# Chapter five

# Equilibrium of Rigid Bodies

- For a rigid body in static equilibrium, the external forces and moments are balanced and will impart no translational or rotational motion to the body.

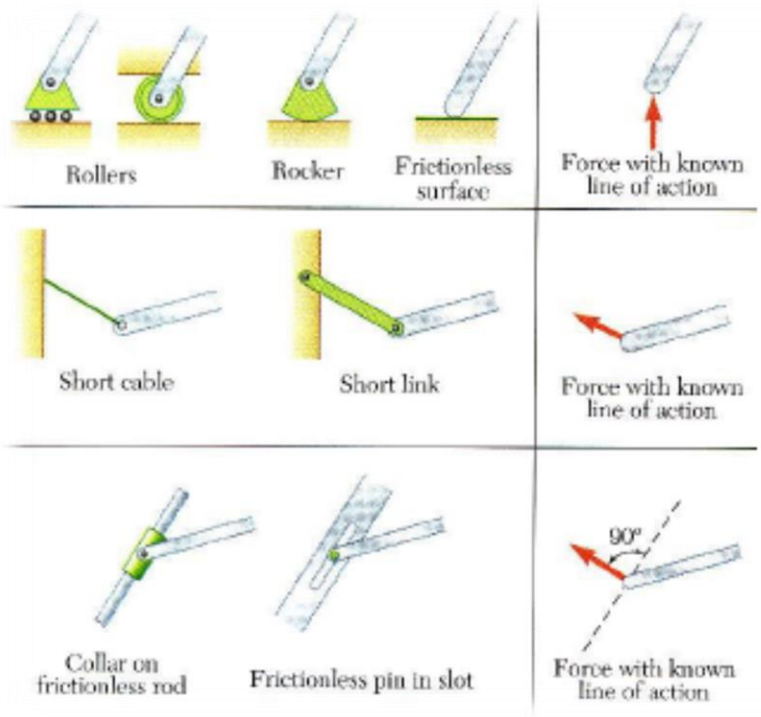
- The conditions for static equilibrium,

$$\sum F_x = 0 \quad \sum F_y = 0$$
$$\sum M_x = 0 \quad \sum M_y = 0$$

## Free-Body Diagram

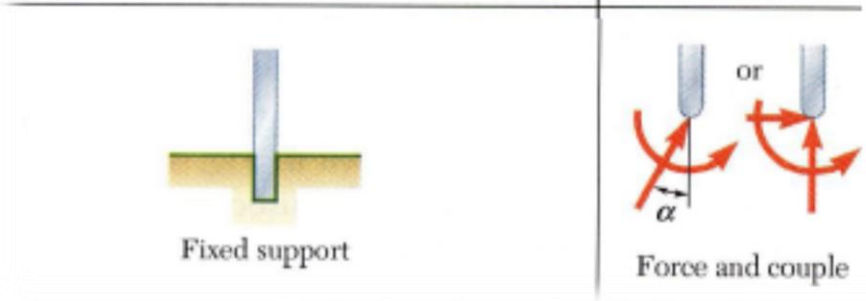
- First step in the static equilibrium analysis of a rigid body is identification of all forces acting on the body with a free-body diagram.

*Reactions at Supports and Connections for a Two-Dimensional Structure*

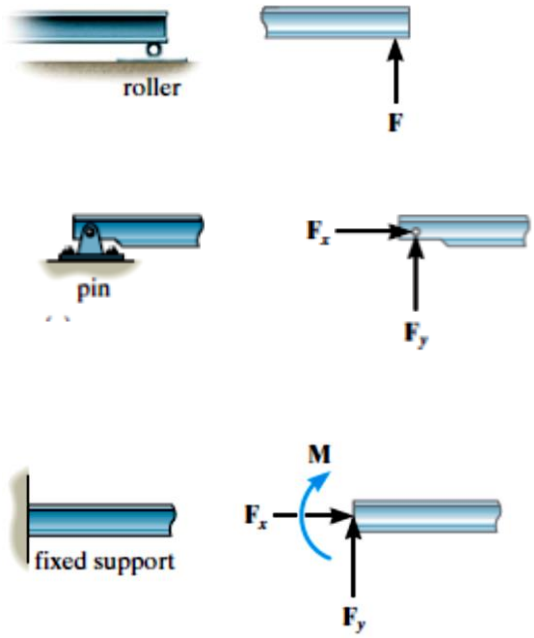



- Reactions equivalent to a force with known line of action.

