Energy and its conservation, work, energy, and power

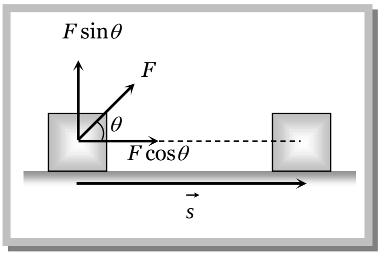
1. Introduction

The terms 'work', 'energy', and 'power' are frequently used in everyday language. A farmer clearing weeds in his field is said to be working hard. A woman carrying water from a well to her house is said to be working. In a drought-affected region, she may be required to carry it over large distances. If she can do so, she is said to have a large stamina or energy. Energy is thus the capacity to do work. The term power is usually associated with speed. In karate, a powerful punch is one delivered at great speed. In physics, we shall define these terms very precisely. We shall find that there is at best a loose correlation between the physical definitions and the physiological pictures these terms generate in our minds.

Work is said to be done when a force applied on the body displaces the body through a certain distance in the direction of force.

2. Work Done by a Constant Force

Let a constant force $\vec{F}$ be applied on the body such that it makes an angle θ with the horizontal and body is displaced through a distance s



By resolving force $\vec{F}$ into two components:

(i) $F\cos\theta$ in the direction of displacement of the body.

(ii) $F\sin\theta$ in the perpendicular direction of displacement of the body.

Since body is being displaced in the direction of $F\cos\theta$, therefore work done by the force in displacing the body through a distance s is given by

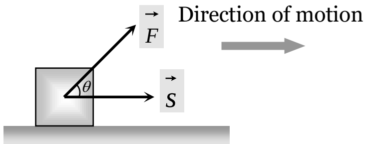
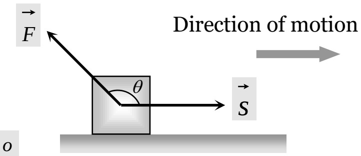
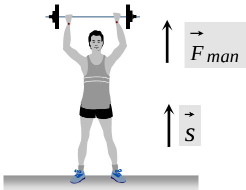
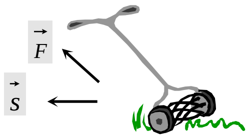
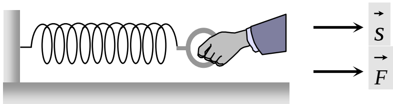
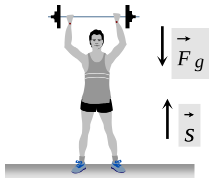
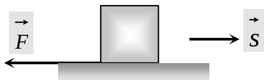
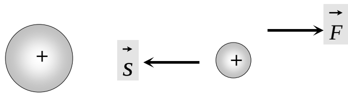
$$W = (F\cos\theta)s = Fs\cos\theta$$

or $$W = \vec{F}.\vec{s}$$

Thus work done by a force is equal to the scalar or dot product of the force and the displacement of the body.

If a number of force $\vec{F}_1, \vec{F}_2, \vec{F}_3 ...... \vec{F}_n$ are acting on a body and it shifts from position vector $\vec{r}_1$ to position vector $\vec{r}_2$ then $W = (\vec{F}_1 + \vec{F}_2 + \vec{F}_3 + ... \vec{F}_n).(\vec{r}_2 - \vec{r}_1)$

3. Nature of Work Done

Positive work	Negative work
Positive work means that force (or its component) is parallel to displacement  $0^o \le \theta < 90^o$ The positive work signifies that the external force favours the motion of the body.	Negative work means that force (or its component) is opposite to displacement *i.e.*  $90^o < \theta \le 180^o$ The negative work signifies that the external force opposes the motion of the body.
Example: (i) When a person lifts a body from the ground, the work done by the (upward) lifting force is positive  (ii) When a lawn roller is pulled by applying a force along the handle at an acute angle, work done by the applied force is positive.  (iii) When a spring is stretched, work done by the external (stretching) force is positive.  	*Example*: (i) When a person lifts a body from the ground, the work done by the (downward) force of gravity is negative.  (ii) When a body is made to slide over a rough surface, the work done by the frictional force is negative.  (iii) When a positive charge is moved towards another positive charge. The work done by electrostatic force between them is negative. 
Maximum work : $W_{max} = F s$ When $\cos\theta = \text{maximum} = 1$ *i.e.* $\theta = 0^o$ It means force does maximum work when angle between force and displacement is zero.	Minimum work : $W_{min} = -F s$ When $\cos\theta = \text{minimum} = -1$ *i.e* $\theta = 180^o$ It means force does minimum [maximum negative] work when angle between force and displacement is 180°.

Zero work

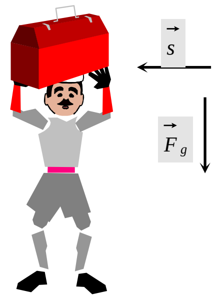
Under three conditions, work done becomes zero $W = Fs\cos\theta = 0$

(1) If the force is perpendicular to the displacement $[\vec{F} \perp \vec{s}\,]$

Example: (i) When a coolie travels on a horizontal platform with a load on his head, work done <u>against gravity</u> by the coolie is zero.(Work done against friction is +ve)

(ii) When a body moves in a circle the work done by the centripetal force is always zero.

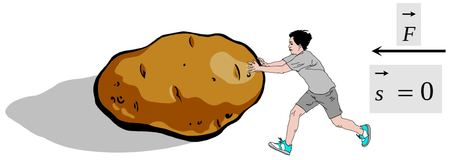
(iii) In case of motion of a charged particle in a magnetic field as force $[\vec{F} = q(\vec{v} \times \vec{B})]$ is always perpendicular to motion, work done by this force is always zero.



