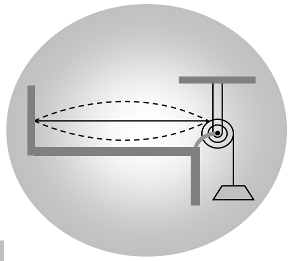
speed of a transverse wave on a string, reflection of a wave at the boundary



Reflection of Mechanical Waves

Medium	Longitudinal wave	Transverse wave	Change in direction	Phase change	Time change	Path change
Reflection from rigid end/denser medium	Compression as rarefaction and vice-versa	Crest as crest and Trough as trough	Reversed	π	$\frac{T}{2}$	$\frac{\lambda}{2}$
Reflection from free end/rarer medium	Compression as compression and rarefaction as rarefaction	Crest as trough and trough as crest	No change	Zero	Zero	Zero

Progressive Wave

(1) These waves propagate in the forward direction of medium with a finite velocity.

(2) Energy and momentum are transmitted in the direction of propagation of waves without actual transmission of matter.

(3) In progressive waves, equal changes in pressure and density occurs at all points of medium.

(4) Various forms of progressive wave function.

(i) $y = A \sin(\omega t - kx)$

(ii) $y = A \sin(\omega t - \frac{2\pi}{\lambda} x)$

(iii) $y = A \sin 2\pi \left[\frac{t}{T} - \frac{x}{\lambda}\right]$

(iv) $y = A \sin \frac{2\pi}{\lambda}(vt - x)$

(v) $y = A \sin \omega \left(t - \frac{x}{v}\right)$

where y = displacement
A = amplitude
ω = angular frequency
n = frequency
k = propagation constant
T = time period
λ = wave length
v = wave velocity
t = instantaneous time
x = position of particle from origin

Important points

(a) If the sign between t and x terms is negative the wave is propagating along positive X-axis and if the sign is positive then the wave moves in negative X-axis direction.

(b) The coefficient of sin or cos functions *i.e.* Argument of sin or cos function *i.e.* $(\omega t - kx)$ = Phase.

(c) The coefficient of t gives angular frequency $\omega = 2\pi n = \frac{2\pi}{T} = vk$.

(d) The coefficient of x gives propagation constant or wave number $k = \frac{2\pi}{\lambda} = \frac{\omega}{v}$.

(e) The ratio of coefficient of t to that of x gives wave or phase velocity. *i.e.* $v = \frac{\omega}{k}$.

(f) When a given wave passes from one medium to another its frequency does not change.

(g) From $v = n\lambda \Rightarrow v \propto \lambda \because n = \text{constant} \Rightarrow \frac{v_1}{v_2} = \frac{\lambda_1}{\lambda_2}$.

(5) Some terms related to progressive waves

(i) Wave number ($\bar{n}$) : The number of waves present in unit length is defined as the wave number ($\bar{n}$) = $\frac{1}{\lambda}$.

Unit = $meter^{-1}$; Dimension = $[L^{-1}]$.

(ii) Propagation constant (k) : $k = \frac{\phi}{x} = \frac{\text{Phase difference between particles}}{\text{Distance between them}}$

$$k = \frac{\omega}{v} = \frac{\text{Angular velocity}}{\text{Wave velocity}} \quad \text{and} \quad k = \frac{2\pi}{\lambda} = 2\pi\bar{\lambda}$$

(iii) Wave velocity (v) : The velocity with which the crests and troughs or compression and rarefaction travel in a medium, is defined as wave velocity $v = \frac{\omega}{k} = n\lambda = \frac{\omega\lambda}{2\pi} = \frac{\lambda}{T}$.

(iv) Phase and phase difference : Phase of the wave is given by the argument of sine or cosine in the equation of wave. It is represented by $\phi(x,t) = \frac{2\pi}{\lambda}(vt - x)$.

(v) At a given position (for fixed value of x) phase changes with time (t).

$$\frac{d\phi}{dt} = \frac{2\pi v}{\lambda} = \frac{2\pi}{T} \Rightarrow d\phi = \frac{2\pi}{T}.dt \Rightarrow \text{Phase difference} = \frac{2\pi}{T} \times \text{Time difference.}$$

(vi) At a given time (for fixed value of t) phase changes with position (x).

$$\frac{d\phi}{dx} = \frac{2\pi}{\lambda} \Rightarrow d\phi = \frac{2\pi}{\lambda} \times dx \Rightarrow \text{Phase difference} = \frac{2\pi}{\lambda} \times \text{Path difference}$$

$$\Rightarrow \text{Time difference} = \frac{T}{\lambda} \times \text{Path difference}$$

Sample problems based on Progressive wave

Problem 1. The speed of a wave in a certain medium is 960 *m/sec*. If 3600 waves pass over a certain point of the medium in 1 minute, the wavelength is

(a) 2 *meters* (b) 4 *meters* (c) 8 *meters* (d) 16 *meters*

Solution : (d) $v = 960\ m/s$; $n = \frac{3600}{60}\ Hz$. So $\lambda = \frac{v}{n} = \frac{960}{60} = 16$ *meters*.

Problem 2. A simple harmonic progressive wave is represented by the equation $y = 8 \sin 2\pi(0.1x - 2t)$ where x and y are in *cm* and t is in seconds. At any instant the Phase difference between two particles separated by 2.0 *cm* in the x-direction is

(a) 18° (b) 36° (c) 54° (d) 72°

Solution : (d) $y = 8 \sin 2\pi\left(\frac{x}{10} - 2t\right)$ given by comparing with standard equation $y = a \sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)$

$\lambda = 10\ cm$

So Phase Difference = $\frac{2\pi}{\lambda}$ × path difference = $\frac{2\pi}{10} \times 2 = \frac{2}{5} \times 180° = 72°$

Problem 3. The frequency of sound wave is n and its velocity is v if the frequency is increased to $4n$ the velocity of the wave will be

(a) v (b) $2v$ (c) $4v$ (d) $v/4$

Solution : (a) Wave velocity does not depends on the frequency. It depends upon the Elasticity and inertia of the medium.

