Basrah University
College of Engineering
Electrical Engineering Department

Introduction to Electrical Networks

Asst. Lect: Hamzah Abdulkareem

Chapter One

Coupling Circuits

Part 1

- ***Syllabus***
- ***References***
- ***Mutual Inductance***
- ***Dot Convention***
- ***Physical Basis of the Dot Convention***

Syllabus:-

***Coupling Circuits*:-**
Magnetic coupling, coefficient of coupling, equivalent linear circuits and ideal transformers.

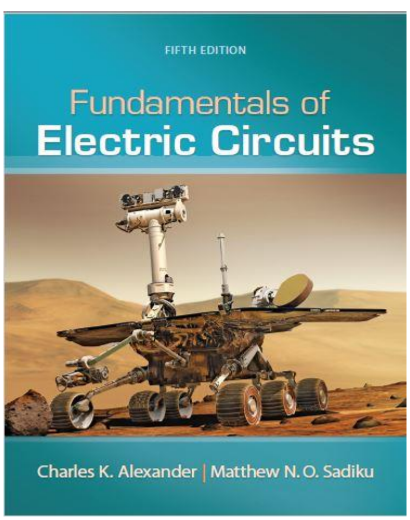
S-domain Circuit Analysis:-
Impedance and admittance in s-domain, circuit analysis in s-domain, poles and zeros of transfer functions, natural response and s-plane.

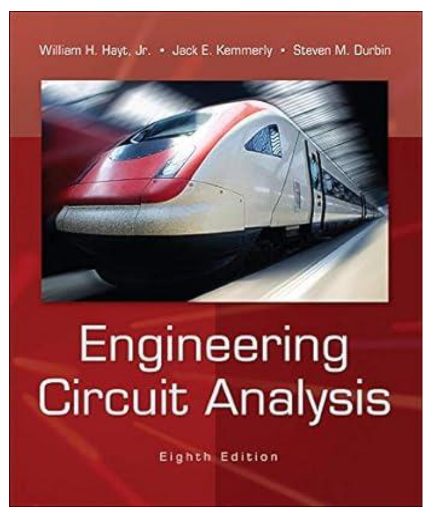
Two-Port Networks:-
Two-port networks, y-z-h and ABCD parameters, attenuation and phase functions, loss of networks.

Filters:-
Constant k-filters, low pass and high pass, modern filter design, Butterworth and Chebyshev filters, network transformations and all pass filters.

References :-

1- *Fundamentals of electric circuits.by Alexander, Charles K. 5th Edition.*
2- *Engineering circuit analysis by William Hayat 8th Edition*





Chapter One

Coupling Circuits

1.1 Mutual Inductance

When two inductors (or coils) are in a close proximity to each other, the magnetic flux caused by current in one coil links with the other coil, thereby inducing voltage in the latter. This phenomenon is known as mutual inductance.

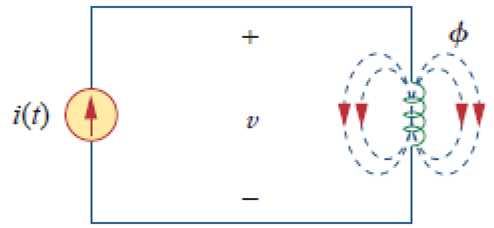


Fig 1.1 Magnetic flux produced by a single coil with N turns.

Let us first consider a single inductor, a coil with ***N*** turns. When current ***i*** flows through the coil, a magnetic flux ϕ is produced around it (Fig. 1.1). According to Faraday's law, the voltage v induced in the coil is proportional to the number of turns ***N*** and the time rate of change of the magnetic flux ϕ ; that is,

$$v = N\frac{d\phi}{dt} \qquad \ldots(1)$$

But the flux ϕ is produced by current ***i*** so that any change in ϕ is caused by a change in the current. Hence, Eq. (1) can be written as

$$v = N\frac{d\phi}{di}\frac{di}{dt} \qquad \text{And as} \qquad v = L\frac{di}{dt} \quad \Rightarrow \quad L = N\frac{d\phi}{di} \qquad \ldots(2)$$

This inductance is commonly called ***self-inductance***, because it relates the voltage induced in a coil by a time-varying current in the same coil.