(2) If there is no displacement [*s* = 0]

Example: (i) When a person tries to displace a wall or heavy stone by applying a force then it does not move, the work done is zero.

(ii) A weight lifter does work in lifting the weight off the ground but does not work in holding it up.



(3) If there is no force acting on the body [*F* = 0]

Example: Motion of an isolated body in free space.

Sample Problems based on work done by constant force

Problem 1. A body of mass 5 *kg* is placed at the origin, and can move only on the *x*-axis. A force of 10 *N* is acting on it in a direction making an angle of 60^o with the *x*-axis and displaces it along the *x*-axis by 4 *metres*. The work done by the force is

(a) 2.5 *J* (b) 7.25 *J* (c) 40 *J* (d) 20 *J*

Solution : (d) Work done $= \vec{F}.\vec{s} = F\,s\cos\theta = 10 \times 4 \times \cos 60^o = 20\,J$

Problem 2. A force $F = (5\hat{i} + 3\hat{j})\,N$ is applied over a particle which displaces it from its origin to the point $r = (2\hat{i} - 1\hat{j})$ *metres*. The work done on the particle is

(a) −7 *J* (b) +13 *J* (c) +7 *J* (d) +11 *J*

Solution : (c) Work done $= \vec{F}.\vec{r} = (5i + 3\hat{j}).(2\hat{i} - \hat{j}) = 10 - 3 = +7\,J$

Problem 3. A horizontal force of 5 *N* is required to maintain a velocity of 2 *m/s* for a block of 10 *kg* mass sliding over a rough surface. The work done by this force in one minute is

(a) 600 *J* (b) 60 *J* (c) 6 *J* (d) 6000 *J*

Solution : (a) Work done = Force × displacement = $F \times s = F \times v \times t = 5 \times 2 \times 60 = 600\,J$.

Problem 4. A box of mass 1 *kg* is pulled on a horizontal plane of length 1 *m* by a force of 8 *N* then it is raised vertically to a height of 2*m*, the net work done is

(a) 28 *J* (b) 8 *J* (c) 18 *J* (d) None of above

Solution : (a) Work done to displace it horizontally = $F \times s = 8 \times 1 = 8\,J$

Work done to raise it vertically $F \times s = mgh = 1 \times 10 \times 2 = 20\,J$

$\therefore$ Net work done = 8 +20 = 28 *J*

Problem 5. A 10 *kg* satellite completes one revolution around the earth at a height of 100 *km* in 108 *minutes*. The work done by the gravitational force of earth will be

(a) $108 \times 100 \times 10\,J$ (b) $\frac{108 \times 10}{100}\,J$ (c) $\frac{100 \times 10}{108}\,J$ (d) Zero

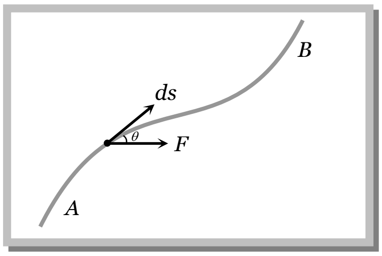
Solution : (d) Work done by centripetal force in circular motion is always equal to zero.

4. Work Done by a Variable Force

When the magnitude and direction of a force varies with position, the work done by such a force for an infinitesimal displacement is given by $dW = \vec{F}.\vec{ds}$

The total work done in going from A to B as shown in the figure is

$$W = \int_A^B \vec{F}.\vec{ds} = \int_A^B (F\cos\theta)ds$$



In terms of rectangular component $\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

$$\vec{ds} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$\therefore\ W = \int_A^B (F_x\hat{i} + F_y\hat{j} + F_z\hat{k}).(dx\hat{i} + dy\hat{j} + dz\hat{k})$$

or $W = \int_{x_A}^{x_B} F_x\,dx + \int_{y_A}^{y_B} F_y\,dy + \int_{z_A}^{z_B} F_z\,dz$

Sample Problems based on work done by variable force

***Problem* 1.** A position dependent force $\vec{F} = (7 - 2x + 3x^2)N$ acts on a small abject of mass 2 *kg* to displace it from $x = 0$ to $x = 5m$. The work done in joule is **[CBSE PMT 1994]**

(a) 70 J (b) 270 J (c) 35 J (d) 135 J

Solution : (d) Work done $= \int_{x_1}^{x_2} F\,dx = \int_0^5 (7 - 2x + 3x^2)dx = [7x - x^2 + x^3]_0^5 = 35 - 25 + 125 = 135\ J$

***Problem* 2.** A particle moves under the effect of a force $F = Cx$ from $x = 0$ to $x = x_1$. The work done in the process is

[CPMT 1982]

(a) Cx_1^2 (b) $\frac{1}{2}Cx_1^2$ (c) Cx_1 (d) Zero

Solution : (b) Work done $= \int_{x_1}^{x_2} F\,dx = \int_0^{x_1} Cx\,dx = C\left[\frac{x^2}{2}\right]_0^{x_1} = \frac{1}{2}C\,x_1^2$

***Problem* 3.** The vessels A and B of equal volume and weight are immersed in water to a depth h. The vessel A has an opening at the bottom through which water can enter. If the work done in immersing A and B are W_A and W_B respectively, then

(a) $W_A = W_B$ (b) $W_A < W_B$ (c) $W_A > W_B$ (d) $W_A >=< W_B$

Solution : (b) When the vessels are immersed in water, work has to be done against up-thrust force but due to opening at the bottom in vessel A, up-thrust force goes on decreasing. So work done will be less in this case.

