Lecture 7

Wave Properties of Light

There are many properties of light that can only be understood in terms of a wave-like description. In this lecture we will examine these in some detail; many of the more mathematical aspects will be omitted.

- General Properties of Light
- General Properties of Waves
- Spherical Waves
- Huygens' Principle
- Reflection and Refraction, Snell's Law
- Dispersion
- Photons (light "particles")
- Photoelectric Effect
- Atomic Radiation
- Problems

General Properties of Light

In vacuum light always travels at the same speed:

$$c = 3.0 \text{ x } 10^8 \ m/s \ .$$

- Until the middle of the 1800's, the generally accepted theory of light was the particle picture. In this viewpoint, advocated by Newton, light was considered to be a stream of tiny particles. However, in the late 1800's, the particle picture was replaced by the wave theory of light. This was because certain phenomena associated with light, namely refraction, diffraction and interference, could only be explained using the wave picture.
- Visible light is just one particular type of electromagnetic radiation. Other types of electromagnetic radiation include radio waves, infrared radiation (heat), ultraviolet radiation, x-rays and γ -rays. The different types of radiation are distinguished by their wavelength, or frequency, as shown in Fig. (7.1).

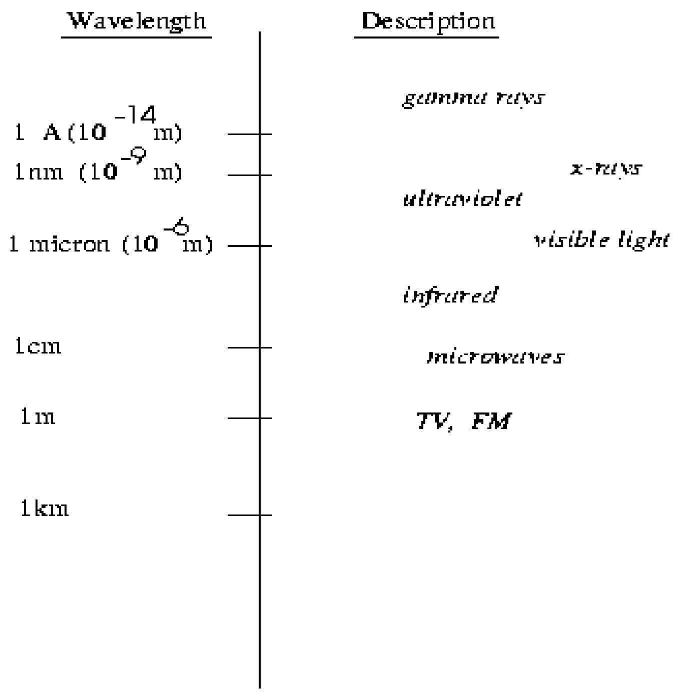


Figure 7.1: The Electromagnetic Spectrum

For example blue light has a wavelength (in vacuum) of 434 x 10^{-9}m = 434 nanometers (nm), while red light has a wavelength of 768 nm. Radiation outside the visible spectrum with wavelengths longer than red light is called infrared, while radiation with wavelength shorter than blue is called ultraviolet. The theory which accurately describes the wave-like properties of all types of electromagnetic radiation is called **Maxwell's Theory of Electromagnetism**.

In the early 20th century, experiments revealed that there were some phenomena associated with light that could only be explained by a particle picture. Thus, light as it is now understood, has attributes of both particles and waves. In this Chapter we will deal mainly with the wave attributes of light. The particle-like behavior of light is described by the modern theory of quantum mechanics.

General Properties of Waves

Definitions

- The **wavelength** (λ) is the distance between neighboring crests or troughs.
- The **speed** (v) is the rate at which the crests (or troughs) move forward.
- The **Period** (T) is the time that elapses between passing crests (or troughs).

  The period can be expressed in terms of the speed and wavelength: $T = \frac{\lambda}{v}$
- The **frequency** (f) is the number of crests (or troughs) that pass by per unit time. It is equal to the inverse of the period: $f = \frac{1}{T}$

  Using the expression for T above we get the useful expression: $v = f\lambda$

Spherical Waves

Another type of ideal light wave. Constant phase fronts are circular, emanating from a point source. Far away from the source, the radius of the circle becomes so large that we can approximate the wave as a plane wave, as shown in Fig. (7.2).

