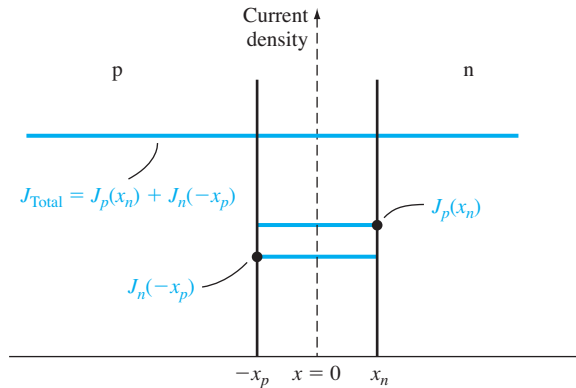
Figure 8.7 | Electron and hole current densities through the space charge region of a pn junction.

We can calculate the minority carrier hole diffusion current density at $x = x_n$ from the relation

$$J_p(x_n) = -eD_p \frac{dp_n(x)}{dx}\bigg|_{x=x_n} \tag{8.20}$$

Since we are assuming uniformly doped regions, the thermal-equilibrium carrier concentration is constant, so the hole diffusion current density may be written as

$$J_p(x_n) = -eD_p \frac{d(\delta p_n(x))}{dx}\bigg|_{x=x_n} \tag{8.21}$$

Taking the derivative of Equation (8.14) and substituting into Equation (8.21), we obtain

$$J_p(x_n) = \frac{eD_p p_{n0}}{L_p}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right] \tag{8.22}$$

The hole current density for this forward-bias condition is in the $+x$ direction, which is from the p to the n region.

Similarly, we may calculate the electron diffusion current density at $x = -x_p$. This may be written as

$$J_n(-x_p) = eD_n \frac{d(\delta n_p(x))}{dx}\bigg|_{x=-x_p} \tag{8.23}$$

Using Equation (8.15), we obtain

$$J_n(-x_p) = \frac{eD_n n_{p0}}{L_n}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right] \tag{8.24}$$

The electron current density is also in the $+x$ direction.

An assumption we made at the beginning was that the individual electron and hole currents were continuous functions and constant through the space charge region. The total current is the sum of the electron and hole currents and is constant through the entire junction. Figure 8.7 again shows a plot of the magnitudes of these currents.

The total current density in the pn junction is then

$$J = J_p(x_n) + J_n(-x_p) = \left[\frac{eD_p p_{n0}}{L_p} + \frac{eD_n n_{p0}}{L_n}\right]\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right] \tag{8.25}$$

Equation (8.25) is the ideal current–voltage relationship of a pn junction.

We may define a parameter J_s as

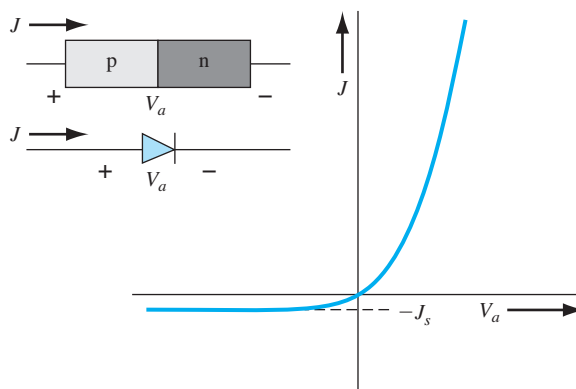$$J_s = \left[\frac{eD_p p_{n0}}{L_p} + \frac{eD_n n_{p0}}{L_n}\right] \tag{8.26}$$

Figure 8.8 | Ideal *I–V* characteristic of a pn junction diode.

so that Equation (8.25) may be written as

$$J = J_s\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

(8.27)

Equation (8.27), known as the ideal-diode equation, gives a good description of the current–voltage characteristics of the pn junction over a wide range of currents and voltages. Although Equation (8.27) was derived assuming a forward-bias voltage ($V_a > 0$), there is nothing to prevent V_a from being negative (reverse bias). Equation (8.27) is plotted in Figure 8.8 as a function of forward-bias voltage V_a. If the voltage V_a becomes negative (reverse bias) by a few kT/eV, then the reverse-biased current density becomes independent of the reverse-biased voltage. The parameter J_s is then referred to as the reverse-saturation current density. The current–voltage characteristics of the pn junction diode are obviously not bilateral.

EXAMPLE 8.2

Objective: Determine the ideal reverse-saturation current density in a silicon pn junction at $T = 300$ K.

Consider the following parameters in a silicon pn junction:

$$N_a = N_d = 10^{16} \text{ cm}^{-3} \qquad n_i = 1.5 \times 10^{10} \text{ cm}^{-3}$$
$$D_n = 25 \text{ cm}^2/\text{s} \qquad \tau_{p0} = \tau_{n0} = 5 \times 10^{-7} \text{ s}$$
$$D_p = 10 \text{ cm}^2/\text{s} \qquad \epsilon_r = 11.7$$

■ **Solution**

The ideal reverse-saturation current density is given by

$$J_s = \frac{eD_n n_{p0}}{L_n} + \frac{eD_p p_{n0}}{L_p}$$

which may be rewritten as

$$J_s = en_i^2 \left(\frac{1}{N_a} \sqrt{\frac{D_n}{\tau_{n0}}} + \frac{1}{N_d} \sqrt{\frac{D_p}{\tau_{p0}}} \right)$$

Then

$$J_s = (1.6 \times 10^{-19})(1.5 \times 10^{10})^2 \left(\frac{1}{10^{16}} \sqrt{\frac{25}{5 \times 10^{-7}}} + \frac{1}{10^{16}} \sqrt{\frac{10}{5 \times 10^{-7}}} \right)$$

or $\quad J_s = 4.16 \times 10^{-11}$ A/cm²

■ Comment

The ideal reverse-biased saturation current density is very small. If the pn junction cross-sectional area were A $= 10^{-4}$ cm², for example, then the ideal reverse-biased diode current would be $I_s = 4.15 \times 10^{-15}$ A.

■ EXERCISE PROBLEM

Ex 8.2   Consider a GaAs pn junction diode at $T = 300$ K. The parameters of the device are $N_d = 2 \times 10^{16}$ cm⁻³, $N_a = 8 \times 10^{15}$ cm⁻³, $D_n = 210$ cm²/s, $D_p = 8$ cm²/s, $\tau_{no} = 10^{-7}$ s, and $\tau_{po} = 5 \times 10^{-8}$ s. Determine the ideal reverse-saturation current density.

(Ans. $J_s = 3.30 \times 10^{-18}$ A/cm²)

If the forward-bias voltage in Equation (8.27) is positive by more than a few kT/eV, then the (-1) term in Equation (8.27) becomes negligible. Figure 8.9 shows the forward-bias current–voltage characteristic when the current is plotted on a log scale. Ideally, this plot yields a straight line when V_a is greater than a few kT/eV. The forward-bias current is an exponential function of the forward-bias voltage.
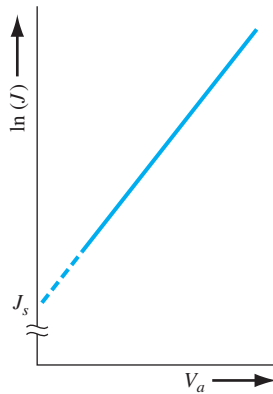

Figure 8.9 | Ideal *I–V* characteristic of a pn junction diode with the current plotted on a log scale.

EXAMPLE 8.3

Objective: Design a pn junction diode to produce particular electron and hole current densities at a given forward-bias voltage.

Consider a silicon pn junction diode at $T = 300$ K. Design the diode such that $J_n = 20$ A/cm^2 and $J_p = 5$ A/cm^2 at $V_a = 0.65$ V. Assume the remaining semiconductor parameters are as given in Example 8.2.

■ **Solution**

The electron diffusion current density is given by Equation (8.24) as

$$J_n = \frac{eD_n n_{p0}}{L_n}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right] = e\sqrt{\frac{D_n}{\tau_{n0}}} \cdot \frac{n_i^2}{N_a}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

Substituting the numbers, we have

$$20 = (1.6 \times 10^{-19})\sqrt{\frac{25}{5 \times 10^{-7}}} \cdot \frac{(1.5 \times 10^{10})^2}{N_a}\left[\exp\left(\frac{0.65}{0.0259}\right) - 1\right]$$

which yields

$$N_a = 1.01 \times 10^{15}\ \text{cm}^{-3}$$

The hole diffusion current density is given by Equation (8.22) as

$$J_p = \frac{eD_p p_{n0}}{L_p}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right] = e\sqrt{\frac{D_p}{\tau_{p0}}} \cdot \frac{n_i^2}{N_d}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]$$

Substituting the numbers, we have

$$5 = (1.6 \times 10^{-19})\sqrt{\frac{10}{5 \times 10^{-7}}} \cdot \frac{(1.5 \times 10^{10})^2}{N_d}\left[\exp\left(\frac{0.65}{0.0259}\right) - 1\right]$$

which yields

$$N_d = 2.55 \times 10^{15}\ \text{cm}^{-3}$$

■ **Comment**

The relative magnitude of the electron and hole current densities through a diode can be varied by changing the doping concentrations in the device.

■ **EXERCISE PROBLEM**

Ex 8.3   Using the parameters given in Ex 8.2 for the GaAs diode, determine the electron and hole current densities at the space charge edges, and determine the total current density in the diode for a forward-bias voltage of $V_a = 1.05$ V.

[Ans. $J_n(-x_p) = 1.20$ A/cm^2, $J_p(x_n) = 0.1325$ A/cm^2, $J_T = 1.33$ A/cm^2]

8.1.6 Summary of Physics

We have been considering the case of a forward-bias voltage being applied to a pn junction. The forward-bias voltage lowers the potential barrier so that electrons and holes are injected across the space charge region. The injected carriers become minority carriers which then diffuse from the junction and recombine with majority carriers.

We calculated the minority carrier diffusion current densities at the edge of the space charge region. We can reconsider Equations (8.14) and (8.15) and determine
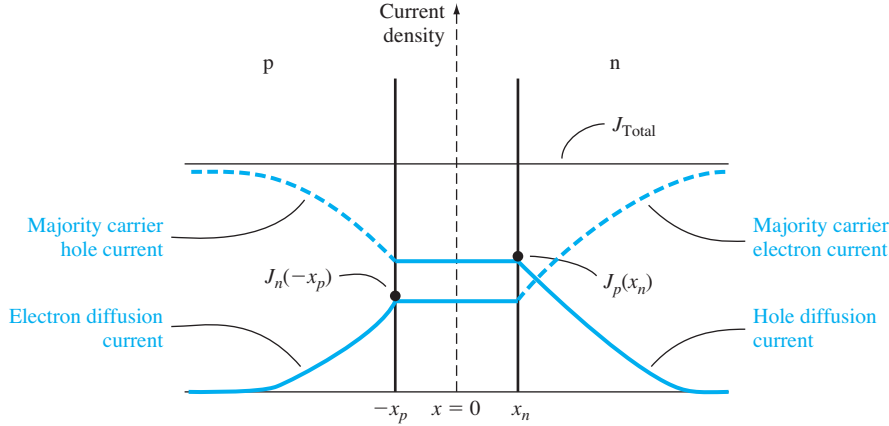
Figure 8.10 | Ideal electron and hole current components through a pn junction under forward bias.

the minority carrier diffusion current densities as a function of distance through the p and n regions. These results are

$$J_p(x) = \frac{eD_p p_{n0}}{L_p} \left[\exp\left(\frac{eV_a}{kT}\right) - 1 \right] \exp\left(\frac{x_n - x}{L_p}\right) \qquad (x \geq x_n) \qquad (8.28)$$

and

$$J_n(x) = \frac{eD_n n_{p0}}{L_n} \left[\exp\left(\frac{eV_a}{kT}\right) - 1 \right] \exp\left(\frac{x_p + x}{L_n}\right) \qquad (x \leq -x_p) \qquad (8.29)$$

The minority carrier diffusion current densities decay exponentially in each region. However, the total current through the pn junction is constant. The difference between total current and minority carrier diffusion current is a majority carrier current. Figure 8.10 shows the various current components through the pn structure. The drift of majority carrier holes in the p region far from the junction, for example, is to supply holes that are being injected across the space charge region into the n region and also to supply holes that are lost by recombination with excess minority carrier electrons. The same discussion applies to the drift of electrons in the n region.

We have seen that excess carriers are created in a forward-biased pn junction. From the results of the ambipolar transport theory derived in Chapter 6, the behavior of the excess carriers is determined by the minority carrier parameters for low injection. In determining the current–voltage relationship of the pn junction, we consider the flow of minority carriers since we know the behavior and characteristics of these particles. It may seem strange, at times, that we concern ourselves so much with minority carriers rather than with the vast number of majority carriers, but the reason for this can be found in the results derived from the ambipolar transport theory.

The fact that we now have drift current densities in the p and n regions implies that the electric field in these regions is not zero as we had originally assumed. We can calculate the electric field in the neutral regions and determine the validity of our zero-field approximation.

EXAMPLE 8.4

Objective: Calculate the electric field in a neutral region of a silicon diode to produce a given majority carrier drift current density.

Consider a silicon pn junction at $T = 300$ K with the parameters given in Example 8.2 and with an applied forward-bias voltage $V_a = 0.65$ V.

■ **Solution**

The total forward-bias current density is given by

$$J = J_s \left[\exp\left(\frac{eV}{kT} \right) - 1 \right]$$

We determined the reverse-saturation current density in Example 8.2, so we can write

$$J = (4.155 \times 10^{-11}) \left[\exp\left(\frac{0.65}{0.0259} \right) - 1 \right] = 3.295 \text{ A/cm}^2$$

The total current far from the junction in the n region will be majority carrier electron drift current, so we can write

$$J = J_n \approx e \mu_n N_d \, \text{E}$$

The doping concentration is $N_d = 10^{16}$ cm^{-3}, and, if we assume $\mu_n = 1350$ cm^2/V-s, then the electric field must be

$$\text{E} = \frac{J_n}{e \mu_n N_d} = \frac{3.295}{(1.6 \times 10^{-19})(1350)(10^{16})} = 1.525 \text{ V/cm}$$

■ **Comment**

We assumed, in the derivation of the current–voltage equation, that the electric field in the neutral p and n regions was zero. Although the electric field is not zero, this example shows that the magnitude is very small—thus the approximation of zero electric field is very good.

■ **EXERCISE PROBLEM**

Ex 8.4   Determine the electric field in the neutral n region and neutral p region for the GaAs pn junction diode described in Ex 8.3.

(Ans. $E_n = 0.0694$ V/cm, $E_p = 3.25$ V/cm)

8.1.7   Temperature Effects

The ideal reverse-saturation current density J_s, given by Equation (8.26), is a function of the thermal-equilibrium minority carrier concentrations n_{p0} and p_{n0}. These minority carrier concentrations are proportional to n_i^2, which is a very strong function of temperature. For a silicon pn junction, the ideal reverse-saturation current density will increase by approximately a factor of 4 for every 10°C increase in temperature.