***Problem* 4.** Work done in time t on a body of mass m which is accelerated from rest to a speed v in time t_1 as a function of time t is given by

(a) $\frac{1}{2}m\frac{v}{t_1}t^2$ (b) $m\frac{v}{t_1}t^2$ (c) $\frac{1}{2}\left(\frac{mv}{t_1}\right)^2 t^2$ (d) $\frac{1}{2}m\frac{v^2}{t_1^2}t^2$

Solution : (d) Work done $= F.s = ma.\left(\frac{1}{2}at^2\right) = \frac{1}{2}ma^2t^2 = \frac{1}{2}m\left(\frac{v}{t_1}\right)^2 t^2$ $\left[\text{As acceleration } (a) = \frac{v}{t_1} \text{ given}\right]$

5. Dimension and Units of Work

Dimension : As work = Force × displacement

$\therefore$ $[W]$ = [Force] × [Displacement]

$= [MLT^{-2}] \times [L] = [ML^2T^{-2}]$

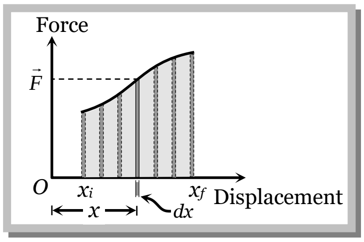
Units : The units of work are of two types

Absolute units	Gravitational units
Joule [S.I.]: Work done is said to be one *Joule*, when 1 *Newton* force displaces the body through 1 *meter* in its own direction. From $W = F.s$ 1 *Joule* = 1 *Newton* × 1 *metre*	*kg-m* [S.I.]: 1 *Kg-m* of work is done when a force of 1*kg-wt*. displaces the body through 1*m* in its own direction. From $W = F\,s$ 1 *kg-m* = 1 *kg-wt* × 1 *metre* = 9.81 *N* × 1 *metre* = 9.81 *Joule*
Erg [C.G.S.] : Work done is said to be one *erg* when 1 *dyne* force displaces the body through 1 *cm* in its own direction. From $W = F\,s$ $1\,Erg = 1 Dyne \times 1 cm$ *Relation between Joule and erg* 1 *Joule* = 1 *N* × 1 *m* = 10^5 *dyne* × 10^2 *cm* = 10^7 *dyne* × *cm* = 10^7 *Erg*	*gm-cm* [C.G.S.] : 1 *gm-cm* of work is done when a force of 1*gm-wt* displaces the body through 1*cm* in its own direction. From $W = F\,s$ 1 *gm-cm* = 1*gm-wt* × 1*cm*. = 981 *dyne* × 1*cm* = 981 *erg*

6. Work Done Calculation by Force Displacement Graph

Let a body, whose initial position is x_i, is acted upon by a variable force (whose magnitude is changing continuously) and consequently the body acquires its final position x_f.

Let $\vec{F}$ be the average value of variable force within the interval *dx* from position *x* to (*x* + *dx*) *i.e.* for small displacement *dx*. The work done will be the area of the shaded strip of width *dx*. The work done on the body in displacing it from position x_i to x_f will be equal to the sum of areas of all the such strips



$dW = \vec{F}\,dx$

$\therefore W = \int_{x_i}^{x_f} dW = \int_{x_i}^{x_f} \vec{F}\,dx$

$\therefore W = \int_{x_i}^{x_f}$ (Area of strip of width dx)

$\therefore W =$ Area under curve Between x_i and x_f

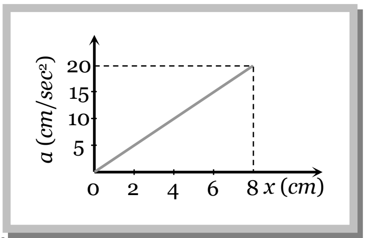
i.e. Area under force displacement curve with proper algebraic sign represents work done by the force.

Sample problems based on force displacement graph

Problem 1. A 10 *kg* mass moves along *x*-axis. Its acceleration as a function of its position is shown in the figure. What is the total work done on the mass by the force as the mass moves from $x = 0$ to $x = 8\ cm$ **[AMU (Med.) 2000]**

(a) $8 \times 10^{-2}\ J$ (b) $16 \times 10^{-2}\ J$

(c) $4 \times 10^{-4}\ J$ (d) $1.6 \times 10^{-3}\ J$



Solution : (a) Work done on the mass = mass × covered area between the graph and displacement axis on *a-t* graph.

$$= 10 \times \frac{1}{2}(8 \times 10^{-2}) \times 20 \times 10^{-2} = 8 \times 10^{-2}\ J.$$

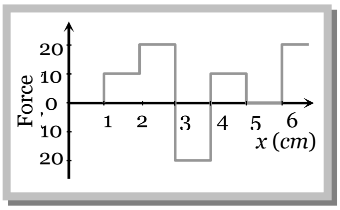
Problem 2. The relationship between force and position is shown in the figure given (in one dimensional case). The work done by the force in displacing a body from $x = 1\ cm$ to $x = 5\ cm$ is

(a) 20 *ergs*

(b) 60 *ergs*

(c) 70 *ergs*

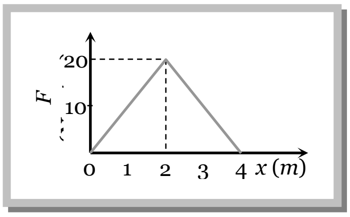
(d) 700 *ergs*



Solution : (a) Work done = Covered area on force-displacement graph = 1 × 10 + 1 × 20 – 1 × 20 + 1 × 10 = 20 *erg*.

Problem 3. The graph between the resistive force *F* acting on a body and the distance covered by the body is shown in the figure. The mass of the body is 25 *kg* and initial velocity is 2 *m/s*. When the distance covered by the body is $5m$, its kinetic energy would be

(a) 50 *J*
(b) 40 *J*
(c) 20 *J*
(d) 10 *J*



Solution : (d) Initial kinetic energy of the body $= \frac{1}{2}mu^2 = \frac{1}{2} \times 25 \times (2)^2 = 50\ J$

Final kinetic energy = Initial energy – work done against resistive force (Area between graph and displacement axis) $= 50 - \frac{1}{2} \times 4 \times 20 = 50 - 40 = 10\ J.$

7. Energy

The energy of a body is defined as its capacity for doing work.

