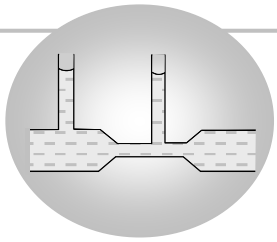


Fluid Mechanics

Fluid is the name given to a substance which begins to flow when external force is applied on it. Liquids and gases are fluids. Fluids do not have their own shape but take the shape of the containing vessel. The branch of physics which deals with the study of fluids at rest is called hydrostatics and the branch which deals with the study of fluids in motion is called hydrodynamics.

Pressure

The normal force exerted by liquid at rest on a given surface in contact with it is called thrust of liquid on that surface.

The normal force (or thrust) exerted by liquid at rest per unit area of the surface in contact with it is called pressure of liquid or hydrostatic pressure.

If F be the normal force acting on a surface of area A in contact with liquid, then pressure exerted by liquid on this surface is $P = F/A$

(1) Units : N/m^2 or Pascal (S.I.) and Dyne/cm^2 (C.G.S.)

(2) Dimension : $[P] = \frac{[F]}{[A]} = \frac{[MLT^{-2}]}{[L^2]} = [ML^{-1}T^{-2}]$

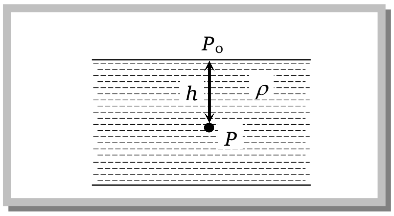
(3) At a point pressure acts in all directions and a definite direction is not associated with it. So pressure is a tensor quantity.

(4) Atmospheric pressure : The gaseous envelope surrounding the earth is called the earth's atmosphere and the pressure exerted by the atmosphere is called atmospheric pressure. Its value on the surface of the earth at sea level is nearly $1.013 \times 10^5\, N/m^2$ or Pascal in S.I. other practical units of pressure are atmosphere, bar and torr (mm of Hg)

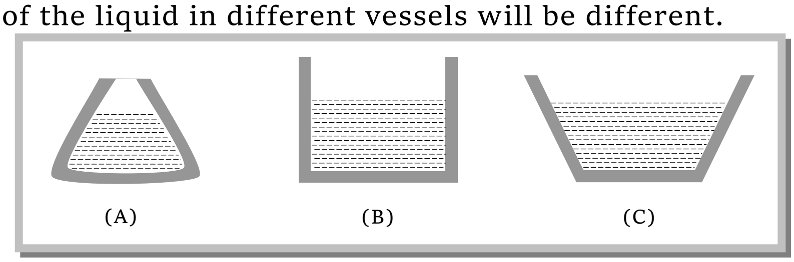
$$1atm = 1.01 \times 10^5\, Pa = 1.01\,\text{bar} = 760\ \text{torr}$$

The atmospheric pressure is maximum at the surface of earth and goes on decreasing as we move up into the earth's atmosphere.

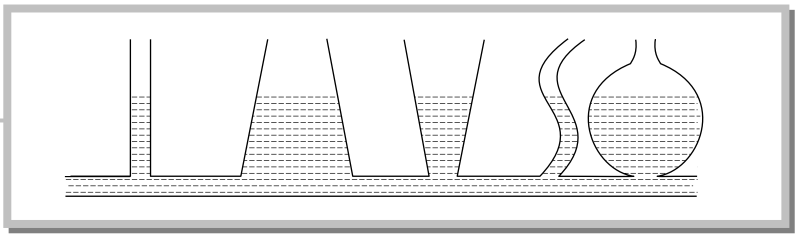
(5) If P_0 is the atmospheric pressure then for a point at depth h below the surface of a liquid of density ρ, hydrostatic pressure P is given by $P =$



(6) Hydrostatic pressure depends on the depth of the point below the surface (h), nature of liquid (ρ) and acceleration due to gravity (g) while it is independent of the amount of liquid, shape of the container or cross-sectional area considered. So if a given liquid is filled in vessels of different shapes to same height, the pressure at the base in each vessel's will be the same, though the volume or weight of the liquid in different vessels will be different.



$P_A = P_B = P_C$ but $W_A < W_B < W_C$



(7) In a liquid at same level, the pressure will be same at all points, if not, due to pressure difference the liquid cannot be at rest. This is why the height of liquid is the same in vessels of different shapes containing different amounts of the same liquid at rest when they are in communication with each other.

(8) Gauge pressure : The pressure difference between hydrostatic pressure P and atmospheric pressure P_o is called gauge pressure. $P - P_0 = h\rho g$

Sample problems based on Pressure

Problem 1. If pressure at half the depth of a lake is equal to 2/3 pressure at the bottom of the lake then what is the depth of the lake

(a) 10 m (b) 20 m (c) 60 m (d) 30 m

Solution : (b) Pressure at bottom of the lake = $P_0 + h\rho g$ and pressure ay half the depth of a lake $= P_0 + \frac{h}{2}\rho g$

According to given condition $P_0 + \frac{1}{2}h\rho g = \frac{2}{3}(P_0 + h\rho g)$ $\Rightarrow$ $\frac{1}{3}P_0 = \frac{1}{6}h\rho g$ $\Rightarrow$ $h = \frac{2P_0}{\rho g} = \frac{2\times 10^5}{10^3 \times 10} = 20m$.

Problem 2. Two bodies are in equilibrium when suspended in water from the arms of a balance. The mass of one body is 36 g and its density is 9 g / cm^3. If the mass of the other is 48 g, its density in g / cm^3 is [CBSE 1994]

(a) $\frac{4}{3}$ (b) $\frac{3}{2}$ (c) 3 (d) 5

Solution : (c) Apparent weight $= V(\rho - \sigma)g = \frac{m}{\rho}(\rho - \sigma)g$

where $m =$ mass of the body, $\rho =$ density of the body and $\sigma =$ density of water

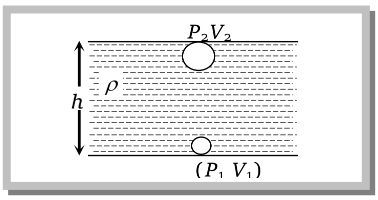
If two bodies are in equilibrium then their apparent weight must be equal.