Now consider two coils with self-inductances L_1 and L_2 that are in close proximity with each other (Fig. 1.2).

Coil 1 has N_1 turns, while coil 2 has N_2 turns. For the sake of simplicity, assume that the second inductor carries no current. The magnetic flux ϕ_1 emanating from coil 1 has two components: one component ϕ_{11} links only coil 1, and another component ϕ_{12} links both coils. Hence,

$$\phi_1 = \phi_{11} + \phi_{12} \qquad \ldots(3)$$

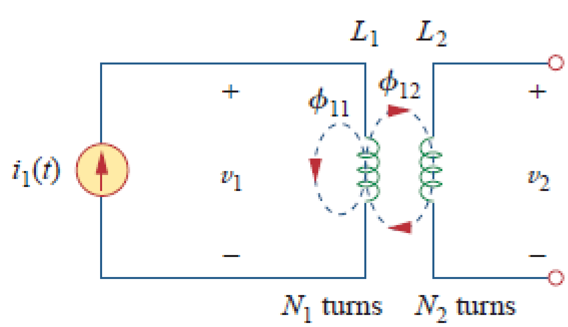


Fig 1.2 Mutual inductance of coil 2 with respect to coil 1.

Although the two coils are physically separated, they are said to be magnetically coupled. Since the entire flux ϕ_1 links coil 1, the voltage induced in coil 1 is

$$v_1 = N_1 \frac{d\phi_1}{dt} \qquad \ldots(4)$$

Only flux ϕ_{12} links coil 2, so the voltage induced in coil 2 is

$$v_2 = N_2 \frac{d\phi_{12}}{dt} \qquad \ldots(5)$$

Again, as the fluxes are caused by the current i_1 flowing in coil 1, Eq. (4) can be written as

$$v_1 = N_1 \frac{d\phi_1}{di_1} \frac{di_1}{dt} = L_1 \frac{di_1}{dt} \qquad \ldots(6)$$

Similarly,

Eq. (5) can be written as

$$v_2 = \boxed{N_2 \frac{d\phi_{12}}{di_1}} \frac{di_1}{dt} = \boxed{M_{21}} \frac{di_1}{dt} \qquad \ldots(7)$$

M_{21} is known as the ***mutual inductance*** of coil 2 with respect to coil 1. Thus, the open-circuit mutual voltage (or induced voltage) across coil 2 is

$$\boxed{v_2 = M_{21} \frac{di_1}{dt}} \qquad \ldots(8)$$

Suppose we now let current i_2 flow in coil 2, while coil 1 carries no current (Fig. 1.3). The magnetic flux ϕ_2 emanating from coil 2 comprises flux ϕ_{22} that links only coil 2 and flux ϕ_{21} that links both coils. Hence ans simlar to the previous case,

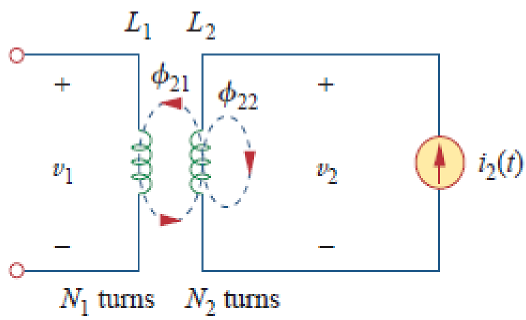


Fig 1.2 Mutual inductance of coil 1 with respect to coil 2.

$$\phi_2 = \phi_{21} + \phi_{22}$$

$$v_2 = N_2 \frac{d\phi_2}{dt} = N_2 \frac{d\phi_2}{di_2} \frac{di_2}{dt} = L_2 \frac{di_2}{dt}$$

$$v_1 = N_1 \frac{d\phi_{21}}{dt} = N_1 \frac{d\phi_{21}}{di_2} \frac{di_2}{dt} = M_{12} \frac{di_2}{dt}$$

$$M_{12} = N_1 \frac{d\phi_{21}}{di_2} \quad \Rightarrow \quad \boxed{v_1 = M_{12} \frac{di_2}{dt}}$$

Like self-inductance L, mutual inductance M is measured in henrys (H). Keep in mind that mutual coupling only exists when the inductors or coils are in close proximity, and the circuits are driven by time-varying sources. We recall that inductors act like **short circuits** to dc.