(1) Since energy of a body is the total quantity of work done therefore it is a scalar quantity.

(2) Dimension: $[ML^2T^{-2}]$ it is same as that of work or torque.

(3) Units : *Joule* [S.I.], *erg* [C.G.S.]

Practical units : *electron volt* (*eV*), Kilowatt hour (*KWh*), Calories (*Cal*)

Relation between different units: $1\ Joule = 10^7\ erg$

$$1\ eV = 1.6 \times 10^{-19}\ Joule$$

$$1\ KWh = 3.6 \times 10^6\ Joule$$

$$1\ Calorie = 4.18\ Joule$$

(4) Mass energy equivalence : Einstein's special theory of relativity shows that material particle itself is a form of energy.

The relation between the mass of a particle *m* and its equivalent energy is given as

$E = mc^2$ where *c* = velocity of light in vacuum.

If $m = 1\ amu = 1.67 \times 10^{-27}\ kg$ then $E = 931\ MeV = 1.5 \times 10^{-10}\ Joule$.

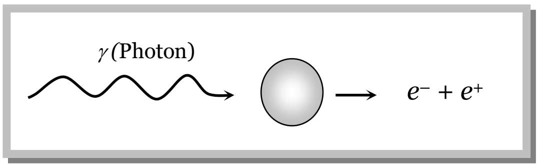
If $m = 1 kg$ then $E = 9 \times 10^{16}\ Joule$

Examples : (i) Annihilation of matter when an electron (e^-) and a positron (e^+) combine with each other, they annihilate or destroy each other. The masses of electron and positron are converted into energy. This energy is released in the form of γ -rays.

$$e^- + e^+ \rightarrow \gamma + \gamma$$

Each γ photon has energy = 0.51 MeV.
Here two γ photons are emitted instead of one γ photon to conserve the linear momentum.

(ii) Pair production : This process is the reverse of annihilation of matter. In this case, a photon (γ) having energy equal to 1.02 MeV interacts with a nucleus and give rise to electron (e^-) and positron (e^+). This energy is converted into matter.

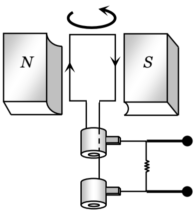
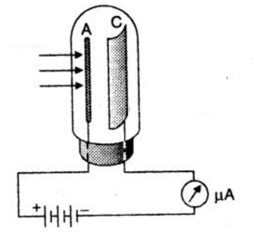
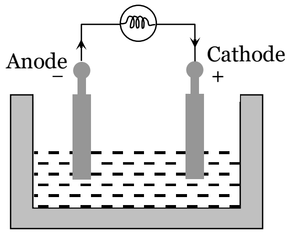
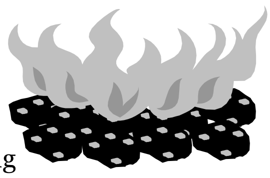
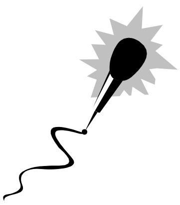
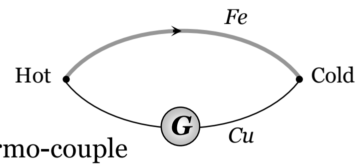
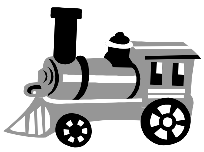
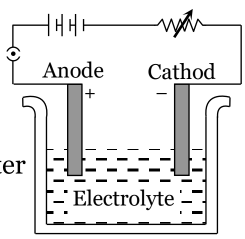
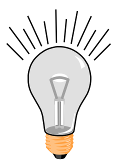


(iii) Nuclear bomb : When the nucleus is split up due to mass defect (The difference in the mass of nucleons and the nucleus) energy is released in the form of γ -radiations and heat.

(5) Various forms of energy

(i) Mechanical energy (Kinetic and Potential) (ii) Chemical energy (iii) Electrical energy
(iv) Magnetic energy (v) Nuclear energy (vi) Sound energy
(vii) Light energy (viii) Heat energy

(6) Transformation of energy : Conversion of energy from one form to another is possible through various devices and processes.

Mechanical → electrical	**Light → Electrical**	**Chemical → electrical**
Dynamo 	Photoelectric cell 	Primary cell 
Chemical → heat	**Sounds → Electrical**	**Heat → electrical**
Coal Burning 	Microphone 	Thermo-couple 
Heat → Mechanical	**Electrical → Mechanical**	**Electrical → Heat**
Engine 	Motor 	Heater 
Electrical → Sound	**Electrical → Chemical**	**Electrical → Light**
Speaker 	Voltameter 	Bulb 

Sample problems based on energy

Problem 1. A particle of mass 'm' and charge 'q' is accelerated through a potential difference of 'V' *volt*. Its energy is

[UPSEAT 2001]

(a) qV (b) mqV (c) $\left(\frac{q}{m}\right)V$ (d) $\frac{q}{mV}$

Solution : (a) Energy of charged particle = charge × potential difference = qV

Problem 2. An ice cream has a marked value of 700 *kcal*. How many kilowatt hour of energy will it deliver to the body as it is digested

(a) 0.81 kWh (b) 0.90 kWh (c) 1.11 kWh (d) 0.71 kWh

Solution : (a) 700 $k\,cal = 700\times 10^3 \times 4.2\,J = \frac{700\times 10^3 \times 4.2}{3.6\times 10^6} = 0.81\,kWh$ [As $3.6\times 10^6\,J = 1\,kWh$]

Problem 3. A metallic wire of length L *metres* extends by l *metres* when stretched by suspending a weight Mg to it. The mechanical energy stored in the wire is

(a) $2Mgl$ (b) Mgl (c) $\frac{Mgl}{2}$ (d) $\frac{Mgl}{4}$

Solution : (c) Elastic potential energy stored in wire $U = \frac{1}{2}Fx = \frac{Mgl}{2}$.

