Basrah University
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Electric Circuits Analysis

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Chapter One

Resistive Circuits with Dependent Sources

- ***Syllabus***
- ***References***
- ***Voltage & Current Sources***
- ***Nodal Analysis***
- ***Mesh Analysis***
- ***Superposition***
- ***Thevenin's Theorem***
- ***Norton's Theorem***
- ***Maximum Power Transfer***

Syllabus:-

Chapter 1-Resistive Circuits with Dependent Sources:
Dependent and independent sources, mesh analysis, super Mesh, nodal analysis, super node, Thevenin and Norton equivalent circuits, superposition analysis, maximum power transfer.

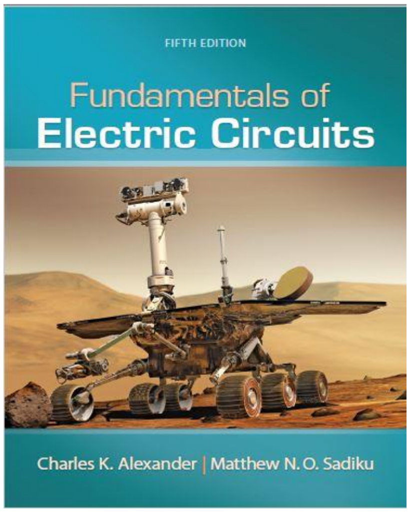
Chapter 2-The Transient Circuits
RL, RC, RLC circuit in parallel and series and their complete response.

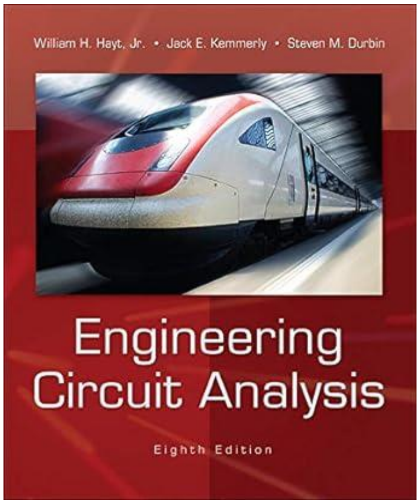
Chapter 3:- Sinusoidal Steady State Analysis
Sinusoidal analysis and phasor, mesh and nodal ac analysis, Thevenin and Norton ac analysis, superposition ac analysis, AC power calculation.

Chapter 4:-Poly-phase Circuits
Single-phase three wire system, 3-phase balance and unbalance systems with star and delta connections, power in 3-phase circuits.

References :-

1- *Fundamentals of electric circuits.by Alexander, Charles K. 5th Edition.*
2- *Engineering circuit analysis by William Hayat 8th Edition*





Chapter One

Resistive Circuits with Dependent Sources

1.1 Voltage & Current Sources

Independent Voltage Source is an active element that provides a specified voltage that is completely ***independent*** of other circuit elements.

In other words, an ideal independent voltage source delivers to the circuit whatever current is necessary to maintain its terminal voltage. Physical sources such as batteries and generators may be regarded as approximations to ideal voltage sources. Fig 1.1 shows the symbols for independent voltage sources. Notice that both symbols in Fig. 1.1 (a) and (b) can be used to represent a dc voltage source, but only the symbol in Fig. 1.1 (a) can be used for a time-varying voltage source.

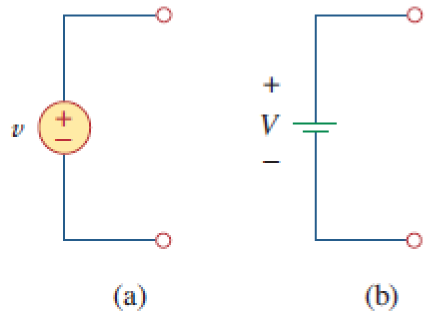


Fig 1.1 Symbols for independent voltage sources: (a) used for constant or time-varying voltage, (b) used for constant voltage (dc).

Independent Current Source is an active element that provides a specified current that is completely ***independent*** of other circuit elements. The symbol is shown in Fig. 1.2.

Like the independent voltage source, the independent current source is at best a reasonable approximation for a physical element. In theory, it can deliver infinite power from its terminals because it produces the same finite current for any voltage across it, no matter how large that voltage may be. It is, however, a good approximation for many practical sources, particularly in electronic circuits.

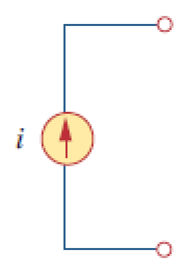


Fig 1.2 Symbol for independent current source.

Dependent Sources:- A dependent (or controlled) source is an active element in which the source quantity is ***dependent or controlled*** by another voltage or current. Sources such as these appear in the equivalent electrical models for many electronic devices, such as transistors, operational amplifiers, and integrated circuits.

Dependent sources are usually designated by diamond-shaped symbols, as shown in Fig. 1. 3. Since the control of the dependent source is achieved by a voltage or current of some other element in the circuit, and the source can be voltage or current, it follows that there are four possible types of dependent sources, namely:

1. A voltage-controlled current source (***VCVS***), as shown in Fig. 1.3 a.
2. A current-controlled current source (***CCVS***), as shown in Fig. 1.3 b.
3. A voltage-controlled voltage source (***VCCS***), as shown in Fig. 1.3 c.
4. A current-controlled voltage source (***CCCS***), as shown in Fig. 1.3 d.

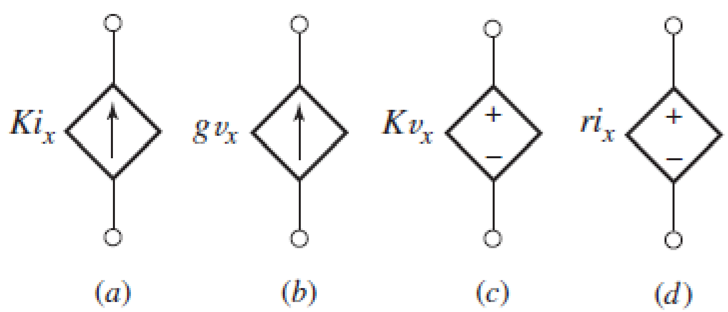


Fig 1..3 The four different types of dependent sources: (a) voltage controlled current source; (b) current - controlled current source; (c) voltage-controlled voltage source; (d) current controlled voltage source.