The forward-bias current–voltage relation is given by Equation (8.27). This relation includes J_s as well as the exp (eV_a/kT) factor, making the forward-bias current–voltage relation a function of temperature also. As temperature increases, less forward-bias voltage is required to obtain the same diode current. If the voltage is held constant, the diode current will increase as temperature increases. The change in forward-bias current with temperature is less sensitive than the reverse-saturation current.

Objective: Determine the change in the forward-bias voltage on a pn junction with a change in temperature to maintain a constant diode current.   **EXAMPLE 8.5**

    Consider a silicon pn junction initially biased at 0.60 V at $T = 300$ K. Assume the temperature increases to $T = 310$ K. Calculate the change in the forward-bias voltage required to maintain a constant current through the junction.

■ **Solution**

The forward-bias current can be written as follows:

$$J \propto \exp\left(\frac{-E_g}{kT}\right) \exp\left(\frac{eV_a}{kT}\right)$$

If the temperature changes, we may take the ratio of the diode currents at the two temperatures. This ratio is

$$\frac{J_2}{J_1} = \frac{\exp(-E_g/kT_2)\exp(eV_{a2}/kT_2)}{\exp(-E_g/kT_1)\exp(eV_{a1}/kT_1)}$$

If current is to be held constant, then $J_1 = J_2$, and we must have

$$\frac{E_g - eV_{a2}}{kT_2} = \frac{E_g - eV_{a1}}{kT_1}$$

For $T_1 = 300$ K, $T_2 = 310$ K, $E_g = 1.12$ eV, and $V_{a1} = 0.60$ V. We then find

$$\frac{1.12 - V_{a2}}{310} = \frac{1.12 - 0.60}{300}$$

which yields

$$V_{a2} = 0.5827 \text{ V}$$

■ **Comment**

The change in the forward-bias voltage is -17.3 mV for a 10°C temperature change.

■ **EXERCISE PROBLEM**

Ex 8.5   Repeat Example 8.5 for a GaAs pn junction diode biased at $V_a = 1.050$ V for $T = 300$ K.

(Ans. -12.3 mV)

8.1.8 The "Short" Diode

We assumed in the previous analysis that both p and n regions were long compared with the minority carrier diffusion lengths. In many pn junction structures, one region may, in fact, be short compared with the minority carrier diffusion length. Figure 8.11 shows one such example: the length W_n is assumed to be much smaller than the minority carrier hole diffusion length, L_p.
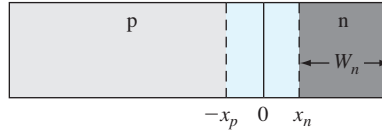
Figure 8.11 | Geometry of a "short" diode.

The steady-state excess minority carrier hole concentration in the n region is determined from Equation (8.9), which is given as

$$\frac{d^2(\delta p_n)}{dx^2} - \frac{\delta p_n}{L_p^2} = 0$$

The original boundary condition at $x = x_n$ still applies, given by Equation (8.11a) as

$$p_n(x_n) = p_{n0} \exp\left(\frac{eV_a}{kT}\right)$$

A second boundary condition needs to be determined. In many cases we assume that an ohmic contact exists at $x = (x_n + W_n)$, implying an infinite surface-recombination velocity and therefore an excess minority carrier concentration of zero. The second boundary condition is then written as

$$p_n(x = x_n + W_n) = p_{n0} \tag{8.30}$$

The general solution to Equation (8.9) is again given by Equation (8.12), which was

$$\delta p_n(x) = p_n(x) - p_{n0} = Ae^{x/L_p} + Be^{-x/L_p} \qquad (x \geq x_n)$$

In this case, because of the finite length of the n region, both terms of the general solution must be retained. Applying the boundary conditions of Equations (8.11b) and (8.30), the excess minority carrier concentration is given by

$$\delta p_n(x) = p_{n0}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\frac{\sinh\left[(x_n + W_n - x)/L_p\right]}{\sinh[W_n/L_p]} \tag{8.31}$$

Equation (8.31) is the general solution for the excess minority carrier hole concentration in the n region of a forward-biased pn junction. If $W_n \gg L_p$, the assumption for the long diode, Equation (8.31) reduces to the previous result given by Equation (8.14). If $W_n \ll L_p$, we can approximate the hyperbolic sine terms by

$$\sinh\left(\frac{x_n + W_n - x}{L_p}\right) \approx \left(\frac{x_n + W_n - x}{L_p}\right) \tag{8.32a}$$

and

$$\sinh\left(\frac{W_n}{L_p}\right) \approx \left(\frac{W_n}{L_p}\right) \tag{8.32b}$$

Then Equation (8.31) becomes

$$\delta p_n(x) = p_{n0}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right]\left(\frac{x_n + W_n - x}{W_n}\right) \tag{8.33}$$

The minority carrier concentration becomes a linear function of distance.

The minority carrier hole diffusion current density is given by

$$J_p = -eD_p \frac{d[\delta p_n(x)]}{dx}$$

so that in the short n region, we have

$$J_p(x) = \frac{eD_p p_{n0}}{W_n}\left[\exp\left(\frac{eV_a}{kT}\right) - 1\right] \tag{8.34}$$

The minority carrier hole diffusion current density now contains the length W_n in the denominator, rather than the diffusion length L_p. The diffusion current density is larger for a short diode than for a long diode since $W_n \ll L_p$. In addition, since the minority carrier concentration is approximately a linear function of distance through the n region, the minority carrier diffusion current density is a constant. This constant current implies that there is no recombination of minority carriers in the short region.

TEST YOUR UNDERSTANDING

TYU 8.1   The doping concentrations in a GaAs pn junction diode at $T = 300$ K are $N_d = 5 \times 10^{15}$ cm^{-3} and $N_a = 5 \times 10^{16}$ cm^{-3}. The minority carrier concentration at either space charge edge is to be no larger than 10 percent of the respective majority carrier concentration. Calculate the maximum forward-bias voltage that can be applied to this junction and still meet the required specifications.

[Ans. $V_a(\text{max}) = 1.067$ V]

TYU 8.2   A silicon pn junction at $T = 300$ K has the following parameters: $N_a = 5 \times 10^{16}$ cm^{-3}, $N_d = 1 \times 10^{16}$ cm^{-3}, $D_n = 25$ cm^2/s, $D_p = 10$ cm^2/s, $\tau_{n0} = 5 \times 10^{-7}$ s, and $\tau_{p0} = 1 \times 10^{-7}$ s. The cross-sectional area is $A = 10^{-3}$ cm^2 and the forward-bias voltage is $V_a = 0.625$ V. Calculate the (a) minority electron diffusion current at the space charge edge, (b) minority hole diffusion current at the space charge edge, and (c) total current in the pn junction diode.

[Ans. (a) 0.154 mA; (b) 1.09 mA; (c) 1.24 mA]

TYU 8.3   Consider the silicon pn junction diode described in TYU 8.2. The p region is long and the n region is short with $W_n = 2$ μm. (a) Calculate the electron and hole currents in the depletion region. (b) Why has the hole current increased compared to that found in TYU 8.2?

[Ans. (a) $I_n = 0.154$ mA, $I_p = 5.44$ mA; (b) The hole density gradient has increased.]

8.2 | GENERATION–RECOMBINATION CURRENTS AND HIGH-INJECTION LEVELS

In the derivation of the ideal current–voltage relationship, we assumed low injection and neglected any effects occurring within the space charge region. High-level injection and other current components generated within the space charge region cause

the *I–V* relationship to deviate from the ideal expression. The additional currents are generated from the recombination processes discussed in Chapter 6.

8.2.1 Generation–Recombination Currents

The recombination rate of excess electrons and holes, given by the Shockley–Read–Hall recombination theory, was written as

$$R = \frac{C_n C_p N_t (np - n_i^2)}{C_n(n + n') + C_p(p + p')} \tag{8.35}$$

The parameters *n* and *p* are, as usual, the concentrations of electrons and holes, respectively.

Reverse-Biased Generation Current For a pn junction under reverse bias, we have argued that the mobile electrons and holes have essentially been swept out of the space charge region. Accordingly, within the space charge region, $n \approx p \approx 0$. The recombination rate from Equation (8.35) becomes

$$R = \frac{-C_n C_p N_t n_i^2}{C_n n' + C_p p'} \tag{8.36}$$

The negative sign implies a negative recombination rate; hence, we are really generating electron–hole pairs within the reverse-biased space charge region. The recombination of excess electrons and holes is the process whereby we are trying to reestablish thermal equilibrium. Since the concentration of electrons and holes is essentially zero within the reverse-biased space charge region, electrons and holes are being generated via the trap level to also try to reestablish thermal equilibrium. This generation process is schematically shown in Figure 8.12. As the electrons and holes
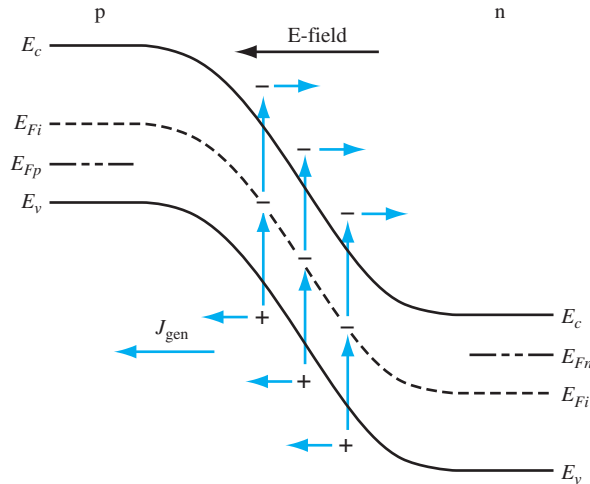

Figure 8.12 | Generation process in a reverse-biased pn junction.

are generated, they are swept out of the space charge region by the electric field. The flow of charge is in the direction of a reverse-biased current. This *reverse-biased generation current,* caused by the generation of electrons and holes in the space charge region, is in addition to the ideal reverse-biased saturation current.

We may calculate the density of the reverse-biased generation current by considering Equation (8.36). If we make a simplifying assumption and let the trap level be at the intrinsic Fermi level, then from Equations (6.92) and (6.97), we have that $n' = n_i$ and $p' = n_i$. Equation (8.36) now becomes

$$R = \frac{-n_i}{\dfrac{1}{N_t C_p} + \dfrac{1}{N_t C_n}} \tag{8.37}$$

Using the definitions of lifetimes from Equations (6.103) and (6.104), we may write Equation (8.37) as

$$R = \frac{-n_i}{\tau_{p0} + \tau_{n0}} \tag{8.38}$$

If we define a new lifetime as the average of τ_{p0} and τ_{n0}, or

$$\tau_0 = \frac{\tau_{p0} + \tau_{n0}}{2} \tag{8.39}$$

then the recombination rate can be written as

$$R = \frac{-n_i}{2\tau_0} \equiv -G \tag{8.40}$$

The negative recombination rate implies a generation rate, so G is the generation rate of electrons and holes in the space charge region.

The generation current density may be determined from

$$J_{\text{gen}} = \int_0^W e\, G dx \tag{8.41}$$

where the integral is over the space charge region. If we assume that the generation rate is constant throughout the space charge region, then we obtain

$$J_{\text{gen}} = \frac{e n_i W}{2\tau_0} \tag{8.42}$$

The total reverse-biased current density is the sum of the ideal reverse saturation current density and the generation current density, or

$$J_R = J_s + J_{\text{gen}} \tag{8.43}$$

The ideal reverse-saturation current density J_s is independent of the reverse-biased voltage. However, J_{gen} is a function of the depletion width W, which in turn is a function of the reverse-biased voltage. The actual reverse-biased current density, then, is no longer independent of the reverse-biased voltage.

EXAMPLE 8.6

Objective: Determine the relative magnitudes of the ideal reverse-saturation current density and the generation current density in a reverse-biased pn junction.

Consider a silicon pn junction at $T = 300$ K with parameters $D_n = 25$ cm²/s, $D_p = 10$ cm²/s, $N_a = N_d = 10^{16}$ cm⁻³, and $\tau_0 = \tau_{n0} = \tau_{p0} = 5 \times 10^{-7}$ s. Assume the diode is reverse biased at $V_R = 5$ V.

■ **Solution**

The ideal reverse-saturation current density was calculated in Example 8.2 and was found to be $J_s = 4.155 \times 10^{-11}$ A/cm².

The built-in potential is found as

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) = (0.0259) \ln\left[\frac{(10^{16})(10^{16})}{(1.5 \times 10^{10})^2}\right] = 0.695 \text{ V}$$

The depletion width is found to be

$$W = \left\{\frac{2 \, \epsilon_s (V_{bi} + V_R)}{e}\left(\frac{N_a + N_d}{N_a N_d}\right)\right\}^{1/2}$$

$$= \left\{\frac{2(11.7)(8.85 \times 10^{-14})(0.695 + 5)}{1.6 \times 10^{-19}}\left[\frac{10^{16} + 10^{16}}{(10^{16})(10^{16})}\right]\right\}^{1/2}$$

$$= 1.214 \times 10^{-4} \text{ cm}$$

The generation current density is then found to be

$$J_{gen} = \frac{en_i W}{2\tau_0} = \frac{(1.6 \times 10^{-19})(1.5 \times 10^{10})(1.214 \times 10^{-4})}{2(5 \times 10^{-7})}$$

or

$$J_{gen} = 2.914 \times 10^{-7} \text{ A/cm}^2$$

The ratio of the two currents is

$$\frac{J_{gen}}{J_s} = \frac{2.914 \times 10^{-7}}{4.155 \times 10^{-11}} \cong 7 \times 10^3$$

■ **Comment**

Comparing the solutions for the two current densities, it is obvious that, for the silicon pn junction diode at room temperature, the generation current density is approximately four orders of magnitude larger than the ideal saturation current density. The generation current is the dominant reverse-biased current in a silicon pn junction diode.

■ **EXERCISE PROBLEM**

Ex 8.6   Consider a GaAs pn junction diode at $T = 300$ K with parameters $N_d = 8 \times 10^{16}$ cm⁻³, $N_a = 2 \times 10^{15}$ cm⁻³, $D_n = 207$ cm²/s, $D_p = 9.80$ cm²/s, and $\tau_0 = \tau_{p0} = \tau_{n0} = 5 \times 10^{-8}$ s. (a) Calculate the ideal reverse-biased saturation current density. (b) Find the reverse-biased generation current density if the diode is reverse biased at $V_R = 5$ V. (c) Determine the ratio of J_{gen} to J_s.

[Ans. (a) 1.677×10^{-17} A/cm²; (b) 6.166×10^{-10} A/cm²; (c) 3.68×10^7]

Forward-Bias Recombination Current   For the reverse-biased pn junction, electrons and holes are essentially completely swept out of the space charge region so that $n \approx p \approx 0$. Under forward bias, however, electrons and holes are injected across the space charge region, so we do, in fact, have some excess carriers in the space charge region. The possibility exists that some of these electrons and holes will recombine within the space charge region and not become part of the minority carrier distribution.

