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Introduction to Electrical Networks

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Chapter Four

The Filters

Part 1

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4.1 Frequency Response

As we discussed in **Chapter Two**, the **transfer function** $\mathbf{H}(\boldsymbol{\omega})$ (also called the **network function**) is a useful analytical tool for finding the frequency response of a circuit. In fact, the frequency response of a circuit is the plot of the circuit's transfer function $\mathbf{H}(\boldsymbol{\omega})$ versus $\boldsymbol{\omega}$,with $\boldsymbol{\omega}$ varying from $\boldsymbol{\omega} = 0$ to $\boldsymbol{\omega} = \infty$. So in the frequency domain The transfer function $\mathbf{H}(\boldsymbol{\omega})$ of a circuit is the frequency-dependent ratio of a phasor output $\mathbf{Y}(\boldsymbol{\omega})$ (an element voltage or current) to a phasor input $\mathbf{X}(\boldsymbol{\omega})$ (source voltage or current). So assuming zero initial conditions, the transfer function in the frequency domain will be

$$\mathbf{H}(\omega) = \frac{\mathbf{Y}(\omega)}{\mathbf{X}(\omega)} \qquad \ldots (1)$$

Since the input and output can be either voltage or current at any place in the circuit, there are four possible transfer functions:

$$\mathbf{H}(\omega) = \text{Voltage gain} = \frac{\mathbf{V}_o(\omega)}{\mathbf{V}_i(\omega)}$$

$$\mathbf{H}(\omega) = \text{Current gain} = \frac{\mathbf{I}_o(\omega)}{\mathbf{I}_i(\omega)}$$

$$\mathbf{H}(\omega) = \text{Transfer Impedance} = \frac{\mathbf{V}_o(\omega)}{\mathbf{I}_i(\omega)}$$

$$\mathbf{H}(\omega) = \text{Transfer Admittance} = \frac{\mathbf{I}_o(\omega)}{\mathbf{V}_i(\omega)} \qquad \ldots (2)$$

Being a complex quantity, $\mathbf{H}(\boldsymbol{\omega})$ has a magnitude $H(\omega)$ and a phase $\emptyset$; that is, $\mathbf{H}(\boldsymbol{\omega}) = H(\omega)\angle\emptyset$. To obtain the transfer function using Eq. (2), we first obtain the frequency-domain equivalent of the circuit by replacing resistors, inductors, and capacitors with their impedances R, $j\omega L$ and $1/j\omega C$ .We then use any circuit technique(s) to obtain the appropriate quantity in Eq. (2). We can obtain the frequency response of the circuit by plotting the magnitude and phase of the transfer function as the frequency varies.

Example 4.1:- For the RC circuit in Fig. 4.1(a), obtain the transfer function V_o/V_i and its frequency response. Let $v_s = V_m \cos \omega t$.

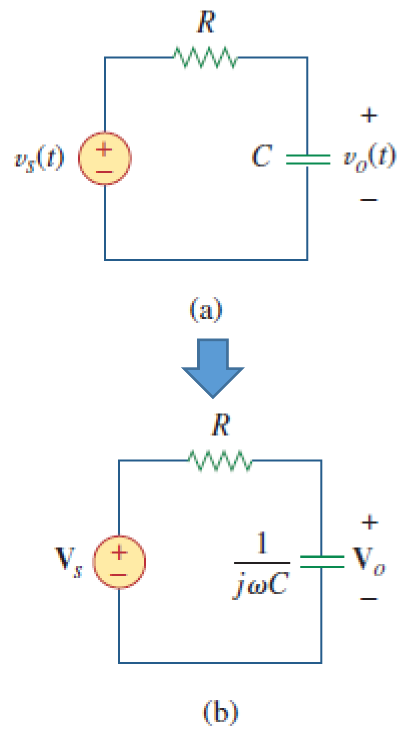

Fig 4.1 For Example 4.1

4.2 Filters

The concept of filters has been an integral part of the evolution of electrical engineering from the beginning. Several technological achievements would not have been possible without electrical filters. Because of this prominent role of filters, much effort has been expended on the theory, design, and construction of filters and many articles and books have been written on them. ***A filter is a circuit capable of passing (or amplifying) certain frequencies while attenuating other frequencies. Thus, a filter can extract important frequencies from signals that also contain undesirable or irrelevant frequencies.***

