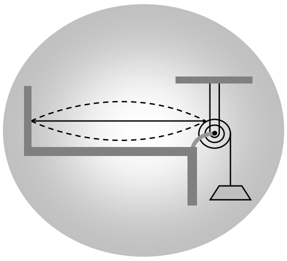


# Wave Motion

## Wave

A wave is a disturbance that propagates energy and momentum from one place to the other without transporting matter.

(1) Necessary properties of the medium for wave propagation:

(i) Elasticity: So that particles can return to their mean position, after having been disturbed.

(ii) Inertia: So that particles can store energy and overshoot their mean position.

(iii) Minimum friction amongst the particles of the medium.

(iv) Uniform density of the medium.

(2) Characteristics of wave motion:

(i) It is a sort of disturbance which travels through a medium.

(ii) Material medium is essential for the propagation of mechanical waves.

(iii) When a wave motion passes through a medium, particles of the medium only vibrate simple harmonically about their mean position. They do leave their position and move with the disturbance.

(iv) There is a continuous phase difference amongst successive particles of the medium *i.e.*, particle 2 starts vibrating slightly later than particle 1 and so on.

(v) The velocity of the particle during their vibration is different at different position.

(vi) The velocity of wave motion through a particular medium is constant. It depends only on the nature of medium not on the frequency, wavelength or intensity.

(vii) Energy is propagated along with the wave motion without any net transport of the medium.

(3) Mechanical waves: The waves which require medium for their propagation are called mechanical waves.

*Example*: Waves on string and spring, waves on water surface, sound waves, seismic waves.

(4) Non-mechanical waves: The waves which do not require medium for their propagation are called non- mechanical or electromagnetic waves.

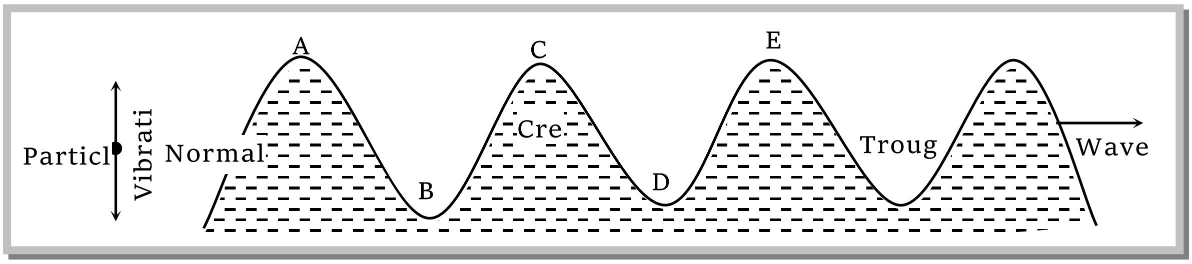
*Examples*: Light, heat (Infrared), radio waves, $\gamma$- rays, $X$-rays *etc*.

أمثلة: الضوء والحرارة (الأشعة تحت الحمراء) وموجات الراديو وأشعة  والأشعة السينية وما إلى ذلك.

(5) Transverse waves: Particles of the medium execute simple harmonic motion about their mean position in a direction perpendicular to the direction of propagation of wave motion.

(i) It travels in the form of crests and troughs.

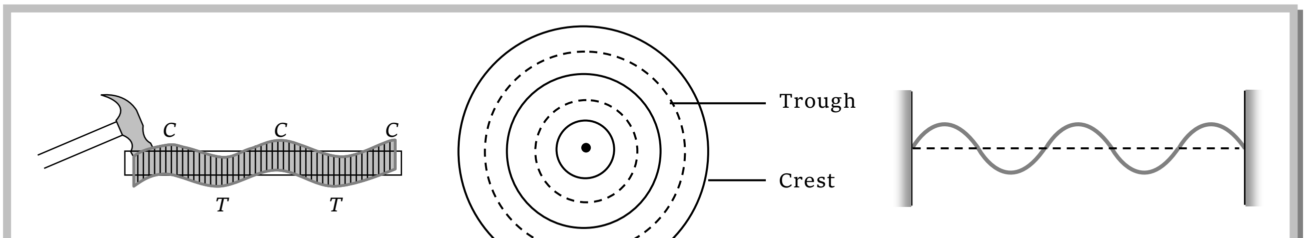
(ii) A crest is a portion of the medium which is raised temporarily above the normal position of rest of the particles of the medium when a transverse wave passes through it.