Example 1.1:- *Determine v_o and i in the circuit shown in Fig 1.4 (a).*

By applying KVL around the loop as shown in Fig. 1.4(b).

The result is

$$-12 + 4i + 2v_o - 4 + 6i = 0 \qquad \ldots(1)$$

Applying Ohm's law to the $6\Omega -$ resistor gives

$$v_o = -6i \qquad \ldots(2)$$

Substituting Eq. (2) into Eq. (1) yields

$$-12 + 4i - 12i - 4 + 6i = 0$$

$$-16 - 2i = 0$$

$$i = -8\ A \quad and \quad v_o = 48\ V$$

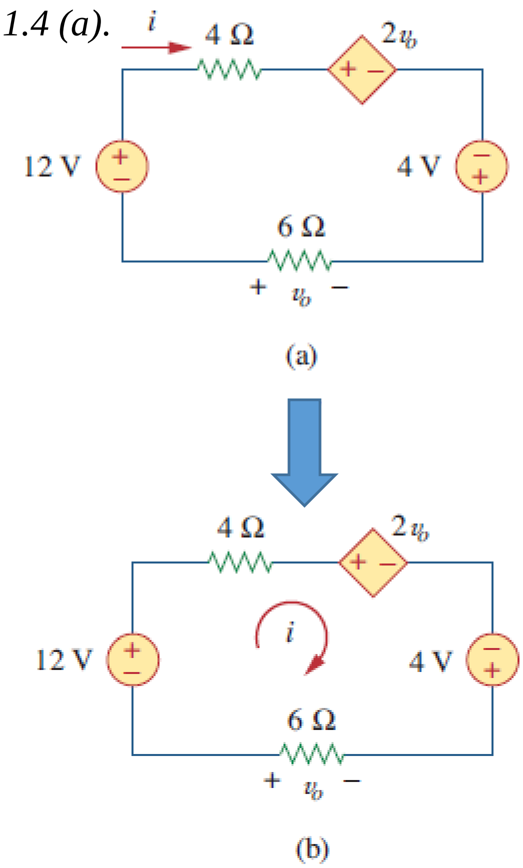


Fig 1.4 For Example 1.1.

Example 1.2:- Calculate the power supplied or absorbed by each element for the circuit shown in Fig 1.5.

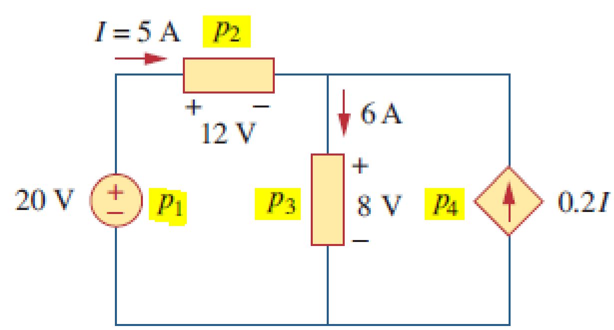


Fig 1.5 For Example 1.2.

For p_1, the $5 - A$ current is out of the positive terminal (or into the negative terminal); hence,

$p_1 = 20(-5) = -100\ W$ **Supplied power**

For p_2 and p_3, the current flows into the positive terminal of the element in each case.

$p_2 = 12(5) = 60\ W \quad and \quad p_3 = 8(6) = 48\ W$ **Absorbed powers**

For p_4, we should note that the voltage is 8 V (positive at the top), the same as the voltage for p_3 since both the passive element and the dependent source are connected to the same terminals.

$p_4 = 8(-0.2\ I) = 8 * (-0.2 * 5) = -8\ W$ **Supplied power**

Homework

Problem1:- Find v_o *and* i in the circuit of Fig. 1. 6.

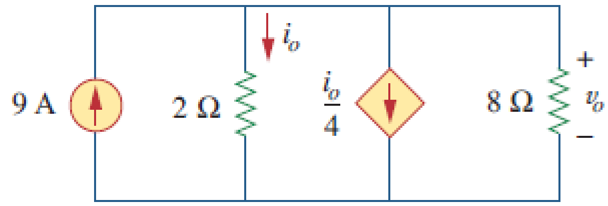


Fig 1.6 For Problem 1.

Problem 2:- Find V_o in the circuit in Fig. 1.7 and the power absorbed by the dependent source.

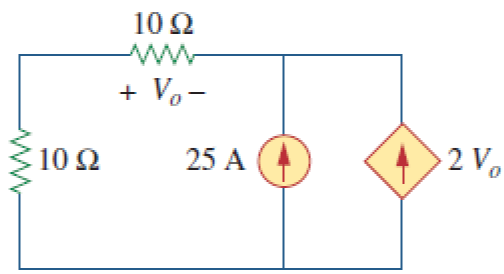


Fig 1.7 For Problem 2.

Problem 3:- Find the power absorbed by each element in the circuit in Fig. 1.8.

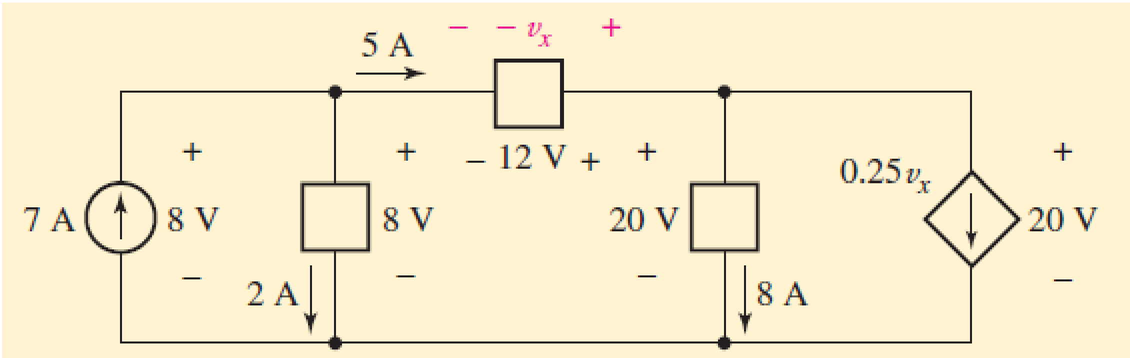


Fig 1.8 For Problem 3.

