Basrah University
College of Engineering
Electrical Engineering Department

Introduction to Electrical Networks

Asst. Lect: Hamzah Abdulkareem

Chapter Four

The Filters

Part 2

- ***Lowpass Filter***
- ***Highpass Filter***
- ***Bandpass Filter***
- ***Bandstop Filter***

4.4 Lowpass Filter

A typical lowpass filter is formed when the output of an *RC* circuit is taken off the capacitor as shown in Fig. 4.5. The transfer function

$$H(\omega) = \frac{V_o}{V_i} = \frac{\frac{1}{j\omega C}}{R+\frac{1}{j\omega C}} = \frac{1}{1+j\omega RC} \qquad \dots (11)$$

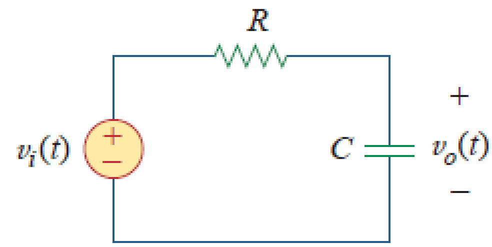

Fig 4.5 A lowpass filter.

Note that $\mathbf{H(0) = H(\infty) = 0}$. Figure 4.6 shows the plot of $|\mathbf{H(\omega)}|$ along with the ideal characteristic. The half-power frequency, which is equivalent to the cutoff frequency $\boldsymbol{\omega_c}$ is obtained by setting the magnitude of $\mathbf{H(\omega)}$ equal to $\mathbf{1/\sqrt{2}}$ thus,

$$H(\omega) = \frac{1}{\sqrt{1+R^2C^2\omega^2}} = \frac{1}{\sqrt{2}} \qquad \dots (12)$$

$$\boldsymbol{\omega_c = 1/\sqrt{RC}} \qquad \dots (13)$$

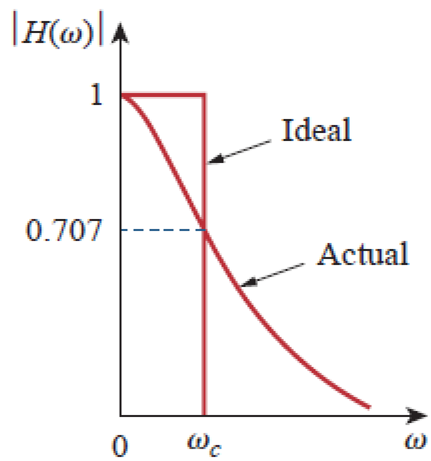

Fig 4.6 Ideal and actual frequency response of a lowpass filter.

So, from the above its clear that a lowpass filter is designed to pass only frequencies from dc up to the cutoff frequency $\boldsymbol{\omega_c}$.

A lowpass filter can also be formed when the output of an RL circuit is taken off the resistor. Of course, there are many other circuits for lowpass filters.

4.5 Highpass Filter

A typical highpass filter is formed when the output of an *RC* circuit is taken off the resistor as shown in Fig. 4.7. The transfer function

$$H(\omega) = \frac{V_o}{V_i} = \frac{R}{R+\frac{1}{j\omega C}} = \frac{j\omega RC}{1+j\omega RC} \qquad \dots (14)$$

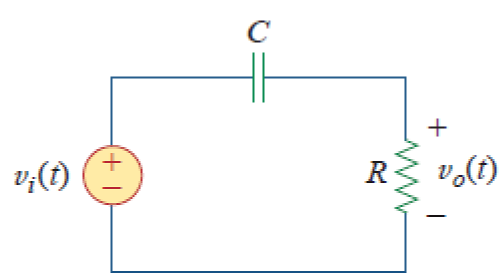

Fig 4.7 A highpass filter.

Note that $\mathbf{H(0) = H(\infty) = 1}$. Figure 4.8 shows the plot of $|\mathbf{H(\omega)}|$. Again the cutoff frequency is

$$\boldsymbol{\omega_c = 1/\sqrt{RC}} \qquad \ldots(15)$$

A highpass filter is designed to pass all frequencies above its cutoff frequency $\boldsymbol{\omega_c}$. Also A highpass filter can also be formed when the output of an RL circuit is taken off the inductor.

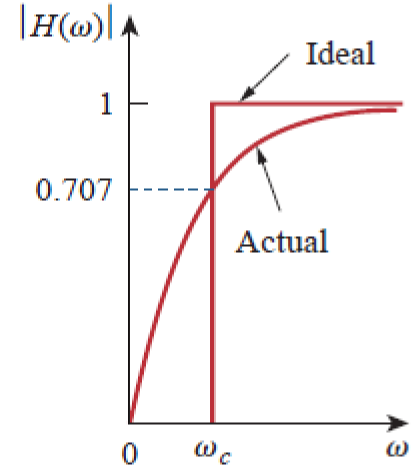

Fig 4.7 Ideal and actual frequency response of a highpass filter.

4.6 Bandpass Filter

The RLC series resonant circuit provides a bandpass filter when the output is taken off the resistor as shown in Fig. 4.8. The transfer function is

$$\mathrm{H}(\omega) = \frac{V_o}{V_i} = \frac{R}{R + j(\omega L - \frac{1}{\omega C})} \qquad \ldots(16)$$

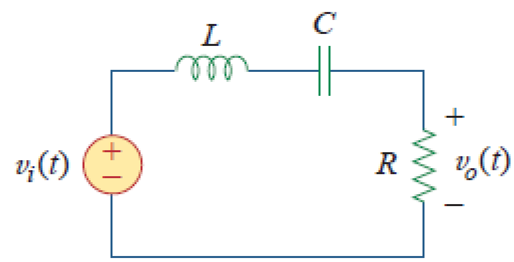

Fig 4.8 A bandpass filter.