(iii) A trough is a portion of the medium which is depressed temporarily below the normal position of rest of the particles of the medium, when transverse wave passes through it. Particle

(iv) Examples of transverse wave motion : Movement of string of a sitar or violin, movement of the membrane of a Tabla or Dholak, movement of kink on a rope, waves set-up on the surface of water.

(v) Transverse waves can be transmitted through solids, they can be setup on the surface of liquids. But they can not be transmitted into liquids and gases.

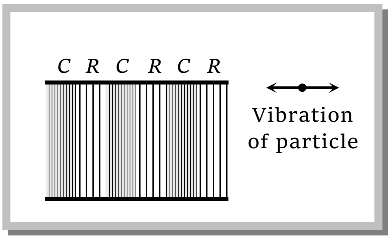



Transverse-wave in a

Transverse wave on water

Transverse wave in a string

(6) Longitudinal waves : If the particles of a medium vibrate in the direction of wave motion the wave is called longitudinal.




(i) It travels in the form of compression and rarefaction.

(ii) A compression ($C$) is a region of the medium in which particles are compressed.

(iii) A rarefaction ($R$) is a region of the medium in which particles are rarefied.

(iv) Examples sound waves travel through air in the form of longitudinal waves, Vibration of air column in organ pipes are longitudinal, Vibration of air column above the surface of water in the tube of resonance apparatus are longitudinal.

(v) These waves can be transmitted through solids, liquids and gases because for these waves propagation, volume elasticity is necessary.

(7) One dimensional wave : Energy is transferred in a single direction only.

*Example* : Wave propagating in a stretched string.

(8) Two-dimensional wave: Energy is transferred in a plane in two mutually perpendicular directions.

*Example:* Wave propagating on the surface of water.

(9) Three dimensional wave : Energy in transferred in space in all direction.

*Example:* Light and sound waves propagating in space.

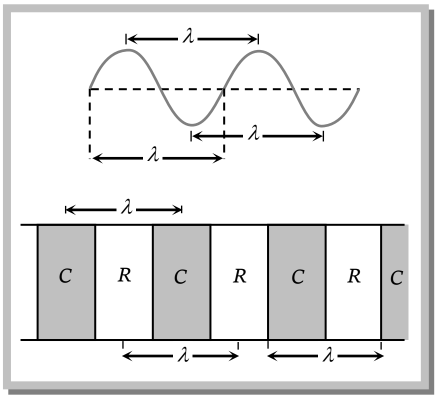
## 16.2 Important Terms Regarding Wave Motion

(1) Wavelength: (i) It is the length of one wave.

(ii) Wavelength is equal to the distance travelled by the wave during the time in which any one particle of the medium completes one vibration about its mean position.

(iii) Wavelength is the distance between any two nearest particles of the medium, vibrating in the same phase.

(iv) Distance travelled by the wave in one time period is known as wavelength.




(v) In transverse wave motion :

$\lambda$ = Distance between the centres of two consecutive crests.

$\lambda$ = Distance between the centres of two consecutive troughs.

$\lambda$ = Distance in which one trough and one crest are contained.

(vi) In longitudinal wave motion :

$\lambda$ = Distance between the centres of two consecutive compression.

$\lambda$ = Distance between the centres of two consecutive rarefaction.

$\lambda$ = Distance in which one compression and one rarefaction contained.

(2) Frequency : (i) Frequency of vibration of a particle is defined as the number of vibrations completed by particle in one second.

(ii) It is the number of complete wavelengths traversed by the wave in one second.

(iii) Units of frequency are hertz ($Hz$) and per second.

(3) Time period : (i) Time period of vibration of particle is defined as the time taken by the particle to complete one vibration about its mean position.

(ii) It is the time taken by the wave to travel a distance equal to one wavelength.

(4) Relation between frequency and time period : Time period = 1/Frequency $\Rightarrow T = 1/n$

(5) Relation between velocity, frequency and wavelength : $v = n\lambda$

Velocity ($v$) of the wave in a given medium depends on the elastic and inertial property of the medium.