1.2 Nodes, Branches, and Loops

Since the elements of an electric circuit can be interconnected in several ways, we need to understand some basic concepts of network topology. These concepts are:-

1- ***Branch:-*** A branch represents a single element such as a voltage source or a resistor. In other words, a branch represents any two-terminal element. The circuit in Fig. 1.9 has five branches, namely, the 10-V voltage source, the 2-A current source, and the three resistors.

2- ***Node:-*** A node is the point of connection between two or more branches. A node is usually indicated by a dot in a circuit. If a short circuit (a connecting wire) connects two nodes, the two nodes constitute a single node. The circuit in Fig. 1.9 has three nodes a, b and c.

3- ***Loop:-*** A loop is any closed path in a circuit.

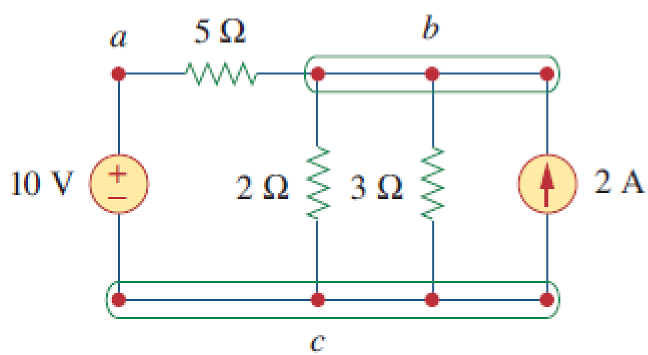


Fig 1.9 Nodes, branches, and loops.

Example 1.3 :- How many branches and nodes does the circuit in Fig. 1.10 have?

Sol/ There are five branches and three nodes.

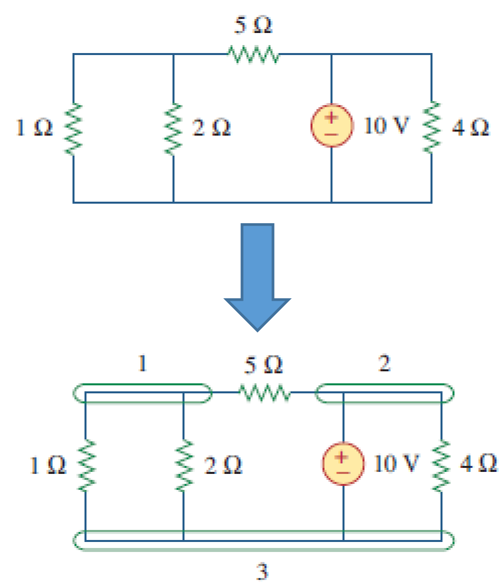


Fig 1.10 For Example 1.3.

1.3 Nodal Analysis

Nodal analysis provides a general procedure for analyzing circuits using node voltages as the circuit variables. Choosing node voltages instead of element voltages as circuit variables is convenient and reduces the number of equations one must solve simultaneously.

In nodal analysis, we are interested in finding the node voltages. Given a circuit with n nodes without voltage sources, the nodal analysis of the circuit involves taking the following three steps:-

1- Select a node as the reference node.

2- Assign voltages $v_1, v_2, \dots, v_{n-1}$ to the remaining $n-1$ nodes. The voltages are referenced with respect to the reference node.

3- Apply ***KCL*** to each of the $n-1$ no reference nodes.

$$\sum currents\ entering\ the\ node\ from\ current\ sources$$
$$= \sum currents\ leaving\ the\ node\ through\ resistors$$

4- Use Ohm's law to express the branch currents in terms of node voltages. You should noted that Current flows from a higher potential to a lower potential in a resistor. We can express this principle as

$$i = \frac{v_{higher} - v_{lower}}{R}$$

5- Solve the resulting simultaneous equations to obtain the unknown node voltages.

Example 1.4:- Calculate the node voltages in the circuit shown in Fig. 1.11

At node 1, applying ***KCL*** and Ohm's law gives

$$i_1 = i_2 + i_3 \quad \Rightarrow \quad 5 = \frac{v_1 - v_2}{4} + \frac{v_1 - 0}{2}$$

$$\Rightarrow 20 = v_1 - v_2 + 2v_1$$

$$\Rightarrow 3v_1 - v_2 = 20 \qquad \ldots (1)$$

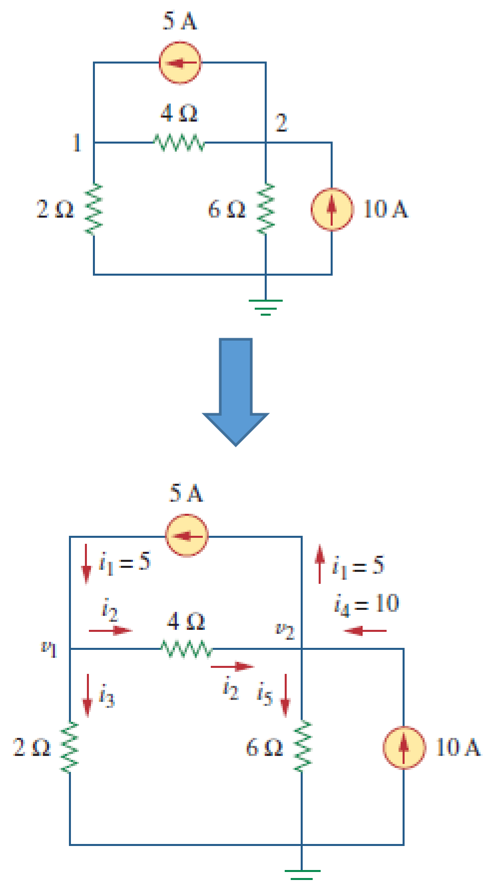


Fig 1.11 For Example 1.4.

