Basrah University
College of Engineering
Electrical Engineering Department

Electric Circuits Analysis

Asst. Lect: Hamzah Abdulkareem

Chapter Four
Part 2

Three Phase Circuits

- ***Balanced Delta -Delta Connection***
- ***Balanced Delta – Wye Connection***
- ***Power in a 3 Phase Balanced System***
- ***Economical Using of Three Phase System***

C- Balanced Delta -Delta Connection

A balanced $\Delta - \Delta$ system is one in which both the balanced source and balanced load are Δ-connected as shown in Fig. 4.15.

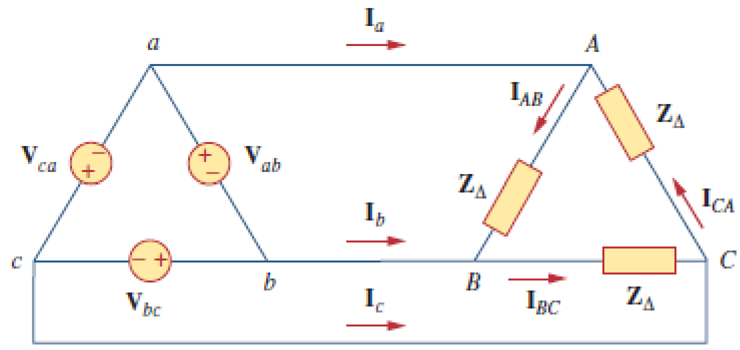

Fig 4.15. Δ-Δ *system.*

Assuming a positive sequence, the phase voltages for a delta-connected source are

$$\mathbf{V}_{ab} = V_p\angle 0^\circ$$
$$\mathbf{V}_{bc} = V_p\angle -120^\circ, \qquad \mathbf{V}_{ca} = V_p\angle +120^\circ \qquad \ldots (4.18)$$

The line voltages are the same as the phase voltages. From Fig. 4.16, assuming there is no line impedances, the phase voltages of the delta connected source are equal to the voltages across the impedances; that is,

$$\mathbf{V}_{ab} = \mathbf{V}_{AB}, \qquad \mathbf{V}_{bc} = \mathbf{V}_{BC}, \qquad \mathbf{V}_{ca} = \mathbf{V}_{CA} \qquad \ldots (4.19)$$

Hence, the phase currents are

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{Z_\Delta} = \frac{\mathbf{V}_{ab}}{Z_\Delta}, \qquad \mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{Z_\Delta} = \frac{\mathbf{V}_{bc}}{Z_\Delta}$$
$$\mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{Z_\Delta} = \frac{\mathbf{V}_{ca}}{Z_\Delta} \qquad \ldots (4.20)$$

The line currents are obtained from the phase currents by applying KCL at nodes A, B, and C, as we did in the previous section:

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA}, \qquad \mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB}, \qquad \mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC} \qquad \ldots (4.21)$$

Also, as shown in the last section, each line current lags the corresponding phase current by 30° the magnitude I_L of the line current is $\sqrt{3}$ times the magnitude I_P of the phase current,

$$I_L = \sqrt{3} I_p \qquad \ldots \quad (4.22)$$

An alternative way of analyzing this circuit is to convert both the source and the load to their Y equivalents.

Example 4.3/ A balanced Δ-connected load having an impedance $(20 - j15)\Omega$ is connected to a Δ -connected, positive-sequence generator having $\boldsymbol{V_{ab} = 330\angle 0^{\circ}\ V}$ Calculate the phase currents of the load and the line currents.

D- Balanced Delta – Wye Connection

A balanced $\Delta - \mathbf{Y}$ system consists of a balanced Δ-connected source feeding a balanced $\mathbf{Y}$ -connected load as shown in Fig. 4.16.