Frequency ($n$) is characterised by the source which produces disturbance. Different sources may produce vibration of different frequencies. Wavelength ($\lambda$) will differ to keep $n\lambda = v$ = constant.

## Prove the Wave equation.

Figure 1.4 shows an element of a sinusoidal waveform produced in a long string that is under tension $F$ (see Fig. 1.3). Let the wave element to be displaced from its equilibrium position $y = 0$. We take a very small part of the string element and consider the length of the part of the string be $\delta x$, as it is shown in Fig. 1.4. The force $\vec{F}$ is executed on either sides of the element $\delta x$ and acted tangentially on both sides but in opposite directions (see Fig. 1.4). Now we find the net force along y direction is

$$\sum F_y = F \sin\theta_2 - F \sin\theta_1. \tag{1.15}$$

Since the element $\delta x$ is very small, we take angles $\theta_1$ and $\theta_2$ are also very small. Then we can consider as $\sin\theta_1 \approx \tan\theta_1$ and $\sin\theta_2 \approx \tan\theta_2$. From Eq. 1.15 we have

$$\sum F_y = F \tan\theta_2 - F \tan\theta_1 = F\ \delta(\tan\theta), \tag{1.16}$$

where $\delta(\tan\theta) = \tan\theta_2 - \tan\theta_1$. Let the mass of the element $\delta x$ be $\delta m (= \mu\ \delta x)$, where $\mu$ is the mass per unit length of the string. Equation 1.16 follows as

$$\sum F_y = \delta m\ a_y = \mu\ \delta x\ a_y = F\ \delta(\tan\theta),$$

$$\text{or} \quad \mu\ \delta x\ a_y = F\ \delta(\tan\theta),$$

$$\text{or} \quad \frac{\delta(\tan\theta)}{\delta x} = \frac{\mu}{F}\ a_y. \tag{1.17}$$