The recombination rate of electrons and holes is again given from Equation (8.35) as

$$R = \frac{C_n C_p N_t (np - n_i^2)}{C_n(n + n') + C_p(p + p')}$$

Dividing both numerator and denominator by $C_n C_p N_t$ and using the definitions of τ_{n0} and τ_{p0}, we may write the recombination rate as

$$R = \frac{np - n_i^2}{\tau_{p0}(n + n') + \tau_{n0}(p + p')} \tag{8.44}$$

Figure 8.13 shows the energy-band diagram of the forward-biased pn junction. Shown in the figure are the intrinsic Fermi level and the quasi-Fermi levels for electrons and holes. From the results of Chapter 6, we may write the electron concentration as

$$n = n_i \exp\left[\frac{E_{Fn} - E_{Fi}}{kT}\right] \tag{8.45}$$

and the hole concentration as

$$p = n_i \exp\left[\frac{E_{Fi} - E_{Fp}}{kT}\right] \tag{8.46}$$

where E_{Fn} and E_{Fp} are the quasi-Fermi levels for electrons and holes, respectively.
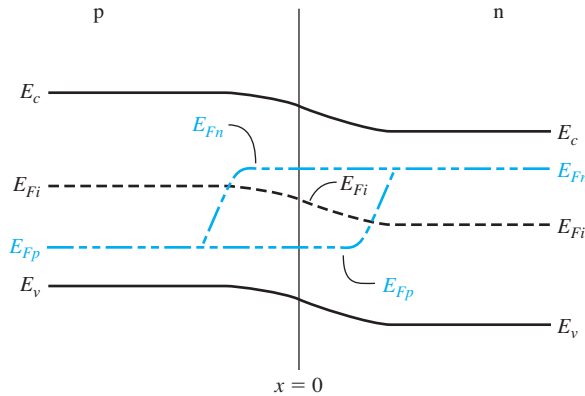

Figure 8.13 | Energy-band diagram of a forward-biased pn junction including quasi-Fermi levels.
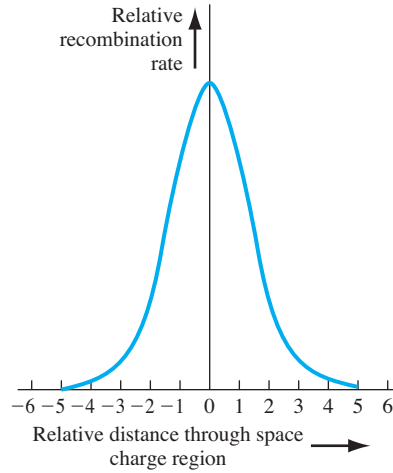
Figure 8.14 | Relative magnitude of the recombination rate through the space charge region of a forward-biased pn junction.

From Figure 8.13, we may note that

$$(E_{Fn} - E_{Fi}) + (E_{Fi} - E_{Fp}) = eV_a \tag{8.47}$$

where V_a is the applied forward-bias voltage. Again, if we assume that the trap level is at the intrinsic Fermi level, then $n' = p' = n_i$. Figure 8.14 shows a plot of the relative magnitude of the recombination rate as a function of distance through the space charge region. This plot was generated using Equations (8.44), (8.45), (8.46), and (8.47). A very sharp peak occurs at the metallurgical junction ($x = 0$).

At the center of the space charge region, we have

$$E_{Fn} - E_{Fi} = E_{Fi} - E_{Fp} = \frac{eV_a}{2} \tag{8.48}$$

Equations (8.45) and (8.46) then become

$$n = n_i \exp\left(\frac{eV_a}{2kT}\right) \tag{8.49}$$

and

$$p = n_i \exp\left(\frac{eV_a}{2kT}\right) \tag{8.50}$$

If we assume that $n' = p' = n_i$ and that $\tau_{n0} = \tau_{p0} = \tau_0$, then Equation (8.44) becomes

$$R_{max} = \frac{n_i}{2\tau_0} \frac{[\exp(eV_a/kT) - 1]}{[\exp(eV_a/2kT) + 1]} \tag{8.51}$$

which is the maximum recombination rate for electrons and holes that occurs at the center of the forward-biased pn junction. If we assume that $V_a \gg kT/e$, we may

neglect the (-1) term in the numerator and the $(+1)$ term in the denominator. Equation (8.51) then becomes

$$R_{\max} = \frac{n_i}{2\tau_0} \exp\left(\frac{eV_a}{2kT}\right) \tag{8.52}$$

The recombination current density may be calculated from

$$J_{\text{rec}} = \int_0^W eR \, dx \tag{8.53}$$

where again the integral is over the entire space charge region. In this case, however, the recombination rate is not a constant through the space charge region. We have calculated the maximum recombination rate at the center of the space charge region, so we may write

$$J_{\text{rec}} = ex'\frac{n_i}{2\tau_0} \exp\left(\frac{eV_a}{2kT}\right) \tag{8.54}$$

where x' is a length over which the maximum recombination rate is effective. However, since τ_0 may not be a well-defined or known parameter, it is customary to write

$$J_{\text{rec}} = \frac{eWn_i}{2\tau_0} \exp\left(\frac{eV_a}{2kT}\right) = J_{r0} \exp\left(\frac{eV_a}{2kT}\right) \tag{8.55}$$

where W is the space charge width.

Total Forward-Bias Current   The total forward-bias current density in the pn junction is the sum of the recombination and the ideal diffusion current densities. Figure 8.15 shows a plot of the minority carrier hole concentration in the neutral
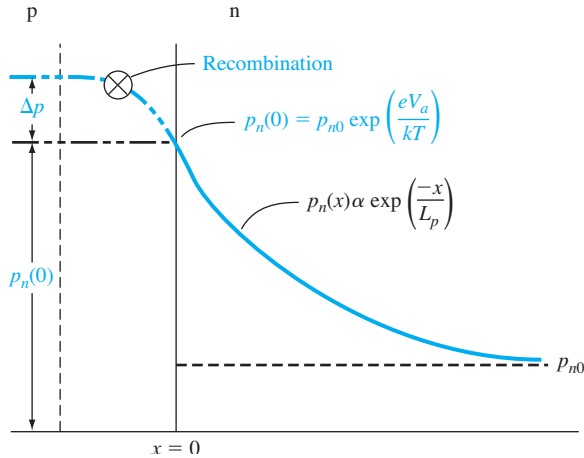

Figure 8.15 | Because of recombination, additional holes from the p region must be injected into the space charge region to establish the minority carrier hole concentration in the n region.

n region. This distribution yields the ideal hole diffusion current density and is a function of the minority carrier hole diffusion length and the applied junction voltage. The distribution is established as a result of holes being injected across the space charge region. If, now, some of the injected holes in the space charge region are lost due to recombination, then additional holes must be injected from the p region to make up for this loss. The flow of these additional injected carriers, per unit time, results in the recombination current. This added component is schematically shown in the figure.

The total forward-bias current density is the sum of the recombination and the ideal diffusion current densities, so we can write

$$J = J_{\text{rec}} + J_D \tag{8.56}$$

where J_{rec} is given by Equation (8.55) and J_D is given by

$$J_D = J_s \exp\left(\frac{eV_a}{kT}\right) \tag{8.57}$$

The (-1) term in Equation (8.27) has been neglected. The parameter J_s is the ideal reverse-saturation current density, and from previous discussion, the value of J_{r0} from the recombination current is larger than the value of J_s.

If we take the natural log of Equations (8.55) and (8.57), we obtain

$$\ln J_{\text{rec}} = \ln J_{r0} + \frac{eV_a}{2kT} = \ln J_{r0} + \frac{V_a}{2V_t} \tag{8.58a}$$

and

$$\ln J_D = \ln J_s + \frac{eV_a}{kT} = \ln J_s + \frac{V_a}{V_t} \tag{8.58b}$$

Figure 8.16 shows the recombination and diffusion current components plotted on a log current scale as a function of V_a/V_t. The slopes of the two curves are not the same. Also shown in the figure is the total current density—the sum of the two current components. We may notice that, at a low current density, the recombination current dominates, and at a higher current density, the ideal diffusion current dominates.

In general, the diode current–voltage relationship may be written as

$$\boxed{I = I_s \left[\exp\left(\frac{eV_a}{nkT}\right) - 1 \right]} \tag{8.59}$$

where the parameter n is called the *ideality factor*. For a large forward-bias voltage, $n \approx 1$ when diffusion dominates, and for low forward-bias voltage, $n \approx 2$ when recombination dominates. There is a transition region where $1 < n < 2$.

8.2.2 High-Level Injection

In the derivation of the ideal diode I–V relationship, we assumed that low injection was valid. Low injection implies that the excess minority carrier concentrations are always much less than the majority carrier concentration.
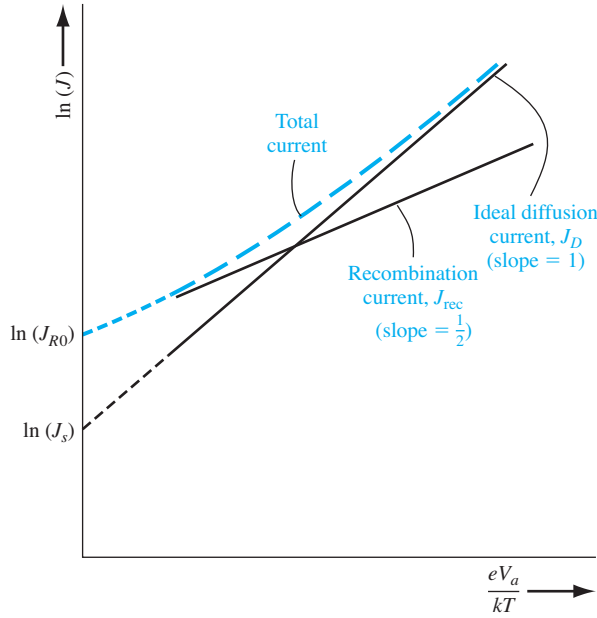
Figure 8.16 | Ideal diffusion, recombination, and total
current in a forward-biased pn junction.

However, as the forward-bias voltage increases, the excess carrier concentrations increase and may become comparable or even greater than the majority carrier concentration. From Equation (8.18), we can write

$$np = n_i^2 \exp\left(\frac{V_a}{V_t}\right)$$

We have that $n = n_o + \delta n$ and $p = p_o + \delta p$, so that

$$(n_o + \delta n)(p_o + \delta p) = n_i^2 \exp\left(\frac{V_a}{V_t}\right) \tag{8.60}$$

Under high-level injection, we may have $\delta n > n_o$ and $\delta p > p_o$ so that Equation (8.60) becomes approximately

$$(\delta n)(\delta p) \cong n_i^2 \exp\left(\frac{V_a}{V_t}\right) \tag{861}$$

Since $\delta n = \delta p$, then

$$\delta n = \delta p \cong n_i \exp\left(\frac{V_a}{2V_t}\right) \tag{8.62}$$

The diode current is proportional to the excess carrier concentration so that, under high-level injection, we have

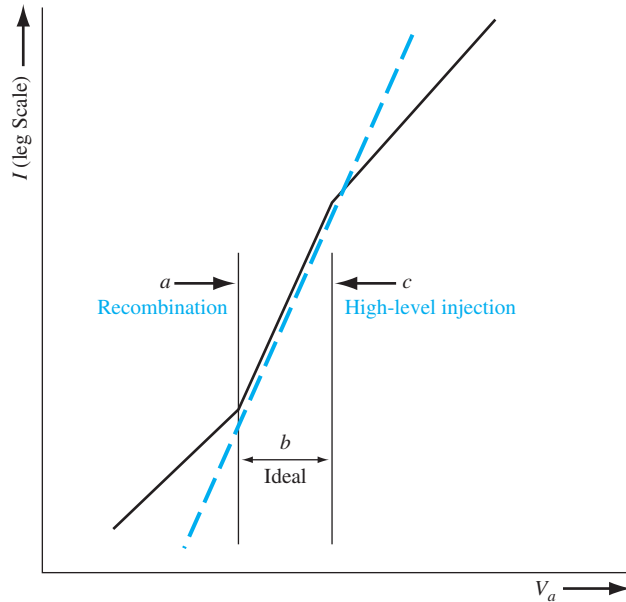$$I \propto \exp\left(\frac{V_a}{2V_t}\right) \tag{8.63}$$

Figure 8.17 | Forward-bias current versus voltage from low
forward bias to high forward bias.

In the high-level injection region, it takes a larger increase in diode voltage to pro-
duce a given increase in diode current.

The diode forward-bias current, from low-bias levels to high-bias levels, is plot-
ted in Figure 8.17. This plot shows the effect of recombination at low-bias voltages
and high-level injection at high-bias voltages.

TEST YOUR UNDERSTANDING

TYU 8.4   Consider a silicon pn junction diode at $T = 300$ K with parameters $N_a = 2 \times 10^{15}$ cm^{-3}, $N_d = 8 \times 10^{16}$ cm^{-3}, $D_p = 10$ cm^2/s, $D_n = 25$ cm^2/s, and $\tau_0 = \tau_{p0} = \tau_{n0} = 10^{-7}$ s. The diode is forward biased at $V_a = 0.35$ V. (*a*) Calculate the ideal diode current density. (*b*) Find the forward-biased recombination current density. (*c*) Determine the ratio of recombination current to the ideal diffusion current.

[Ans. (*a*) 2.137×10^{-4} A/cm^2; (*b*) 5.020×10^{-4} A/cm^2; (*c*) 2.35]

8.3 | SMALL-SIGNAL MODEL OF THE pn JUNCTION

We have been considering the dc characteristics of the pn junction diode. When
semiconductor devices with pn junctions are used in linear amplifier circuits, for
example, sinusoidal signals are superimposed on the dc currents and voltages, so that
the small-signal characteristics of the pn junction become important.

8.3.1 Diffusion Resistance

The ideal current–voltage relationship of the pn junction diode was given by Equation (8.27), where J and J_s are current densities. If we multiply both sides of the equation by the junction cross-sectional area, we have

$$I_D = I_s \left[\exp\left(\frac{eV_a}{kT}\right) - 1 \right] \tag{8.64}$$

where I_D is the diode current and I_s is the diode reverse-saturation current.

Assume that the diode is forward-biased with a dc voltage V_0 producing a dc diode current I_{DQ}. If we now superimpose a small, low-frequency sinusoidal voltage as shown in Figure 8.18, then a small sinusoidal current will be produced, superimposed on the dc current. The ratio of sinusoidal current to sinusoidal voltage is called the incremental conductance. In the limit of a very small sinusoidal current and voltage, the small-signal incremental conductance is just the slope of the dc current–voltage curve, or

$$g_d = \left.\frac{dI_D}{dV_a}\right|_{V_a=V_0} \tag{8.65}$$

The reciprocal of the incremental conductance is the incremental resistance, defined as

$$r_d = \left.\frac{dV_a}{dI_D}\right|_{I_D=I_{DQ}} \tag{8.66}$$
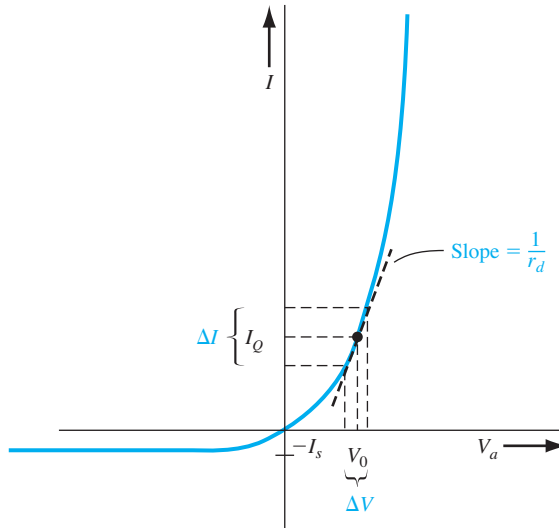
where I_{DQ} is the dc quiescent diode current.