As a frequency-selective device, a filter can be used to limit the frequency spectrum of a signal to some specified band of frequencies. Filters are the circuits used in radio and TV receivers to allow us to select one desired signal out of a multitude of broadcast signals in the environment. A filter is a **passive filter** if it consists of only passive elements **R, L**, and **C**. It is said to be an **active filter** if it consists of active elements (such as **transistors** and **op amps**) in addition to passive elements **R, L**, and **C**.

In the field of electronics, there are many practical applications for filters. Examples include:

- Radio communications: Filters enable radio receivers to only "see" the desired signal while rejecting all other signals (assuming that the other signals have different frequency content).
- DC power supplies: Filters are used to eliminate undesired high frequencies (i.e., noise) that are present on AC input lines. Additionally, filters are used on a power supply's output to reduce ripple.
- Audio electronics: A crossover network is a network of filters used to channel low-frequency audio to woofers, mid-range frequencies to midrange speakers, and high-frequency sounds to tweeters.

As shown in Fig. 4.2, there are four types of filters whether passive or active:

1- **A lowpass filter** passes low frequencies and stops high frequencies, as shown ideally in Fig. 4.2(a).
2- **A highpass filter** passes high frequencies and rejects low frequencies, as shown ideally in Fig. 4.2(b).
3- A *bandpass filter* passes frequencies within a frequency band and blocks or attenuates frequencies outside the band, as shown ideally in Fig. 4.2(c).
4- **A bandstop filter (Notch filter)** passes frequencies outside a frequency band and blocks or attenuates frequencies within the band, as shown ideally in Fig. 4.2(d).

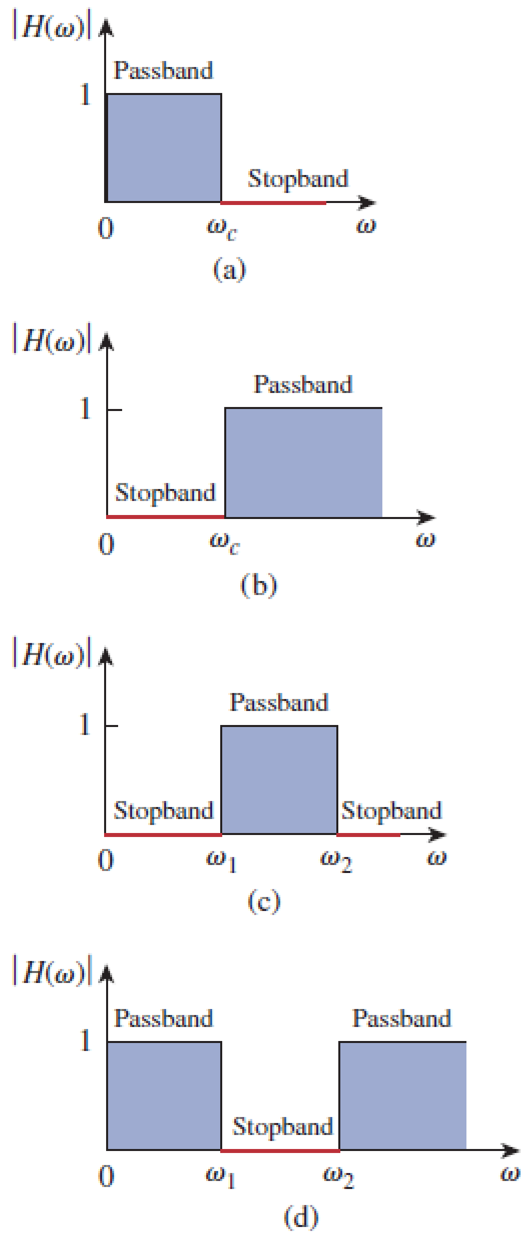

Fig 4.2 (a) Ideal frequency response of four types of filters: (a) lowpass filter, (b) highpass filter, (c) bandpass filter, (d) bandstop filter.