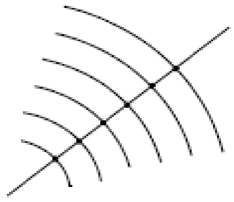
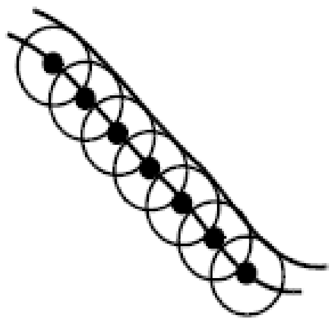


Fig.7.2: plane wave

Huygens' Principle

Very useful model for wave propagation.

- Every point on a wave front is regarded as a secondary point source generating a spherical wavelet.
- The advance of the wave front is found at the envelope of all these wavelets

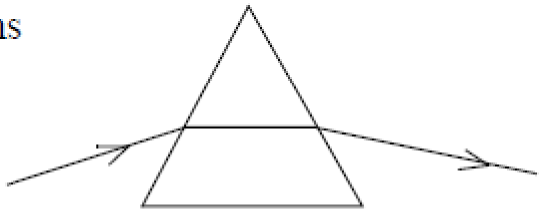


- Generally, this seems to give parallel wave fronts. But things get interesting at edges. This leads to diffraction.

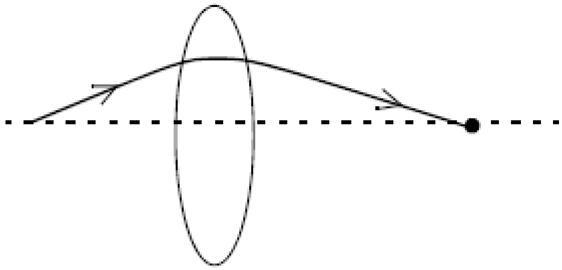
Rays

- Follow a point on the wave front. As the wave front advances the point traces a straight line. This is a ray of light.
- For many cases, we can forget the waves and just trace rays in optical systems. This allows a vast simplification of our analysis and design processes. Virtually all optical design is done with rays. Highly sophisticated optical design CAD programs are available for ray tracing.

Prisms



Lenses



Reflection and Refraction, Snell's Law

An important element of optics is the interface between **two** materials with different index of refraction

Index of Refraction

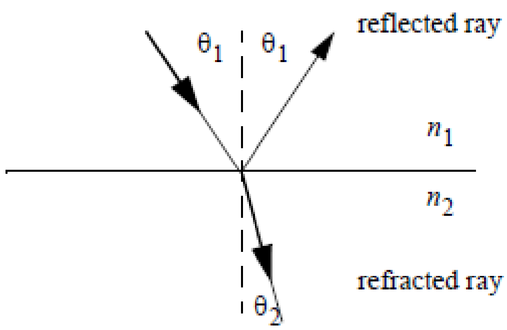
- When light travels in materials, the speed is modified:

$$v = \frac{c}{n} = \lambda f$$

Usually $n \geq 1$. (It can be < 1)

If we define the vacuum wavelength. $\boldsymbol{\lambda_{vac}} = \frac{\boldsymbol{c}}{\boldsymbol{f}}$ then in the material

The wavelength becomes shorter, if $n > 1$ $\quad \lambda = \frac{c}{nf} = \frac{\lambda_{vac}}{n}$

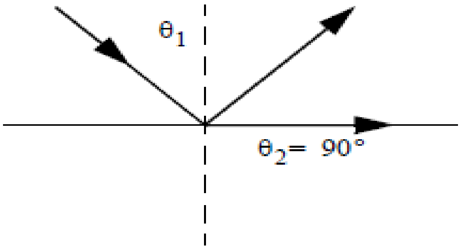


$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1} \rightarrow \begin{array}{ll} \text{if } n_2 > n_1 & \theta_2 < \theta_1 \\ \text{if } n_1 > n_2 & \theta_2 > \theta_1 \end{array}$$

Total Reflection

If $n_1 > n_2$, then we can have

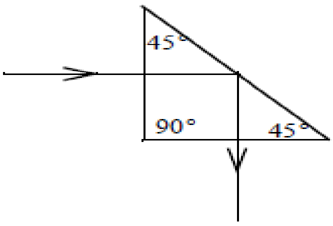


$$\theta_2 = 90^\circ$$
$$\sin\theta_c = \frac{n_2}{n_1}$$
$$\theta_c = \sin^{-1}(n_2/n_1)$$