From the two cases in Figs. 1.2 and 1.3, we conclude that ***mutual inductance*** results if a voltage is induced by a time-varying current in another circuit. It is the property of an inductor to produce a voltage in reaction to a time-varying current in another inductor near it.

Although mutual inductance M is always a positive quantity, the mutual voltage $M\ di/dt$ may be negative or positive, just like the self induced voltage $L\ di/dt$ .However, unlike the self-induced $L\ di/dt$,whose polarity is determined by the reference direction of the current and the reference polarity of the voltage (according to the passive sign convention), the polarity of mutual voltage $M\ di/dt$ is not easy to determine, because four terminals are involved. For the sematic circuit, we can find the polarity of the mutual inductance by signing the coils with dots to define how are physically placed so:-

1- A current entering the dotted terminal of one coil produces an open circuit voltage with a positive voltage reference at the dotted terminal of the second coil

2- A current entering the undotted terminal of one coil provides a voltage that is positively sensed at the undotted terminal of the second coil.

Application of the dot convention is illustrated in the four pairs of mutually coupled coils in Fig. 1.3.

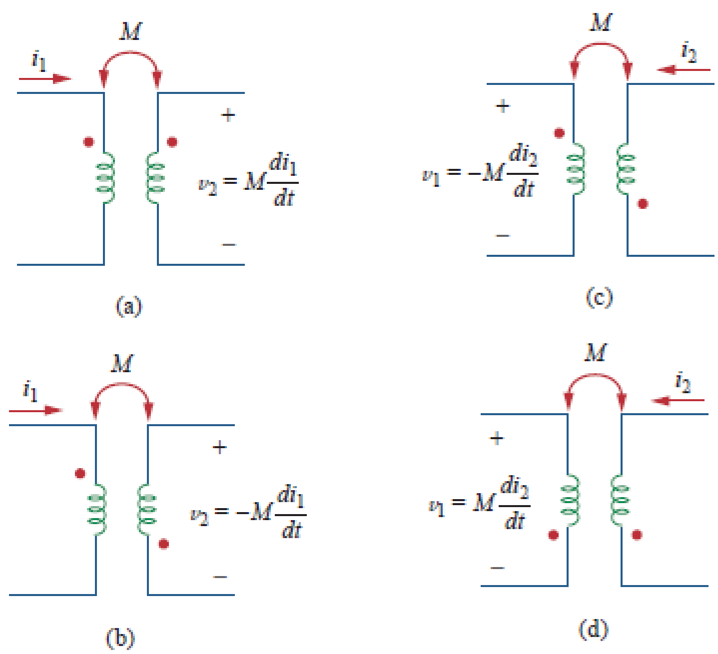


Fig 1.3 Examples illustrating how to apply the dot convention.

Figure 1.4 shows the dot convention for coupled coils in series. For the coils in Fig. 1.4(a,b), the total inductance are