4.3Frequency Response Curve

Response curves are used to describe how a filter behaves. A response curve is simply a graph showing an attenuation ratio (V_{OUT}/V_{IN}) versus frequency (see Fig 4.3). Attenuation is commonly expressed in units of decibels ($\boldsymbol{dB}$). Frequency can be expressed in two forms: either the angular form ω (units are $\boldsymbol{rad/s}$) or the more common form of f (units of $\boldsymbol{Hz}$, i.e., cycles per second). These two forms are related by $\boldsymbol{\omega = 2\pi f}$. Finally, filter response curves may be plotted

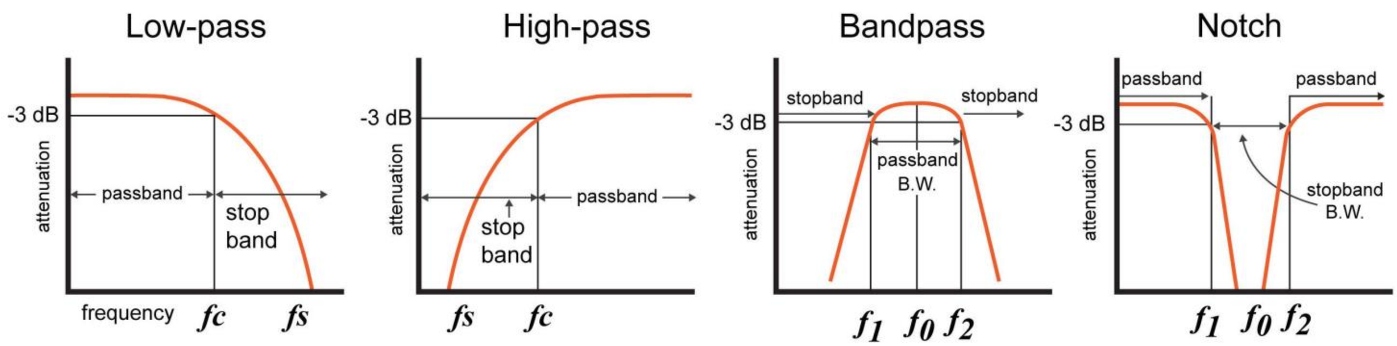

Fig 4.3 Response curves for the four major filter types.

in linear-linear, log-linear, or log-log form. The most common approach is to have decibels on the y-axis and logarithmic frequency on the x-axis.

There are some technical terms that are commonly used when describing filter response curves:

- **The Decibel Scale:-** Historically, the $\boldsymbol{bel}$ is used to measure the ratio of two levels of power or power gain $\boldsymbol{G}$; that is,

$$\boldsymbol{G = Number\ of\ bels = \log_{10}\frac{P_2}{P_1}} \qquad \ldots (3)$$

The decibel (dB) provides us with a unit of less magnitude. It is $\boldsymbol{1/10th}$ of a bel and is given by

$$\boldsymbol{G_{dB} = 10\log_{10}\frac{P_2}{P_1}} \qquad \ldots (4)$$

When $\boldsymbol{P_2 = P_1}$, there is no change in power and the gain is $\boldsymbol{0\ dB}$. If $\boldsymbol{P_2 = 2P_1}$, the gain is

$$\boldsymbol{G_{dB} = 10\log_{10}2 \cong 3dB} \qquad \ldots (5)$$

If $\boldsymbol{P_2 = 0.5P_1}$, the gain is

$$\boldsymbol{G_{dB} = 10\log_{10}0.5 \cong -3dB} \qquad \ldots (6)$$

Equations (5) and (6) show another reason why logarithms are greatly used: The logarithm of the reciprocal of a quantity is simply negative the logarithm of that quantity.

