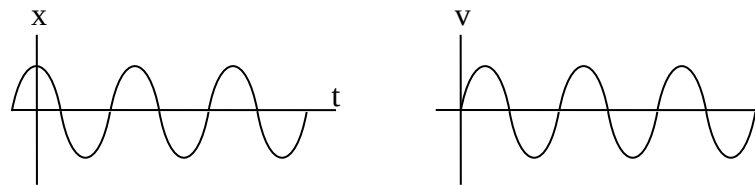


Simple Harmonic Motion (Periodic Motion and Simple Harmonic motion)

A pendulum, a mass on a spring, and many other kinds of oscillators exhibit a special kind of oscillatory motion called Simple Harmonic Motion (SHM).

SHM occurs whenever:

- there is a restoring force proportional to the displacement from equilibrium: $F \propto -x$
- the potential energy is proportional to the square of the displacement: $PE \propto x^2$
- the period T or frequency $f = 1 / T$ is independent of the amplitude of the motion.
- the position x , the velocity v , and the acceleration a are all sinusoidal in time.

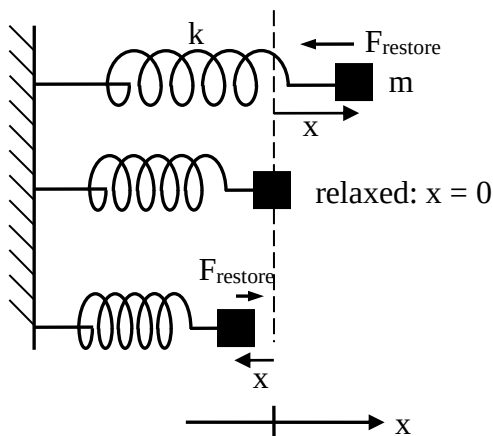


(*Sinusoidal* means sine, cosine, or anything in between.)

As we will see, any one of these four properties guarantees the other three. If one of these 4 things is true, then the oscillator is a simple harmonic oscillator and all 4 things must be true.

Not every kind of oscillation is SHM. For instance, a perfectly elastic ball bouncing up and down on a floor: the ball's position (height) is oscillating up and down, but none of the 4 conditions above is satisfied, so this is not an example of SHM.

A mass on a spring is the simplest kind of Simple Harmonic Oscillator.



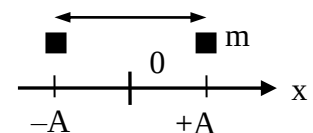
Hooke's Law: $\mathbf{F}_{\text{spring}} = -k \mathbf{x}$

(-) sign because direction of $\mathbf{F}_{\text{spring}}$ is opposite to the direction of displacement vector $\mathbf{x}$ (**bold** font indicates vector)

k = spring constant = stiffness,
units $[k] = \text{N} / \text{m}$

Big k = stiff spring

Definition: *amplitude* $A = |x_{\text{max}}| = |x_{\text{min}}|$.



Mass oscillates between extreme positions $x = +A$ and $x = -A$

Notice that Hooke's Law ($F = -kx$) is condition i : restoring force proportional to the displacement from equilibrium. We showed previously (Work and Energy Chapter) that for a spring obeying Hooke's Law, the potential energy is $U = (1/2)kx^2$, which is condition ii. Also, in the chapter on Conservation of Energy, we showed that $F = -dU/dx$, from which it follows that condition ii implies condition i. Thus, Hooke's Law and quadratic PE ($U \propto x^2$) are equivalent.

We now show that Hooke's Law guarantees conditions iii (period independent of amplitude) and iv (sinusoidal motion).

We begin by deriving the *differential equation* for SHM. A differential equation is simply an equation containing a derivative. Since the motion is 1D, we can drop the vector arrows and use sign to indicate direction.

$$F_{\text{net}} = ma \quad \text{and} \quad F_{\text{net}} = -kx \quad \Rightarrow \quad ma = -kx$$

$$a = dv/dt = d^2x/dt^2 \quad \Rightarrow \quad \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

The constants k and m are both positive, so the k/m is always positive, always. For notational convenience, we write $k/m = \omega^2$. (The square on the ω reminds us that ω^2 is always positive.) The differential equation becomes

$$\boxed{\frac{d^2x}{dt^2} = -\omega^2 x} \quad \text{(equation of SHM)}$$

This is the *differential equation* for SHM. We seek a solution $x = x(t)$ to this equation, a function $x = x(t)$ whose second time derivative is the function $x(t)$ multiplied by a negative constant ($-\omega^2 = -k/m$). The way you solve differential equations is the same way you solve integrals: you *guess* the solution and then check that the solution works.

Based on observation, we guess a sinusoidal solution: $x(t) = A \cos(\omega t + \phi)$,

where A , ϕ are any constants and (as we'll show) $\omega = \sqrt{\frac{k}{m}}$.