$\therefore$ $\frac{m_1}{\rho_1}(\rho_1 - \sigma)g = \frac{m_2}{\rho_2}(\rho_2 - \sigma)g$ $\Rightarrow$ $\frac{36}{9}(9-1) = \frac{48}{\rho_2}(\rho_2 - 1)g$. By solving we get $\rho_2 = 3$.

Problem 3. An inverted bell lying at the bottom of a lake 47.6 m deep has 50 cm^3 of air trapped in it. The bell is brought to the surface of the lake. The volume of the trapped air will be (atmospheric pressure = 70 cm of Hg and density of Hg = 13.6 g/cm^3)

(a) 350 cm^3 (b) 300 cm^3 (c) 250 cm^3 (d) 22 cm^3

Solution : (b) According to Boyle's law, pressure and volume are inversely proportional to each other *i.e.* $P \propto \frac{1}{V}$



$\Rightarrow P_1V_1 = P_2V_2$

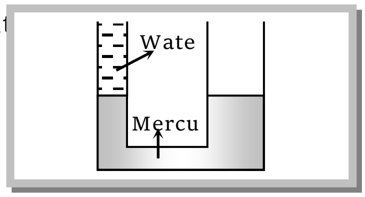
$\Rightarrow (P_0 + h\rho_w g)V_1 = P_0V_2$

$\Rightarrow V_2 = \left(1 + \frac{h\rho_w g}{P_0}\right)V_1$

$\Rightarrow V_2 = \left(1 + \frac{47.6\times 10^2 \times 1 \times 1000}{70 \times 13.6 \times 1000}\right)V_1$ [As $P_2 = P_0 = 70\ cm$ of Hg $= 70 \times 13.6 \times 1000$]

$\Rightarrow V_2 = (1+5)50\ cm^3 = 300\ cm^3$.

Problem 4. A U-tube in which the cross-sectional area of the limb on the left is one quarter, the limb on the right contains mercury (density 13.6 g/cm^3). The level of mercury in the narrow limb is at a distance of 36 cm from the upper end of the tube. What will be the rise in the level of mercury in the right limb if the left limb is filled to the top wi

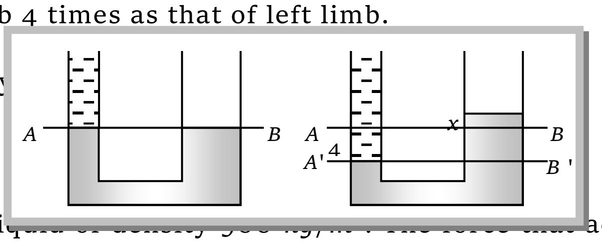


(a) 1.2 cm
(b) 2.35 cm
(c) 0.56 cm
(d) 0.8 cm

Solution : (c) If the rise of level in the right limb be x cm. the fall of level of mercury in left limb be $4x$ cm because the area of cross section of right limb 4 times as that of left limb.

$\therefore$ Level of water in left limb is $(36 + 4x)$ cm.

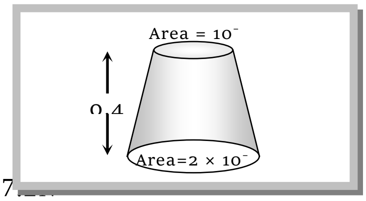
Now equating pressure at interface of mercury and water (at $A'B'$)



$(36 + 4x) \times 1 \times g = 5x \times 13.6 \times g$

By solving we get $x = 0.56$ cm.

Problem 5. A uniformly tapering vessel is filled with a li... acts on the base of the vessel due to the liquid is $(g = 10\ ms^{-2})$



(a) 3.6 N
(b) 7.2 N
(c) 9.0 N
(d) 14.4 N

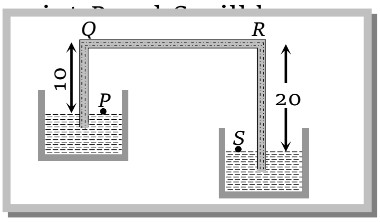
Solution : (b) Force acting on the base $F = P \times A = hdgA = 0.4 \times 900 \times 10 \times 2 \times 10^{-3} = 7...$

Problem 6. A tank 5 m high is half filled with water and then is filled to the top with oil of density 0.85 g/cm^3. The pressure at the bottom of the tank, due to these liquids is

(a) 1.85 g/cm^2 (b) 89.25 g/cm^2 (c) 462.5 g/cm^2 (d) 500 g/cm^2

Solution : (c) Pressure at the bottom $P = (h_1 d_1 + h_2 d_2)\frac{g}{cm^2} = [250 \times 1 + 250 \times 0.85] = 250\ [1.85]\frac{g}{cm^2} = 462.5\frac{g}{cm^2}$

Problem 7. A siphon in use is demonstrated in the following figure. The density of the liquid flowing in siphon is 1.5 gm/cc. The pressure difference between the



(a) 10^5 N/m
(b) 2×10^5 N/m
(c) Zero
(d) Infinity

Solution : (c) As the both points are at the surface of liquid and these points are in the open atmosphere. So both point possess similar pressure and equal to 1 atm. Hence the pressure difference will be zero.

Problem 8. The height of a mercury barometer is 75 cm at sea level and 50 cm at the top of a hill. Ratio of density of mercury to that of air is 10^4. The height of the hill is

(a) 250 m (b) 2.5 km (c) 1.25 km (d) 750 m

Solution : (b) Difference of pressure between sea level and the top of hill

$$\Delta P = (h_1 - h_2) \times \rho_{Hg} \times g = (75 - 50) \times 10^{-2} \times \rho_{Hg} \times g \qquad ......(i)$$

and pressure difference due to h meter of air $\Delta P = h \times \rho_{air} \times g$ ......(ii)

By equating (i) and (ii) we get $h \times \rho_{air} \times g = (75 - 50) \times 10^{-2} \times \rho_{Hg} \times g$

$\therefore h = 25 \times 10^{-2}\left(\frac{\rho_{Hg}}{\rho_{air}}\right) = 25 \times 10^{-2} \times 10^4 = 2500\ m$ $\therefore$ Height of the hill = 2.5 km.