Figure 8.18 | Curve showing the concept of the small-signal diffusion resistance.

If we assume that the diode is biased sufficiently far in the forward-bias region, then the (-1) term can be neglected and the incremental conductance becomes

$$g_d = \frac{dI_D}{dV_a}\Bigg|_{V_a=V_0} = \left(\frac{e}{kT}\right)I_s \exp\left(\frac{eV_0}{kT}\right) \approx \frac{I_{DQ}}{V_t} \tag{8.67}$$

The small-signal incremental resistance is then the reciprocal function, or

$$r_d = \frac{V_t}{I_{DQ}} \tag{8.68}$$

The incremental resistance decreases as the bias current increases, and is inversely proportional to the slope of the *I–V* characteristic as shown in Figure 8.18. The incremental resistance is also known as the *diffusion resistance.*

8.3.2   Small-Signal Admittance

In the last chapter, we considered the pn junction capacitance as a function of the reverse-biased voltage. When the pn junction diode is forward-biased, another capacitance becomes a factor in the diode admittance. The small-signal admittance, or impedance, of the pn junction under forward bias is derived using the minority carrier diffusion current relations we have already considered.

Qualitative Analysis   Before we delve into the mathematical analysis, we can qualitatively understand the physical processes that lead to a diffusion capacitance, which is one component of the junction admittance. Figure 8.19a schematically shows a pn junction forward biased with a dc voltage. A small ac voltage is also
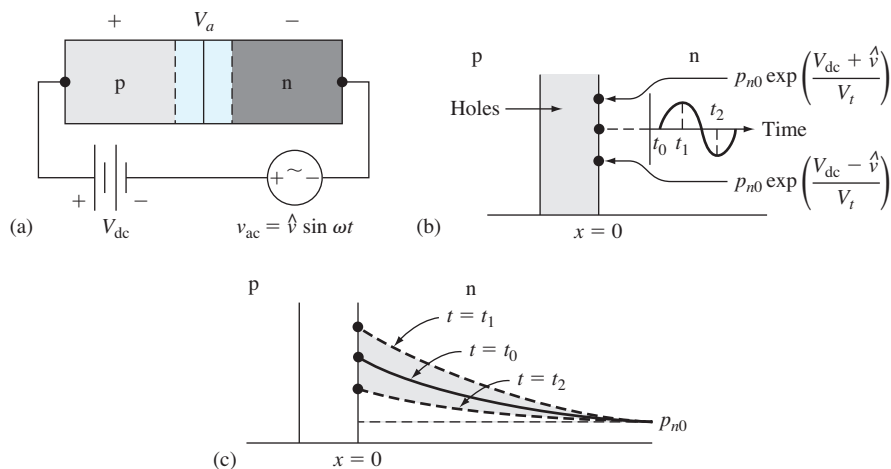

Figure 8.19 | (a) A pn junction with an ac voltage superimposed on a forward-biased dc value; (b) the hole concentration versus time at the space charge edge; (c) the hole concentration versus distance in the n region at three different times.

superimposed on the dc voltage so that the total forward-biased voltage can be written as $V_a = V_{dc} + \hat{v} \sin \omega t$.

As the voltage across the junction changes, the number of holes injected across the space charge region into the n region also changes. Figure 8.19b shows the hole concentration at the space charge edge as a function of time. At $t = t_0$, the ac voltage is zero so that the concentration of holes at $x = 0$ is just given by $p_n(0) = p_{n0} \exp(V_{dc}/V_t)$, which is what we have seen previously.

Now, as the ac voltage increases during its positive half cycle, the concentration of holes at $x = 0$ will increase and reach a peak value at $t = t_1$, which corresponds to the peak value of the ac voltage. When the ac voltage is on its negative half cycle, the total voltage across the junction decreases so that the concentration of holes at $x = 0$ decreases. The concentration reaches a minimum value at $t = t_2$, which corresponds to the time that the ac voltage reaches its maximum negative value. The minority carrier hole concentration at $x = 0$, then, has an ac component superimposed on the dc value as indicated in Figure 8.19b.

As previously discussed, the holes at the space charge edge ($x = 0$) diffuse into the n region where they recombine with the majority carrier electrons. We assume that the period of the ac voltage is large compared to the time it takes carriers to diffuse into the n region. The hole concentration as a function of distance into the n region can then be treated as a steady-state distribution. Figure 8.19c shows the steady-state hole concentrations at three different times. At $t = t_0$, the ac voltage is zero, so the $t = t_0$ curve corresponds to the hole distribution established by the dc voltage. The $t = t_1$ curve corresponds to the distribution established when the ac voltage has reached its peak positive value, and the $t = t_2$ curve corresponds to the distribution established when the ac voltage has reached its maximum negative value. The shaded areas represents the charge ΔQ that is alternately charged and discharged during the ac voltage cycle.

Exactly the same process is occurring in the p region with the electron concentration. The mechanism of charging and discharging of holes in the n region and electrons in the p region leads to a capacitance. This capacitance is called *diffusion capacitance*. The physical mechanism of this diffusion capacitance is different from that of the junction capacitance discussed in the last chapter. We show that the magnitude of the diffusion capacitance in a forward-biased pn junction is usually substantially larger than the junction capacitance.

Mathematical Analysis   The minority carrier distribution in the pn junction will be derived for the case when a small sinusoidal voltage is superimposed on the dc junction voltage. We can then determine small signal, or ac, diffusion currents from these minority carrier functions. Figure 8.20 shows the minority carrier distribution in a pn junction when a forward-biased dc voltage is applied. The origin, $x = 0$, is set at the edge of the space charge region on the n side for convenience. The minority carrier hole concentration at $x = 0$ is given by Equation (8.7) as $p_n(0) = p_{n0} \exp(eV_a/kT)$, where V_a is the applied voltage across the junction.

Now let
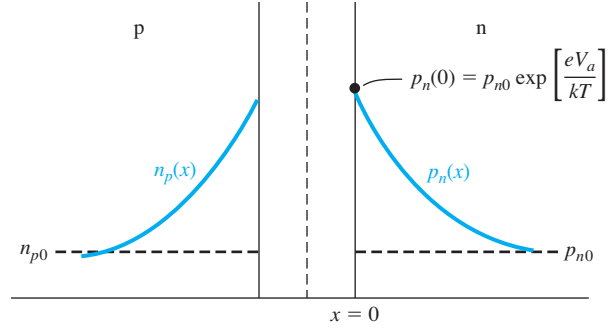
$$V_a = V_0 + v_1(t) \tag{8.69}$$

Figure 8.20 | The dc characteristics of a forward-biased
pn junction used in the small-signal admittance calculations.

where V_0 is the dc quiescent bias voltage and $v_1(t)$ is the ac signal voltage that is su-
perimposed on this dc level. We may now write

$$p_n(x = 0) = p_{n0} \exp\left\{\frac{e[V_0 + v_1(t)]}{kT}\right\} = p_n(0, t) \tag{8.70}$$

Equation (8.70) may be written as

$$p_n(0, t) = p_{dc} \exp\left[\frac{ev_1(t)}{kT}\right] \tag{8.71}$$

where

$$p_{dc} = p_{n0} \exp\left(\frac{eV_0}{kT}\right) \tag{8.72}$$

If we assume that $|v_1(t)| \ll (kT/e) = V_t$, then the exponential term in Equation (8.71)
may be expanded into a Taylor series retaining only the linear terms, and the minor-
ity carrier hole concentration at $x = 0$ can be written as

$$p_n(0, t) \approx p_{dc}\left[1 + \frac{v_1(t)}{V_t}\right] \tag{8.73}$$

If we assume that the time-varying voltage $v_1(t)$ is a sinusoidal signal, we can
write Equation (8.73) as

$$p_n(0, t) = p_{dc}\left(1 + \frac{\hat{V}_1}{V_t} e^{j\omega t}\right) \tag{8.74}$$

where $\hat{V}_1$ is the phasor of the applied sinusoidal voltage. Equation (8.74) will be used
as the boundary condition in the solution of the time-dependent diffusion equation
for the minority carrier holes in the n region.

In the neutral n region ($x > 0$), the electric field is assumed to be zero; thus, the
behavior of the excess minority carrier holes is determined from the equation

$$D_p \frac{\partial^2(\delta p_n)}{\partial x^2} - \frac{\delta p_n}{\tau_{p0}} = \frac{\partial(\delta p_n)}{\partial t} \tag{8.75}$$

where δp_n is the excess hole concentration in the n region. We are assuming that the ac signal voltage $v_1(t)$ is sinusoidal. We then expect the steady-state solution for δp_n to be of the form of a sinusoidal solution superimposed on the dc solution, or

$$\delta p_n(x, t) = \delta p_0(x) + p_1(x)e^{j\omega t} \tag{8.76}$$

where $\delta p_0(x)$ is the dc excess carrier concentration and $p_1(x)$ is the magnitude of the ac component of the excess carrier concentration. The expression for $\delta p_0(x)$ is the same as that given in Equation (8.14).

Substituting Equation (8.76) into the differential Equation (8.75), we obtain

$$D_p \left\{ \frac{\partial^2[\delta p_0(x)]}{\partial x^2} + \frac{\partial^2 p_1(x)}{\partial x^2} e^{j\omega t} \right\} - \frac{\delta p_0(x) + p_1(x)e^{j\omega t}}{\tau_{p0}} = j\omega p_1(x)e^{j\omega t} \tag{8.77}$$

We may rewrite this equation, combining the time-dependent and time-independent terms, as

$$\left\{ \frac{D_p \, \partial^2[\delta p_0(x)]}{\partial x^2} - \frac{\delta p_0(x)}{\tau_{p0}} \right\} + \left[D_p \frac{\partial^2 p_1(x)}{\partial x^2} - \frac{p_1(x)}{\tau_{p0}} - j\omega p_1(x) \right] e^{j\omega t} = 0 \tag{8.78}$$

If the ac component, $p_1(x)$, is zero, then the first bracketed term is just the differential Equation (8.10), which is identically zero. Then we have, from the second bracketed term,

$$D_p \frac{d^2 p_1(x)}{dx^2} - \frac{p_1(x)}{\tau_{p0}} - j\omega p_1(x) = 0 \tag{8.79}$$

Noting that $L_p^2 = D_p\tau_{p0}$, Equation (8.79) may be rewritten in the form

$$\frac{d^2 p_1(x)}{dx^2} - \frac{(1 + j\omega\tau_{p0})}{L_p^2} p_1(x) = 0 \tag{8.80}$$

or

$$\frac{d^2 p_1(x)}{dx^2} - C_p^2 \, p_1(x) = 0 \tag{8.81}$$

where

$$C_p^2 = \frac{(1 + j\omega\tau_{p0})}{L_p^2} \tag{8.82}$$

The general solution to Equation (8.81) is

$$p_1(x) = K_1 e^{-C_p x} + K_2 e^{+C_p x} \tag{8.83}$$

One boundary condition is that $p_1(x \to +\infty) = 0$, which implies that the coefficient $K_2 = 0$. Then

$$p_1(x) = K_1 e^{-C_p x} \tag{8.84}$$

Applying the boundary condition at $x = 0$ from Equation (8.74), we obtain

$$p_1(0) = K_1 = p_{dc}\left(\frac{\hat{V}_1}{V_t} \right) \tag{8.85}$$

The hole diffusion current density can be calculated at $x = 0$. This is given by

$$J_p = -eD_p \frac{\partial p_n}{\partial x}\bigg|_{x=0} \qquad (8.86)$$

If we consider a homogeneous semiconductor, the derivative of the hole concentration will be just the derivative of the excess hole concentration. Then

$$J_p = -eD_p \frac{\partial(\delta p_n)}{\partial x}\bigg|_{x=0} = -eD_p \frac{\partial[\delta p_0(x)]}{\partial x}\bigg|_{x=0} - eD_p \frac{\partial p_1(x)}{\partial x}\bigg|_{x=0} e^{j\omega t} \qquad (8.87)$$

We can write this equation in the form

$$J_p = J_{p0} + j_p(t) \qquad (8.88)$$

where

$$J_{p0} = -eD_p \frac{\partial[\delta p_0(x)]}{\partial x}\bigg|_{x=0} = \frac{eD_p p_{n0}}{L_p}\left[\exp\left(\frac{eV_0}{kT}\right) - 1\right] \qquad (8.89)$$

Equation (8.89) is the dc component of the hole diffusion current density and is exactly the same as in the ideal *I–V* relation derived previously.

The sinusoidal component of the diffusion current density is then found from

$$j_p(t) = \hat{J}_p e^{j\omega t} = -eD_p \frac{\partial p_1(x)}{\partial x} e^{j\omega t}\bigg|_{x=0} \qquad (8.90)$$

where $\hat{J}_p$ is the current density phasor. Combining Equations (8.90), (8.84), and (8.85), we have

$$\hat{J}_p = -eD_p(-C_p)\left[p_{dc}\left(\frac{\hat{V}_1}{V_t}\right)\right]e^{-C_p x}\bigg|_{x=0} \qquad (8.91)$$

We can write the total ac hole current phasor as

$$\hat{I}_p = A\hat{J}_p = eAD_p C_p p_{dc}\left(\frac{\hat{V}_1}{V_t}\right) \qquad (8.92)$$

where A is the cross-sectional area of the pn junction. Substituting the expression for C_p, we obtain

$$\hat{I}_p = \frac{eAD_p p_{dc}}{L_p}\sqrt{1 + j\omega\tau_{p0}}\left(\frac{\hat{V}_1}{V_t}\right) \qquad (8.93)$$

If we define

$$I_{p0} = \frac{eAD_p p_{dc}}{L_p} = \frac{eAD_p p_{n0}}{L_p}\exp\left(\frac{eV_0}{kT}\right) \qquad (8.94)$$

then Equation (8.93) becomes

$$\hat{I}_p = I_{p0}\sqrt{1 + j\omega\tau_{p0}}\left(\frac{\hat{V}_1}{V_t}\right) \qquad (8.95)$$

Going through the same type of analysis for the minority carrier electrons in the p region, we obtain

$$\hat{I}_n = I_{n0}\sqrt{1 + j\omega\tau_{n0}}\left(\frac{\hat{V}_1}{V_t}\right) \qquad (8.96)$$

where

$$I_{n0} = \frac{eAD_n n_{p0}}{L_n} \exp\left(\frac{eV_0}{kT}\right) \tag{8.97}$$

The total ac current phasor is the sum of $\hat{I}_p$ and $\hat{I}_n$. The pn junction admittance is the total ac current phasor divided by the ac voltage phasor, or

$$Y = \frac{\hat{I}}{\hat{V}_1} = \frac{\hat{I}_p + \hat{I}_n}{\hat{V}_1} = \left(\frac{1}{V_t}\right)\left[I_{p0}\sqrt{1 + j\omega\tau_{p0}} + I_{n0}\sqrt{1 + j\omega\tau_{n0}}\right] \tag{8.98}$$