8. Law of Conservation of Energy

(1) **Law of conservation of energy**

For a body or an isolated system by work-energy theorem we have $K_2 - K_1 = \int \vec{F}.d\vec{r}$ .....(i)

But according to definition of potential energy in a conservative field $U_2 - U_1 = -\int \vec{F}.d\vec{r}$ .....(ii)

So from equation (i) and (ii) we have

$$K_2 - K_1 = -(U_2 - U_1)$$

or $$K_2 + U_2 = K_1 + U_1$$

i.e. $K + U$ = constant.

For an isolated system or body in presence of conservative forces the sum of kinetic and potential energies at any point remains constant throughout the motion. It does not depends upon time. This is known as the law of conservation of mechanical energy.

$$\Delta(K+U) = \Delta E = 0$$ [As E is constant in a conservative field]

$\therefore$ $$\Delta K + \Delta U = 0$$

i.e. if the kinetic energy of the body increases its potential energy will decrease by an equal amount and vice-versa.

(2) **Law of conservation of total energy :** If some non-conservative force like friction is also acting on the particle, the mechanical energy is no more constant. It changes by the amount of work done by the frictional force.

$$\Delta(K+U) = \Delta E = W_f$$ [where W_f is the work done against friction]

The lost energy is transformed into heat and the heat energy developed is exactly equal to loss in mechanical energy.

We can, therefore, write $\Delta E + Q = 0$ [where Q is the heat produced]

This shows that if the forces are conservative and non-conservative both, it is not the mechanical energy alone which is conserved, but it is the total energy, may be heat, light, sound or mechanical *etc.*, which is conserved.

In other words : "*Energy may be transformed from one kind to another but it cannot be created or destroyed. The total energy in an isolated system is constant*". This is the law of conservation of energy.

Sample problems based on conservation of energy

Problem 1. Two stones each of mass $5kg$ fall on a wheel from a height of $10m$. The wheel stirs $2kg$ water. The rise in temperature of water would be

(a) $2.6° C$ (b) $1.2° C$ (c) $0.32° C$ (d) $0.12° C$

Solution : (d) For the given condition potential energy of the two masses will convert into heat and temperature of water will increase $W = JQ \Rightarrow 2m \times g \times h = J(m_w\ S\ \Delta t) \Rightarrow$

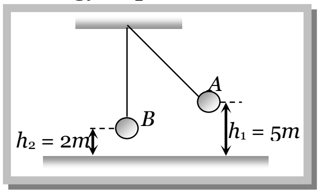
$$2 \times 5 \times 10 \times 10 = 4.2(2 \times 10^3 \times \Delta t)$$

$$\therefore\ \Delta t = \frac{1000}{8.4 \times 10^3} = 0.119\,^oC = 0.12\,^oC.$$

Problem 2. A boy is sitting on a swing at a maximum height of $5m$ above the ground. When the swing passes through the mean position which is $2m$ above the ground its velocity is approximately

(a) $7.6\ m/s$ (b) $9.8\ m/s$ (c) $6.26\ m/s$ (d) None of these

Solution : (a) By the conservation of energy Total energy at point A = Total energy at point B

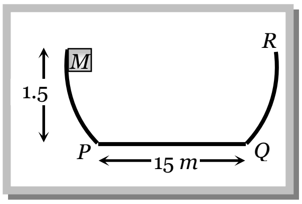


$$\Rightarrow\ mgh_1 = mgh_2 + \frac{1}{2}mv^2$$

$$\Rightarrow\ 9.8 \times 5 = 9.8 \times 2 + \frac{1}{2}v^2$$

$$\Rightarrow\ v^2 = 58.8\ \ \therefore\ v = 7.6\,m/s$$

Problem 3. A block of mass M slides along the sides of a bowl as shown in the figure. The walls of the bowl are frictionless and the base has coefficient of friction 0.2. If the block is released from the top of the side, which is 1.5 m high, where will the block come to rest ?
Given that the length of the base is 15 m



(a) 1 m from P
(b) Mid point
(c) 2 m from P
(d) At Q

Solution : (b) Potential energy of block at starting point = Kinetic energy at point P = Work done against friction in traveling a distance s from point P.

$$\therefore\quad mgh = \mu mgs\ \Rightarrow s = \frac{h}{\mu} = \frac{1.5}{0.2} = 7.5\,m$$

i.e. block come to rest at the mid point between P and Q.

Problem 4. If we throw a body upwards with velocity of $4\,ms^{-1}$ at what height its kinetic energy reduces to half of the initial value ? Take $g = 10\ m/s^2$

(a) $4m$ (b) $2\ m$ (c) $1\ m$ (d) None of these

Solution : (d) We know kinetic energy $K = \frac{1}{2}mv^2\ \ \therefore\ v \propto \sqrt{K}$

When kinetic energy of the body reduces to half its velocity becomes $v = \frac{u}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}\ m/s$

From the equation $v^2 = u^2 - 2gh \Rightarrow (2\sqrt{2})^2 = (4)^2 - 2 \times 10\,h\ \ \ \therefore\ h = \frac{16-8}{20} = 0.4\,m.$

Problem 5. A $2kg$ block is dropped from a height of 0.4 m on a spring of force constant $K = 1960\ Nm^{-1}$. The maximum compression of the spring is

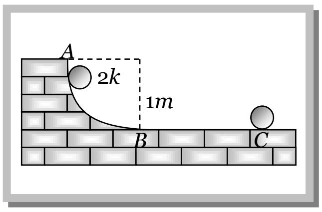
(a) $0.1\ m$ (b) $0.2\ m$ (c) $0.3\ m$ (d) $0.4\ m$

Solution : (a) When a block is dropped from a height, its potential energy gets converted into kinetic energy and finally spring get compressed due to this energy.