Density

In a fluid, at a point, density ρ is defined as: $\rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V} = \frac{dm}{dV}$

(1) In case of homogenous isotropic substance, it has no directional properties, so is a scalar.

(2) It has dimensions $[ML^{-3}]$ and S.I. unit kg/m^3 while C.G.S. unit g/cc with $1 g/cc = 10^3 kg/m^3$

(3) Density of substance means the ratio of mass of substance to the volume occupied by the substance while density of a body means the ratio of mass of a body to the volume of the body. So for a solid body.

Density of body = Density of substance

While for a hollow body, density of body is lesser than that of substance [As $V_{\text{body}} > V_{\text{sub.}}$]

(4) When immiscible liquids of different densities are poured in a container the liquid of highest density will be at the bottom while that of lowest density at the top and interfaces will be plane.

(5) Sometimes instead of density we use the term relative density or specific gravity which is defined as :

$$RD = \frac{\text{Density of body}}{\text{Density of water}}$$

(6) If m_1 mass of liquid of density ρ_1 and m_2 mass of density ρ_2 are mixed, then as

$$m = m_1 + m_2 \text{ and } V = (m_1/\rho_1) + (m_2/\rho_2) \qquad [\text{As } V = m/\rho]$$

$$\rho = \frac{m}{V} = \frac{m_1 + m_2}{(m_1/\rho_1) + (m_2/\rho_2)} = \frac{\sum m_i}{\sum (m_i/p_i)}$$

If $m_1 = m_2$ $\qquad \rho = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2} =$ Harmonic mean

(7) If V_1 volume of liquid of density ρ_1 and V_2 volume of liquid of density ρ_2 are mixed, then as:

$$m = \rho_1 V_1 + \rho_2 V_2 \text{ and } V = V_1 + V_2 \qquad [\text{As } \rho = m/V]$$

If $V_1 = V_2 = V$ $\qquad \rho = (\rho_1 + \rho_2)/2$ = Arithmetic Mean

(8) With rise in temperature due to thermal expansion of a given body, volume will increase while mass will remain unchanged, so density will decrease, *i.e.*,

$$\frac{\rho}{\rho_0} = \frac{(m/V)}{(m/V_0)} = \frac{V_0}{V} = \frac{V_0}{V_0(1+\gamma\Delta\theta)} \qquad [\text{As } V = V_0(1+\gamma\Delta\theta)]$$

or $$\rho = \frac{\rho_0}{(1+\gamma\Delta\theta)} \simeq \rho_0(1-\gamma\Delta\theta)$$

(9) With increase in pressure due to decrease in volume, density will increase, *i.e.,*

$$\frac{\rho}{\rho_0} = \frac{(m/V)}{(m/V_0)} = \frac{V_0}{V} \qquad [\text{As } \rho = \frac{m}{V}]$$

But as by definition of bulk-modulus

$$B = -V_0 \frac{\Delta p}{\Delta V} \;\; i.e., \;\; V = V_0\left[1 - \frac{\Delta p}{B}\right]$$

So $$\rho = \rho_0\left(1 - \frac{\Delta p}{B}\right)^{-1} \simeq \rho_0\left(1 + \frac{\Delta p}{B}\right)$$

Sample problems based on Density

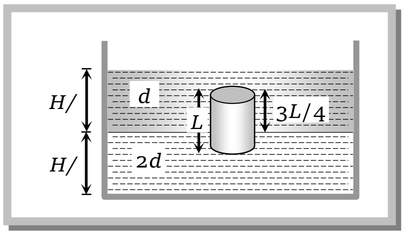
Problem 9. A homogeneous solid cylinder of length $L (L < H/2)$. Cross-sectional area $A/5$ is immersed such that it floats with its axis vertical at the liquid-liquid interface with length $L/4$ in the denser liquid as shown in the fig. The lower density liquid is open to atmosphere having pressure P_0. Then density D of solid is given by [IIT-JEE1995]

(a) $\frac{5}{4}d$

(b) $\frac{4}{5}d$

(c) Ad

(d) $\frac{d}{5}$



Solution : (a) Weight of cylinder = upthrust due to both liquids

$$V \times D \times g = \left(\frac{A}{5}.\frac{3}{4}L\right) \times d \times g + \left(\frac{A}{5}.\frac{L}{4}\right) \times 2d \times g \Rightarrow \left(\frac{A}{5}.L\right) D.g = \frac{A\,L\,d\,g}{4} \Rightarrow \frac{D}{5} = \frac{d}{4} \quad \therefore D = \frac{5}{4}d$$

Problem 10. Density of ice is ρ and that of water is σ. What will be the decrease in volume when a mass M of ice melts

(a) $\frac{M}{\sigma - \rho}$ (b) $\frac{\sigma - \rho}{M}$ (c) $M\left[\frac{1}{\rho} - \frac{1}{\sigma}\right]$ (d) $\frac{1}{M}\left[\frac{1}{\rho} - \frac{1}{\sigma}\right]$

Solution : (c) Volume of ice $= \frac{M}{\rho}$, volume of water $= \frac{M}{\sigma}$ $\therefore$ Change in volume $= \frac{M}{\rho} - \frac{M}{\sigma} = M\left(\frac{1}{\rho} - \frac{1}{\sigma}\right)$

Problem 11. Equal masses of water and a liquid of density 2 are mixed together, then the mixture has a density of

(a) 2/3 (b) 4/3 (c) 3/2 (d) 3

Solution : (b) If two liquid of equal masses and different densities are mixed together then density of mixture

$$\rho = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2} = \frac{2 \times 1 \times 2}{1 + 2} = \frac{4}{3}$$

Problem 12. Two substances of densities ρ_1 and ρ_2 are mixed in equal volume and the relative density of mixture is 4. When they are mixed in equal masses, the relative density of the mixture is 3. The values of ρ_l and ρ_2 are

(a) $\rho_1 = 6$ and $\rho_2 = 2$ (b) $\rho_1 = 3$ and $\rho_2 = 5$ (c) $\rho_1 = 12$ and $\rho_2 = 4$ (d)

Solution : (a) When substances are mixed in equal volume then density $= \frac{\rho_1 + \rho_2}{2} = 4 \Rightarrow \rho_1 + \rho_2 = 8$ ....(i)

When substances are mixed in equal masses then density $= \frac{2\rho_1\rho_2}{\rho_1 + \rho_2} = 3 \Rightarrow 2\rho_1\rho_2 = 3(\rho_1 + \rho_2)$ ....(ii)

By solving (i) and (ii) we get $\rho_1 = 6$ and $\rho_2 = 2$.