At node 2, we do the same thing and get

$$i_4 + i_2 = i_1 + i_5 \quad \Rightarrow \quad 10 + \frac{v_1 - v_2}{4} = 5 + \frac{v_2 - 0}{6}$$

$$\Rightarrow -3v_1 + 5v_2 = 60 \qquad \ldots (2)$$

To solve the above equations, we will use Cramer's rule, we need to put Eqs. (1) and (2) in matrix form as

$$\begin{bmatrix} 3 & -1 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 60 \end{bmatrix}$$

The determinant of the matrix is

$$\Delta = \begin{vmatrix} 3 & -1 \\ -3 & 5 \end{vmatrix} = 15 - 3 = 12$$

We now obtain v_1 and v_2 as

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{\begin{vmatrix} 20 & -1 \\ 60 & 5 \end{vmatrix}}{\Delta} = \frac{100 + 60}{12} = 13.333 \text{ V}$$

$$v_2 = \frac{\Delta_2}{\Delta} = \frac{\begin{vmatrix} 3 & 20 \\ -3 & 60 \end{vmatrix}}{\Delta} = \frac{180 + 60}{12} = 20 \text{ V}$$

If we need the currents, we can easily calculate them from the values of the nodal voltages.

$$i_1 = 5 \text{ A}, \qquad i_2 = \frac{v_1 - v_2}{4} = -1.6668 \text{ A}, \qquad i_3 = \frac{v_1}{2} = 6.666 \text{ A}$$

$$i_4 = 10 \text{ A}, \qquad i_5 = \frac{v_2}{6} = 3.333 \text{ A}$$

Example 1.5:- Determine the voltages at the nodes in Fig. 1.12

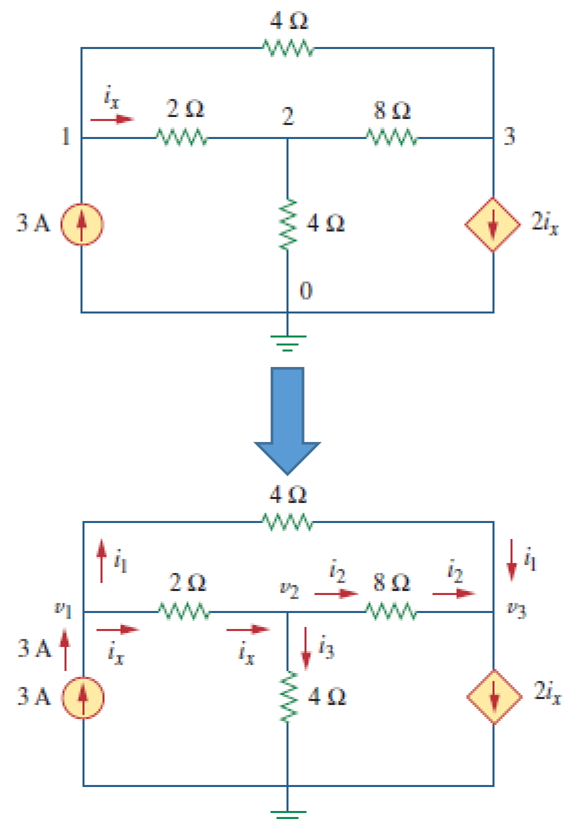


Fig 1.12 :For Example 1.5.

1.3.1 ***Nodal Analysis with Voltage Sources***

We now consider how voltage sources affect nodal analysis. We use the circuit in Fig. 1.13 for illustration. Consider the following two possibilities.

CASE 1 If a voltage source is connected between the reference node and a nonreference node, we simply set the voltage at the nonreference node equal to the voltage of the voltage source. In Fig. 1.13,for example,

$$v_1 = 10 \qquad \ldots (1)$$

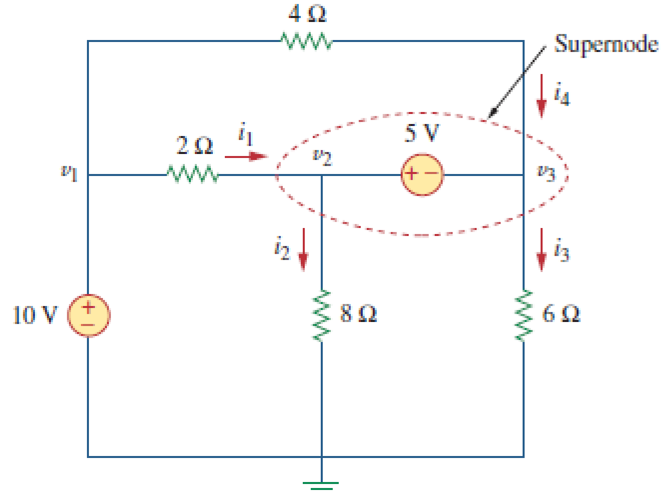


Fig 1.13 : A circuit with a supernode.

CASE 2 If the voltage source (dependent or independent) is connected between two nonreference nodes, the two nonreference nodes form a **supernode**. **A supernode** is formed by enclosing a (dependent or independent) voltage source connected between two nonreference nodes and anyelements connected in parallel with it. We apply both KCL and KVL to determine the node voltages at supernode.

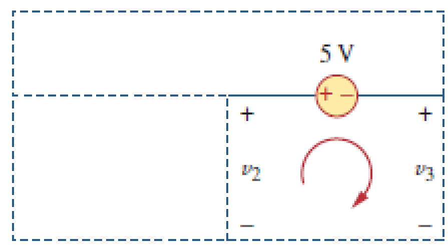


Fig 1.14 : Applying KVL to a supernode.

However, KCL must be satisfied at a **supernode** like any other node. Hence, at the **supernode**

$$i_1 + i_4 = i_2 + i_3 \quad \Rightarrow \quad \frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{4} = \frac{v_2}{8} + \frac{v_3}{6}$$

$$\Rightarrow \quad 18v_1 - 15v_2 - 10v_3 = 0 \qquad \ldots (2)$$

To apply Kirchhoff's voltage law to the supernode in Fig. 1.14, we redraw the circuit as shown in Fig. 1.14.

$$-v_2 + 5 + v_3 = 0 \quad \Rightarrow \quad v_2 - v_3 = 5 \qquad \ldots (3)$$

Example 1.6:- For the circuit shown in Fig. 1.15, find the node voltages.