A = amplitude: x oscillates between $+A$ and $-A$

ϕ = phase constant (more on this later)

Danger: ωt and ϕ have units of radians (not degrees). So set your calculators to radians when using this formula.

Just as with circular motion, the angular frequency ω for SHM is related to the period by

$\omega = 2\pi f = \frac{2\pi}{T}, \quad T = \text{period.}$	simple harmonic motion: periodic motion and simple harmonic motion
--	--

(What does SHM have to do with circular motion? We'll see later.)

Let's check that $x(t) = A \cos(\omega t + \phi)$ is a solution of the SHM equation.

Taking the first derivative dx/dt , we get $v(t) = \frac{dx}{dt} = -A \omega \sin(\omega t + \phi)$.

Here, we've used the Chain Rule: $\frac{d}{dt} \cos(\omega t + \phi) = \frac{d \cos(\theta)}{d \theta} \frac{d \theta}{d t}, \quad (\theta = \omega t + \phi)$
 $= -\sin \theta \cdot \omega = -\omega \sin(\omega t + \phi)$

Taking a second derivative, we get

$$a(t) = \frac{d^2 x}{dt^2} = \frac{dv}{dt} = \frac{d}{dt}(-A \omega \sin(\omega t + \phi)) = -A \omega^2 \cos(\omega t + \phi)$$

$$\frac{d^2 x}{dt^2} = -\omega^2 [A \cos(\omega t + \phi)]$$

$$\frac{d^2 x}{dt^2} = -\omega^2 x$$

This is the SHM equation, with $\omega^2 = \frac{k}{m}$, $\omega = \sqrt{\frac{k}{m}}$

We have shown that our assumed solution is indeed a solution of the SHM equation. (I leave to the mathematicians to show that this solution is unique. Physicists seldom worry about that kind of thing, since we know that nature usually provides only one solution for physical systems, such as masses on springs.)

We have also shown condition iv: x , v , and a are all sinusoidal functions of time:

$$x(t) = A \cos(\omega t + \phi)$$

$$v(t) = -A \omega \sin(\omega t + \phi)$$

$$a(t) = -A \omega^2 \cos(\omega t + \phi)$$

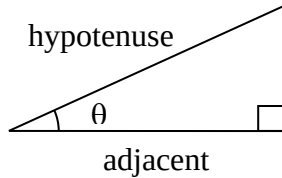
The period T is given by $\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T} \Rightarrow T = 2\pi \sqrt{\frac{m}{k}}$. We see that T does not depend on the amplitude A (condition iii).

Let's first try to make sense of $\omega = \sqrt{k/m}$: big ω means small T which means rapid oscillations. According to the formula, we get a big ω when k is big and m is small. This

makes sense: a big k (stiff spring) and a small mass m will indeed produce very rapid oscillations and a big ω .

A closer look at $x(t) = A \cos(\omega t + \phi)$

Let's review the sine and cosine functions and their relation to the *unit circle*. We often define the sine and cosine functions this way:

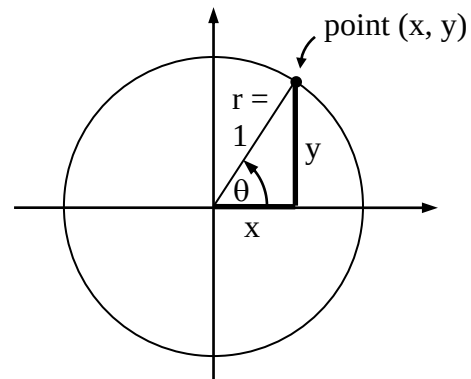


$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

This way of defining sine and cosine is correct but incomplete. It is hard to see from this definition how to get the sine or cosine of an angle greater than 90° .

A more complete way of defining sine and cosine, a way that gives the value of the sine and cosine for *any* angle, is this: Draw a *unit circle* (a circle of radius $r = 1$) centered on the origin of the x - y axes as shown:



Define sine and cosine as

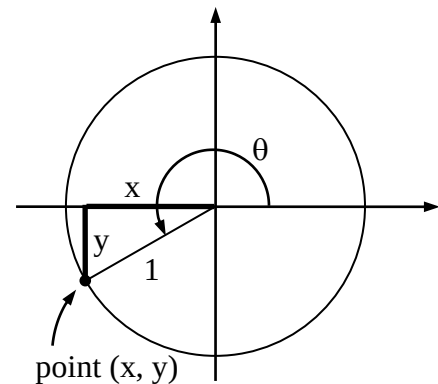
$$\cos \theta = \frac{\text{adj}}{\text{hyp}} = \frac{x}{1} = x$$

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} = \frac{y}{1} = y$$

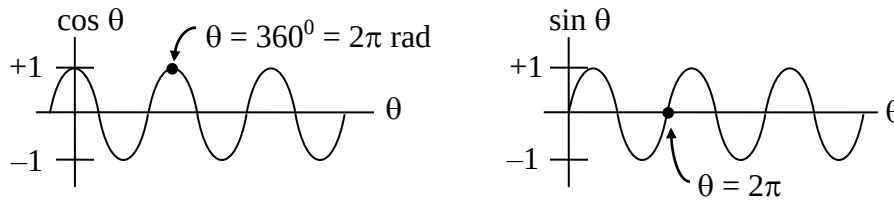
This way of defining sin and cos allows us to compute the sin or cos of *any* angle at all.