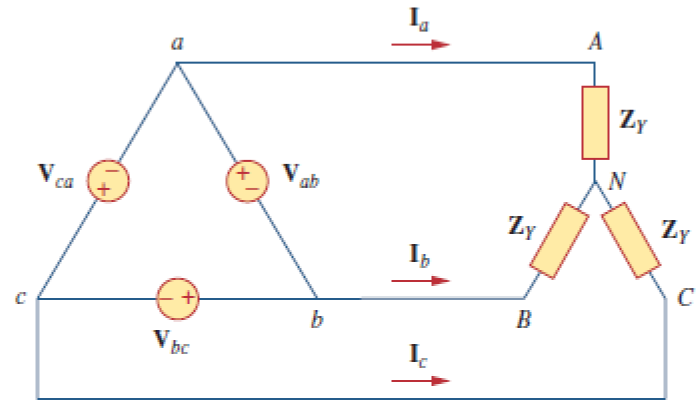

Fig 4.16. Δ- Y system.

Again, assuming the abcsequence, the phase voltages of a delta-connected source are

$$\mathbf{V}_{ab} = V_p\angle 0^\circ$$
$$\mathbf{V}_{bc} = V_p\angle -120^\circ, \qquad \mathbf{V}_{ca} = V_p\angle +120^\circ \qquad \dots (4.23)$$

These are also the line voltages as well as the phase voltages. We can obtain the line currents in many ways. One way is to apply KVL to loop aANBba in Fig. 4.16, writing

$$-\mathbf{V}_{ab} + \mathbf{Z}_Y\mathbf{I}_a - \mathbf{Z}_Y\mathbf{I}_b = 0$$

or

$$\mathbf{Z}_Y(\mathbf{I}_a - \mathbf{I}_b) = \mathbf{V}_{ab} = V_p\angle 0^\circ$$

Thus,

$$\mathbf{I}_a - \mathbf{I}_b = \frac{V_p\angle 0^\circ}{\mathbf{Z}_Y} \qquad \dots (4.24)$$

But I_b lags I_a by 120° since we assumed the abc sequence; that is, $I_b = I_a\angle -\mathbf{120}^\circ$. Hence,

$$\mathbf{I}_a - \mathbf{I}_b = \mathbf{I}_a(1 - 1\angle -120^\circ)$$
$$= \mathbf{I}_a\left(1 + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = \mathbf{I}_a\sqrt{3}\angle 30^\circ \qquad \dots (4.25)$$

Substituting Eq. (4.25) into Eq. (4.24) gives,

$$\mathbf{I}_a = \frac{V_p/\sqrt{3}\angle -30^\circ}{\mathbf{Z}_Y} \qquad \dots (4.26)$$

and from it, we obtain the other line currents I_b & I_c.

Another way to obtain the line currents is to replace the delta connected source with its equivalent wye-connected source, as shown in Fig. 4.17.

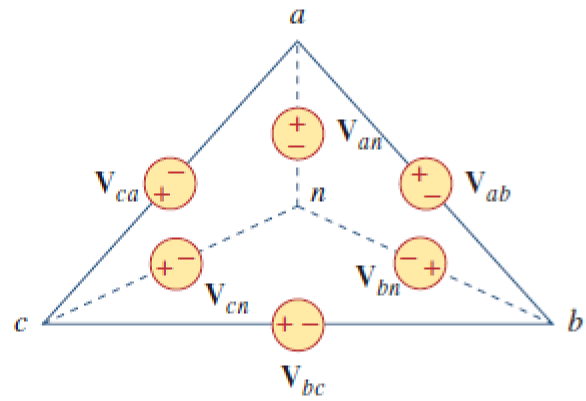

Fig 4.17. Transforming a Δ -connected source to an equivalent Y-connected source.

In previous section , we found that the line-to-line voltages of a wye-connected source lead their corresponding phase voltages by 30° .Therefore, we obtain each phase voltage of the equivalent wye connected source by dividing the corresponding line voltage of the delta-connected source by $\sqrt{3}$ and shifting its phase by -30° .Thus, the equivalent wye-connected source has the phase voltages

$$\mathbf{V}_{an} = \frac{V_p}{\sqrt{3}}\angle -30^\circ$$

$$\mathbf{V}_{bn} = \frac{V_p}{\sqrt{3}}\angle -150^\circ, \qquad \mathbf{V}_{cn} = \frac{V_p}{\sqrt{3}}\angle +90^\circ \qquad \ldots (4.27)$$

If the delta-connected source has source impedance Z_s per phase, the equivalent wye-connected source will have a source impedance of $Z_s/3$ per phase.