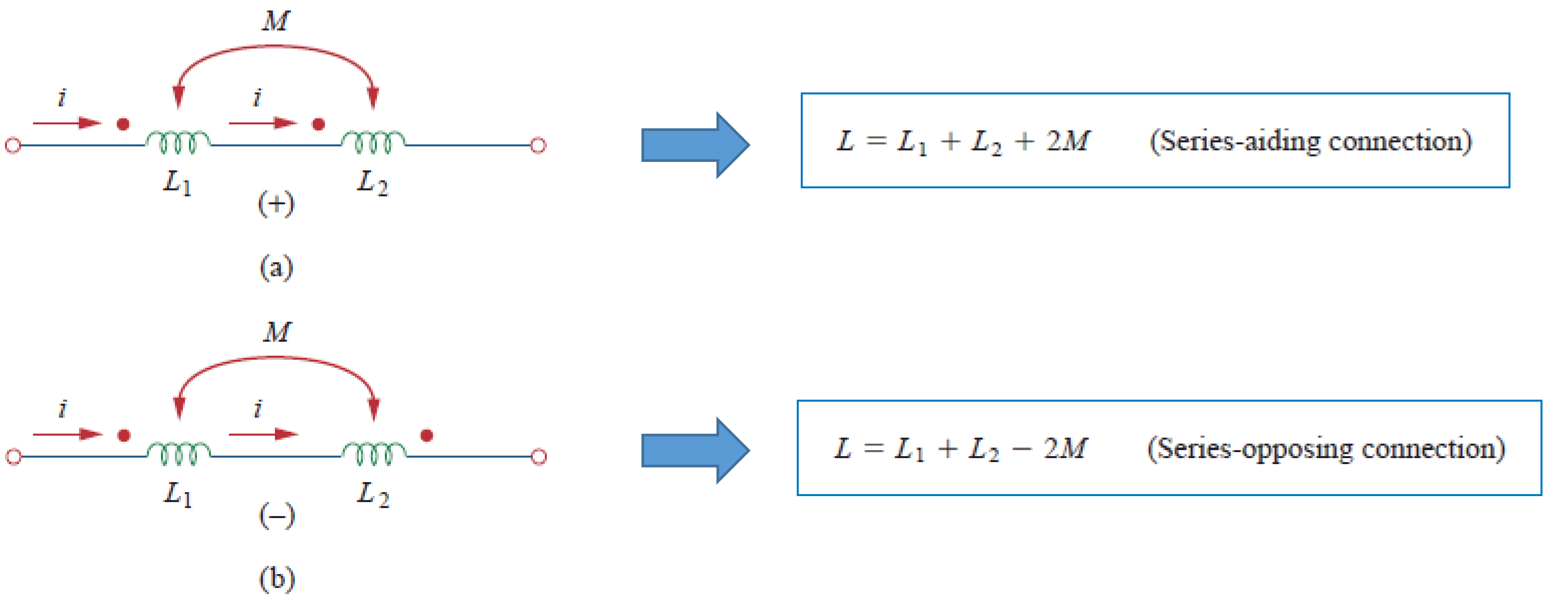


Fig 1.4 Dot convention for coils in series and their equivalent inductance; the sign indicates the polarity of the mutual voltage: (a) series-aiding connection, (b) series-opposing connection.

Now that we know how to determine the polarity of the mutual voltage, we are prepared to analyze circuits involving mutual inductance. As the first example, consider the circuit in Fig. 1.5. Applying KVL to coil 1 gives

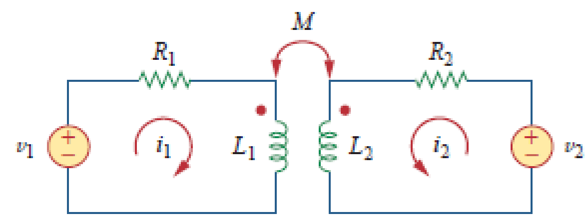


Fig 1.5 Time-domain analysis of a circuit containing coupled coils

$$v_1 = i_1 R_1 + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} \qquad \ldots (9)$$

For coil 2, KVL gives

$$v_2 = i_2 R_2 + L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} \qquad \ldots (10)$$

We can write Eqs. (9 &10) in the frequency domain as

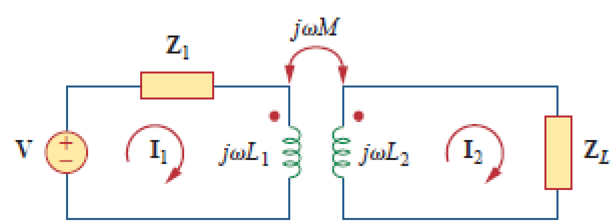


Fig 1.6 Frequency-domain analysis of a circuit containing coupled coils.

$$\mathbf{V}_1 = (R_1 + j\omega L_1)\mathbf{I}_1 + j\omega M \mathbf{I}_2$$
$$\mathbf{V}_2 = j\omega M \mathbf{I}_1 + (R_2 + j\omega L_2)\mathbf{I}_2 \qquad \ldots (11)$$

As a second example, consider the circuit in Fig. 13.8. We analyze this in the frequency domain. Applying KVL to coil 1 and coil 2, we get respectively;

$$\mathbf{V} = (\mathbf{Z}_1 + j\omega L_1)\mathbf{I}_1 - j\omega M \mathbf{I}_2 \qquad \ldots (12)$$
$$0 = -j\omega M \mathbf{I}_1 + (\mathbf{Z}_L + j\omega L_2)\mathbf{I}_2$$

1.2 Physical Basis of the Dot Convention

The meaning of the dots is now interpreted in terms of magnetic flux. Two coils are shown wound on a cylindrical form in Fig. 1.7, and the direction of each winding is evident.