For instance, suppose the angle is $\theta = 210^\circ$. Then the diagram looks like this:

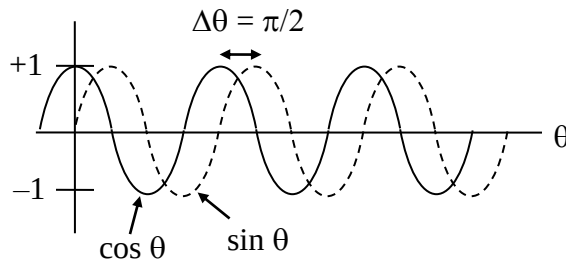
The point on the unit circle is in the third quadrant, where both x and y are negative. So both $\cos \theta = x$ and $\sin \theta = y$ are negative



For any angle θ , even angles bigger than 360° (more than once around the circle), we can always compute $\sin$ and $\cos$. When we plot $\sin$ and $\cos$ vs angle θ , we get functions that oscillate between $+1$ and -1 like so:



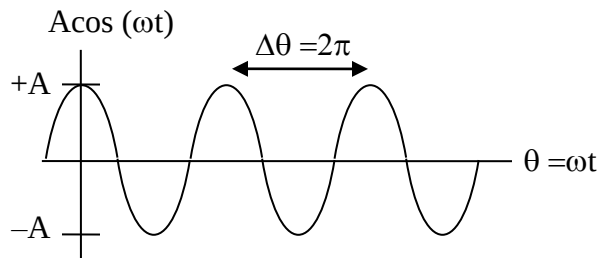
We will almost always measure angle θ in radians. Once around the circle is 2π radians, so sine and cosine functions are periodic and repeat every time θ increases by 2π rad. The sine and cosine functions have exactly the same shape, except that $\sin$ is shifted to the right compared to $\cos$ by $\Delta\theta = \pi/2$. Both these functions are called *sinusoidal* functions.



The function $\cos(\theta + \phi)$ can be made to be anything in between $\cos(\theta)$ and $\sin(\theta)$ by adjusting the size of the *phase* ϕ between 0 and -2π .

$$\cos \theta, (\phi = 0) \rightarrow \sin \theta = \cos\left(\theta - \frac{\pi}{2}\right), (\phi = -\pi/2)$$

The function $\cos(\omega t + \phi)$ oscillates between $+1$ and -1 , so the function $A\cos(\omega t + \phi)$ oscillates between $+A$ and $-A$.

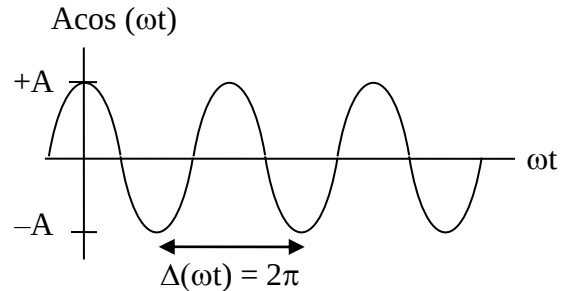
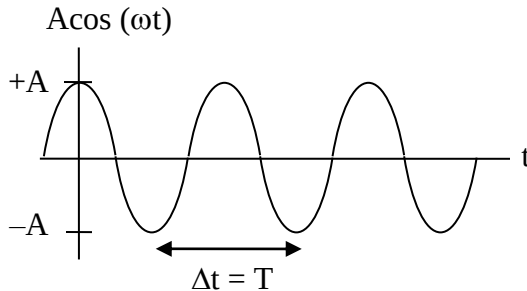


Why $\omega = \frac{2\pi}{T}$? The function $f(\theta) = \cos\theta$ is periodic with period $\Delta\theta = 2\pi$. Since

$\theta = \omega t + \phi$, and ϕ is some constant, we have $\Delta\theta = \omega \Delta t$. One complete cycle of the cosine function corresponds to $\Delta\theta = 2\pi$ and $\Delta t = T$, (T is the period). So we have $2\pi = \omega T$ or

simple harmonic motion; periodic motion and simple harmonic motion

$\omega = \frac{2\pi}{T}$. Here is another way to see it: $\cos(\omega t) = \cos\left(2\pi \frac{t}{T}\right)$ is periodic with period $\Delta t = T$. To see this, notice that when t increases by T , the fraction t/T increases by 1 and the fraction $2\pi t/T$ increases by 2π .