$\therefore$ Gravitational potential energy of block = Elastic potential energy of spring

$$\Rightarrow\ mgh = \frac{1}{2}Kx^2\ \Rightarrow x = \sqrt{\frac{2mgh}{K}} = \sqrt{\frac{2 \times 2 \times 10 \times 0.4}{1960}} = 0.09\,m \simeq 0.1\,m.$$

Problem 6. A block of mass $2kg$ is released from A on the track that is one quadrant of a circle of radius $1m$. It slides down the track and reaches B with a speed of $4\,ms^{-1}$ and finally stops at C at a distance of $3m$ from B. The work done against the force of friction is



(a) $10\,J$
(b) $20\,J$
(c) $2\,J$
(d) $6\,J$

Solution : (b) Block possess potential energy at point $A = mgh = 2 \times 10 \times 1 = 20\ J$

Finally block stops at point *C*. So its total energy goes against friction *i.e.* work done against friction is 20 *J*.

***Problem* 7.** A stone projected vertically upwards from the ground reaches a maximum height *h*. When it is at a height $\frac{3h}{4}$, the ratio of its kinetic and potential energies is

(a) 3 : 4 (b) 1 : 3 (c) 4 : 3 (d) 3 : 1

Solution : (b) At the maximum height, Total energy = Potential energy = *mgh*

At the height $\frac{3h}{4}$, Potential energy = $mg\frac{3h}{4} = \frac{3}{4}mgh$

and Kinetic energy = Total energy – Potential energy $= mgh - 3\frac{mgh}{4} = \frac{1}{4}mgh$

$\therefore \quad \frac{\text{Kinetic energy}}{\text{Potential energy}} = \frac{1}{3}$.

9. Power

Power of a body is defined as the rate at which the body can do the work.

Average power $(P_{av.}) = \frac{\Delta W}{\Delta t} = \frac{W}{t}$

Instantaneous power $(P_{inst.}) = \frac{dW}{dt} = \frac{\vec{F}.d\vec{s}}{dt}$ [As $dW = \vec{F}.d\vec{s}$]

$P_{inst} = \vec{F}.\vec{v}$ [As $\vec{v} = \frac{d\vec{s}}{dt}$]

i.e. power is equal to the scalar product of force with velocity.

Important points

(1) Dimension : $[P] = [F][v] = [MLT^{-2}][LT^{-1}]$

$\therefore \quad [P] = [ML^2T^{-3}]$

(2) Units : *Watt* or *Joule/sec* [S.I.]

Erg/sec [C.G.S.]

Practical units : *Kilowatt* (*kW*), *Mega watt* (*MW*) and Horse power (*hp*)

Relations between different units : $1\,watt = 1\,Joule/\sec = 10^7\,erg/\sec$

$$1\,hp = 746\ Watt$$

$$1\,MW = 10^6\ Watt$$

$$1\,kW = 10^3\ Watt$$

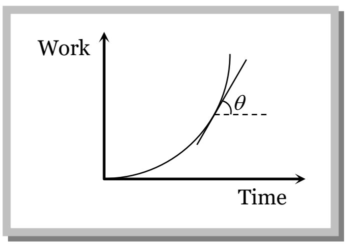
(3) If work done by the two bodies is same then power $\propto \frac{1}{\text{time}}$

i.e. the body which perform the given work in lesser time possess more power and vice-versa.

(4) As power = work/time, any unit of power multiplied by a unit of time gives unit of work (or energy) and not power, *i.e.* Kilowatt-hour or watt-day are units of work or energy.

$$1\,KWh = 10^3\frac{J}{sec}\times(60\times 60\,sec) = 3.6\times 10^6\ Joule$$

(5) The slope of work time curve gives the instantaneous power. As *P* = *dW*/*dt* = $\tan\theta$



(6) Area under power time curve gives the work done as $P = \frac{dW}{dt}$

$\therefore\ W = \int P\, dt$

$\therefore$ W = Area under P-t curve

Sample problems based on power

Problem 1. A car of mass 'm' is driven with acceleration 'a' along a straight level road against a constant external resistive force 'R'. When the velocity of the car is 'v', the rate at which the engine of the car is doing work will be **[MP PMT/PET 1998; JIMPER 2000]**

(a) Rv (b) mav (c) $(R + ma)v$ (d) $(ma - R)v$

Solution : (c) The engine has to do work against resistive force R as well as car is moving with acceleration a.

Power = Force × velocity = $(R+ma)v$.

Problem 2. A wind-powered generator converts wind energy into electrical energy. Assume that the generator converts a fixed fraction of the wind energy intercepted by its blades into electrical energy. For wind speed v, the electrical power output will be proportional to

(a) v (b) v^2 (c) v^3 (d) v^4

Solution : (c) Force $= v\frac{dm}{dt} = v\frac{d}{dt}(V \times \rho) = v\rho\frac{d}{dt}[A \times l] = v\rho A\frac{dl}{dt} = \rho Av^2$

Power = $F \times v = \rho Av^2 \times v = \rho Av^3 \quad \therefore\ P \propto v^3$.

Problem 3. A pump motor is used to deliver water at a certain rate from a given pipe. To obtain twice as much water from the same pipe in the same time, power of the motor has to be increased to

(a) 16 times (b) 4 times (c) 8 times (d) 2 times

Solution : (d) $P = \frac{\text{work done}}{\text{time}} = \frac{mgh}{t} \quad \therefore P \propto m$

i.e. To obtain twice water from the same pipe in the same time, the power of motor has to be increased to 2 times.