There is not a linear, lumped, finite, passive, bilateral network that can be synthesized to give this admittance function. However, we may make the following approximations. Assume that

$$\omega\tau_{p0} \ll 1 \tag{8.99a}$$

and

$$\omega\tau_{n0} \ll 1 \tag{8.99b}$$

These two assumptions imply that the frequency of the ac signal is not too large. Then we may write

$$\sqrt{1 + j\omega\tau_{p0}} \approx 1 + \frac{j\omega\tau_{p0}}{2} \tag{8.100a}$$

and

$$\sqrt{1 + j\omega\tau_{n0}} \approx 1 + \frac{j\omega\tau_{n0}}{2} \tag{8.100b}$$

Substituting Equations (8.100a) and (8.100b) into the admittance Equation (8.98), we obtain

$$Y = \left(\frac{1}{V_t}\right)\left[I_{p0}\left(1 + \frac{j\omega\tau_{p0}}{2}\right) + I_{n0}\left(1 + \frac{j\omega\tau_{n0}}{2}\right)\right] \tag{8.101}$$

If we combine the real and imaginary portions, we get

$$Y = \left(\frac{1}{V_t}\right)(I_{p0} + I_{n0}) + j\omega\left[\left(\frac{1}{2V_t}\right)(I_{p0}\tau_{p0} + I_{n0}\tau_{n0})\right] \tag{8.102}$$

Equation (8.102) may be written in the form

$$Y = g_d + j\omega C_d \tag{8.103}$$

The parameter g_d is called the *diffusion conductance* and is given by

$$g_d = \left(\frac{1}{V_t}\right)(I_{p0} + I_{n0}) = \frac{I_{DQ}}{V_t} \tag{8.104}$$

where I_{DQ} is the dc bias current. Equation (8.104) is exactly the same conductance as we obtained previously in Equation (8.67). The parameter C_d is called the *diffusion capacitance* and is given by

$$\boxed{C_d = \left(\frac{1}{2V_t}\right)(I_{p0}\tau_{p0} + I_{n0}\tau_{n0})} \tag{8.105}$$
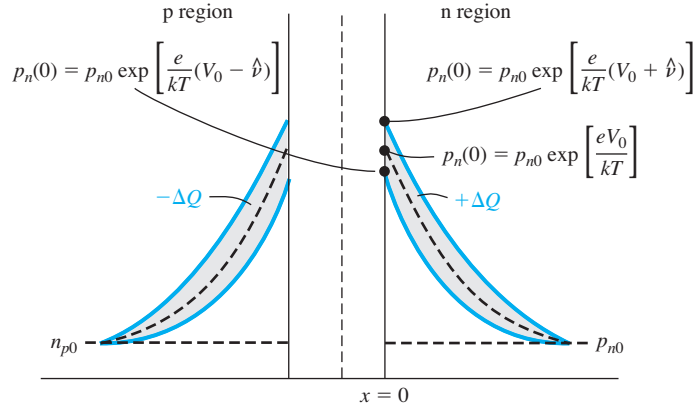
Figure 8.21 | Minority carrier concentration changes with changing forward-bias voltage.

The physics of the diffusion capacitance may be seen in Figure 8.21. The dc values of the minority carrier concentrations are shown along with the changes due to the ac component of voltage. The ΔQ charge is alternately being charged and discharged through the junction as the voltage across the junction changes. The change in the stored minority carrier charge as a function of the change in voltage is the diffusion capacitance. One consequence of the approximations $\omega \tau_{p0} \ll 1$ and $\omega \tau_{n0} \ll 1$ is that there are no "wiggles" in the minority carrier curves. The sinusoidal frequency is low enough so that the exponential curves are maintained at all times.

EXAMPLE 8.7

Objective: Calculate the small-signal admittance parameters of a pn junction diode.

This example is intended to give an indication of the magnitude of the diffusion capacitance as compared with the junction capacitance considered in the last chapter. The diffusion resistance will also be calculated. Assume that $N_a \gg N_d$ so that $p_{n0} \gg n_{p0}$. This assumption implies that $I_{p0} \gg I_{n0}$. Let $T = 300$ K, $\tau_{p0} = 10^{-7}$ s, and $I_{p0} = I_{DQ} = 1$ mA.

■ **Solution**

The diffusion capacitance, with these assumptions, is given by

$$C_d \approx \left(\frac{1}{2V_t}\right)(I_{p0}\tau_{p0}) = \frac{1}{(2)(0.0259)}(10^{-3})(10^{-7}) = 1.93 \times 10^{-9} \text{ F}$$

The diffusion resistance is

$$r_d = \frac{V_t}{I_{DQ}} = \frac{0.0259 \text{ V}}{1 \text{ mA}} = 25.9 \ \Omega$$

■ **Comment**

The value of 1.93 nF for the diffusion capacitance of a forward-biased pn junction is three to four orders of magnitude larger than the junction capacitance of the reverse-biased pn junction, which we calculated in Example 7.5.

■ EXERCISE PROBLEM

Ex 8.7 A silicon pn junction diode at $T = 300$ K has the following parameters: $N_d = 8 \times 10^{16}$ cm^{-3}, $N_a = 2 \times 10^{15}$ cm^{-3}, $D_n = 25$ cm^2/s, $D_p = 10$ cm^2/s, $\tau_{n0} = 5 \times 10^{-7}$s, and $\tau_{p0} = 10^{-7}$s. The cross-sectional area is $A = 10^{-3}$ cm^2. Determine the diffusion resistance and diffusion capacitance if the diode is forward biased at (a) $V_a = 0.550$ V and (b) $V_a = 0.610$ V.

[Ans. (a) $r_d = 118$ Ω, $C_d = 2.07$ nF; (b) $r_d = 11.6$ Ω, $C_d = 20.9$ nF]

The diffusion capacitance tends to dominate the capacitance terms in a forward-biased pn junction. The small-signal diffusion resistance can be fairly small if the diode current is a fairly large value. As the diode current decreases, the diffusion resistance increases. We will consider the impedance of forward-biased pn junctions again when we discuss bipolar transistors.

8.3.3 Equivalent Circuit

The small-signal equivalent circuit of the forward-biased pn junction is derived from Equation (8.103). This circuit is shown in Figure 8.22a. We need to add the junction capacitance, which will be in parallel with the diffusion resistance and diffusion capacitance. The last element we add, to complete the equivalent circuit, is a series resistance. The neutral n and p regions have finite resistances so the actual pn junction will include a series resistance. The complete equivalent circuit is given in Figure 8.22b.

The voltage across the actual junction is V_a and the total voltage applied to the pn diode is given by V_{app}. The junction voltage V_a is the voltage in the ideal current–voltage expression. We can write the expression

$$V_{app} = V_a + Ir_s \tag{8.106}$$

Figure 8.23 is a plot of the current–voltage characteristic from Equation (8.106) showing the effect of the series resistance. A larger applied voltage is required to
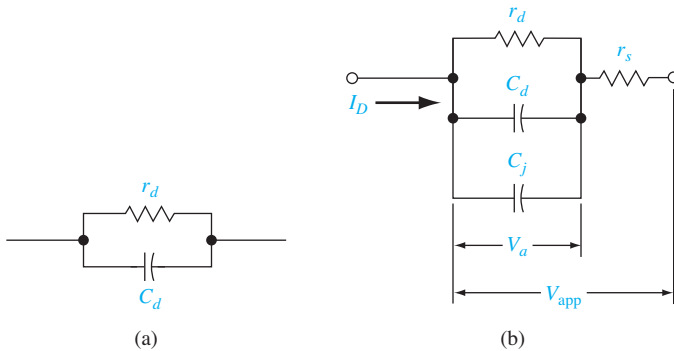

Figure 8.22 | (a) Small-signal equivalent circuit of ideal forward-biased pn junction diode; (b) complete small-signal equivalent circuit of pn junction.
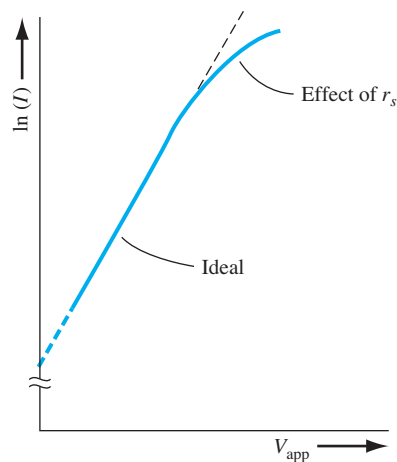
Figure 8.23 | Forward-biased *I–V* characteristics of a pn junction diode showing the effect of series resistance.

achieve the same current value when a series resistance is included. In most diodes, the series resistance will be negligible. In some semiconductor devices with pn junctions, however, the series resistance will be in a feedback loop; in these cases, the resistance is multiplied by a gain factor and becomes non-negligible.

TEST YOUR UNDERSTANDING

TYU 8.5   A GaAs pn junction diode at $T = 300$ K has the same parameters given in Ex 8.7 except that $D_n = 207$ cm^2/s and $D_p = 9.80$ cm^2/s. Determine the diffusion resistance and diffusion capacitance if the diode is forward biased at (*a*) $V_a = 0.970$ V and (*b*) $V_a = 1.045$ V.

[Ans. (*a*) $r_d = 263$ Ω, $C_d = 0.940$ nF; (*b*) $r_d = 14.6$ Ω, $C_d = 17.0$ nF]

TYU 8.6   A silicon pn junction diode at $T = 300$ K has the same parameters as those described in Ex 8.7. The neutral n-region and neutral p-region lengths are 0.01 cm. Estimate the series resistance of the diode (neglect ohmic contacts).

(Ans. $R = 66$ Ω)

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*8.4 | CHARGE STORAGE AND DIODE TRANSIENTS

The pn junction is typically used as an electrical switch. In forward bias, referred to as the *on* state, a relatively large current can be produced by a small applied voltage; in reverse bias, referred to as the *off* state, only a very small current will exist. Of primary interest in circuit applications is the speed of the pn junction diode in switching states. We qualitatively discuss the transients that occur and the charge storage effects. We simply state the equations that describe the switching times without any mathematical derivations.
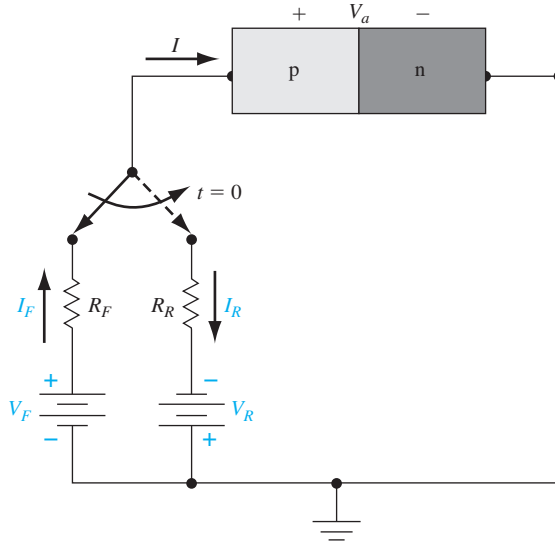
Figure 8.24 | Simple circuit for switching a diode from forward to reverse bias.

8.4.1   The Turn-off Transient

Suppose we want to switch a diode from the forward bias on state to the reverse-biased off state. Figure 8.24 shows a simple circuit that will switch the applied bias at $t = 0$. For $t < 0$, the forward-bias current is

$$I = I_F = \frac{V_F - V_a}{R_F} \tag{8.107}$$

The minority carrier concentrations in the device, for the applied forward voltage V_F, are shown in Figure 8.25a. There is excess minority carrier charge stored in both the p and n regions of the diode. The excess minority carrier concentrations at the space charge edges are supported by the forward-bias junction voltage V_a. When the voltage is switched from the forward- to the reverse-biased state, the excess minority carrier concentrations at the space charge edges can no longer be supported and they start to decrease, as shown in Figure 8.25b.

The collapse of the minority carrier concentrations at the edges of the space charge region leads to large concentration gradients and diffusion currents in the reverse-biased direction. If we assume, for the moment, that the voltage across the diode junction is small compared with V_R, then the reverse-biased current is limited to approximately

$$I = -I_R \approx \frac{-V_R}{R_R} \tag{8.108}$$

The junction capacitances do not allow the junction voltage to change instantaneously. If the current I_R were larger than this value, there would be a forward-bias voltage across the junction, which would violate our assumption of a reverse-biased current.
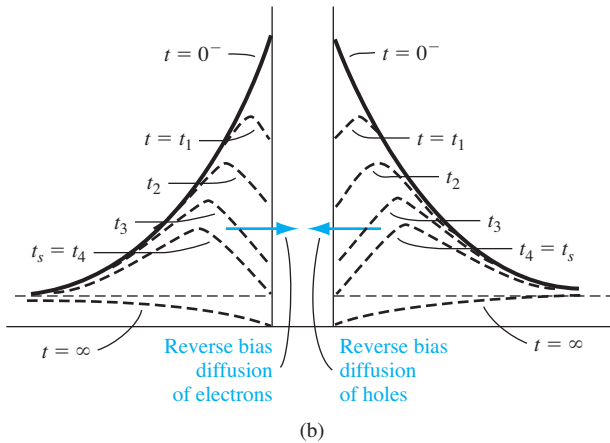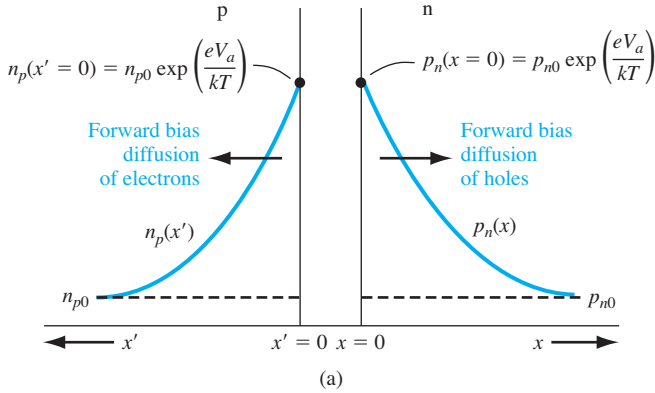
Figure 8.25 | (a) Steady-state forward-bias minority carrier concentrations; (b) minority carrier concentrations at various times during switching.
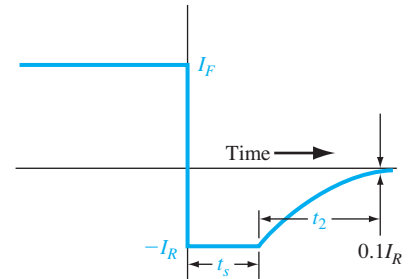

Figure 8.26 | Current characteristic versus time during diode switching.

If the current I_R were smaller than this value, there would be a reverse-biased voltage across the junction, which means that the junction voltage would have changed instantaneously. Since the reverse current is limited to the value given by Equation (8.108), the reverse-biased density gradient is constant; thus, the minority carrier concentrations at the space charge edge decrease with time as shown in Figure 8.25b.

This reverse current I_R will be approximately constant for $0^+ \leq t \leq t_s$, where t_s is called the *storage time*. The storage time is the length of time required for the minority carrier concentrations at the space charge edge to reach the thermal-equilibrium values. After this time, the voltage across the junction will begin to change. The current characteristic is shown in Figure 8.26. The reverse current is the flow of the stored minority carrier charge, which is the difference between the minority carrier concentrations at $t = 0^-$ and $t = \infty$, as shown in Figure 8.25b.