The slope of the string to be written

$$\tan\theta = \partial y/\partial x.$$

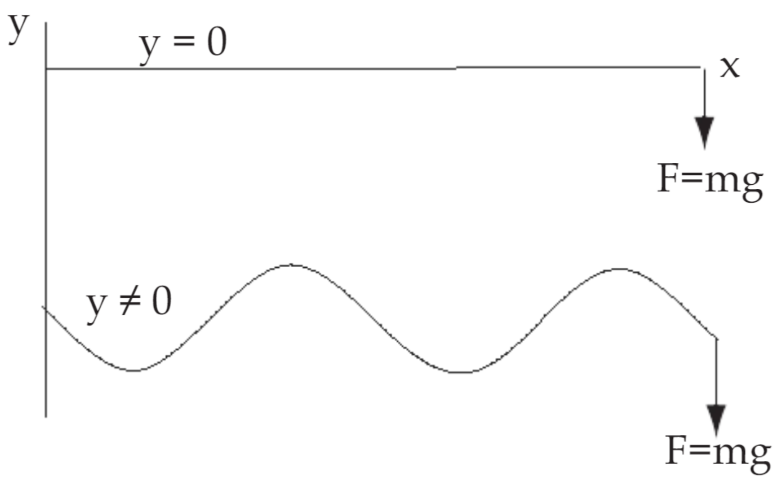



***Figure 1.3: A string under tension (upper-line) has produced a waveform (lower-line).***

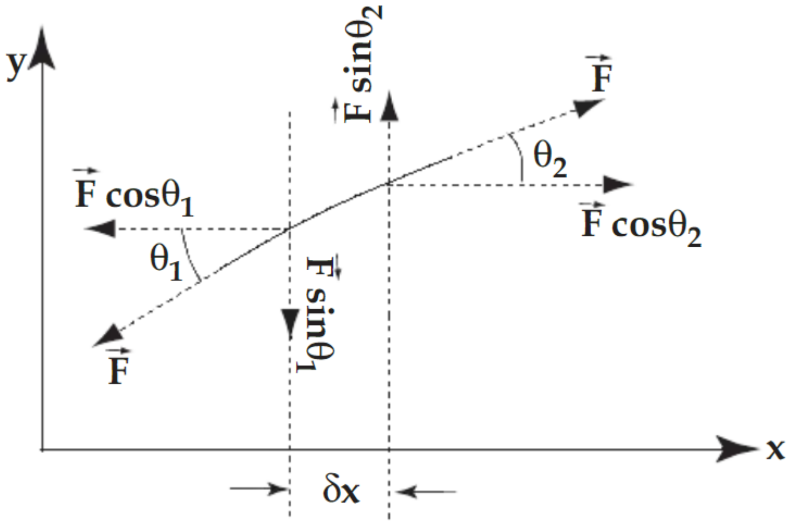



***Figure 1.4: A small element of a sinusoidal waveform.***

$$\therefore \quad \frac{\delta(\tan\theta)}{\delta x} = \lim_{\delta x\to 0}\frac{\delta(\partial y/\partial x)}{\delta x} = \frac{d^2y}{dx^2}. \tag{1.18}$$

For the displacement of a particle along y axis, the y component of the acceleration

$$a_y = \frac{d^2y}{dt^2}. \tag{1.19}$$

Finally from Eqs. 1.17, 1.19 and 1.18 we find the equation of a wave in a string is

$$\frac{d^2y}{dx^2} = \frac{\mu}{F}\frac{d^2y}{dt^2}, \tag{1.20}$$

$$\text{or} \quad \frac{d^2y}{dx^2} = \frac{1}{F/\mu}\frac{d^2y}{dt^2}. \tag{1.21}$$

Let us try to find the dimension of the term $F/\mu$ which follows

$$\frac{F}{\mu} = \frac{force}{mass\ per\ unit\ lenght} = \frac{kg\ m\ s^{-2}}{kg\ m^{-1}} = \frac{m^2}{s^2} = (speed)^2 = v^2.$$

$$\therefore \quad v = \sqrt{\frac{F}{\mu}}. \tag{1.22}$$

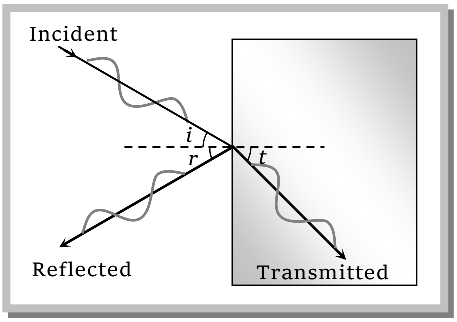
Therefore, substituting $v = \sqrt{F/\mu}$, the general equation of a wave passing with a speed $v$ is given by the equation as follows

$$\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2}, \tag{1.23}$$

which describes that the second derivative of the wave displacement along $y$ with respect to the coordinate $x$ in the direction of propagation of wave is equal to $1/v^2$ times the second derivative of displacement $y$ with respect to time.

## Reflection and Refraction of Waves

When sound waves are incident on a boundary between two media, a part of the incident waves returns into the initial medium (reflection) while the remaining is partly absorbed and partly transmitted into the second medium (refraction) In the case of reflection and refraction of sound




(1) The frequency of the wave remains unchanged which means

$$\omega_i = \omega_r = \omega_t = \omega = \text{constant}$$

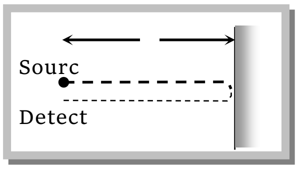
(2) The incident ray, reflected ray, normal and refracted ray all lie in the same plane.

(3) For reflection angle of incidence ($i$) = Angle of reflection ($r$)

(4) For refraction $\frac{\sin i}{\sin t} = \frac{v_i}{v_t}$

(5) In reflection from a denser medium or rigid support, phase changes by 180° and direction reverses if incident wave is $y = A_1 \sin(\omega t - kx)$ then reflected wave becomes $y = A_r \sin(\omega t + kx + \pi) = -A_r \sin(\omega t + kx)$.

(6) In the reflection from a rarer medium or free end, the phase does not change and direction reverses if the incident wave is $y = A_I \sin(\omega t - kx)$ then reflected wave becomes $y = A_r \sin(\omega t + kx)$




(7) Echo is an example of reflection.

If there is a sound reflector at a distance $d$ from the source then the time interval between the original sound and its echo at the site of the source will be $t = \frac{2d}{v}$