Problem 13. A body of density d_1 is counterpoised by Mg of weights of density d_2 in air of density d. Then the true mass of the body is

(a) M (b) $M\left(1 - \frac{d}{d_2}\right)$ (c) $M\left(1 - \frac{d}{d_1}\right)$ (d) $\frac{M(1 - d/d_2)}{(1 - d/d_1)}$

Solution : (d) Let M_0 = mass of body in vacuum.

Apparent weight of the body in air = Apparent weight of standard weights in air

$\Rightarrow$ Actual weight – upthrust due to displaced air = Actual weight – upthrust due to displaced air

$$\Rightarrow \quad M_0 g - \left(\frac{M_0}{d_1}\right) dg = Mg - \left(\frac{M}{d_2}\right) dg \Rightarrow M_0 = \frac{M\left[1 - \frac{d}{d_2}\right]}{\left[1 - \frac{d}{d_1}\right]}.$$

Pascal's Law

It states that if gravity effect is neglected, the pressure at every point of liquid in equilibrium of rest is same.

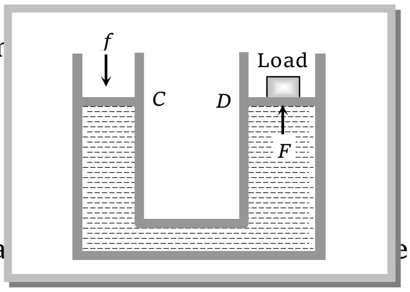
or

The increase in pressure at one point of the enclosed liquid in equilibrium of rest is transmitted equally to all other points of the liquid and also to the walls of the container, provided the effect of gravity is neglected.

Example : Hydraulic lift, hydraulic press and hydraulic brakes

Working of hydraulic lift : It is used to lift the heavy loads. If a small force f is applied on piston of C then the pressure exerted on the liquid

$P = f / a$ [a = Area of cross section of the piston ir



This pressure is transmitted equally to piston of cylinder D.

Hence the upward force acting on piston of cylinder D.

$$F = P\,A = \frac{f}{a} A = f\left(\frac{A}{a}\right)$$

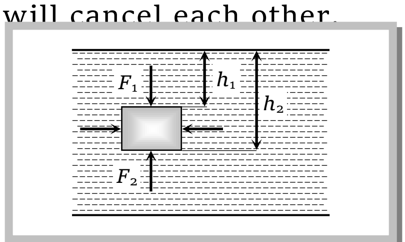
As $A >> a$, therefore $F >> f$. So heavy load placed on the la ed upwards by applying a small force.

Archimedes Principle

Accidentally Archimedes discovered that when a body is immersed partly or wholly in a fluid, in rest it is buoyed up with a force equal to the weight of the fluid displaced by the body. This principle is called Archimedes principle and is a necessary consequence of the laws of fluid statics.

When a body is partly or wholly dipped in a fluid, the fluid exerts force on the body due to hydrostatic pressure. At any small portion of the surface of the body, the force exerted by the fluid is perpendicular to the surface and is equal to the pressure at that point multiplied by the area. The resultant of all these constant forces is called upthrust or buoyancy.

To determine the magnitude and direction of this force consider a body immersed in a fluid of density σ as shown in fig. The forces on the vertical sides of the body will cancel each other. The top surface of the body will experience a downward force.



$F_1 = AP_1 = A(h_1\sigma g + P_0)$ [As $P = h\sigma g + P_0$]

While the lower face of the body will experience an upward force.

$F_2 = AP_2 = A(h_2\sigma g + P_0)$

As $h_2 > h_1, F_2$ will be greater than F_1, so the body will experience a net upward force

$F = F_2 - F_1 = A\sigma g(h_2 - h_1)$

If L is the vertical height of the body $F = A\sigma gL = V\sigma g$ [As $V = AL = A(h_2 - h_1)$]]

i.e., F = Weight of fluid displaced by the body.

This force is called upthrust or buoyancy and acts vertically upwards (opposite to the weight of the body) through the centre of gravity of displaced fluid (called centre of buoyancy). Though we have derived this result for a body fully submerged in a fluid, it can be shown to hold good for partly submerged bodies or a body in more than one fluid also.

(1) Upthrust is independent of all factors of the body such as its mass, size, density etc. except the volume of the body inside the fluid.

(2) Upthrust depends upon the nature of displaced fluid. This is why upthrust on a fully submerged body is more in sea water than in fresh water because its density is more than fresh water.

(3) Apparent weight of the body of density (ρ) when immersed in a liquid of density (σ).

Apparent weight = Actual weight – Upthrust $= W - F_{up} = V\rho g - V\sigma g = V(\rho - \sigma)g = V\rho g\left(1 - \dfrac{\sigma}{\rho}\right)$

$$\therefore \qquad W_{APP} = W\left(1 - \frac{\sigma}{\rho}\right)$$

(4) If a body of volume V is immersed in a liquid of density σ then its weight reduces.