The storage time t_s can be determined by solving the time-dependent continuity equation. If we consider a one-sided p^+n junction, the storage time is determined from the equation

$$\text{erf} \sqrt{\frac{t_s}{\tau_{p0}}} = \frac{I_F}{I_F + I_R} \tag{8.109}$$

where erf (x) is known as the error function. An approximate solution for the storage time can be obtained as

$$t_s \approx \tau_{p0} \ln\left(1 + \frac{I_F}{I_R}\right) \tag{8.110}$$

The recovery phase for $t > t_s$ is the time required for the junction to reach its steady-state reverse-biased condition. The remainder of the excess charge is being removed and the space charge width is increasing to the reverse-biased value. The decay time t_2 is determined from

$$\text{erf} \sqrt{\frac{t_2}{\tau_{p0}}} + \frac{\exp(-t_2/\tau_{p0})}{\sqrt{\pi t_2/\tau_{p0}}} = 1 + 0.1 \left(\frac{I_R}{I_F}\right) \tag{8.111}$$

The total turn-off time is the sum of t_s and t_2.

To switch the diode quickly, we need to be able to produce a large reverse current as well as have a small minority carrier lifetime. In the design of diode circuits, then, the designer must provide a path for the transient reverse-biased current pulse in order to be able to switch the diode quickly. These same effects will be considered when we discuss the switching of bipolar transistors.

8.4.2 The Turn-on Transient

The turn-on transient occurs when the diode is switched from its "off" state into the forward-bias "on" state. The turn-on can be accomplished by applying a forward-bias current pulse. The first stage of turn-on occurs very quickly and is the length of time required to narrow the space charge width from the reverse-biased value to its thermal-equilibrium value when $V_a = 0$. During this time, ionized donors and acceptors are neutralized as the space charge width narrows.

The second stage of the turn-on process is the time required to establish the minority carrier distributions. During this time the voltage across the junction is increasing toward its steady-state value. A small turn-on time is achieved if the minority carrier lifetime is small and if the forward-bias current is small.

TEST YOUR UNDERSTANDING

TYU 8.7 A one-sided p^+n silicon diode, which has a forward-bias current of $I_F = 1.75$ mA, is switched to reverse bias with an effective reverse-biased voltage of $V_R = 2$ V and an effective series resistance of $R_R = 4$ kΩ. The minority carrier hole lifetime is 10^{-7} s. (a) Determine the storage time t_s. (b) Calculate the decay time t_2. (c) What is the turn-off time of the diode?

[Ans. (a) 0.746×10^{-7} s; (b) 1.25×10^{-7} s; (c) $\approx 2 \times 10^{-7}$ s]

*8.5 | THE TUNNEL DIODE

The *tunnel diode* is a pn junction in which both the n and p regions are degenerately doped. As we discuss the operation of this device, we will find a region that exhibits a negative differential resistance. The tunnel diode was used in oscillator circuits in the past, but other types of solid-state devices are now used as high-frequency oscillators; thus, the tunnel diode is really only of academic interest. Nevertheless, this device does demonstrate the phenomenon of tunneling we discussed in Chapter 2.

Recall the degenerately doped semiconductors we discussed in Chapter 4: the Fermi level is in the conduction band of a degenerately doped n-type material and in the valence band of a degenerately doped p-type material. Then, even at $T = 0$ K, electrons will exist in the conduction band of the n-type material, and holes (empty states) will exist in the p-type material.

Figure 8.27 shows the energy-band diagram of a pn junction in thermal equilibrium for the case when both the n and p regions are degenerately doped. The depletion region width decreases as the doping increases and may be on the order of approximately 100 Å for the case shown in Figure 8.27. The potential barrier at the junction can be approximated by a triangular potential barrier, as shown in Figure 8.28. This potential barrier is similar to the potential barrier used in Chapter 2 to illustrate the tunneling phenomenon. The barrier width is small and the electric field in the space charge region is quite large; thus, a finite probability exists that an electron may tunnel through the forbidden band from one side of the junction to the other.

We may qualitatively determine the current–voltage characteristics of the tunnel diode by considering the simplified energy-band diagrams in Figure 8.29.
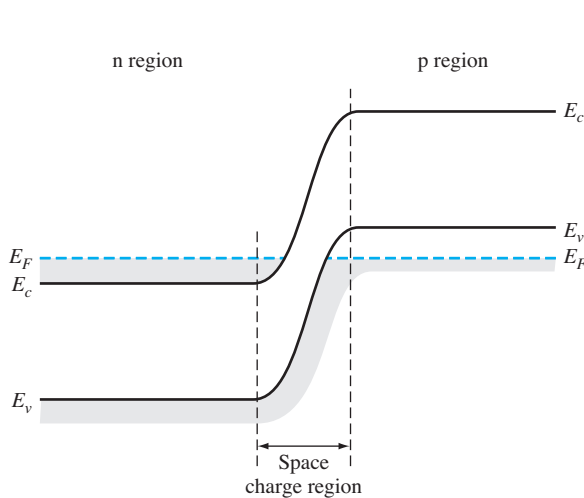

Figure 8.27 | Energy-band diagram of a pn junction in thermal equilibrium in which both the n and p regions are degenerately doped.
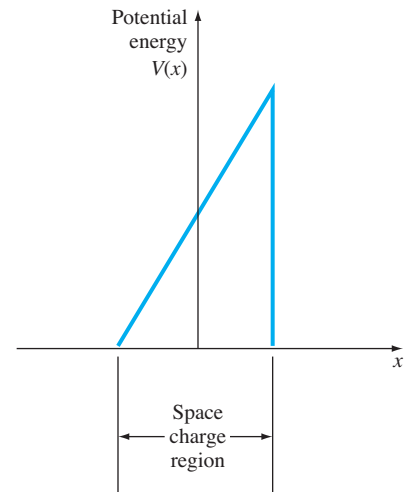

Figure 8.28 | Triangular potential barrier approximation of the potential barrier in the tunnel diode.
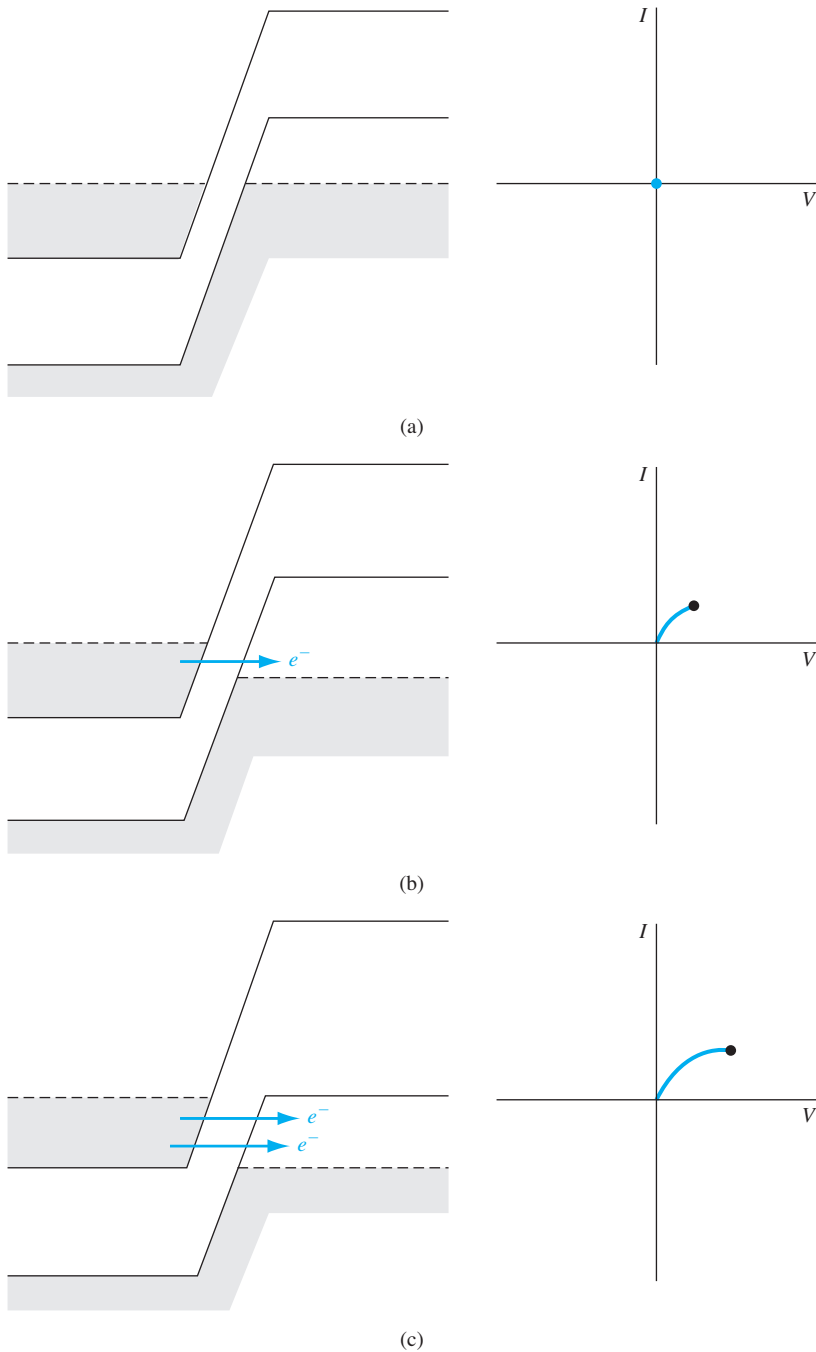
(a)

(b)

(c)

Figure 8.29 | Simplified energy-band diagrams and *I–V* characteristics of the tunnel diode at (a) zero bias; (b) a slight forward bias; (c) a forward bias producing maximum tunneling current. (*continued*)
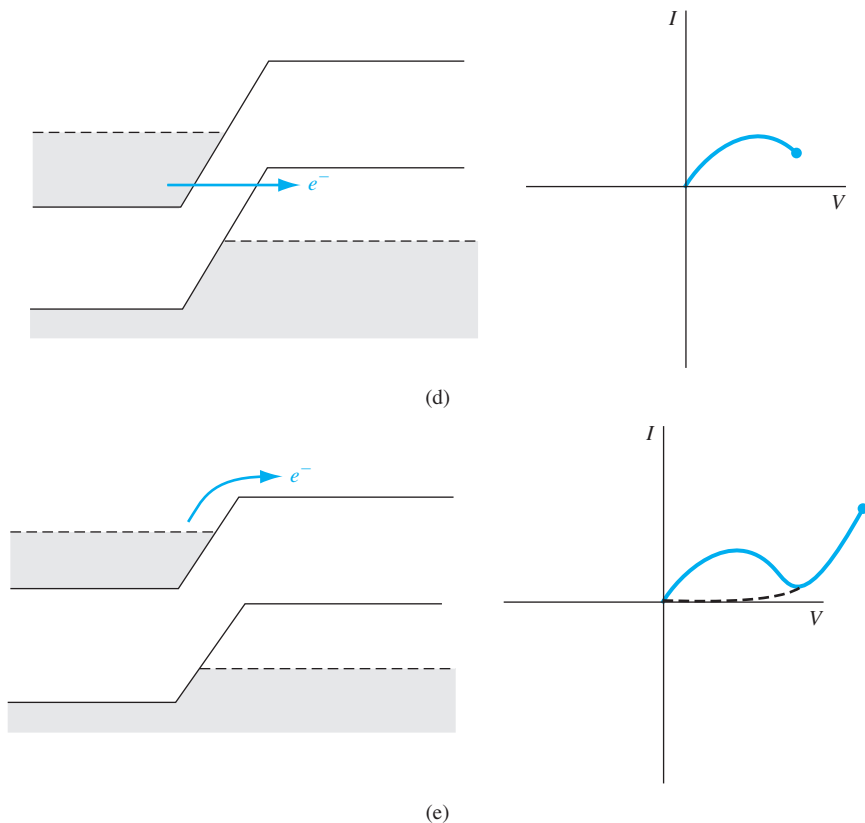
(d)

(e)

Figure 8.29 | *(concluded)* (d) A higher forward bias showing less tunneling current; (e) a forward bias for which the diffusion current dominates.

Figure 8.29a shows the energy-band diagram at zero bias, which produces zero current on the *I–V* diagram. If we assume, for simplicity, that we are near 0 K, then all energy states are filled below E_F on both sides of the junction.

Figure 8.29b shows the situation when a small forward-bias voltage is applied to the junction. Electrons in the conduction band of the n region are directly opposite to empty states in the valence band of the p region. There is a finite probability that some of these electrons will tunnel directly into the empty states, producing a forward-bias tunneling current as shown. With a slightly larger forward-bias voltage, as in Figure 8.29c, the maximum number of electrons in the n region will be opposite the maximum number of empty states in the p region; this will produce a maximum tunneling current.

As the forward-bias voltage continues to increase, the number of electrons on the n side directly opposite empty states on the p side decreases, as in Figure 8.29d, and the tunneling current will decrease. In Figure 8.29e, there are no electrons on the n side directly opposite to available empty states on the p side. For this forward-bias voltage, the tunneling current will be zero and the normal ideal diffusion current will exist in the device as shown in the *I–V* characteristics.
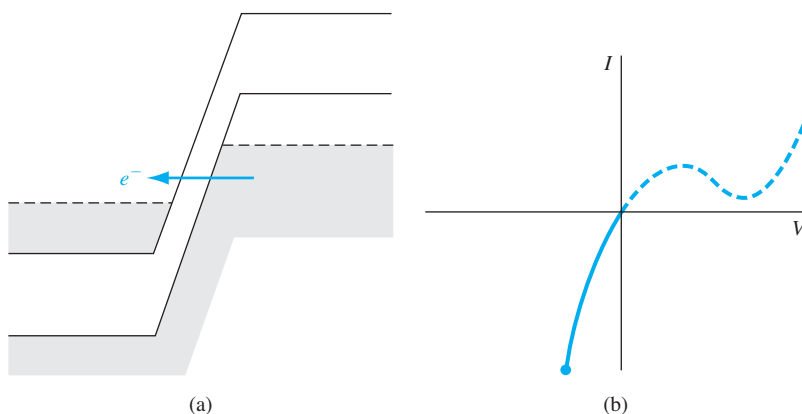
Figure 8.30 | (a) Simplified energy-band diagram of a tunnel diode with a reverse-biased voltage; (b) *I–V* characteristic of a tunnel diode with a reverse-biased voltage.

The portion of the curve showing a decrease in current with an increase in voltage is the region of differential negative resistance. The range of voltage and current for this region is quite small; thus, any power generated from an oscillator using this negative resistance property would also be fairly small.

A simplified energy-band diagram of the tunnel diode with an applied reverse-biased voltage is shown in Figure 8.30a. Electrons in the valence band on the p side are directly opposite empty states in the conduction band on the n side, so electrons can now tunnel directly from the p region into the n region, resulting in a large reverse-biased tunneling current. This tunneling current will exist for any reverse-biased voltage. The reverse-biased current will increase monotonically and rapidly with reverse-biased voltage as shown in Figure 8.30b.