We observe that $\mathbf{H(0) = H(\infty) = 0}$. Figure 4.9 shows the plot of $|\mathbf{H(\omega)}|$. The bandpass filter passes a band of frequencies $(\boldsymbol{\omega_1 < \omega < \omega_2})$ centered on $\boldsymbol{\omega_o}$, the center frequency, which is given by

$$\boldsymbol{\omega_o = 1/\sqrt{LC}} \qquad \ldots(17)$$

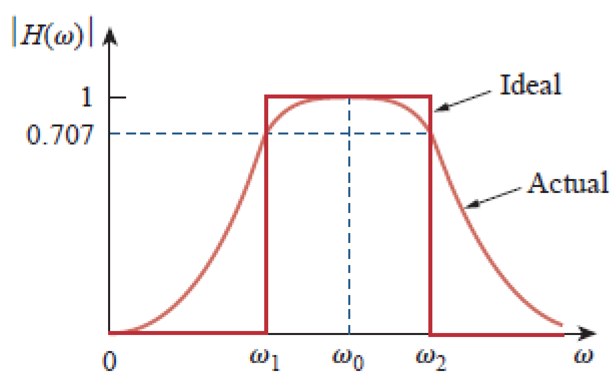

Fig 4.9 Ideal and actual frequency response of a bandpass filter.

Since the bandpass filter in Fig. 4.8 is a series resonant circuit, the half-power frequencies, the bandwidth, and the quality factor are determined.

A bandpass filter can also be formed by cascading the lowpass filter (where $\boldsymbol{\omega_2 = \omega_c}$) in Fig. 4.5 with the highpass filter (where $\boldsymbol{\omega_1 = \omega_c}$) in Fig. 4.7. However, the result would not be the same as just adding the output of the lowpass filter to the input of the highpass filter, because one circuit loads the other and alters the desired transfer function.

4.7 Bandstop Filter

A filter that prevents a band of frequencies between two designated values (ω_1 and ω_2) from passing is variably known as a bandstop, bandreject, or notch filter. A bandstop filter is formed when the output RLC series resonant circuit is taken off the LC series combination as shown in Fig. 4.10. The transfer function is

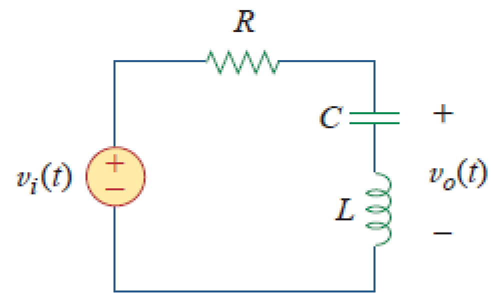

Fig 4.9 A bandstop filter.

$$H(\omega) = \frac{V_o}{V_i} = \frac{j(\omega L - \frac{1}{\omega C})}{R + j(\omega L - \frac{1}{\omega C})} \quad \ldots (18)$$

Note that $\mathbf{H(0) = H(\infty) = 1}$. Figure 4.11 shows the plot of $|\mathbf{H(\omega)}|$. Again the center frequency is

$$\boldsymbol{\omega_o = 1/\sqrt{LC}} \quad \ldots(19)$$

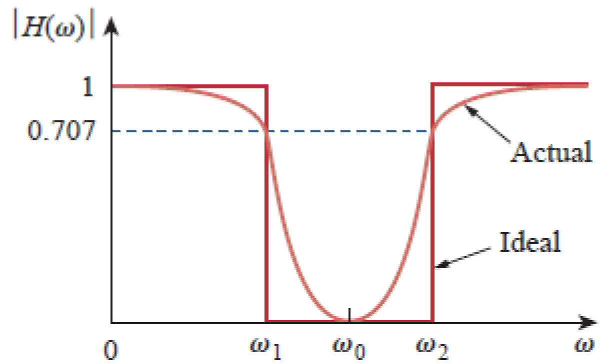

Fig 4.11 Ideal and actual frequency response of a bandstop filter.

Here $\boldsymbol{\omega_o}$, is called the frequency of rejection, while the corresponding bandwidth ($\beta = \boldsymbol{\omega_2 - \omega_1}$) is known as the bandwidth of rejection. Thus, A bandstop filter is designed to stop or eliminate all frequencies within a band of frequencies, $\boldsymbol{\omega_1 < \omega < \omega_2}$.

In concluding this section, we should note that:

1. From Eqs. (11), (14), (16), and (18), the maximum gain of a passive filter is unity. To generate a gain greater than unity, one should use an active filter .

2. There are other ways to get the types of filters treated in this section.

3. The filters treated here are the simple types. Many other filters have sharper and complex frequency responses.

Example 4.2:- Show that a series LR circuit is a lowpass filter if the output is taken across the resistor.

Example 4.3:- Determine what type of filter is shown in Fig. 4.12. Calculate the cutoff frequency. Take $R = 2\ K\Omega,\ L = 2\ H$ & $C = 2\ \mu F$.

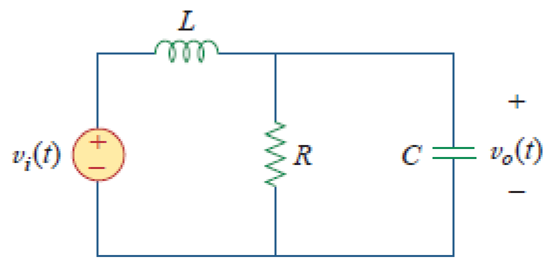

Fig 4.12 For Example 4.2