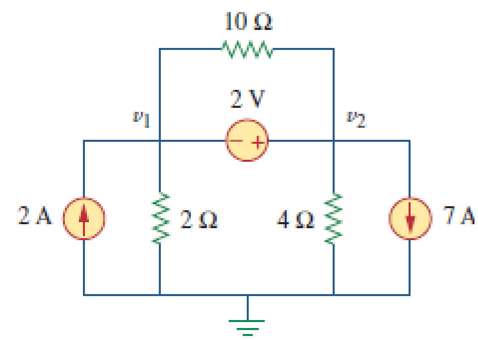


Fig 1.15 : For Example 1.6.

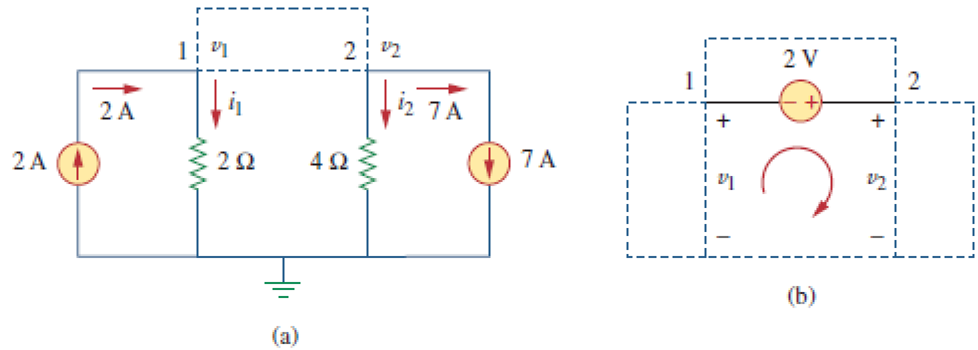


Fig 1.16 : Applying: (a) KCL to the supernode, (b) KVL to the loop.

1.4 Mesh Analysis

Mesh analysis provides another general procedure for analyzing circuits, using mesh currents as the circuit variables. ***A mesh*** is a loop which does not contain any other loops within it. Using mesh currents instead of element currents as circuit variables is convenient and reduces the number of equations that must be solved simultaneously. Nodal analysis applies KCL to find unknown voltages in a given circuit, while mesh analysis applies KVL to find unknown currents.

Mesh analysis is not quite as general as nodal analysis because it is only applicable to a circuit that is planar. A planar circuit is one that can be drawn in a plane with no branches crossing one another; otherwise it is nonplanar. A circuit may have crossing branches and still be planar if it can be redrawn such that it has no crossing branches as shown in Fig. 1.17.

Anymore in the mesh analysis of a circuit with n meshes, we take the following three steps:-

1- Assign mesh currents $i_1, i_2, \ldots, i_n$ to the n meshes.

2- Apply KVL to each of the n meshes.

3- Use Ohm's law to express the voltages in terms of the mesh currents.

4- Solve the resulting n simultaneous equations to get the mesh currents.

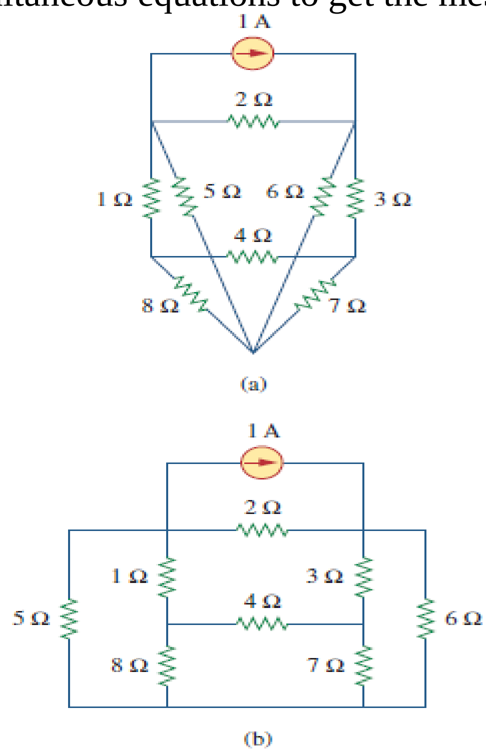


Fig 1.17 : (a) A planar circuit with crossing branches, (b) the same circuit redrawn with no crossing branches.

Example 1.7:- For the circuit in Fig. 1.18, find the branch currents i_1 and i_2 using mesh analysis.

Slo/ We first obtain the mesh currents using KVL. For mesh 1,

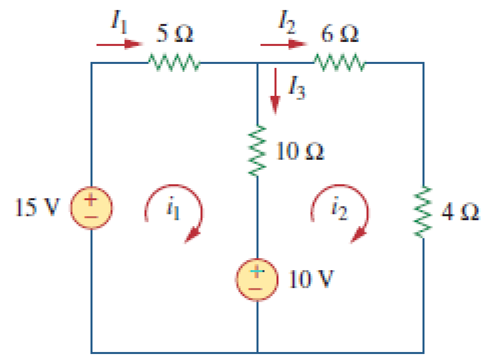


Fig 1.18 : For Example 1.7.

$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0 \qquad \ldots (1)$$

or

$$3i_1 - 2i_2 = 1$$

For mesh 2,

$$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0$$

or

$$i_1 = 2i_2 - 1 \qquad \ldots (2)$$

By substituting Eq. (2) into Eq. (1), and write

$6i_2 - 3 - 2i_2 = 1$ $\Rightarrow$ $i_2 = 1\ A$

By substituting $i_2 = 1\ A$ into Eq (2), we will get

$i_1 = 1\ A$

Thus;

$$I_1 = i_1 = 1\text{ A}, \qquad I_2 = i_2 = 1\text{ A}, \qquad I_3 = i_1 - i_2 = 0$$

Homework:- Calculate the mesh currents i_1 and i_2 of the circuit of Fig. 1.19.

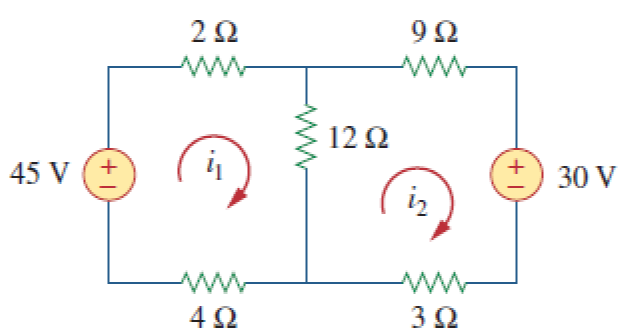


Fig 1.19 : For homework.