8.6 | SUMMARY

■ When a forward-bias voltage is applied across a pn junction (p region positive with respect to the n region), the potential barrier is lowered so that holes from the p region and electrons from the n region can flow across the junction.

■ The boundary conditions relating the minority carrier hole concentration in the n region at the space charge edge and the minority carrier electron concentration in the p region at the space charge edge were derived.

■ The holes that are injected into the n region and the electrons that are injected into the p region now become excess minority carriers. The behavior of the excess minority carrier is described by the ambipolar transport equation developed and described in Chapter 6. Solving the ambipolar transport equation and using the boundary conditions, the steady-state minority carrier hole and electron concentrations in the n region and p region, respectively, were derived.

■ Gradients exist in the minority carrier hole and electron concentrations so that minority carrier diffusion currents exist in the pn junction. These diffusion currents yield the ideal current–voltage relationship of the pn junction diode.

■ Excess carriers are generated in the space charge region of a reverse-biased pn junction. These carriers are swept out by the electric field and create the reverse-biased generation current that is another component of the reverse-biased diode current. Excess carriers recombine in the space charge region of a forward-biased pn junction. This recombination process creates the forward-bias recombination current that is another component of the forward-bias diode current.

■ The small-signal equivalent circuit of the pn junction diode was developed. The two parameters of interest are the diffusion resistance and the diffusion capacitance.

■ When a pn junction is switched from forward bias to reverse bias, the stored excess minority carrier charge must be removed from the junction. The time required to remove this charge is called the storage time and is a limiting factor in the switching speed of a diode.

■ The *I–V* characteristics of a tunnel diode were developed showing a region of negative differential resistance.

GLOSSARY OF IMPORTANT TERMS

carrier injection   The flow of carriers across the space charge region of a pn junction when a voltage is applied.

diffusion capacitance   The capacitance of a forward-biased pn junction due to minority carrier storage effects.

diffusion conductance   The ratio of a low-frequency, small-signal sinusoidal current to voltage in a forward-biased pn junction.

diffusion resistance   The inverse of diffusion conductance.

forward bias   The condition in which a positive voltage is applied to the p region with respect to the n region of a pn junction so that the potential barrier between the two regions is lowered below the thermal-equilibrium value.

generation current   The reverse-biased pn junction current produced by the thermal generation of electron–hole pairs within the space charge region.

high-level injection   The condition in which the excess carrier concentration becomes comparable to or greater than the majority carrier concentration.

"long" diode   A pn junction diode in which both the neutral p and n regions are long compared with the respective minority carrier diffusion lengths.

recombination current   The forward-bias pn junction current produced as a result of the flow of electrons and holes that recombine within the space charge region.

reverse saturation current   The ideal reverse-biased current in a pn junction.

"short" diode   A pn junction diode in which at least one of the neutral p or n regions is short compared to the respective minority carrier diffusion length.

storage time   The time required for the excess minority carrier concentrations at the space charge edge to go from their steady-state values to zero when the diode is switched from forward to reverse bias.

CHECKPOINT

After studying this chapter, the reader should have the ability to:

■ Describe the mechanism of charge flow across the space charge region of a pn junction when a forward-bias voltage is applied.

■ State the boundary conditions for the minority carrier concentrations at the edge of the space charge region.

■ Derive the expressions for the steady-state minority carrier concentrations in the pn junction.

■ Derive the ideal current–voltage relationship for a pn junction diode.

■ Describe the characteristics of a "short" diode.

■ Describe generation and recombination currents in a pn junction.

■ Define high-level injection and describe its effect on the diode *I–V* characteristics.

■ Describe what is meant by diffusion resistance and diffusion capacitance.

■ Describe the turn-off transient response in a pn junction.

REVIEW QUESTIONS

1. Sketch the energy bands in a zero-biased, reverse-biased, and forward-biased pn junction.

2. Write the boundary conditions for the excess minority carriers in a pn junction (a) under forward bias and (b) under reverse bias.

3. Sketch the steady-state minority carrier concentrations in a forward-biased pn junction.

4. Explain the procedure that is used in deriving the ideal current–voltage relationship in a pn junction diode.

5. Sketch the electron and hole currents through a forward-biased pn junction diode. Are currents near the junction primarily due to drift or diffusion? What about currents far from the junction?

6. What is the temperature dependence of the ideal reverse-saturation current?

7. What is meant by a "short" diode?

8. Explain the physical mechanism of the (a) generation current and (b) recombination current.

9. Sketch the forward-bias *I–V* characteristics of a pn junction diode showing the effects of recombination and high-level injection.

10. (a) Explain the physical mechanism of diffusion capacitance. (b) What is diffusion resistance?

11. If a forward-biased pn junction is switched off, explain what happens to the stored minority carriers. In which direction is the current immediately after the diode is switched off?

PROBLEMS

[**Note:** In the following problems, assume $T = 300$ K and the following parameters unless otherwise stated. For silicon pn junctions: $D_n = 25$ cm²/s, $D_p = 10$ cm²/s, $\tau_{n0} = 5 \times 10^{-7}$ s, $\tau_{p0} = 10^{-7}$ s. For GaAs pn junctions: $D_n = 205$ cm²/s, $D_p = 9.8$ cm²/s, $\tau_{n0} = 5 \times 10^{-8}$ s, $\tau_{p0} = 10^{-8}$ s.]

Section 8.1   pn Junction Current

8.1 (*a*) Consider an ideal pn junction diode at $T = 300$ K operating in the forward-bias region. Calculate the change in diode voltage that will cause a factor of 10 increase in current. (*b*) Repeat part (*a*) for a factor of 100 increase in current.

8.2 A silicon pn junction has impurity doping concentrations of $N_d = 2 \times 10^{15}$ cm^{-3} and $N_a = 8 \times 10^{15}$ cm^{-3}. Determine the minority carrier concentrations at the edges of the space charge region for (a) $V_a = 0.45$ V, (b) $V_a = 0.55$ V, and (c) $V_a = -0.55$ V.

8.3 The doping concentrations in a GaAs pn junction are $N_d = 10^{16}$ cm^{-3} and $N_a = 4 \times 10^{16}$ cm^{-3}. Find the minority carrier concentrations at the edges of the space charge region for (a) $V_a = 0.90$ V, (b) $V_a = 1.10$ V, and (c) $V_a = -0.95$ V.

8.4 (a) The doping concentrations in a silicon pn junction are $N_d = 5 \times 10^{15}$ cm^{-3} and $N_a = 5 \times 10^{16}$ cm^{-3}. The minority carrier concentration at either space charge edge is to be no larger than 10 percent of the respective majority carrier concentration. (i) Determine the maximum forward-bias voltage that can be applied to the junction and still meet the required specifications. (ii) Is the n-region or p-region concentration the factor that limits the forward-bias voltage? (b) Repeat part (a) if the doping concentrations are $N_d = 3 \times 10^{16}$ cm^{-3} and $N_a = 7 \times 10^{15}$ cm^{-3}.

8.5 Consider a GaAs pn junction with doping concentrations $N_a = 5 \times 10^{16}$ cm^{-3} and $N_d = 10^{16}$ cm^{-3}. The junction cross-sectional area is $A = 10^{-3}$ cm^2 and the applied forward-bias voltage is $V_a = 1.10$ V. Calculate the (a) minority electron diffusion current at the edge of the space charge region, (b) minority hole diffusion current at the edge of the space charge region, and (c) total current in the pn junction diode.

8.6 An n$^+$p silicon diode at $T = 300$ K has the following parameters: $N_d = 10^{18}$ cm^{-3}, $N_a = 10^{16}$ cm^{-3}, $D_n = 25$ cm^2/s, $D_p = 10$ cm^2/s, $\tau_{n0} = \tau_{p0} = 1$ μs, and $A = 10^{-4}$ cm^2. Determine the diode current for (a) a forward-bias voltage of 0.5 V and (b) a reverse-biased voltage of 0.5 V.

8.7 An ideal germanium pn junction diode has the following parameters: $N_a = 4 \times 10^{15}$ cm^{-3}, $N_d = 2 \times 10^{17}$ cm^{-3}, $D_p = 48$ cm^2/s, $D_n = 90$ cm^2/s, $\tau_{p0} = \tau_{n0} = 2 \times 10^{-6}$ s, and $A = 10^{-4}$ cm^2. Determine the diode current for (a) a forward-bias voltage of 0.25 V and (b) a reverse-biased voltage of 0.25 V.

8.8 A one-sided p$^+$n silicon diode has doping concentrations of $N_a = 5 \times 10^{17}$ cm^{-3} and $N_d = 8 \times 10^{15}$ cm^{-3}. The minority carrier lifetimes are $\tau_{n0} = 10^{-7}$ s and $\tau_{p0} = 8 \times 10^{-8}$ s. The cross-sectional area is $A = 2 \times 10^{-4}$ cm^2. Calculate the (a) reverse-biased saturation current, and (b) the forward-bias current at (i) $V_a = 0.45$ V, (ii) $V_a = 0.55$ V, and (iii) $V_a = 0.65$ V.

8.9 Calculate the applied reverse-biased voltage at which the ideal reverse current in a pn junction diode at $T = 300$ K reaches 90 percent of its reverse-saturation current value.

8.10 Fill in the missing data in the following table.

Case	V_a (V)	I(mA)	I_s (mA)	J_s (mA/cm^2)	A (cm^2)
1	0.65	0.50			2×10^{-4}
2	0.70		2×10^{-12}		1×10^{-3}
3		0.80		1×10^{-7}	1×10^{-4}
4	0.72	1.20		2×10^{-8}	

8.11 Consider an ideal silicon pn junction diode. (a) What must be the ratio of N_d/N_a so that 90 percent of the current in the depletion region is due to the flow of electrons? (b) Repeat part (a) if 80 percent of the current in the depletion region is due to the flow of holes.

8.12 A silicon pn junction diode is to be designed to operate at $T = 300$ K such that the diode current is $I = 10$ mA at a diode voltage of $V_D = 0.65$ V. The ratio of electron current to total current is to be 0.10 and the maximum current density is to be no more than 20 A/cm^2. Use the semiconductor parameters given in Example 8.2.

8.13 An ideal silicon pn junction at $T = 300$ K is under forward bias. The minority carrier lifetimes are $\tau_{n0} = 10^{-6}$ s and $\tau_{p0} = 10^{-7}$ s. The doping concentration in the n region is $N_d = 10^{16}$ cm^{-3}. Plot the ratio of hole current to the total current crossing the space charge region as the p-region doping concentration varies over the range $10^{15} \le N_a \le 10^{18}$ cm^{-3}. (Use a log scale for the doping concentrations.)

8.14 For a silicon pn junction at $T = 300$ K, assume $\tau_{p0} = 0.1\tau_{n0}$ and $\mu_n = 2.4\,\mu_p$. The ratio of electron current crossing the depletion region to the total current is defined as the electron injection efficiency. Determine the expression for the electron injection efficiency as a function of (a) N_d/N_a and (b) the ratio of n-type conductivity to p-type conductivity.

8.15 A silicon pn junction with a cross-sectional area of 10^{-4} cm^2 has the following properties at $T = 300$ K:

n region	p region
$N_d = 10^{17}$ cm^{-3}	$N_a = 5 \times 10^{15}$ cm^{-3}
$\tau_{p0} = 10^{-7}$ s	$\tau_{n0} = 10^{-6}$ s
$\mu_n = 850$ cm^2/V-s	$\mu_n = 1250$ cm^2/V-s
$\mu_p = 320$ cm^2/V-s	$\mu_p = 420$ cm^2/V-s

(a) Sketch the thermal equilibrium energy-band diagram of the pn junction, including the values of the Fermi level with respect to the intrinsic level on each side of the junction. (b) Calculate the reverse-saturation current I_s and determine the forward-bias current I at a forward-bias voltage of 0.5 V. (c) Determine the ratio of hole current to total current at the space charge edge x_n.

8.16 Consider an ideal silicon pn junction diode with the geometry shown in Figure P8.16. The doping concentrations are $N_a = 5 \times 10^{16}$ cm^{-3} and $N_d = 1.5 \times 10^{16}$ cm^{-3}, and the minority carrier lifetimes are $\tau_{n0} = 2 \times 10^{-7}$ s and $\tau_{p0} = 8 \times 10^{-8}$ s. The cross-sectional area is $A = 5 \times 10^{-4}$ cm^2. Calculate (a) the ideal reverse-saturation current due to holes, (b) the ideal reverse-saturation current due to electrons, (c) the hole concentration at $x = x_n$ for $V_a = 0.8V_{bi}$, (d) the electron current at $x = x_n$ for $V_a = 0.8\,V_{bi}$, and (e) the electron current at $x = x_n + (1/2)L_p$ for $V_a = 0.8\,V_{bi}$.

8.17 Consider the ideal long silicon pn junction shown in Figure P8.17. $T = 300$ K. The n region is doped with 10^{16} donor atoms per cm^3 and the p region is doped with 5×10^{16} acceptor atoms per cm^3. The minority carrier lifetimes are $\tau_{n0} = 0.05$ μs and $\tau_{p0} = 0.01$ μs. The minority carrier diffusion coefficients are $D_n = 23$ cm^2/s and $D_p = 8$ cm^2/s. The forward-bias voltage is $V_a = 0.610$ V. Calculate (a) the excess hole concentration as a function of x for $x \ge 0$, (b) the hole diffusion current density at $x = 3 \times 10^{-4}$ cm, and (c) the electron current density at $x = 3 \times 10^{-4}$ cm.

8.18 The limit of low injection is normally defined to be when the minority carrier concentration at the edge of the space charge region in the low-doped region becomes equal
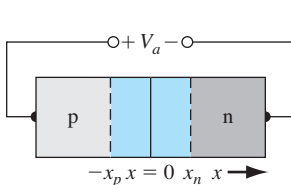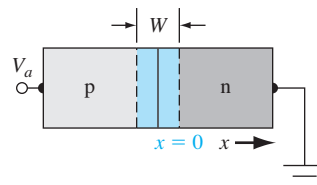
Figure P8.16 | Figure for Problem 8.16.

Figure P8.17 | Figure for Problem 8.17.

to one-tenth the majority carrier concentration in this region. Determine the value of the forward-bias voltage at which the limit of low injection is reached for the diode described in (*a*) Problem 8.7 and (*b*) Problem 8.8.

8.19   The cross-sectional area of a silicon pn junction is 10^{-3} cm². The temperature of the diode is $T = 300$ K, and the doping concentrations are $N_d = 10^{16}$ cm⁻³ and $N_a = 8 \times 10^{15}$ cm⁻³. Assume minority carrier lifetimes of $\tau_{n0} = 10^{-6}$ s and $\tau_{p0} = 10^{-7}$ s. Calculate the total number of excess electrons in the p region and the total number of excess holes in the n region for (*a*) $V_a = 0.3$ V, (*b*) $V_a = 0.4$ V, and (*c*) $V_a = 0.5$ V.

8.20   Consider two ideal pn junctions at $T = 300$ K, having exactly the same electrical and physical parameters except for the bandgap energy of the semiconductor materials. The first pn junction has a bandgap energy of 0.525 eV and a forward-bias current of 10 mA with $V_a = 0.255$ V. For the second pn junction, "design" the bandgap energy so that a forward-bias voltage of $V_a = 0.32$ V will produce a current of 10 μA.