Problem 4. A force applied by an engine of a train of mass $2.05 \times 10^6\, kg$ changes its velocity from 5 m/s to 25 m/s in 5 minutes. The power of the engine is **[EAMCET 2001]**

(a) 1.025 MW (b) 2.05 MW (c) 5MW (d) 5 MW

Solution : (b) Power $= \frac{\text{Work done}}{\text{time}} = \frac{\text{Increase in kinetic energy}}{\text{time}} = \frac{\frac{1}{2}m(v_2^2 - v_1^2)}{t} = \frac{\frac{1}{2} \times 2.05 \times 10^6 \times [25^2 - 5^2]}{5 \times 60}$

$= 2.05 \times 10^6\, watt = 2.05\ MW$

Problem 5. From a water fall, water is falling at the rate of 100 kg/s on the blades of turbine. If the height of the fall is 100m then the power delivered to the turbine is approximately equal to

(a) 100kW (b) 10 kW (c) 1kW (d) 1000 kW

Solution : (a) Power $= \frac{\text{Work done}}{t} = \frac{mgh}{t} = 100 \times 10 \times 100 = 10^5\, watt = 100\ kW$ $\left[\text{As } \frac{m}{t} = 100\ \frac{kg}{\text{sec}} \text{(given)}\right]$

Problem 6. A particle moves with a velocity $\vec{v} = 5\hat{i} - 3\hat{j} + 6\hat{k}\ ms^{-1}$ under the influence of a constant force $\vec{F} = 10\hat{i} + 10\hat{j} + 20\hat{k}\ N$. The instantaneous power applied to the particle is

(a) 200 J-s^{-1} (b) 40 J-s^{-1} (c) 140 J-s^{-1} (d) 170 J-s^{-1}

Solution : (c) $P = \vec{F}.\vec{v} = (10\hat{i} + 10\hat{j} + 20\hat{k}).(5\hat{i} - 3\hat{j} + 6\hat{k}) = 50 - 30 + 120 = 140\ J\text{-}s^{-1}$

Problem 8. A car of mass 1250 kg experience a resistance of 750 N when it moves at $30 ms^{-1}$. If the engine can develop $30 kW$ at this speed, the maximum acceleration that the engine can produce is

(a) $0.8 ms^{-2}$ (b) $0.2 ms^{-2}$ (c) $0.4 ms^{-1}$ (d) $0.5 ms^{-2}$

Solution : (b) Power = Force × velocity = (Resistive force + Accelerating force) × velocity

$$\Rightarrow \; 30 \times 10^3 = (750 + ma) \times 30 \Rightarrow ma = 1000 - 750 \Rightarrow a = \frac{250}{1250} = 0.2\, ms^{-2}.$$

Problem 9. A bus weighing 100 quintals moves on a rough road with a constant speed of $72 km/h$. The friction of the road is 9% of its weight and that of air is 1% of its weight. What is the power of the engine. Take $g = 10 m/s^2$

(a) 50 kW (b) 100 kW (c) 150 kW (d) 200 kW

Solution : (d) Weight of a bus = mass × g $= 100 \times 100\, kg \times 10\, m/s^2 = 10^5\, N$

Total friction force = 10% of weight = 10^4 N

$\therefore$ Power = Force × velocity $= 10^4\, N \times 72\, km/h = 10^4 \times 20\, watt = 2 \times 10^5\, watt = 200\, kW$.

Problem 10. Two men with weights in the ratio 5 : 3 run up a staircase in times in the ratio 11 : 9. The ratio of power of first to that of second is

(a) $\frac{15}{11}$ (b) $\frac{11}{15}$ (c) $\frac{11}{9}$ (d) $\frac{9}{11}$

Solution : (a) Power $(P) = \frac{mgh}{t}$ or $P \propto \frac{m}{t} \Rightarrow \frac{P_1}{P_2} = \frac{m_1}{m_2}\frac{t_2}{t_1} = \left(\frac{5}{3}\right)\left(\frac{9}{11}\right) = \frac{45}{33} = \frac{15}{11}$ (g and h are constants)

Problem 11. A dam is situated at a height of 550 *metre* above sea level and supplies water to a power house which is at a height of 50 *metre* above sea level. 2000 kg of water passes through the turbines per second. The maximum electrical power output of the power house if the whole system were 80% efficient is

(a) 8 MW (b) 10 MW (c) 12.5 MW (d) 16 MW

Solution : (a) Power $= \frac{\text{work done}}{\text{time}} = \frac{mg\Delta h}{t} = \frac{2000 \times 10 \times (550 - 50)}{1} = 10\, MW$

But the system is 80% efficient $\therefore$ Power output = 10 × 80% = 8 MW.

Problem 12. A constant force F is applied on a body. The power (P) generated is related to the time elapsed (t) as

(a) $P \propto t^2$ (b) $P \propto t$ (c) $P \propto \sqrt{t}$ (d) $P \propto t^{3/2}$

Solution : (b) $F = \frac{mdv}{dt} \; \therefore F\,dt = mdv \Rightarrow v = \frac{F}{m}t$

Now $P = F \times v = F \times \frac{F}{m}t = \frac{F^2 t}{m}$. If force and mass are constants then $P \propto t$.