Once the source is transformed to wye, the circuit becomes a wye-wye system. Therefore, we can use the equivalent single-phase circuit shown in Fig. 4.18, from which the line current for phase a is

$$\mathbf{I}_a = \frac{V_p/\sqrt{3}\angle -30^\circ}{\mathbf{Z}_Y} \qquad \ldots (4.28)$$

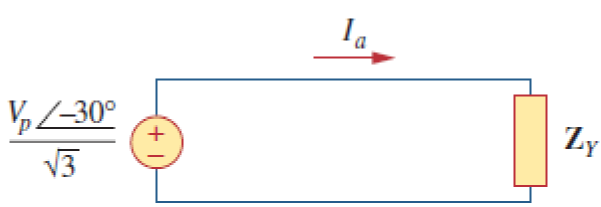

Fig 4.18. The single-phase equivalent circuit.

Example 4.4/ A balanced Y-connected load with a phase impedance of $(40 + j25)\Omega$ is supplied by a balanced, positive sequence Δ -connected source with a line voltage of 210 V. Calculate the phase currents. Use V_{ab} as a reference.

Table 4.1 presents a summary of the formulas for phase currents and voltages and line currents and voltages for the four connections. Students are advised not to memorize the formulas but to understand how they are derived. The formulas can always be obtained by directly applying KCL and KVL to the appropriate three phase circuits.

Table 4.1. Summary of phase and line voltages/currents for balanced three-phase systems.

Connection	Phase voltages/currents	Line voltages/currents
Y-Y	$\mathbf{V}_{an} = V_p\angle 0^\circ$	$\mathbf{V}_{ab} = \sqrt{3}V_p\angle 30^\circ$
	$\mathbf{V}_{bn} = V_p\angle -120^\circ$	$\mathbf{V}_{bc} = \mathbf{V}_{ab}\angle -120^\circ$
	$\mathbf{V}_{cn} = V_p\angle +120^\circ$	$\mathbf{V}_{ca} = \mathbf{V}_{ab}\angle +120^\circ$
	Same as line currents	$\mathbf{I}_a = \mathbf{V}_{an}/\mathbf{Z}_Y$
		$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$
		$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Y-Δ	$\mathbf{V}_{an} = V_p\angle 0^\circ$	$\mathbf{V}_{ab} = \mathbf{V}_{AB} = \sqrt{3}V_p\angle 30^\circ$
	$\mathbf{V}_{bn} = V_p\angle -120^\circ$	$\mathbf{V}_{bc} = \mathbf{V}_{BC} = \mathbf{V}_{ab}\angle -120^\circ$
	$\mathbf{V}_{cn} = V_p\angle +120^\circ$	$\mathbf{V}_{ca} = \mathbf{V}_{CA} = \mathbf{V}_{ab}\angle +120^\circ$
	$\mathbf{I}_{AB} = \mathbf{V}_{AB}/\mathbf{Z}_\Delta$	$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$
	$\mathbf{I}_{BC} = \mathbf{V}_{BC}/\mathbf{Z}_\Delta$	$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$
	$\mathbf{I}_{CA} = \mathbf{V}_{CA}/\mathbf{Z}_\Delta$	$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Δ-Δ	$\mathbf{V}_{ab} = V_p\angle 0^\circ$	Same as phase voltages
	$\mathbf{V}_{bc} = V_p\angle -120^\circ$	
	$\mathbf{V}_{ca} = V_p\angle +120^\circ$	
	$\mathbf{I}_{AB} = \mathbf{V}_{ab}/\mathbf{Z}_\Delta$	$\mathbf{I}_a = \mathbf{I}_{AB}\sqrt{3}\angle -30^\circ$
	$\mathbf{I}_{BC} = \mathbf{V}_{bc}/\mathbf{Z}_\Delta$	$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$
	$\mathbf{I}_{CA} = \mathbf{V}_{ca}/\mathbf{Z}_\Delta$	$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$
Δ-Y	$\mathbf{V}_{ab} = V_p\angle 0^\circ$	Same as phase voltages
	$\mathbf{V}_{bc} = V_p\angle -120^\circ$	
	$\mathbf{V}_{ca} = V_p\angle +120^\circ$	
	Same as line currents	$\mathbf{I}_a = \dfrac{V_p\angle -30^\circ}{\sqrt{3}\mathbf{Z}_Y}$
		$\mathbf{I}_b = \mathbf{I}_a\angle -120^\circ$
		$\mathbf{I}_c = \mathbf{I}_a\angle +120^\circ$