Now back to simple harmonic motion. Instead of a circle of radius 1, we have a circle of radius A (where A is the amplitude of the Simple Harmonic Motion).

SHM and Conservation of Energy:

Recall $PE_{\text{elastic}} = (1/2) k x^2 = \text{work done to compress or stretch a spring by distance } x$.

If there is no friction, then the total energy $E_{\text{tot}} = KE + PE = \text{constant during oscillation}$. The value of E_{tot} depends on initial conditions – where the mass is and how fast it is moving initially. But once the mass is set in motion, E_{tot} stays constant (assuming no dissipation.)

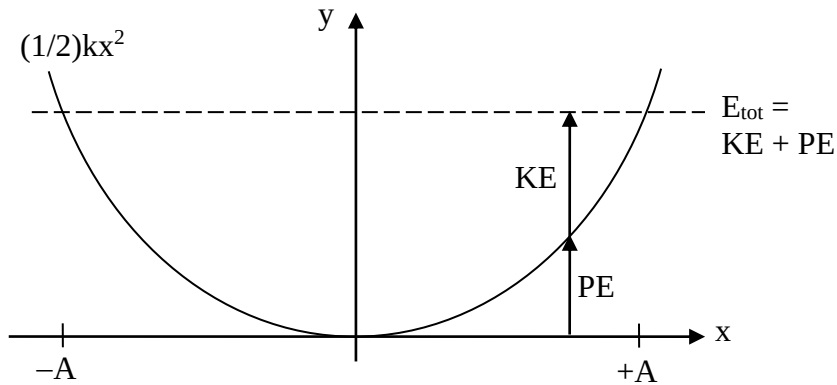
At any position x , speed v is such that $\boxed{\frac{1}{2} m v^2 + \frac{1}{2} k x^2 = E_{\text{tot}}}$.

When $|x| = A$, then $v = 0$, and all the energy is PE: $\underset{0}{KE} + \underset{(1/2)kA^2}{PE} = E_{\text{tot}}$

So total energy $E_{\text{tot}} = \frac{1}{2} k A^2$

When $x = 0$, $v = v_{\text{max}}$, and all the energy is KE: $\underset{(1/2)mv_{\text{max}}^2}{KE} + \underset{0}{PE} = E_{\text{tot}}$

So, total energy $E_{\text{tot}} = \frac{1}{2} m v_{\text{max}}^2$.



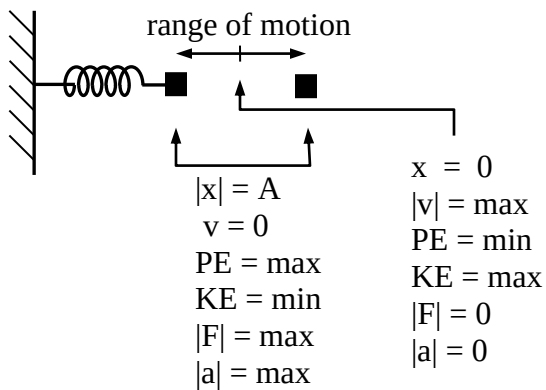
So, we can relate $v_{\max}$ to amplitude A : $PE_{\max} = KE_{\max} = E_{\text{tot}} \Rightarrow \frac{1}{2}kA^2 = \frac{1}{2}mv_{\max}^2 \Rightarrow$

$$v_{\max} = \sqrt{\frac{k}{m}} A$$

Example Problem: A mass m on a spring with spring constant k is oscillating with amplitude A . Derive a general formula for the speed v of the mass when its position is x .

Answer: $v(x) = A \sqrt{\frac{k}{m}} \sqrt{1 - \left(\frac{x}{A}\right)^2}$

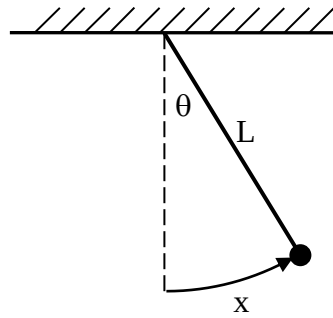
Be sure you understand these things:



Pendulum Motion

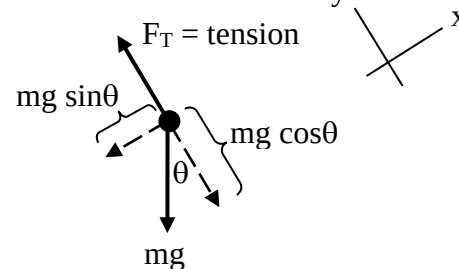
simple harmonic motion: periodic motion and simple harmonic motion