Example 1.8:- Use mesh analysis to find the current I_O in the circuit of Fig. 1.20.

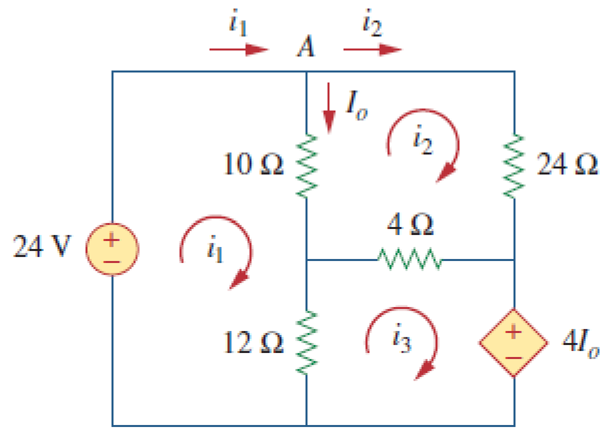


Fig 1.20 : For Example 1.8.

1.4.1 *Mesh Analysis with Current Sources*

Applying mesh analysis to circuits containing current sources (dependent or independent) may appear complicated. But it is actually much easier than what we encountered in the previous section, because the presence of the current sources reduces the number of equations. Consider the following two possible cases.

CASE1 When a current source exists only in one mesh: Consider the circuit in Fig. 1.21, for example. We set $i_2 = -5\ A$ and write a mesh equation for the other mesh in the usual way; that is,

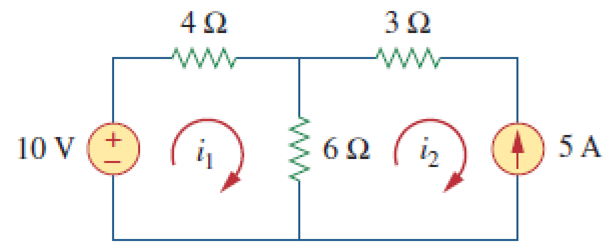


Fig 1.21 : For Example 1.8.

$$-10 + 4i_1 + 6(i_1 - i_2) = 0 \Rightarrow i_1 = -2\ A$$

CASE 2 When a current source exists between two meshes: Consider the circuit in Fig. 1.22(a), for example. We create a **supermesh** by excluding the current source and any elements connected in series with it, as shown in Fig. 1.22(b). Thus,

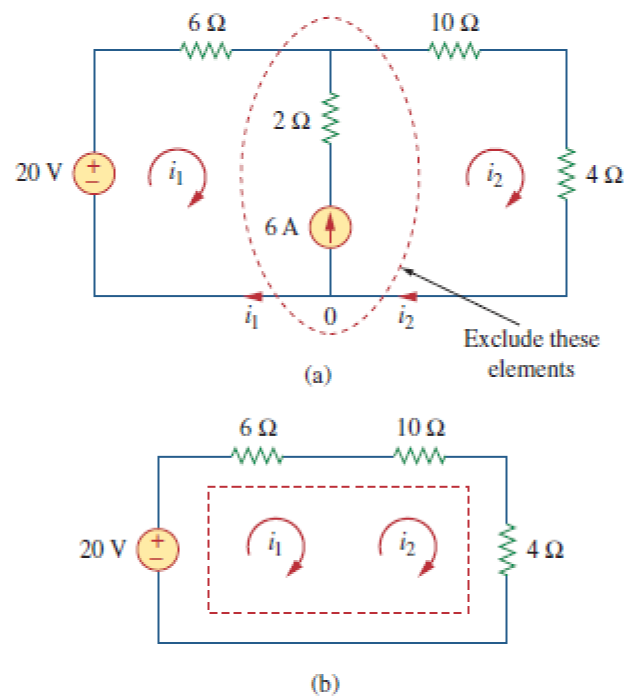


Fig 1.22 : (a) Two meshes having a current source in common, (b) A supermesh, created by excluding the current source.

$$-20 + 6i_1 + 10i_2 + 4i_2 = 0$$

$$\Rightarrow 6i_1 + 14i_2 = 20 \qquad \ldots(1)$$

We apply KCL to a node in the branch where the two meshes intersect.

$$i_2 = i_1 + 6 \qquad \ldots(2)$$

Solving Eq1 & Eq2, we will get

$$i_1 = -3.2\ A$$

$$i_2 = 2.8\ A$$

Example 1.9:- For the circuit in Fig. 1.23, find i_1 to i_4 using mesh analysis.

Sol/ Note that meshes 1 and 2 form a supermesh since they have an independent current source in common. Also, meshes 2 and 3 form another supermesh because they have a dependent current source incommon. The two supermeshes intersect and form a larger supermesh as shown. Applying KVL to the larger supermesh,

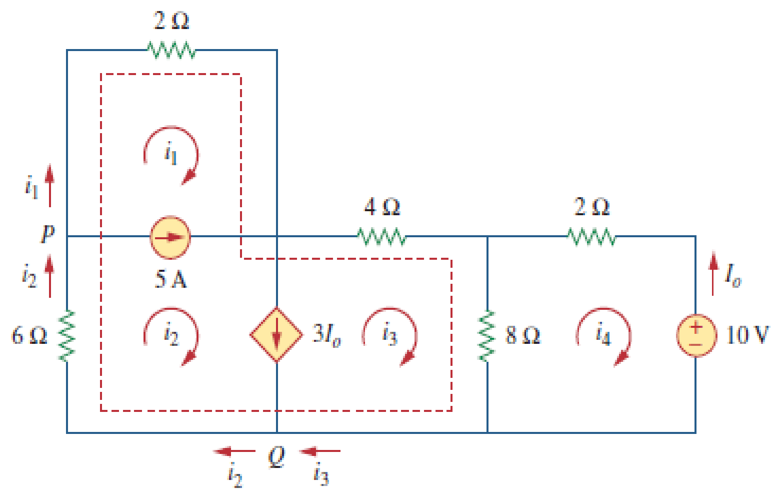


Fig 1.23 : For Example 1.9.