4.3 Power in a Balanced System

Let us now consider the power in a balanced three-phase system. We begin by examining the instantaneous power absorbed by the load. This requires that the analysis be done in the time domain. For a Y-connected load, the phase voltages are

$$v_{AN} = \sqrt{2}V_p \cos \omega t, \qquad v_{BN} = \sqrt{2}V_p \cos(\omega t - 120^\circ)$$
$$v_{CN} = \sqrt{2}V_p \cos(\omega t + 120^\circ) \qquad \ldots (4.29)$$

The phase currents lag behind their corresponding phase voltages by θ Thus,

$$i_a = \sqrt{2}I_p \cos(\omega t - \theta), \qquad i_b = \sqrt{2}I_p \cos(\omega t - \theta - 120^\circ)$$
$$i_c = \sqrt{2}I_p \cos(\omega t - \theta + 120^\circ) \qquad \ldots (4.30)$$

The total instantaneous power in the load is the sum of the instantaneous powers in the three phases; that is,

$$\begin{aligned} p = p_a + p_b + p_c &= v_{AN}i_a + v_{BN}i_b + v_{CN}i_c \\ &= 2V_pI_p[\cos \omega t \cos(\omega t - \theta) \\ &\quad + \cos(\omega t - 120^\circ)\cos(\omega t - \theta - 120^\circ) \\ &\quad + \cos(\omega t + 120^\circ)\cos(\omega t - \theta + 120^\circ)] \end{aligned} \qquad \ldots (4.31)$$

Applying the trigonometric identity

$$\cos A \cos B = \frac{1}{2}[\cos(A + B) + \cos(A - B)] \qquad \ldots (4.32)$$

Gives

$$\begin{aligned} p &= V_pI_p[3\cos\theta + \cos(2\omega t - \theta) + \cos(2\omega t - \theta - 240^\circ) \\ &\quad + \cos(2\omega t - \theta + 240^\circ)] \\ &= V_pI_p[3\cos\theta + \cos\alpha + \cos\alpha\cos 240^\circ + \sin\alpha\sin 240^\circ \\ &\quad + \cos\alpha\cos 240^\circ - \sin\alpha \sin 240^\circ] \\ &\quad \text{where } \alpha = 2\omega t - \theta \\ &= V_pI_p\left[3\cos\theta + \cos\alpha + 2\left(-\frac{1}{2}\right)\cos\alpha\right] = 3V_pI_p\cos\theta \end{aligned} \qquad \ldots (4.33)$$

Thus the total instantaneous power in a balanced three-phase system is ***constant***—it does not change with time as the instantaneous power of each phase does. This result is true whether the load is Y- or Δ -connected.

Since the total instantaneous power is independent of time, the average power per phase P_p and the reactive power per phase Q_p and the apparent power per phase S_p and complex power per phase S_p either the Δ-connected load or the Y-connected load are

$$P_p = V_p I_p \cos\theta \quad \& \quad Q_p = V_p I_p \sin\theta \quad \& \quad S_p = V_p I_p \quad \& \quad \mathbf{S}_p = P_p + jQ_p$$

The total average, reactive, and complex power are the sum of the corresponding powers in the phases:

$$P = P_a + P_b + P_c = 3P_p = 3V_p I_p \cos\theta = \sqrt{3} V_L I_L \cos\theta$$
$$Q = 3V_p I_p \sin\theta = 3Q_p = \sqrt{3} V_L I_L \sin\theta$$
$$\mathbf{S} = P + jQ \qquad \dots (4.34)$$