W_1 = Weight of the body in air, W_2 = Weight of the body in water

Then apparent (loss of weight) $W_1 - W_2 = V\sigma g \quad \therefore \quad V = \dfrac{W_1 - W_2}{\sigma g}$

(5) Relative density of a body (R.D.)$= \dfrac{\text{density of body}}{\text{density of water}} = \dfrac{\text{Weight of body}}{\text{Weight of equal volume of water}} = \dfrac{\text{Weight of body}}{\text{Water thrust}}$

$$= \frac{\text{Weight of body}}{\text{Loss of weight in water}} = \frac{\text{Weight of body in air}}{\text{Weight in air} - \text{weight in water}} = \frac{W_1}{W_1 - W_2}$$

(6) If the loss of weight of a body in water is 'a ' while in liquid is 'b'

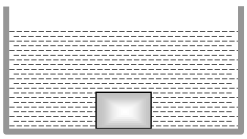
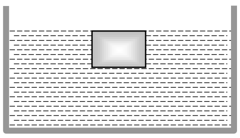
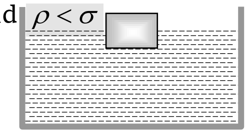
$$\therefore \qquad \frac{\sigma_L}{\sigma_W} = \frac{\text{Upthrust on body in liquid}}{\text{Upthrust on body in water}} = \frac{\text{Loss of weight in liquid}}{\text{Loss of weight in water}} = \frac{a}{b} = \frac{W_{\text{air}} - W_{\text{liquid}}}{W_{\text{air}} - W_{\text{water}}}$$

Floatation

(1) Translatory equilibrium : When a body of density ρ and volume V is immersed in a liquid of density σ, the forces acting on the body are

Weight of body $W = mg = V\rho g$, acting vertically downwards through centre of gravity of the body.

Upthrust force = $V\sigma g$ acting vertically upwards through the centre of gravity of the displaced liquid *i.e.*, centre of buoyancy.

If density of body is greater than that of liquid $\rho > \sigma$	If density of body is equal to that of liquid $\rho = \sigma$	If density of body is lesser than that of liquid $\rho < \sigma$
		
Weight will be more than upthrust so the body will sink	Weight will be equal to upthrust so the body will float fully submerged in neutral equilibrium anywhere in the liquid.	Weight will be less than upthrust so the body will move upwards and in equilibrium will float partially immersed in the liquid Such that, $W = V_{in}\sigma g \Rightarrow$ $V\rho g = V_{in}\sigma g$ $V\rho = V_{in}\sigma$ Where V_{in} is the volume of body in the liquid

Important points

(i) A body will float in liquid only and only if $\rho \le \sigma$

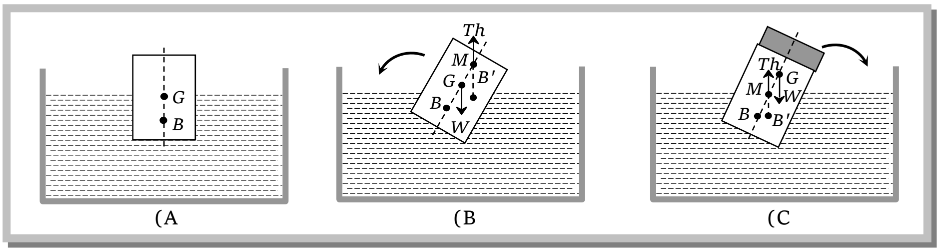
(ii) In case of floating as weight of body = upthrust

So W_{App} = Actual weight – upthrust = 0

(iii) In case of floating $V\rho g = V_{in}\sigma g$

So the equilibrium of floating bodies is unaffected by variations in g though both thrust and weight depend on g.

(2) Rotatory Equilibrium : When a floating body is slightly tilted from equilibrium position, the centre of buoyancy B shifts. The vertical line passing through the new centre of buoyancy B' and initial vertical line meet at a point M called meta-centre. If the meta-centre M is above the centre of gravity the couple due to forces at G (weight of body W) and at B' (upthrust) tends to bring the body back to its original position. So for rotational equilibrium of floating body the meta-centre must always be higher than the centre of gravity of the body.



However, if meta-centre goes below CG, the couple due to forces at G and B' tends to topple the floating body.

That is why a wooden log cannot be made to float vertical in water or a boat is likely to capsize if the sitting passengers stand on it. In these situations CG becomes higher than MC and so the body will topple if slightly tilted.

(3) Application of floatation

(i) When a body floats then the weight of body = Upthrust

$$V\rho g = V_{in}\sigma g \Rightarrow V_{in} = \left(\frac{\rho}{\sigma}\right)V \quad \therefore \quad V_{\text{out}} = V - V_{in} = \left(1-\frac{\rho}{\sigma}\right)V$$

i.e., Fraction of volume outside the liquid $f_{\text{out}} = \frac{V_{\text{out}}}{\text{V}} = \left[1-\frac{\rho}{\sigma}\right]$

(ii) For floatation $V\rho = V_{in}\sigma \Rightarrow \rho = \frac{V_{in}}{V}\sigma = f_{in}\sigma$

If two different bodies A and B are floating in the same liquid then $\frac{\rho_A}{\rho_B} = \frac{(f_{in})_A}{(f_{in})_B}$

(iii) If the same body is made to float in different liquids of densities σ_A and σ_B respectively.

$$V\rho = (V_{in})_A\sigma_A = (V_{in})_B\sigma_B \qquad \therefore \quad \frac{\sigma_A}{\sigma_B} = \frac{(V_{in})_B}{(V_{in})_A}$$

(iv) If a platform of mass M and cross-section A is floating in a liquid of density σ with its height h inside the liquid

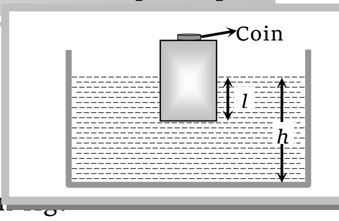
$$Mg = hA\sigma g \qquad ......(i)$$

Now if a body of mass m is placed on it and the platform sinks by y then

$$(M+m)g = (y+h)A\sigma g \qquad ......(ii)$$

Subtracting equation (i) and (ii), $mg = A\sigma y g$, *i.e.,* $W \propto y$ ......(iii)

So we can determine the weight of a body by placing it on a floating platform and noting the depression of the platform in the liquid by it.



Sample problems based on Archimedes principle

Problem 14. A wooden block, with a coin placed on its top, floats in water as shown in fig. the distance l and h are shown there. After some time the coin falls into the water. Then

(a) l decreases and h increases (b) l increases and h decreases

(c) Both l and h increase (d) Both l and h decrease

Solution : (d) As the block moves up with the fall of coin, l decreases, similarly h will also decrease because when the coin is in water, it displaces water equal to its own volume only.