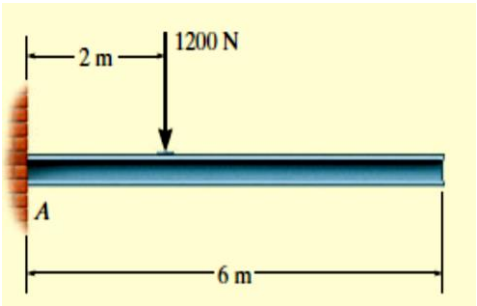

- Reactions equivalent to a force of unknown direction and magnitude.

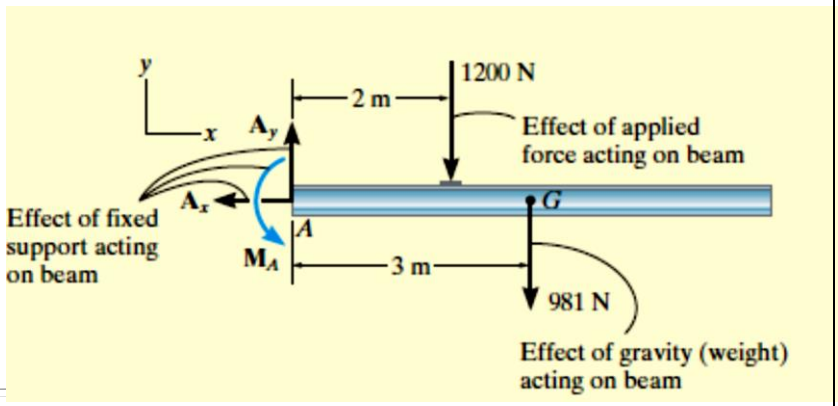



- Reactions equivalent to a force of unknown direction and magnitude and a couple of unknown magnitude




## Example of Free Body Diagram

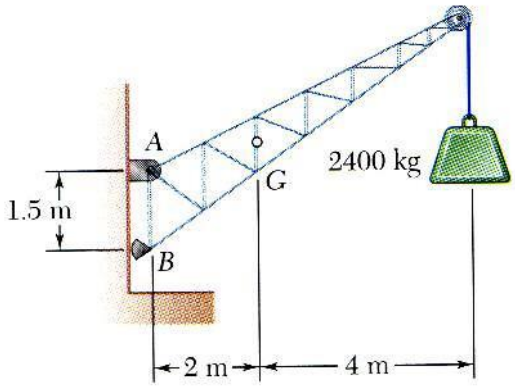


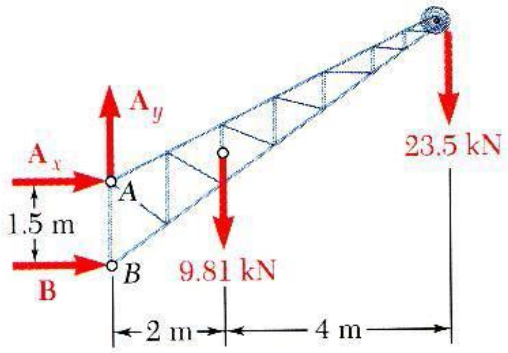



## Equations of Equilibrium

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M_O = 0$$

**Ex 1:-** A fixed crane has a mass of 1000 kg and is used to lift a 2400 kg crate. It is held in place by a pin at *A* and a rocker at *B*. The center of gravity of the crane is located at *G*. Determine the components of the reactions at *A* and *B*




- Determine *B* by solving the equation for the sum of the moments of all forces about *A*.

$$\sum M_A = 0: \quad +B(1.5\text{m}) - 9.81\,\text{kN}(2\text{m}) - 23.5\,\text{kN}(6\text{m}) = 0$$

$$B = +107.1\,\text{kN}$$

- Determine the reactions at *A* by solving the equations for the sum of all horizontal forces and all vertical forces.