1.5 Superposition

If a circuit has two or more independent sources, one way to determine the value of a specific variable (voltage or current) is to use nodal or mesh analysis. Another way is to determine the contribution of each independent source to the variable and then add them up. The latter approach is known as the superposition. The principle of superposition helps us to analyze a linear circuit with more than one independent source by calculating the contribution of each independent source separately. However, to apply the superposition principle, we must follow the below steps:

1- Turn off all independent sources except one source . We replace every voltage source by $0\ V$ (or a short circuit), and every current source by $0\ A$ (or an open circuit).
2- Find the output (voltage or current) due to that active source
3- Repeat step 1 for each of the other independent sources.
4- Find the total contribution by adding algebraically all the contributions due to the independent sources.

Keep in mind that superposition is based on **linearity**. For this reason, it is not applicable to the effect on **power** due to each source, because the power absorbed by a resistor depends on the **square** of the voltage or current. If the power value is needed, the current through (or voltage across) the element must be calculated first using superposition.

Example 1.10:- Use the superposition theorem to find v in the circuit of Fig. 1.24.

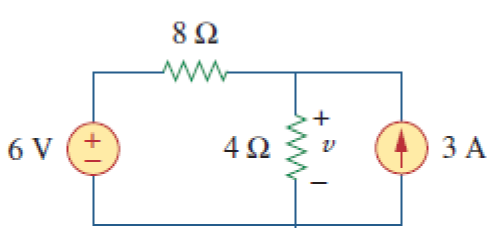


Fig 1.24 : For Example 1.10.

Sol/ Since there are two sources, let

$$v = v_1 + v_2$$

where v_1 and v_2 are the contributions due to the $6 - V$ voltage source and the $3 - A$ current source, respectively. To obtain we set the current source to zero, as shown in Fig. 1.25(a). Applying KVL to the loop in Fig. 1.25(a) gives

$$12i_1 - 6 = 0 \quad \Rightarrow \quad i_1 = 0.5\ A$$

Thus,

$$v_1 = 4i_1 = 2\text{ V}$$

We may also use voltage division to get v_1 by writing

$$v_1 = \frac{4}{4+8}(6) = 2\text{ V}$$

To get we set the voltage source to zero, as in Fig. 1.25(b). Using current division,

$$i_3 = \frac{8}{4+8}(3) = 2\text{ A}$$

Hence,

$$v_2 = 4i_3 = 8\text{ V}$$

And we find

$$v = v_1 + v_2 = 2 + 8 = 10\text{ V}$$

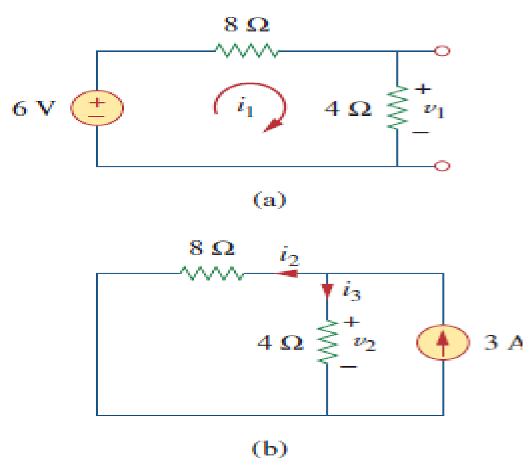


Fig 1.25 : For Example 1.10: (a) calculating v_1 (b) calculating v_2.

Example 1.11:- Use the superposition theorem to find i in the circuit of Fig. 1.26.

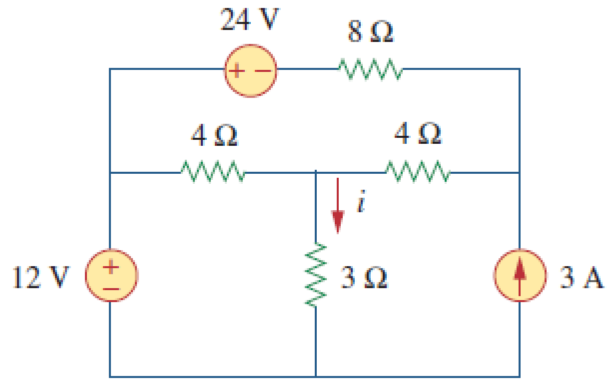


Fig 1.26 : For Example 1.11.

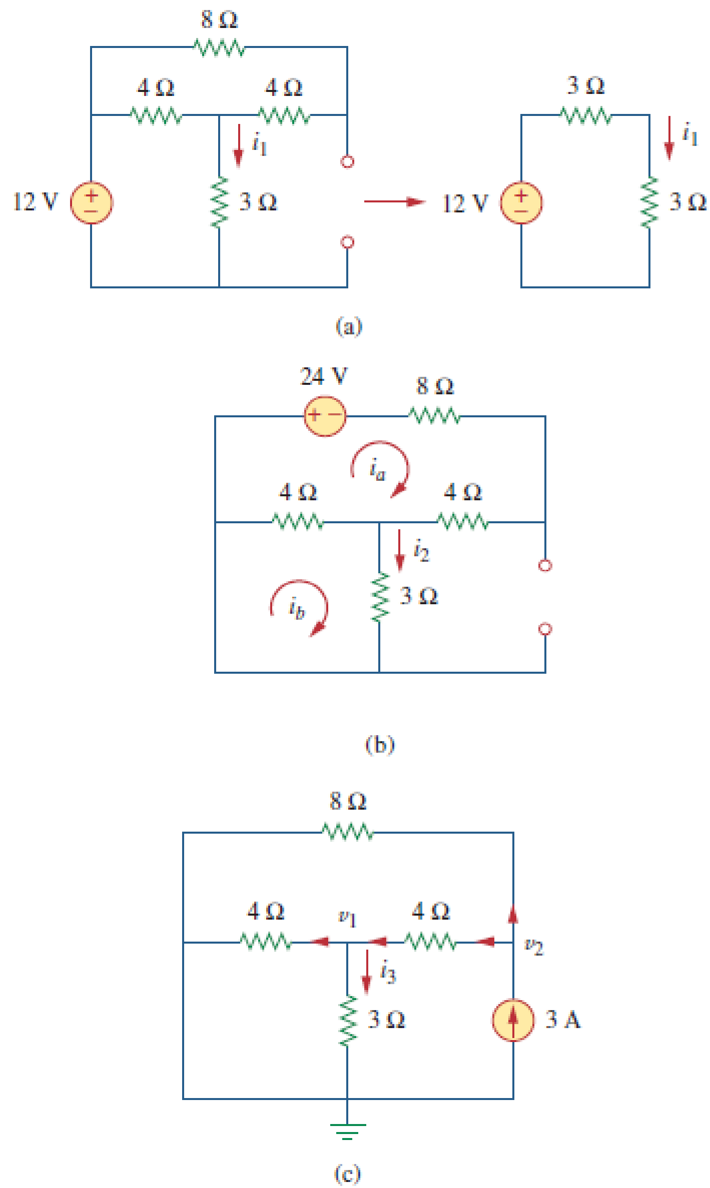


Fig 1.27 : For Example 1.11.