The refracted ray disappears! The light is totally reflected. This usually occurs inside a prism , and is called total internal reflection. θ_c = "critical angle". For a typical glass with $n = 1.5$, the critical angle is:

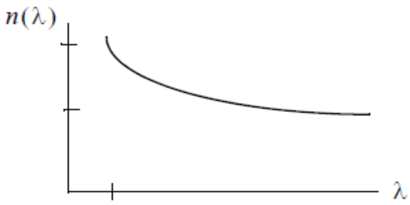
$$\boxed{\sin^{-1}\left(\frac{1}{1.5}\right) = 41.8^\circ}$$

So for $\theta = 45$, the light is reflected. A very common prism is the right angle prism



Dispersion

The index of refraction in most materials depends on wavelength. $n(\lambda)$.
This is called dispersion.



In air – the index depends also on air pressure, humidity, and temperature which leads to many beautiful atmospheric effects.

Wavelength units (length)

We commonly use Angstrom units (Å) for light wavelength.

1Å = 10^{-8}cm = 0.1nm

This is of the order of the size of an atom. We also use standard metric units: m, cm, mm, nm

Visible light ~ 4000 → 7000A°, 400 → 700 nm , 0.4 → 0.7 μm

Photons (light "particles")

• This picture has light represented by tiny bundles of energy (or quanta), following straight line paths along the rays.

• The coexistence of electromagnetic wave physics and photon physics is the central paradox of quantum mechanics.

• Each photon has an energy given by

$E = hv$

h = 6.62×10^{-34} J-s

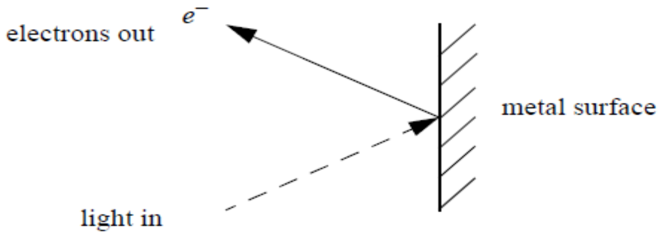
1W= 1J per second

h is Planck's constant

For 2eV visible photons, $1W = 6.3 \times 10^{18} eV/s = 3.15 \times 10^{18}$ photons/sec

• Light power → photons/sec

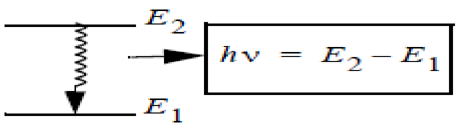
Photoelectric Effect



- The electron energy is directly related to the photon energy.
- When the photon energy is below threshold value, no electrons are emitted. The threshold depends on the metal. It is called the work function.
- When the light power is low$\sim 10^2 \rightarrow 10^3$ photons/sec →each individual electron can be separated and counted. This called photon counting

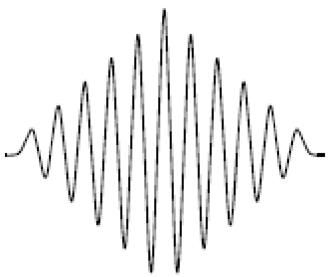
Atomic Radiation

- Atoms have energy states corresponding to electron orbits.



- One atom "jumps" from a higher energy state to a lower energy state and emits one photon.
- Photons are not point particles. They have a wave-like property.

A useful picture is the wave-packet.

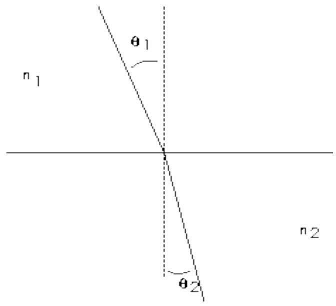


Many photon packets can be thought of as superimposing to make up a plane wave, spherical wave or any other wave.
The typical photon energy unit is the electron-Volt. This is defined as the energy required to push one electron across a one-Volt potential,
$1eV = 1.6 \times 10^{-19} J$
Typical visible photon energy ~ $1.2 \rightarrow 2.3\ eV$

Problem 7.1

A light ray of wavelength $\lambda = 589$ nm is incident on glass with an angle of incidence of 30^{o}. The index of refraction of glass is 1.52. a)What is the angle of refraction? b) What are the speed and wavelength of the light inside the glass?