QUICK REFERENCE

Important Terms

conservative force
: a force which does work on an object which is independent of the path taken by the object between its starting point and its ending point

conserved properties
: any properties which remain constant during a process

energy
: the non-material quantity which is the ability to do work on a system

joule
: the unit for energy equal to one Newton-meter

law of conservation of energy
: the total energy of a system remains constant during a process

power
: the rate at which work is done or energy is dissipated

watt
: the SI unit for power equal to one joule of energy per second

work
: the scalar product of force and displacement

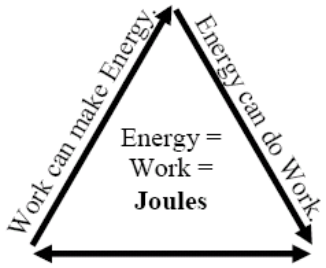
Summary

Energy and work are interconnected—one can make the other.

Energy

Energy is stored work.
A battery can store energy to make things work whenever you want.

Energy can cause forces,
which can cause motion,
which can do work.



Energy is Work.

Work

Work uses energy.
It takes energy to move things.
Energy can make things work.

Work can create energy.
A generator uses work to make energy, which can be stored to do more work.

Work

Work is defined as a force applied (moved) through a distance.

Work (in Joules) → $W = Fd$ ← Force (in newtons), distance (in meters)

Work equals force times distance.

If you push harder (**more force**) you do **more work**.

If you push longer (**more distance**) you do **more work**.

Power

How fast you do work is called **power**. If you work faster, you use more power.

Power (in watts) → $P = \frac{W}{t}$ ← Work (in joules), Time (in seconds)

Power equals work divided by time.

Putting in the work equation: $P = \frac{Fd}{t}$

A machine that **works faster** (in less time) is **more powerful**.

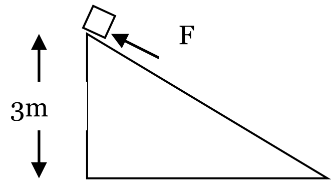
A **more powerful** light bulb gives off the **same** amount of light (**work**), it **just** does it **faster**.

H.W. *For each of the multiple choice questions below, choose the best answer.*
Unless otherwise noted, use $g = 10\ m/s^2$ and neglect air resistance.

1. Which of the following is NOT true of work?
(A) It is the scalar product of force and displacement.
(B) It is measured in joules.
(C) It has the same units as energy.
(D) It is a vector which is always in the same direction as the displacement.
(E) It takes energy to perform work

2. A 4-kg box is pushed across a level floor with a force of 60 N for a displacement of 20 m, then lifted to a height of 2 m. What is the total work done on the box?
(A) 200 J
(B) 400 J
(C) 1120 J
(D) 1200 J
(E) 1280 J

3. A 20-kg cart is pushed up the inclined plane shown by a force **F** to a height of 3 m. What is the potential energy of the cart when it reaches the top of the inclined plane?
(A) 1500 J
(B) 630 J
(C) 600 J
(D) 300 J
(E) 150 J



4. A ball falls from a height *h* from a tower. Which of the following statements is true?
(A) The potential energy of the ball is conserved as it falls.
(B) The kinetic energy of the ball is conserved as it falls.
(C) The difference between the potential energy and kinetic energy is a constant as the ball falls.
(D) The sum of the kinetic and potential energies of the ball is a constant.
(E) The velocity of the ball is constant as the ball falls.

5. A 0.5-kg ball is dropped from a third story window which is 20 m above the sidewalk. What is the speed of the ball just before it strikes the sidewalk?
(A) 5 m/s
(B) 10 m/s
(C) 14 m/s
(D) 20 m/s
(E) 200 m/s

ANSWERS AND EXPLANATIONS

Multiple Choice

1. D
Work is not a vector quantity.

2. E
$W_{total} = Fs + mgh = (60\,N)(20\,m) + (4\,kg)(10\,m/s^2)(2\,m) = 1280\,J$

3. C
The potential energy is equal to the work done against gravity:
$W = mgh = (20\,kg)(10\,m/s^2)(3\,m) = 600\,J$

4. D
As the ball falls, the potential energy decreases and the kinetic energy increases, but the sum of the two remains constant.

5. D
Conservation of energy:
$U_{top} = K_{bottom}$

$mgh = \frac{1}{2}mv^2$

$v = \sqrt{2gh} = \sqrt{2(10\,m/s^2)(20\,m)} = 20\,m/s$

Short Questions

Q.1 A person holds a bag of groceries while standing still, talking to a friend. A car is stationery with its engine running. From the stand point of work, how are these two situations similar?

Ans. A man holding a bag and standing still, and a car is stationery with its engine running, have zero value of displacements. In both of these two cases work done is zero. In this respect both the cases are similar.

Q.2 Calculate the work done in kilo joules in lifting a mass of 10kg (at a steady velocity) through a vertical height of 10m.

Ans. The work done against gravity is

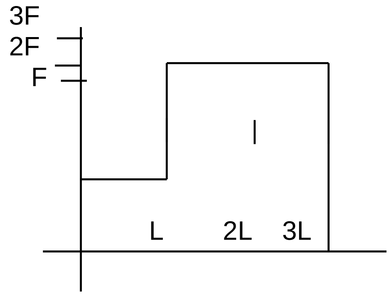
$$W = mgh$$

$$W = 10 \times 9.8 \times 10$$

$$W = 980\ J$$

Q.3 A force *F* acts through a distance L. the force is then increased to 3*F*, and then acts through a further distance of 2L. Draw the work diagram to scale.

Ans. The work diagram is shown in the graph.



Q.4 In which case is more work done? When a 50kg bag of books is lifted through 50cm, or when a 50kg crate is pushed through 2m across the floor with a force of 50N?

Ans. In first case

$W_1 = 50 \times 9.8 \times 0.5$

$W_1 = 245\ J$

In Second case

$W_2 = 2 \times 50$

$W_2 = 100\ J$

This shows that work done in the first case is larger.

Q.5 An object has 1J of potential energy. Explain what it means.

Ans. An object having 1J potential energy means that it has capacity to do work, 1 J work.