Problem 15. A hemispherical bowl just floats without sinking in a liquid of density $1.2 \times 10^3 kg/m^3$. If outer diameter and the density of the bowl are 1 m and 2×10^4 kg/m^3 respectively, then the inner diameter of the bowl will be

(a) 0.94 m (b) 0.97 m (c) 0.98 m (d) 0.99 m

Solution : (c) Weight of the bowl = $mg = V\rho g = \frac{4}{3}\pi\left[\left(\frac{D}{2}\right)^3 - \left(\frac{d}{2}\right)^3\right]\rho g$

where D is the outer diameter , d is the inner diameter and ρ is the density of bowl

Weight of the liquid displaced by the bowl $= V\sigma g = \frac{4}{3}\pi\left(\frac{D}{2}\right)^3 \sigma g$

where σ is the density of the liquid.

For the flotation $\frac{4}{3}\pi\left(\frac{D}{2}\right)^3 \sigma g = \frac{4}{3}\pi\left[\left(\frac{D}{2}\right)^3 - \left(\frac{d}{2}\right)^3\right]\rho g \Rightarrow \left(\frac{1}{2}\right)^3 \times 1.2 \times 10^3 = \left[\left(\frac{1}{2}\right)^3 - \left(\frac{d}{2}\right)^3\right] 2 \times 10^4$

By solving we get $d = 0.98$ m.

Problem 16. In making an alloy, a substance of specific gravity s_1 and mass m_1 is mixed with another substance of specific gravity s_2 and mass m_2; then the specific gravity of the alloy is

(a) $\left(\frac{m_1 + m_2}{s_1 + s_2}\right)$ (b) $\left(\frac{s_1 s_2}{m_1 + m_2}\right)$ (c) $\frac{m_1 + m_2}{\left(\frac{m_1}{s_1} + \frac{m_2}{s_2}\right)}$ (d) $\frac{\left(\frac{m_1}{s_1} + \frac{m_2}{s_2}\right)}{m_1 + m_2}$

Solution : (c) Specific gravity of alloy $= \frac{\text{Density of alloy}}{\text{Density of water}} = \frac{\text{Mass of alloy}}{\text{Volume of alloy} \times \text{density of water}}$

$$= \frac{m_1 + m_2}{\left(\frac{m_1}{\rho_1} + \frac{m_2}{p_2}\right) \times \rho_w} = \frac{m_1 + m_2}{\frac{m_1}{\rho_1/\rho_w} + \frac{m_2}{\rho_2/\rho_w}} = \frac{m_1 + m_2}{\frac{m_1}{s_1} + \frac{m_2}{s_2}} \quad \left[\text{As specific gravity of substance} = \frac{\text{density of substance}}{\text{density of water}}\right]$$

Problem 17. A concrete sphere of radius R has a cavity of radius r which is packed with sawdust. The specific gravities of concrete and sawdust are respectively 2.4 and 0.3 for this sphere to float with its entire volume submerged under water. Ratio of mass of concrete to mass of sawdust will be [AIIMS 1995]

(a) 8 (b) 4 (c) 3 (d) Zero

Solution : (b) Let specific gravities of concrete and saw dust are ρ_1 and ρ_2 respectively.

According to principle of floatation weight of whole sphere = upthrust on the sphere

$$\frac{4}{3}\pi(R^3 - r^3)\rho_1 g + \frac{4}{3}\pi r^3 \rho_2 g = \frac{4}{3}\pi R^3 \times 1 \times g \Rightarrow R^3\rho_1 - r^3\rho_1 + r^3\rho_2 = R^3$$

$$\Rightarrow R^3(\rho_1 - 1) = r^3(\rho_1 - \rho_2) \Rightarrow \frac{R^3}{r^3} = \frac{\rho_1 - \rho_2}{\rho_1 - 1} \Rightarrow \frac{R^3 - r^3}{r^3} = \frac{\rho_1 - \rho_2 - \rho_1 + 1}{\rho_1 - 1} \Rightarrow$$

$$\frac{(R^3 - r^3)\rho_1}{r^3\rho_2} = \left(\frac{1 - \rho_2}{\rho_1 - 1}\right)\frac{\rho_1}{\rho_2}$$

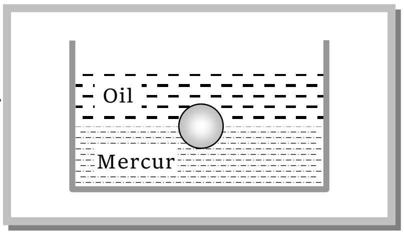
$$\Rightarrow \frac{\text{Mass of concrete}}{\text{Mass of saw dust}} = \left(\frac{1 - 0.3}{2.4 - 1}\right) \times \frac{2.4}{0.3} = 4$$

Problem 18. A vessel contains oil (density = 0.8 gm/cm^3) over mercury (density = 13.6 gm/cm^3). A homogeneous sphere floats with half of its volume immersed in mercury and the other half in oil. The density of the material of the sphere in gm/cm^3 is

(a) 3.3 (b) 6.4 (c) 7.2 (d) 12.8

Solution : (c) As the sphere floats in the liquid. Therefore its weight will be equal to the upthrust force on it

Weight of sphere $= \frac{4}{3}\pi R^3 \rho g$

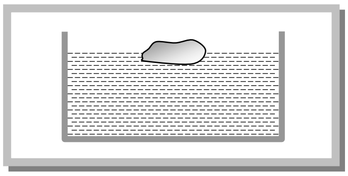


Upthrust due to oil and mercury $= \frac{2}{3}\pi R^3 \times \sigma_{oil} g + \frac{2}{3}\pi R^3 \sigma_{Hg} g$ ...