A simple pendulum consists of a small mass m suspended at the end of a massless string of length L . A pendulum executes SHM, if the amplitude is not too large.



$$\theta = x / L \text{ (rads)}$$

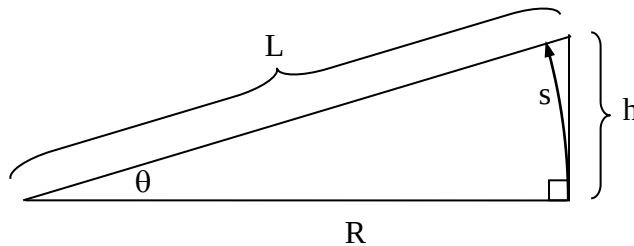
Forces on mass :



The restoring force is the component of the force along the direction of motion:

$$\text{restoring force} = -mg \sin \theta \cong -mg \theta = -mg \frac{x}{L}$$

Claim: $\sin \theta \cong \theta$ (rads) when θ is small. $\sin \theta = \frac{h}{L}$



$$\theta = \frac{s}{R}$$

If θ small, then $h \approx s$, and $L \approx R$, so $\sin \theta \approx \theta$.

Try it on your calculator: $\theta = 5^\circ = 0.087266.. \text{ rad}$, $\sin \theta = 0.087156..$

$F_{\text{restore}} = -\left(\frac{mg}{L}\right)x$ is exactly like Hooke's Law $F_{\text{restore}} = -kx$, except we have

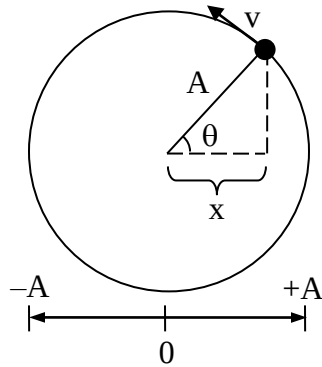
replaced the constant k with another constant (mg/L) . The math is exactly the same as with a mass on a spring; all results are the same, except we replace k with (mg/L) .

$$T_{\text{spring}} = 2\pi \sqrt{\frac{m}{k}} \Rightarrow T_{\text{pend}} = 2\pi \sqrt{\frac{m}{(mg/L)}} = 2\pi \sqrt{\frac{L}{g}}$$

Notice that the period is independent of the amplitude; the period depends only on length L and acceleration of gravity. (But this is true only if θ is not too large.)

Appendix: SHM and circular motion

There is an exact analogy between SHM and *circular motion*. Consider a particle moving with constant speed v around the rim of a circle of radius A . The x-component of the position of the particle has *exactly* the same mathematical form as the motion of a mass on a spring executing SHM with amplitude A .



$$\text{angular velocity } \omega = \frac{d\theta}{dt} = \text{const} \Rightarrow$$

$$\theta = \omega t \text{ so}$$

$$x = A \cos \theta = A \cos \omega t$$

This same formula also describes the *sinusoidal* motion of a mass on a spring.

That the same formula applies for two different situations (mass on a spring & circular motion) is no accident. The two situations have the same solution because they both obey the same equation. As Feynman said, "The same equations have the same

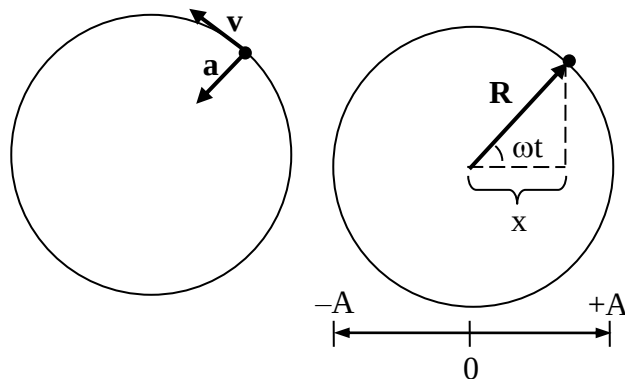
solutions". The equation of SHM is $\frac{d^2x}{dt^2} = -\omega^2 x$. We now show that a particle in circular motion obeys this same SHM equation.

Recall that for circular motion with angular speed ω , the acceleration of a the particle is toward the center and has magnitude $|\vec{a}| = \frac{v^2}{R}$. Since $v = \omega R$, we can rewrite this as