8.21   The reverse-biased saturation current is a function of temperature. (*a*) Assuming that I_s varies with temperature only from the intrinsic carrier concentration, show that we can write $I_s = CT^3 \exp\left(-E_g/kT\right)$ where C is a constant and a function only of the diode parameters. (*b*) Determine the increase in I_s as the temperature increases from $T = 300$ K to $T = 400$ K for a (*i*) germanium diode and (*ii*) silicon diode.

8.22   Assume that the mobilities, diffusion coefficients, and minority carrier lifetime parameters are independent of temperature (use the $T = 300$ K values). Assume that $\tau_{n0} = 10^{-6}$ s, $\tau_{p0} = 10^{-7}$ s, $N_d = 5 \times 10^{15}$ cm⁻³, and $N_a = 5 \times 10^{16}$ cm⁻³. Plot the ideal reverse saturation current density from $T = 200$ K to $T = 500$ K for (*a*) silicon, (*b*) germanium, and (*c*) gallium arsenide ideal pn junctions. (Use a log scale for the current density.)

8.23   An ideal silicon pn junction diode has a cross-sectional area of $A = 5 \times 10^{-4}$ cm². The doping concentrations are $N_a = 4 \times 10^{15}$ cm⁻³ and $N_d = 2 \times 10^{17}$ cm⁻³. Assume that $E_g = 1.12$ eV as well as the diffusion coefficients and lifetimes are independent of temperature. The ratio of the magnitude of forward- to reverse-biased currents is to be no less than 2×10^4 with forward- and reverse-biased voltages of 0.50 V, and the maximum reverse-biased current is to be limited to 1.2 μA. Determine the maximum temperature at which the diode will meet these specifications and determine which specification is the limiting factor.

8.24   (*a*) A silicon pn junction diode has the geometry shown in Figure 8.11 in which the n region is "short" with a length $W_n = 0.7$ μm. The doping concentrations are $N_a = 2 \times 10^{17}$ cm⁻³ and $N_d = 2 \times 10^{15}$ cm⁻³. The cross-sectional area is $A = 10^{-3}$ cm². Determine (*i*) the maximum forward-bias voltage such that low injection is still valid, and (*ii*) the resulting current at this forward-bias voltage. (*b*) Repeat part (*a*) if the doping concentrations are reversed such that $N_a = 2 \times 10^{15}$ cm⁻³ and $N_d = 2 \times 10^{17}$ cm⁻³.

*8.25   A p⁺n silicon diode is fabricated with a narrow n region as shown in Figure 8.11, in which $W_n < L_p$. Assume the boundary condition of $p_n = p_{n0}$ at $x = x_n + W_n$. (*a*) Derive the expression for the excess hole concentration $\delta p_n(x)$ as given by Equation (8.31). (*b*) Using the results of part (*a*), show that the current density in the diode is given by

$$J = \frac{eD_p p_{n0}}{L_p} \coth\left(\frac{W_n}{L_p}\right)\left[\exp\left(\frac{eV}{kT}\right) - 1\right]$$

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*Asterisks next to problems indicate problems that are more difficult.

8.26 A silicon diode can be used to measure temperature by operating the diode at a fixed forward-bias current. The forward-bias voltage is then a function of temperature. At $T = 300$ K, the diode voltage is found to be 0.60 V. Determine the diode voltage at (a) $T = 310$ K and (b) $T = 320$ K.

8.27 A forward-biased silicon diode is to be used as a temperature sensor. The diode is forward biased with a constant current source and V_a is measured as a function of temperature. (a) Derive an expression for $V_a(T)$ assuming that D/L for electrons and holes, and E_g are independent of temperature. (b) If the diode is biased at $I_D = 0.1$ mA and if $I_s = 10^{-15}$ A at $T = 300$ K, plot V_a versus T for $20°C < T < 200°C$. (c) Repeat part (b) if $I_D = 1$ mA. (d) Determine any changes in the results of parts (a) through (c) if the change in bandgap energy with temperature is taken into account.

Section 8.2   Generation–Recombination Currents

8.28 Consider a silicon pn junction diode with an applied reverse-biased voltage of $V_R = 5$V. The doping concentrations are $N_a = N_d = 4 \times 10^{16}$ cm^{-3} and the cross-sectional area is $A = 10^{-4}$ cm^2. Assume minority carrier lifetimes of $\tau_0 = \tau_{n0} = \tau_{p0} = 10^{-7}$ s. Calculate the (a) ideal reverse-saturation current, (b) reverse-biased generation current, and (c) the ratio of the generation current to ideal saturation current.

8.29 Consider the diode described in Problem 8.28. Assume that all parameters except n_i are independent of temperature. (a) Determine the temperature at which I_s and I_{gen} will be equal. What are the values of I_s and I_{gen} at this temperature? (b) Calculate the forward-bias voltage at which the ideal diffusion current is equal to the recombination current at $T = 300$ K.

8.30 Consider a GaAs pn junction diode with a cross-sectional area of $A = 2 \times 10^{-4}$ cm^2 and doping concentrations of $N_a = N_d = 7 \times 10^{16}$ cm^{-3}. The electron and hole mobility values are $\mu_n = 5500$ cm^2/V-s and $\mu_p = 220$ cm^2/V-s, respectively, and the lifetime values are $\tau_0 = \tau_{n0} = \tau_{p0} = 2 \times 10^{-8}$ s. (a) Calculate the ideal diode current at a (i) reverse-biased voltage of $V_R = 3$ V, (ii) forward-bias voltage of $V_a = 0.6$ V, (iii) forward-bias voltage of $V_a = 0.8$ V, and (iv) forward-bias voltage of $V_a = 1.0$ V. (b) (i) Calculate the generation current at $V_R = 3$ V. Assuming the recombination current extrapolated to $V_a = 0$ is $I_{ro} = 6 \times 10^{-14}$ A, determine the generation current at (ii) $V_a = 0.6$ V, (iii) $V_a = 0.8$ V, and (iv) $V_a = 1.0$ V.

8.31 Consider the pn junction diode described in Problem 8.30. Plot the diode recombination current and the ideal diode current (on a log scale) versus forward-bias voltage over the range $0.1 \leq V_a \leq 1.0$ V.

8.32 A silicon pn junction diode at $T = 300$ K has the following parameters: $N_a = N_d = 10^{16}$ cm^{-3}, $\tau_{p0} = \tau_{n0} = \tau_0 = 5 \times 10^{-7}$ s, $D_p = 10$ cm^2/s, $D_n = 25$ cm^2/s, and a cross-sectional area of 10^{-4} cm^2. Plot the diode recombination current and the ideal diode current (on a log scale) versus forward-bias voltage over the range $0.1 \leq V_a \leq 0.6$ V.

8.33 Consider a GaAs pn diode at $T = 300$ K with $N_a = N_d = 10^{17}$ cm^{-3} and with a cross-sectional area of 5×10^{-3} cm^2. The minority carrier mobilities are $\mu_n = 3500$ cm^2/V-s and $\mu_p = 220$ cm^2/V-s. The electron–hole lifetimes are $\tau_{n0} = \tau_{p0} = \tau_0 = 10^{-8}$ s. Plot the diode forward-bias current including recombination current between diode voltages of $0.1 \leq V_D \leq 1.0$ V. Compare this plot to that for an ideal diode.

*8.34 Starting with Equation (8.44) and using the suitable approximations, show that the maximum recombination rate in a forward-biased pn junction is given by Equation (8.52).
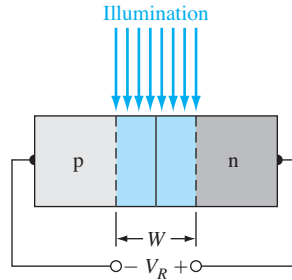
Figure P8.35 | Figure for
Problem 8.35 and 8.36.

8.35 Consider, as shown in Figure P8.35, a uniformly doped silicon pn junction at
$T = 300$ K with impurity doping concentrations of $N_a = N_d = 5 \times 10^{15}$ cm^{-3} and
minority carrier lifetimes of $\tau_{n0} = \tau_{p0} = \tau_0 = 10^{-7}$ s. A reverse-biased voltage of
$V_R = 10$ V is applied. A light source is incident only on the space charge region, pro-
ducing an excess carrier generation rate of $g' = 4 \times 10^{19}$ cm^{-3} s^{-1}. Calculate the gen-
eration current density.

8.36 A long silicon pn junction diode has the following parameters: $N_d = 10^{18}$ cm^{-3},
$N_a = 3 \times 10^{16}$ cm^{-3}, $\tau_{n0} = \tau_{p0} = \tau_0 = 10^{-7}$ s, $D_n = 18$ cm^2/s, and $D_p = 6$ cm^2/s. A
light source is incident on the space charge region such as shown in Figure P8.35, pro-
ducing a generation current density of $J_G = 25$ mA/cm^2. The diode is open circuited.
The generation current density forward biases the junction, inducing a forward-bias
current in the opposite direction to the generation current. A steady-state condition is
reached when the generation current density and forward-bias current density are equal
in magnitude. What is the induced forward-bias voltage at this steady-state condition?

Section 8.3   Small-Signal Model of the pn Junction

8.37 (a) Calculate the small-signal diffusion capacitance and diffusion resistance of a sili-
con pn junction diode biased at $I_{DQ} = 1.2$ mA. Assume the minority carrier lifetimes
are 0.5 μs in both the n and p regions. (b) Repeat part (a) for the case when the diode
is biased at $I_{DQ} = 0.12$ mA.

8.38 Consider the diode described in Problem 8.37. A sinusoidal signal voltage with a peak
value of 50 mV is superimposed on the dc forward-bias voltage. Determine the mag-
nitude of charge that is being alternately charged and discharged in the n region.

8.39 Consider a p$^+$n silicon diode at $T = 300$ K. The diode is forward biased at a current
of 1 mA. The hole lifetime in the n region is 10^{-7} s. Neglecting the depletion capaci-
tance, calculate the diode impedance at frequencies of 10 kHz, 100 kHz, 1 MHz, and
10 MHz.

8.40 Consider a silicon pn junction with parameters as described in Problem 8.8. (a) Cal-
culate and plot the depletion capacitance and diffusion capacitance over the voltage
range $-10 \leq V_a \leq 0.75$ V. (b) Determine the voltage at which the two capacitances
are equal.

8.41 Consider a p$^+$n silicon diode at $T = 300$ K. The slope of the diffusion capacitance
versus forward-bias current is 2.5×10^{-6} F/A. Determine the hole lifetime and the
diffusion capacitance at a forward-bias current of 1 mA.

8.42 A one-sided p^+n silicon diode has doping concentrations of $N_a = 4 \times 10^{17}$ cm^{-3} and $N_d = 8 \times 10^{15}$ cm^{-3}. The diode cross-sectional area is $A = 5 \times 10^{-4}$ cm^2. (a) The maximum diffusion capacitance is to be limited to 1 nF. Determine (i) the maximum current through the diode, (ii) the maximum forward-bias voltage, and (iii) the diffusion resistance. (b) Repeat part (a) if the maximum diffusion capacitance is limited to 0.25 nF.

8.43 A silicon pn junction diode at $T = 300$ K has a cross-sectional area of 10^{-2} cm^2. The length of the p region is 0.2 cm and the length of the n region is 0.1 cm. The doping concentrations are $N_d = 10^{15}$ cm^{-3} and $N_a = 10^{16}$ cm^{-3}. Determine (a) approximately the series resistance of the diode and (b) the current through the diode that will produce a 0.1 V drop across this series resistance.

8.44 We want to consider the effect of a series resistance on the forward-bias voltage required to achieve a particular diode current. (a) Assume the reverse-saturation current in a diode is $I_s = 10^{-10}$ A at $T = 300$ K. The resistivity of the n region is 0.2 Ω-cm and the resistivity of the p region is 0.1 Ω-cm. Assume the length of each neutral region is 10^{-2} cm and the cross-sectional area is 2×10^{-5} cm^2. Determine the required applied voltage to achieve a current of (i) 1 mA and (ii) 10 mA. (b) Repeat part (a) neglecting the series resistance.

8.45 (a) The reverse-saturation current in a diode is $I_s = 5 \times 10^{-12}$ A. The maximum small-signal diffusion resistance is to be $r_d = 32$ Ω. Determine the minimum forward-bias voltage that can be applied to meet this specification. (b) Repeat part (a) if the maximum small-signal diffusion resistance is to be $r_d = 60$ Ω.

8.46 (a) An ideal silicon pn junction diode at $T = 300$ K is forward biased at $V_a = +20$ mV. The reverse-saturation current is $I_s = 10^{-13}$ A. Calculate the small-signal diffusion resistance. (b) Repeat part (a) for an applied reverse-biased voltage of $V_a = -20$ mV.

Section 8.4 Charge Storage and Diode Transients

8.47 (a) In switching a pn junction from forward to reverse bias, assume that the ratio of reverse current, I_R, to forward current, I_F, is 0.2. Determine the ratio of storage time to minority carrier lifetime, t_s/τ_{p0}. (b) Repeat part (a) if the ratio of I_R to I_F is 1.0.

8.48 A pn junction is switched from forward to reverse bias. The storage time is specified to be $\tau_s = 0.3\,\tau_{p0}$. (a) Determine the required ratio of I_R to I_F to meet this specification. (b) Determine t_2/τ_{p0}.

8.49 Consider a diode with a junction capacitance of 18 pF at zero bias and 4.2 pF at a reverse-biased voltage of $V_R = 10$ V. The minority carrier lifetimes are 10^{-7} s. The diode is switched from a forward bias with a current of 2 mA to a reverse-biased voltage of 10 V applied through a 10 kΩ resistor. Estimate the turn-off time.

Section 8.5 The Tunnel Diode

8.50 Consider a silicon pn junction at $T = 300$ K with doping concentration of $N_d = N_a = 5 \times 10^{19}$ cm^{-3}. Assuming the abrupt junction approximation is valid, determine the space charge width at a forward-bias voltage of $V_a = 0.40$ V.

8.51 Sketch the energy-band diagram of an abrupt pn junction under zero bias in which the p region is degenerately doped and $E_C = E_F$ in the n region. Sketch the forward- and reverse-biased current–voltage characteristics. This diode is sometimes called a *backward diode*. Why?

Summary and Review

8.52   (*a*) Explain physically why the diffusion capacitance is not important in a reverse-biased pn junction. (*b*) Consider a silicon, a germanium, and gallium arsenide pn junction. If the total current density is the same in each diode under forward bias, discuss the expected relative values of electron and hole current densities.

*8.53   A silicon p^+n junction diode is to be designed to have a breakdown voltage of at least 60 V and to have a forward-bias current of $I_D = 50$ mA while still operating under low injection. The minority carrier lifetimes are $\tau_0 = \tau_{n0} = \tau_{p0} = 2 \times 10^{-7}$ s. Determine the doping concentrations and the minimum cross-sectional area.

*8.54   The donor and acceptor concentrations on either side of a silicon step junction are equal. (*a*) Derive an expression for the breakdown voltage in terms of the critical electric field and doping concentration. (*b*) If the breakdown voltage is to be $V_B = 50$ V, specify the range of allowed doping concentrations.

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