4.3.1 ***Economical Using of Three Phase System***

A second major advantage of three-phase systems for power distribution is that the three-phase system uses a lesser amount of wire than the single-phase system for the same line voltage V_L and the same absorbed power P_L. For the two-wire single-phase system in Fig. 4.19(a), $I_L = P_L/V_L$ so the power loss in the two wires is

$$P_{\text{loss}} = 2I_L^2 R = 2R\frac{P_L^2}{V_L^2} \qquad \dots (4.35)$$

For the three-wire three-phase system in Fig. 14.19 (b), $I'_L = |I_b| = |I_c| = P_L/(\sqrt{3}V_L)$ from Eq. (4.34). The power loss in the three wires is

$$P'_{\text{loss}} = 3(I'_L)^2 R' = 3R'\frac{P_L^2}{3V_L^2} = R'\frac{P_L^2}{V_L^2} \qquad \dots (4.36)$$

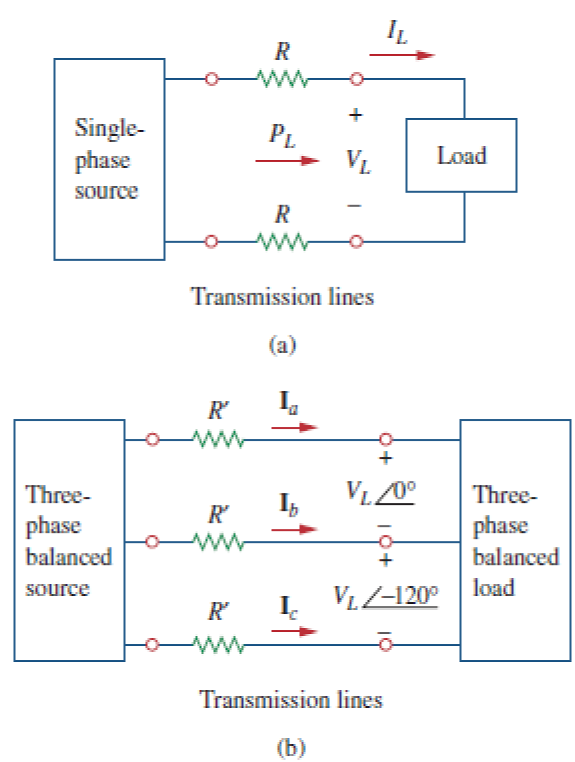

Fig 4.19. Comparing the power loss in (a) a single-phase system, and (b) a three-phase system.

Equations (3.35) and (3.36) show that for the same total power delivered P_L and same line voltage V_L

$$\frac{P_{\text{loss}}}{P'_{\text{loss}}} = \frac{2R}{R'} \quad \ldots (4.37)$$

As $R = \rho l/\pi r^2$ & $R' = \rho l/\pi r'^2$, where r and r' are the radii of the wires. Thus,

$$\frac{P_{\text{loss}}}{P'_{\text{loss}}} = \frac{2r'^2}{r^2} \quad \ldots (4.38)$$

If the same power loss is tolerated in both systems, then $r^2 = 2r'^2$ .The ratio of material required is determined by the number of wires and their volumes, so

$$\frac{\text{Material for single-phase}}{\text{Material for three-phase}} = \frac{2(\pi r^2 \ell)}{3(\pi r'^2 \ell)} = \frac{2r^2}{3r'^2}$$
$$= \frac{2}{3}(2) = 1.333 \quad \ldots (4.39)$$

Equation (4.39) shows that the single-phase system uses ***33 percent more material*** than the three-phase system or that the three-phase system uses only ***75 percent of the material*** used in the equivalent single-phase system. In other words, considerably less material is needed to deliver the same power with a three-phase system than is required for a single-phase system.

Example 4.5/ A three-phase motor can be regarded as a balanced Y-load. A three phase motor draws 5.6 kW when the line voltage is 220 V and the line current is 18.2 A. Determine the power factor of the motor.