Problem 4. The equation of a transverse wave travelling on a rope is given by $y = 10 \sin \pi(0.01\, x - 2.00\, t)$ where y and x are in cm and t in seconds. The maximum transverse speed of a particle in the rope is about [MP PET 1999]

(a) 63 cm/sec (b) 75 cm/s (c) 100 cm/sec (d) 121 cm/sec

Solution : (a) Standard eq. of travelling wave $y = A \sin (kx - \omega t)$

By comparing with the given equation $y = 10 \sin (0.01\pi x - 2\pi t)$

$A = 10\ cm$, $\omega = 2\pi$

Maximum particle velocity = $A\omega = 2\pi \times 10 = 63\ cm/sec$

Problem 5. In a wave motion $y = a \sin (kx - \omega t)$, y can represents

(a) Electric Field (b) magnetic field (c) Displacement (d) Pressure

Solution : (a,b,c,d)

Problem 6. The displacement x (in metres) of a particle performing simple harmonic motion is related to time t (in seconds) as $x = 0.05 \cos\left(4\pi t + \frac{\pi}{4}\right)$. The frequency of the motion will be

(a) 0.5 Hz (b) 1.0 Hz (c) 1.5 Hz (d) 2.0 Hz

Solution : (d) From the given equation, coefficient of t = $\omega = 4\pi$

$\therefore \quad n = \frac{\omega}{2\pi} = \frac{4\pi}{2\pi} = 2\ Hz$

Problem 7. A wave is represented by the equation $Y = 7 \sin\left(7\pi t - 0.04\pi x + \frac{\pi}{3}\right)$ x is in meters and t is in seconds. The speed of the wave is

(a) 175 m/sec (b) 49 $\pi\ m/s$ (c) $\frac{49}{\pi}\ m/s$ (d) 0.28 $\pi\ m/s$

Solution : (a) Standard equation $y = A \sin (\omega t - kx + \phi_0)$

In a given equation $\omega = 7\pi, k = 0.04\pi$

$$v = \frac{\omega}{k} = \frac{7\pi}{.04\pi} = 175\ m/sec$$

Problem 8. A wave is represented by the equation y = 0.5 sin (10 t + x)m. It is a travelling wave propagating along the x direction with velocity.

(a) 10 m/s (b) 20 m/s (c) 5 m/s (d) None of these

Solution : (a) $v = \omega / k = 10/1 = 10\ m/s$

Problem 9. A transverse progressive wave on a stretched string has a velocity of 10 ms^{-1} and a frequency of 100 Hz. The phase difference between two particles of the string which are 2.5 cm apart will be [MP PMT 1994]

(a) $\pi/8$ (b) $\pi/4$ (c) $3\pi/8$ (d) $\pi/2$

Solution : (d) $\lambda = v/n = \frac{10}{100} = 0.1\,m = 10\,cm$

Phase difference = $\frac{2\pi}{\lambda}$ × path difference $= \frac{2\pi}{10}$ × 2.5 = $\frac{\pi}{2}$

Problem 10. In a stationary wave, all particles are [MP PMT 1994]

(a) At rest at the same time twice in every period of oscillation

(b) At rest at the same time only once in every period of oscillation

(c) Never at rest at the same time

(d) Never at rest at all

Solution : (a)

Problem 11. A wave represented by the given equation $y = A \sin\left(10\pi x + 15\pi t + \frac{\pi}{3}\right)$ where x is in meter and t is in second. The expression represents

(a) A wave travelling in the positive x-direction with a velocity of 1.5 *m/sec*

(b) A wave travelling in the negative x-direction with a velocity of 1.5 *m/sec*

(c) A wave travelling in the negative x-direction with a wavelength of 0.2 *m*

(d) A wave travelling in the positive x-direction with a wavelength of 0.2 *m*

Solution : (b, c) By comparing with standard equation $Y = A \sin\left(kx + \omega t + \pi/3\right)$

$K = 10\pi,\ \omega = 15\pi$

We know that : $v = \frac{\omega}{k}$ = 1.5 *m/sec*; $\lambda = \frac{2\pi}{k}$ = 0.2 *meter*.

Problem 12. The equation of a wave travelling in a string can be written as $y = 3\cos\pi(100t - x)$ Its wavelength is

[MP PMT 1991, 94, 97; MNR 1985]

(a) 100 *cm* (b) 2 *cm* (c) 5 *cm* (d) None of these

Solution : (b) $y = A\cos(\omega t - kx)$ – standard equation

$y = 3\cos(100\pi t - \pi x)$ – given equation

So $K = \pi$ and $\lambda = \frac{2\pi}{k} = 2\ cm$

Problem 13. A plane wave is represented by $x = 1.2 \sin(314t + 12.56y)$ where x and y are distances measured along in x and y direction in meter and t is time in seconds. This wave has

(a) A wave length of 0.25 *m* and travels m + *ve* x-direction

(b) A wavelength of 0.25 *m* and travels in + *ve* y-direction

(c) A wavelength of 0.5 *m* and travels in – *ve* y-direction

(d) A wavelength of 0.5 *m* and travels in – *ve* x-direction

Solution : (c) From given equation $k = 12.56$

$\lambda = \frac{2\pi}{k}$ 0.5 *m* direction = – y

Problem 14. A wave is reflected from a rigid support. The change in phase on reflection will be

(a) $\pi/4$ (b) $\pi/2$ (c) π (d) 2π

Solution : (c)