$$|\vec{a}| = \frac{(\omega R)^2}{R} = \omega^2 R$$

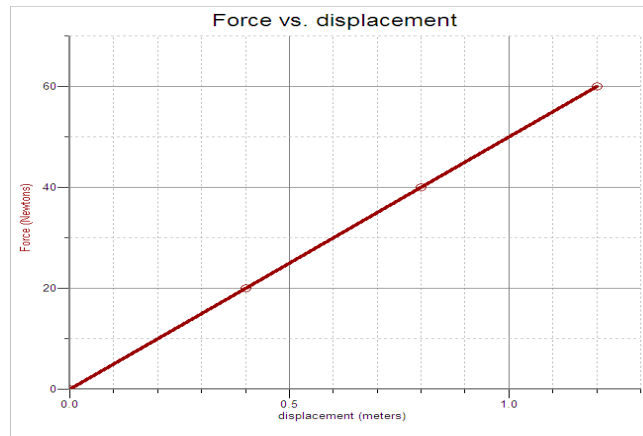
Let's set the origin at the center of the circle so the position vector $\mathbf{R}$ is along the radius. Notice that the acceleration vector $\mathbf{a}$ is always in the direction opposite the position vector $\mathbf{R}$. Since $|\vec{a}| = \omega^2 |\vec{R}|$, the vectors $\mathbf{a}$ and $\mathbf{R}$ are related by $\vec{a} = -\omega^2 \vec{R}$. The x-component of this vector equation is: $a_x = -\omega^2 R_x$. If we write $R_x = x$, then we have

$$\frac{d^2x}{dt^2} = -\omega^2 x, \text{ which is the SHM equation. Done.}$$



Example 1

A mass of 0.5 kg oscillates on the end of a spring on a horizontal surface with negligible friction according to the equation $x = A\cos(\omega t)$. The graph of F vs. x for this motion is shown below.



The last data point corresponds to the maximum displacement of the mass.

Determine the

- angular frequency ω of the oscillation,
- frequency f of oscillation,
- amplitude of oscillation,
- displacement from equilibrium position ($x = 0$) at a time of 2 s.

Solution:

(a) We know that the spring constant $k = 50 \text{ N/m}$ from when we looked at this graph earlier. So,

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{50 \text{ N/m}}{0.5 \text{ kg}}} = 10 \frac{\text{rad}}{\text{s}}$$

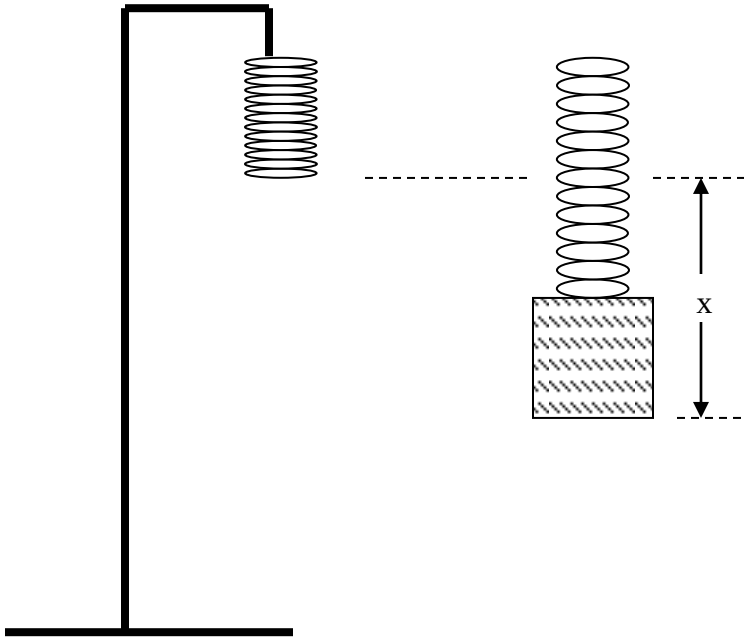
$$(b) \quad f = \frac{\omega}{2\pi} = \frac{10 \text{ rad/s}}{2\pi} = 1.6 \text{ Hz}$$

(c) The amplitude corresponds to the last displacement on the graph, $A = 1.2 \text{ m}$.

$$(d) \quad x = A\cos(\omega t) = (1.2 \text{ m})\cos[(10 \text{ rad/s})(2 \text{ s})] = 0.5 \text{ m}$$

Example 2

A spring of constant $k = 100 \text{ N/m}$ hangs at its natural length from a fixed stand. A mass of 3 kg is hung on the end of the spring, and slowly let down until the spring and mass hang at their new equilibrium position.



- Find the value of the quantity x in the figure above. The spring is now pulled down an additional distance x and released from rest.
- What is the potential energy in the spring at this distance?
- What is the speed of the mass as it passes the equilibrium position?
- How high above the point of release will the mass rise?
- What is the period of oscillation for the mass?