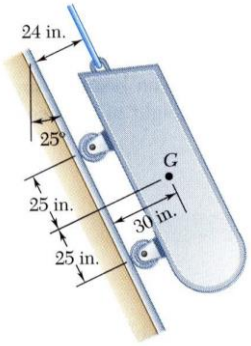
$$\sum F_x = 0: \quad A_x + B = 0$$

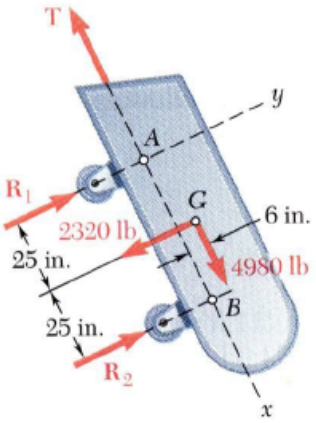
$$A_x = -107.1\,\text{kN}$$

$$\sum F_y = 0: \quad A_y - 9.81\,\text{kN} - 23.5\,\text{kN} = 0$$

$$A_y = +33.3\,\text{kN}$$

**Ex 2:-** A loading car is at rest on an inclined track. The gross weight of the car and its load is 5500 lb, and it is applied at at G. The cart is held in position by the cable. Determine the tension in the cable and the reaction at each pair of wheels.




- Create a free-body diagram

$$W_x = +(5500\,\text{lb})\cos 25^\circ$$
$$= +4980\,\text{lb}$$
$$W_y = -(5500\,\text{lb})\sin 25^\circ$$
$$= -2320\,\text{lb}$$

- Determine the reactions at the wheels.

$$\Sigma M_A = 0: \quad -(2320\,\text{lb})25\text{in.} - (4980\,\text{lb})6\text{in.} + R_2(50\text{in.}) = 0$$

$$\boxed{R_2 = 1758\,\text{lb}}$$

$$\Sigma M_B = 0: \quad +(2320\,\text{lb})25\text{in.} - (4980\,\text{lb})6\text{in.} - R_1(50\text{in.}) = 0$$

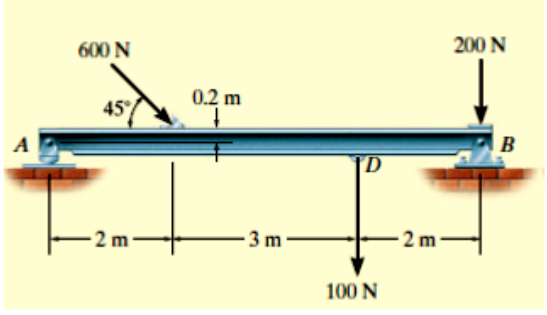
$$\boxed{R_1 = 562\,\text{lb}}$$

- Determine the cable tension.

$$\Sigma F_x = 0: \quad +4980\,\text{lb} - \text{T} = 0$$

$$\boxed{T = +4980\,\text{lb}}$$

**Example -1:** Determine the horizontal and vertical components of reaction on the beam caused by the pin at B and the rocker at A as shown in Fig. 5–12 a . Neglect the weight of the beam.



Equations of equilibrium:

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M_O = 0$$

Summing forces in x-direction:

+

$\rightarrow$

$\sum Fx=0$

$600 \cos 45\ N - B_x = 0$

$B_x = 424\ N$

To find $A_y$ applying moment equation about B

$$\sum M_B = 0$$

$100\ N * 2\ m + 600 \sin 45\ N * 5\ m - 600 \cos 45\ N * 0.2\ m - A_y * 7\ m = 0$

$A_y = 319N$

Summing forces in y –direction:

$$\sum F_y = 0$$

$319\ N - 600 \sin 45\ N - 100\ N - 300\ N + B_y = 0$

$B_y = 405\ N$