Solution:



$$sin\theta_2 = \frac{n_1}{n_2} sin\theta_1 = \frac{1.00}{1.52} sin30 = 0.329$$

$$\theta_2 = sin^{-} 0.329 = 19.2^{o}$$

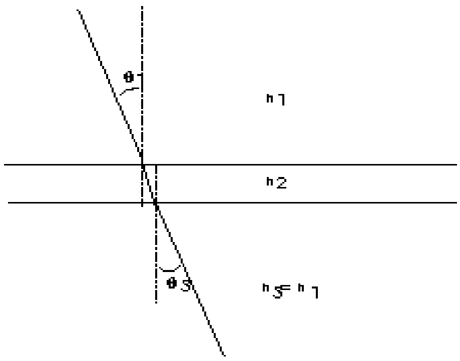
$$\lambda_2 = \frac{n_1}{n_2}\lambda_1 = \frac{1.00}{1.52} \times 589 \times 10^{-6} m = 387 \times 10^{-9} m$$
$$= 387\ nm$$

$$v_2 = \frac{c}{n_2} = \frac{3 \times 10^{8}\ m/s}{1.52} = 2 \times 10^{8}\ m/s$$

Problem 7.2

Light passes through a flat slab of glass. The angle of incidence of the light onto the glass is 30^{o}. What is the angle with which the light emerges on the other side of the slab?

Solution:



First calculate the angle of refraction inside the glass:

$$sin\theta_2 = \frac{n_1}{n_2} \sin\theta_1$$

Then use as the angle of incidence onto the boundary between the glass and the air on the other side, so the angle of refraction in the air is:

$$sin\theta_3 = \frac{n_2}{n_3}\sin\theta_2 = \frac{n_2}{n_3}\frac{n_1}{n_2}sin\theta_1 = \frac{n_1}{n_3}sin\theta_1$$

Since $n_1 = n_3$ because there is air on both sides of the slab, $\theta_3 = \theta_1$ and the ray emerges parallel to the incoming ray.

Problem 7.3

An unknown glass has an index of refraction of ***n*=1.5**. For a beam of light originated in the glass, at what angles the light **100%** reflected back into the glass. (The index of refraction of air is $\boldsymbol{n_{air}}$ **=1.00**).

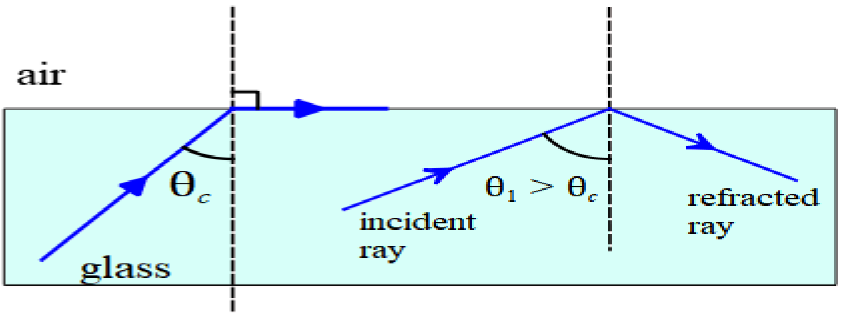
Solution:

When light moves through a medium with an index of refraction n_1 and strike a boundary of a region with an index of refraction n_2 such that $n_1 > n_2$, then total internal reflection can occur if the incident angle equals or is greater than a critical angle θc given by the following formula

$$sin\ \theta_c = \frac{n_2}{n_1}$$

If the incident ray equals the critical angle, then the refracted ray exit along the boundary at the angle of 90^o

For incident angles greater than the critical angle, $\theta 1 > \theta c$, the refracted ray vanishes at the boundary, and 100% is reflected back into the original medium. This process is called total internal reflection.



In this problem, we are told that light is initially in a medium with a higher index of refraction and wants to enter a medium with a lower index of refraction. So, the necessary condition for occurring total internal reflection is satisfied.

Thus, we must first find the critical angle whose magnitude is obtained as below

$$sin\theta_c = \frac{n_{air}}{n_{glass}} = \frac{1}{1.5}\ , \qquad \theta_c = sin^{-1}\left(\frac{1}{1.5}\right) = 41^o$$

Hence, if the angle of incidence in the glass exceeds about 42^o, then all incident rays completely return back into the same medium.