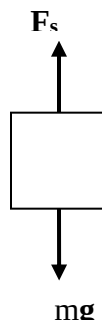
Solution:

(a) As it hangs in equilibrium, the upward spring force must be equal and opposite to the downward weight of the block.

$$F_s = mg$$

$$kx = mg$$

$$x = \frac{mg}{k} = \frac{(3\text{ kg})(10 \text{ m/s}^2)}{100 \text{ N/m}} = 0.3 \text{ m}$$



(b) The potential energy in the spring is related to the displacement from equilibrium position by the equation

$$U = \frac{1}{2}kx^2 = \frac{1}{2}(100 \text{ N/m})(0.3\text{ m})^2 = 4.5 \text{ J}$$

(c) Since energy is conserved during the oscillation of the mass, the kinetic energy of the mass as it passes through the equilibrium position is equal to the potential energy at the amplitude. Thus,

$$K = U = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2U}{m}} = \sqrt{\frac{2(4.5 J)}{3 kg}} = 1.7 m/s$$

(d) Since the amplitude of the oscillation is 0.3 m, it will rise to 0.3 m above the equilibrium position.

$$(e) T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{3 kg}{100 N/m}} = 1.1 s$$

Example 3

A pendulum of mass 0.4 kg and length 0.6 m is pulled back and released from an angle of 10° to the vertical.

(a) What is the potential energy of the mass at the instant it is released. Choose potential energy to be zero at the bottom of the swing.

(b) What is the speed of the mass as it passes its lowest point?

This same pendulum is taken to another planet where its period is 1.0 second.

(c) What is the acceleration due to gravity on this planet?

Solution

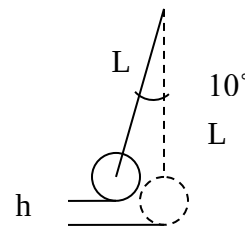
(a) First we must find the height above the lowest point in the swing at the instant the pendulum is released.

Recall from chapter 1 of this study guide that $h = L - L \cos \theta$.

Then

$$U = mg(L - L \cos \theta)$$

$$U = (0.4 kg)(10 m/s^2)(0.6 m - 0.6 m \cos 10^\circ) = 0.4 J$$



(b) Conservation of energy:

$$U_{\max} = K_{\max} = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2U}{m}} = \sqrt{\frac{2(0.4 J)}{0.4 kg}} = 1.4 m/s$$

(c)

$$T = 2\pi \sqrt{\frac{L}{g}}$$

$$g = \frac{4\pi^2 L}{T^2} = \frac{4\pi^2 (0.6\text{ m})}{(1.0\text{ s})^2} = 23.7 \frac{\text{m}}{\text{s}^2}$$

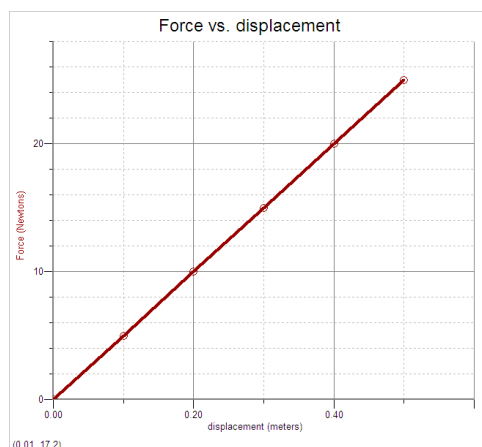
REVIEW QUESTIONS

For each of the multiple-choice questions below, choose the best answer.

Unless otherwise noted, use $g = 10 \text{ m/s}^2$ and neglect air resistance.

1. According to Hooke's law for an ideal spring, doubling the stretch distance will
 - (A) double the velocity of the mass.
 - (B) double the force that the spring exerts on the mass.
 - (C) quadruple the force the spring exerts on the mass.
 - (D) double the period.
 - (E) double the frequency.

Questions 2 – 3: Consider the *force vs displacement* graph shown for an ideal spring.



2. The work done in stretching the spring from 0.1 m to 0.5 m is

- (A) 1 J
- (B) 4 J
- (C) 6 J
- (D) 12 J
- (E) 24 J

3. The spring constant k is equal to

- (A) 5 N/m
- (B) 10 N/m
- (C) 20 N/m
- (D) 25 N/m
- (E) 50 N/m

Questions 4 – 6:

A pendulum of length L swings with an amplitude θ and a frequency f as shown above.

4. If the amplitude is increased and the pendulum is released from a greater angle,
 - (A) the period will decrease.
 - (B) the period will increase.
 - (C) the period will not change.
 - (D) the frequency will increase.
 - (E) the frequency will decrease.

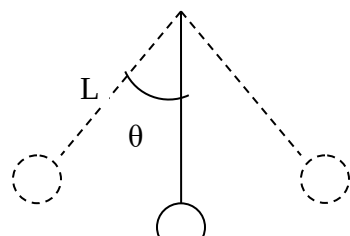
5. If the mass and the length of the pendulum are both doubled, the frequency of vibration will be

- (A) f
- (B) $2f$
- (C) $4f$
- (D) $\frac{1}{2}f$
- (E) $\frac{1}{4}f$

6. Which of the following statements is true about the swinging pendulum?

- I. The greatest restoring force and the greatest velocity occur at the same point.
- II. The greatest restoring force and the greatest acceleration occur at the same point.
- III. The greatest acceleration and the greatest velocity occur at the same point.