Alternatively, the gain G can be expressed in terms of voltage or current ratio. To do so, consider the network shown in Fig. 4.4. If $\boldsymbol{P_1}$ is the input power, $\boldsymbol{P_2}$ is the output (load) power, $\boldsymbol{R_1}$ is the input resistance, and $\boldsymbol{R_2}$ is the load resistance,

$$G_{dB} = 10 \log_{10} \frac{P_2}{P_1} = 10 \log_{10} \frac{V_2^2/R_2}{V_1^2/R_1}$$

$$= 10 \log_{10} \left(\frac{V_2}{V_1}\right)^2 + 10 \log_{10} \frac{R_1}{R_2}$$

$$G_{dB} = 20 \log_{10} \frac{V_2}{V_1} - 10 \log_{10} \frac{R_2}{R_1} \quad \ldots (7)$$

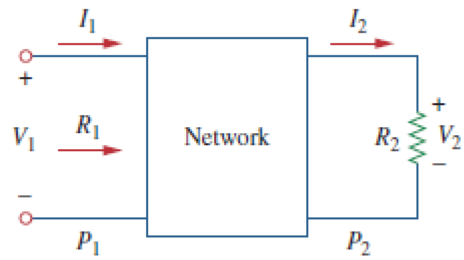

Fig 4.4 Voltage-current relationships for a four terminal network.

For the case when $\boldsymbol{R_1 = R_2}$, a condition that is often assumed when comparing voltage levels, Eq. (7) becomes

$$\boldsymbol{G_{dB} = 20 \log_{10} \frac{V_2}{V_1}} \quad \ldots (8)$$

Instead, if $\boldsymbol{P_1 = I_1^2 R_1}$ and $\boldsymbol{P_2 = I_2^2 R_2}$, for $\boldsymbol{R_1 = R_2}$ we obtain

$$\boldsymbol{G_{dB} = 20 \log_{10} \frac{I_2}{I_1}} \quad \ldots (9)$$

From equeations (4, 8 & 9) its clear that $\mathbf{10 \log_{10}}$ is used for power, while $\mathbf{20 \log_{10}}$ is used for voltage or current, because of the square relationship between them.

- **-3 $\boldsymbol{dB}$ *frequency* ($\boldsymbol{f_{3dB}}$) :-** This term, pronounced "minus 3dB frequency", corresponds to the input frequency that causes the output signal to drop by $-3dB$ relative to the input signal. The $-3\ dB$ frequency is also referred to as the ***cutoff frequency***. It is the frequency at which the output power is reduced by one-half (which is why this frequency is also called the "half-power frequency"), or the output voltage is the input voltage multiplied by $\mathbf{1/\sqrt{2}}$. For low-pass and high-pass filters, there is only one $-3\ dB$ frequency. However, there are two $-3\ dB$ frequencies for band-pass and notch filters—normally referred to as f_1 and f_2.
- **Center frequency ($\boldsymbol{f_0}$) :-** The center frequency, a term used for band-pass and notch filters, is a central frequency that lies between the upper and lower cutoff frequencies. The center frequency is commonly defined as either the arithmetic mean or the geometric mean of the lower cutoff frequency and the upper cutoff frequency.
- **Bandwidth ($\boldsymbol{\beta}$ *or* $\boldsymbol{B.W}$) :-** The bandwidth is the width of the passband, and the passband is the band of frequencies that do not experience significant attenuation when moving from the input of the filter to the output of the filter.

- **Stopband frequency (f_s) :-** This is a particular frequency at which the attenuation reaches a specified value. So this term has two meaning according to the type of filter :-
    1- For **low-pass and high-pass filters**, frequencies beyond the stopband frequency are referred to as the stopband.
    2- For **band-pass and notch filters**, two stopband frequencies exist. The frequencies between these two stopband frequencies are referred to as the stopband.
- **Quality factor (Q) :-** The quality factor of a filter conveys its damping characteristics. In the time domain, damping corresponds to the amount of oscillation in the system's step response. In the frequency domain, higher Q corresponds to more (positive or negative) peaking in the system's magnitude response. For a bandpass or notch filter, Q represents the ratio between the center frequency and the $-3dB$ bandwidth (i.e., the distance between f_1 and f_2). For both band-pass and notch filters:

$$Q = f_o / (f_2 - f_1) \qquad \dots (10)$$