Let us assume that the current i_1 is positive and increasing with time. The magnetic flux that i_1 produces within the form has a direction, which may be found by the right hand rule: when the right hand is wrapped around the coil with the fingers pointing in the direction of current flow, the thumb indicates the direction of the flux within the coil. Thus i_1 produces a flux which is directed downward; since i_1 is increasing with time, the flux, which is proportional to i_1, is also increasing with time. Turning now to the second coil, let us also think of i_2 as positive and increasing; the application of the right-hand rule shows that i_2 also produces a magnetic flux which is directed downward and is increasing. In other words, the assumed currents i_1 and i_2 produce additive fluxes.

The voltage across the terminals of any coil results from the time rate of change of the flux within that coil. The voltage across the terminals of the first coil is therefore greater with i_2 flowing than it would be if i_2 were zero; i_2 induces a voltage in the first coil which has the same sense as the self-induced voltage in that coil. The sign of the self-induced voltage is known from the passive sign convention, and the sign of the mutual voltage is thus obtained. The dot convention enables us to suppress the physical construction of the coils by placing a dot at one terminal of each coil such that currents entering dot-marked terminals produce additive fluxes. It is apparent that there are always two possible locations for the dots, because both dots may always be moved to the other ends of the coils and additive fluxes will still result.

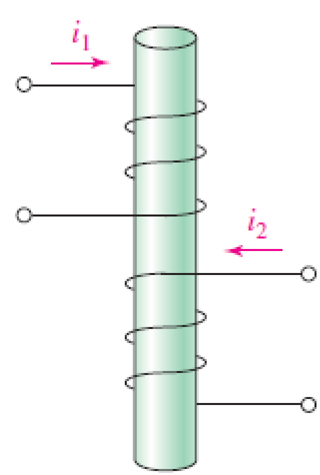


Fig 1.7 The physical construction of two mutually coupled coils

Example1.1:- Calculate the phasor currents I1 and I2 in the circuit of Fig. 1.8

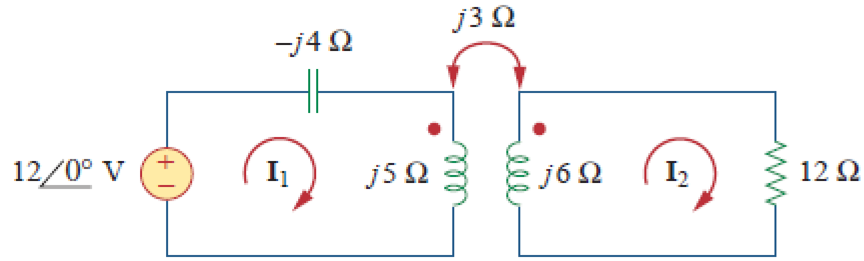


Fig 1.8 For Example 1.1

Example1.2:- Calculate the mesh currents in the circuit of Fig. 1.9.

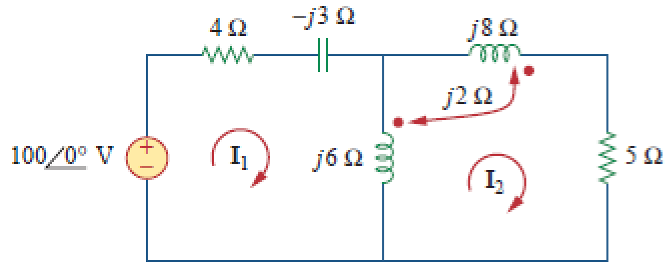


Fig 1.9 For Example 1.2