Equating (i) and (ii)

$\frac{4}{3}\pi R^3 \rho g = \frac{2}{3}\pi R^3 0.8 g + \frac{2}{3}\pi R^3 \times 13.6 g \Rightarrow 2\rho = 0.8 + 13.6 = 14.4 \Rightarrow \rho$

Problem 19. A body floats in a liquid contained in a beaker. The whole system as shown falls freely under gravity. The upthrust on the body due to the liquid is



(a) Zero
(b) Equal to the weight of the liquid displaced
(c) Equal to the weight of the body in air
(d) Equal to the weight of the immersed position of the body

Solution : (a) Upthrust $= V\rho_{\text{liquid}}(g - a)$; where, a = downward acceleration, V = volume of liquid displaced

But for free fall $a = g$ $\therefore$ Upthrust = 0

Problem 20. A metallic block of density 5 gm cm^{-3} and having dimensions 5 cm × 5 cm × 5 cm is weighed in water. Its apparent weight will be

(a) 5 × 5 × 5 × 5 gf (b) 4 × 4 × 4 × 4 gf (c) 5 × 4 × 4 × 4 gf (d) 4 × 5 × 5 × 5 gf

Solution : (d) Apparent weight $= V(\rho - \sigma)g = l \times b \times h \times (5-1) \times g = 5 \times 5 \times 5 \times 4 \times g$ *Dyne* or $4 \times 5 \times 5 \times 5$ gf.

Problem 21. A wooden block of volume 1000 cm^3 is suspended from a spring balance. It weighs 12 N in air. It is suspended in water such that half of the block is below the surface of water. The reading of the spring balance is

(a) 10 N (b) 9 N (c) 8 N (d) 7 N

Solution : (d) Reading of the spring balance = Apparent weight of the block = Actual weight – upthrust

$$= 12 - V_{in}\sigma g = 12 - 500 \times 10^{-6} \times 10^3 \times 10 = 12 - 5 = 7N.$$

Problem 22. An iceberg is floating in sea water. The density of ice is 0.92 gm/cm^3 and that of sea water is 1.03g/cm^3. What percentage of the iceberg will be below the surface of water

(a) 3% (b) 11% (c) 89% (d) 92%

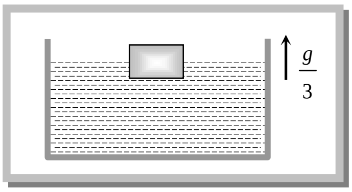
Solution : (c) For the floatation of ice-berg, Weight of ice = upthrust due to displaced water

$V\rho g = V_{in}\sigma g \Rightarrow V_{in} = \left(\frac{\rho}{\sigma}\right)V = \left(\frac{0.92}{1.03}\right)V = 0.89V$ $\therefore \frac{V_{in}}{V} = 0.89$ or 89%.

Problem 23. A cubical block is floating in a liquid with half of its volume immersed in the liquid. When the whole system accelerates upwards with acceleration of $g/3$, the fraction of volume immersed in the liquid will be

(a) $\frac{1}{2}$ (b) $\frac{3}{8}$

(c) $\frac{2}{3}$ (d) $\frac{3}{4}$



Solution : (a) Fraction of volume immersed in the liquid $V_{in} = \left(\frac{\rho}{\sigma}\right)V$ *i.e.* it depends upon the densities of the block and liquid.

So there will be no change in it if system moves upward or downward with constant velocity or some acceleration.

Problem 24. A silver ingot weighing 2.1 *kg* is held by a string so as to be completely immersed in a liquid of relative density 0.8. The relative density of silver is 10.5. The tension in the string in *kg-wt* is

(a) 1.6 (b) 1.94 (c) 3.1 (d) 5.25

Solution : (b) Apparent weight $= V(\rho - \sigma)g = \frac{M}{\rho}(\rho - \sigma)g = M\left(1 - \frac{\sigma}{\rho}\right)g = 2.1\left(1 - \frac{0.8}{10.5}\right)g = 1.94\, g$ *Newton* = 1.94 *Kg-wt*

Problem 25. A sample of metal weighs 210 *gm* in air, 180 *gm* in water and 120 *gm* in liquid. Then relative density (RD) of

(a) Metal is 3 (b) Metal is 7 (c) Liquid is 3 (d) Liquid is $\frac{1}{3}$

Solution : (b, c) Let the density of metal is ρ and density of liquid is σ.

If *V* is the volume of sample then according to problem

$210 = V\rho g$ ....(i)

$180 = V(\rho - 1)g$ ....(ii)

$120 = V(\rho - \sigma)g$ ....(iii)

By solving (i), (ii) and (iii) we get $\rho = 7$ and $\sigma = 3$.

Problem 26. Two solids *A* and *B* float in water. It is observed that *A* floats with half its volume immersed and *B* floats with 2/3 of its volume immersed. Compare the densities of *A* and *B*

(a) 4 : 3 (b) 2 : 3 (c) 3 : 4 (d) 1 : 3

Solution : (c) If two different bodies *A* and *B* are floating in the same liquid then $\frac{\rho_A}{\rho_B} = \frac{(f_{in})_A}{(f_{in})_B} = \frac{1/2}{2/3} = \frac{3}{4}$

Problem 27. The fraction of a floating object of volume V_0 and density d_0 above the surface of a liquid of density *d* will be

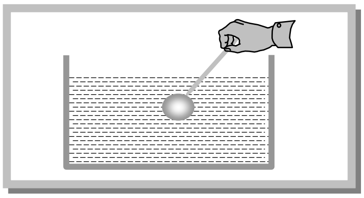
(a) $\frac{d_0}{d}$ (b) $\frac{dd_0}{d + d_0}$ (c) $\frac{d - d_0}{d}$ (d) $\frac{dd_0}{d - d_0}$

Solution : (c) For the floatation $V_0 d_0 g = V_{in}\, d\, g \Rightarrow V_{in} = V_0 \frac{d_0}{d}$

$$\therefore \; V_{out} = V_0 - V_{in} = V_0 - V_0\frac{d_0}{d} = V_0\left[\frac{d - d_0}{d}\right] \Rightarrow \frac{V_{out}}{V_0} = \frac{d - d_0}{d}.$$

Problem 28. A vessel with water is placed on a weighing pan and reads 600 *g*. Now a ball of 40 *g* and density 0.80 *g/cc* is sunk into the water with a pin as shown in fig. keeping it sunk. The weighing pan will show a reading

(a) 600 *g*
(b) 550 *g*
(c) 650 *g*
(d) 632 *g*



Solution : (c) Upthrust on ball = weight of displaced water

$$= V\sigma g = \left(\frac{m}{\rho}\right)\sigma g = \frac{40}{0.8} \times 1 \times g = 50g \text{ Dyne or 50 gm}$$

As the ball is sunk into the water with a pin by applying downward force equal to upthrust on it.