- (A) I only
- (B) I and II only
- (C) II only
- (D) I and III only
- (E) I, II, and III



Questions 7 – 9:

The equation which describes the motion of a mass oscillating on an ideal spring is

$$x = 6 \cos 3t$$

where x is in centimeters and t is in seconds.

7. The amplitude of the harmonic motion is

- (A) 3 cm
- (B) 6 cm
- (C) 9 cm
- (D) 18 cm
- (E) 30 cm

8. The period of vibration for this mass on a spring is most nearly

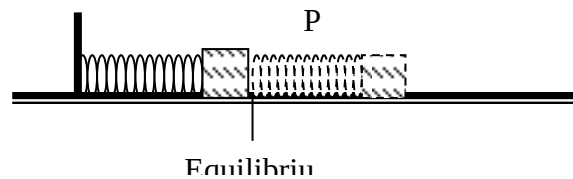
- (A) 1 s
- (B) 2 s
- (C) 3 s
- (D) 6 s
- (E) 9 s

9. The total distance traveled by the mass during one full oscillation is

- (A) 3 cm
- (B) 6 cm
- (C) 12 cm
- (D) 18 cm
- (E) 24 cm

10. A mass vibrates on an ideal spring as shown above. The total energy of the spring is 100 J. What is the kinetic energy of the mass at point P, halfway between the equilibrium point and the amplitude?

- (A) 25 J
- (B) 50 J
- (C) 75 J
- (D) 100 J
- (E) 200 J



ANSWERS AND EXPLANATIONS TO REVIEW QUESTIONS

Multiple Choice

1. B

Since the force is proportional to the stretch distance, the force would double if the stretch distance doubled.

2. C

The work done by the spring from 0.1 m to 0.5 m is equal to the area under the graph between those two points.

$$A = \frac{1}{2}(20\text{ N})(0.4\text{ m}) + (5\text{ N})(0.4\text{ m}) = 6\text{ J}$$

3. E

$$k = \text{slope} = \frac{25\text{ N}}{0.5\text{ m}} = 50 \frac{\text{N}}{\text{m}}$$

4. C

The period of a pendulum does not depend on the amplitude of swing for small swings.

5. D

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{L}}. \text{ Thus, } f \propto \sqrt{\frac{1}{L}}, \text{ and } \frac{1}{2} f \propto \sqrt{\frac{1}{4L}}$$

6. C

Newton's second law states that the acceleration is proportional to the force. The restoring force and the acceleration are the greatest at the amplitude of swing.

7. B

The amplitude is the the constant which appears in front of the cosine of the angle.

8. B

$$T = \frac{2\pi}{\omega} \approx \frac{2(3)}{3} = 2\text{ s}$$

9. E

The total distance moved during one full oscillation would be four times the amplitude of the motion: $4(6\text{ cm}) = 24\text{ cm}$.

10. B

At point B, the mass is halfway between the equilibrium position, where the kinetic energy is 100 J, and the amplitude, where the kinetic energy is zero. Thus, the kinetic energy at point B is 50 J.

QUICK REFERENCE

Important Terms

amplitude

maximum displacement from equilibrium position; the distance from the midpoint of a wave to its crest or trough.

equilibrium position

the position about which an object in harmonic motion oscillates; the center of vibration

frequency

the number of vibrations per unit of time

Hooke's law

law that states that the restoring force applied by a spring is proportional to the displacement of the spring and opposite in direction

ideal spring

any spring that obeys Hooke's law and does not dissipate energy within the spring.

mechanical resonance

condition in which natural oscillation frequency equals frequency of a driving force

period

the time for one complete cycle of oscillation

periodic motion

motion that repeats itself at regular intervals of time

restoring force

the force acting on an oscillating object which is proportional to the displacement and always points toward the equilibrium position.

simple harmonic motion

regular, repeated, friction-free motion in which the restoring force has the mathematical form $F = -kx$.

Equations and Symbols

$$F_s = -kx$$

$$PE_{\text{elastic}} = \frac{1}{2}kx^2$$

$$x = A \cos \omega t$$

$$\omega = \frac{2\pi}{T} = 2\pi f$$

$$T_s = 2\pi \sqrt{\frac{m}{k}}$$

$$T_p = 2\pi \sqrt{\frac{l}{g}}$$

$$T = \frac{1}{f}$$

where

F_s = the restoring force of the spring

k = spring constant

x = displacement from equilibrium position

PE_{elastic} = elastic (spring) potential energy (denoted U_s on the AP exam)

A = *amplitude*

ω = angular frequency

T = period

f = frequency

m = mass

T_p = period of a pendulum

T_s = period of a mass on a spring

g = acceleration due to gravity

Week-3-