So reading of weighing pan = weight of water + downward force against upthrust = 600 + 50 = 650 *gm*.

Some Conceptual Questions

Que.1 Why a small iron needle sinks in water while a large iron ship floats

Ans. *For floatation, the density of body must be lesser or equal to that of liquid. In case of iron needle, the density of needle, i.e., iron is more than that of water, so it will sink. However, the density of a ship due to its large volume is lesser than that of water, so it will float.*

Que.2 A man is sitting in a boat which is floating in a pond. If the man drinks some water from the pond, what will happen to the level of water in the pond

Ans. *If the man drinks m g of water from the pond, the weight of (boat + man) system will increase by mg and so the system will displace mg more water for floating. So due to removal of water from pond, the water level in pond will fall but due to water displaced by the floating system the water level in the pond will rise and so the water removed from the pond is equal to the water displaced by the system; the level of water in the pond will remain unchanged.*

Que.3 A boy is carrying a fish in one hand and a bucket full of water in the other hand. He then places the fish in the bucket thinking that in accordance with Archimedes' principle he is now carrying less weight as the weight of the fish will reduce due to upthrust. Is he right

Ans. *No, when he places the fish in water in the bucket, no doubt the weight of fish is reduced due to upthrust, but the weight of (water + bucket) system is increased by the same amount, so that the total weight carried by him remains unchanged.*

Que. 4 A bucket of water is suspended from a spring balance. Does the reading of balance change (a) when a piece of stone suspended from a string is immersed in the water without touching the bucket? (b) when a piece of iron or cork is put in the water in the bucket?

Ans. (a) *Yes, the reading of the balance will increase but the increase in weight will be equal to the loss in weight of the stone* $(V\sigma g)$ *and not the weight of stone* $(V\rho g)[> V\sigma g$ as $\rho > \sigma]$.

(b) *Yes, the reading of the balance will increase but the increase in weight will be equal to the weight of iron or cork piece.*

Que. 5 Why a soft plastic bag weighs the same when empty or when filled with air at atmospheric pressure? Would the weight be the same if measured in vacuum

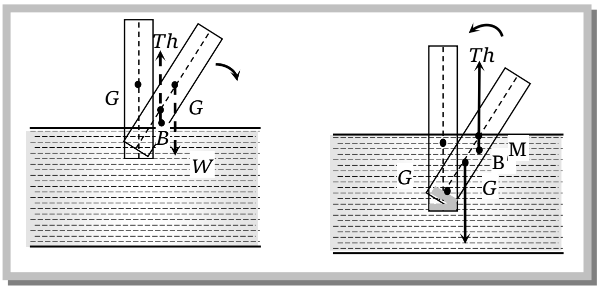
Ans. *If the weight of empty bag is* W_0 *and the volume of bag is V, when the bag is filled with air of density* ρ *at NTP, its weights will increase by* $V\rho g$. *Now when the bag filled with air is weighed in air, the thrust of air* $V\rho g$ *will decrease its weight; so* $W = W_0 + V\rho g - V\rho g = W_0$

i.e., the weight of the bag remains unchanged when it is filled with air at NTP and weighed in air. However if the bag is weighed in vacuum will be W_0 *when empty and* $(W_0 + V\rho g)$ *when filled with air (as there is no upthrust), i.e., in vacuum an air-filled bag will weigh more than an empty bag.*

Que.6 Why does a uniform wooden stick or log float horizontally? If enough iron is added to one end, it will float vertically; explain this also.

Ans. *When a wooden stick is made to float vertically, its rotational equilibrium will be unstable as its meta-centre will be lower than its CG and with a slight tilt it will rotate under the action of the couple formed by thrust and weight in the direction of tilt, till it becomes horizontal.*

However, due to loading at the bottom, the CG of the stick (or log) will be lowered and so may be lower than the meta-centre. In this situation the equilibrium will be stable and if the stick (or log) is tilted, it will come back to its initial vertical position.



Que.7 A boat containing some pieces of material is floating in a pond. What will happen to the level of water in the pond if on unloading the pieces in the pond, the piece (*a*) floats (*b*) sinks?

Ans. *If M is the mass of boat and m of pieces in it, then initially as the system is floating* $M + m = V_D\sigma_W$

i.e., the system displaces water $V_D = \dfrac{M}{\sigma_W} + \dfrac{m}{\sigma_W}$ ......*(i)*

When the pieces are dropped in the pond, the boat will still float, so it displaces water $M = V_1\sigma_W$,

i.e, $V_1 = (M / \sigma_W)$

(a) Now if the unloaded pieces floats in the pond, the water displaced by them $m = V_2\sigma_W$,

i.e, $V_2 = (m / \sigma_W)$

So the total water displaced by the boat and the floating pieces

$$V_1 + V_2 = \frac{M}{\sigma_W} + \frac{m}{\sigma_W} \qquad .....(ii)$$

Which is same as the water displaced by the floating system initially (eqn. 1); so the level of water in the pond will remain unchanged.

(b) Now if the unloaded pieces sink the water displaced by them will be equal to their own volume, i.e,

$$V_2' = \frac{m}{\rho} \qquad \left[as\ \rho = \frac{m}{V}\right]$$

and so in this situation the total volume of water displaced by boat and sinking pieces will be

$$V_1 + V_2' = \left(\frac{M}{\sigma_W} + \frac{m}{\rho}\right) \qquad .....(iii)$$

Now as the pieces are sinking $\rho > \sigma_W$, *so this volume will be lesser than initial water displaced by the floating system (eq. 1); so the level of water in the pond will go down (or fall)*

In this problem if the pieces (either sinking or floating) are unloaded on the ground, the water displaced after unloading, $V_2 = M / \sigma_W$, *will be lesser than before unloading.* $V = (M + m) / \sigma_W$; *so the level of water in the pond will fall.*