1.6 Thevenin's Theorem

Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.

According to Thevenin's theorem, the linear circuit in Fig. 1.28(a) can be replaced by that in Fig. 1.28(b). (The load in Fig. 1.28 may be a single resistor or another circuit.) The circuit to the left of the terminals in Fig. 1.28(b) is known as the Thevenin equivalent circuit.

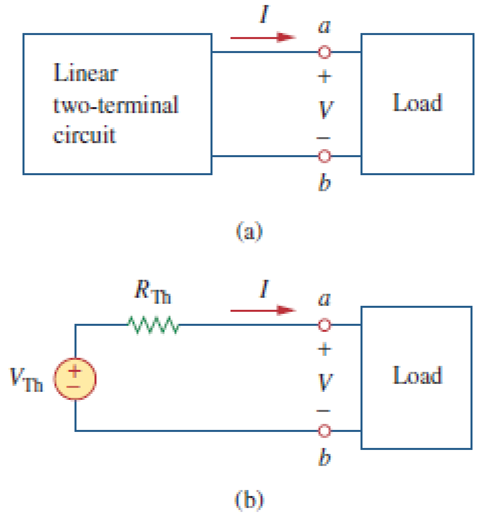


Fig 1.28 : Replacing a linear two-terminal circuit by its Thevenin equivalent: (a) original circuit, (b) the Thevenin equivalent circuit.

So to apply Thevenin to any circuit, we should compute V_{Th} & R_{Th}:-

A- Computing Thevenin Voltage V_{Th}:- V_{Th} is the open-circuit voltage across the terminals as shown in Fig. 1.29; that is,

$$V_{Th} = v_{oc}$$

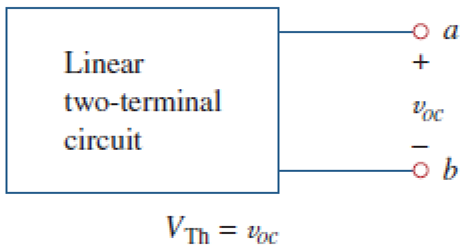
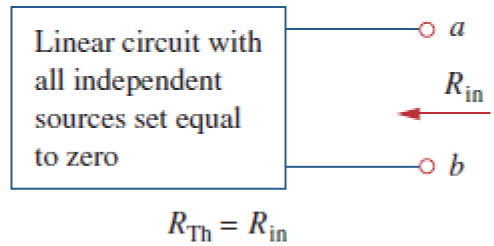


Fig 1.29 : Finding V_{Th} .

B- Computing Thevenin Resistance R_{Th} :- There are two cases that should be noted when we compute R_{Th}, these cases are :-1

CASE 1:- If the network has no dependent sources, we turn off all independent sources. is the input resistance of the network looking between terminals a and b, as shown in Fig. 1.30.



$$R_{Th} = R_{in}$$

CASE 2:- If the network has dependent sources, we turn off all independent sources. As with superposition, dependent sources are not to be turned off because they are controlled by circuit variables. We apply a voltage source v_o or current source i_o at terminals a and b and determine the resulting current or voltage . Then $R_{Th} = \frac{v_o}{i_o}$, as shown in Fig. 1.31(a & b).

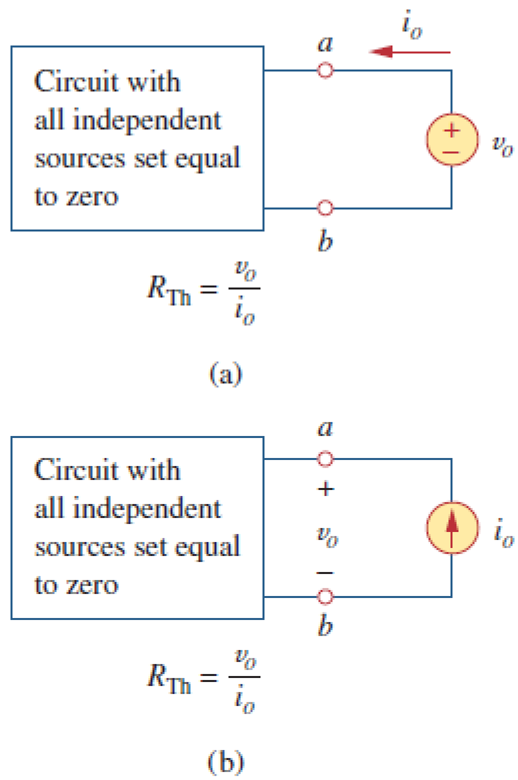


Fig 1.31 : Finding R_{Th} when circuit has dependent sources..

After computing V_{Th} & R_{Th}, now we can configure Thevenin circuit as shown in Fig. 1.32, so thus;

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

$$V_L = R_L I_L = \frac{R_L}{R_{Th} + R_L} V_{Th}$$

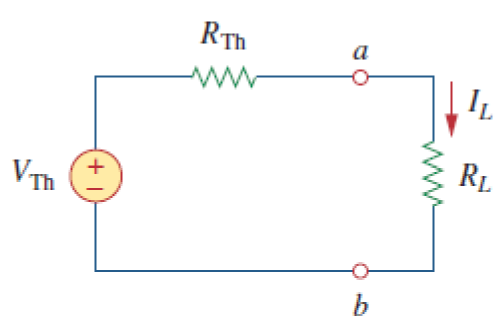


Fig 1.32 : Thevenin equivalent.

Example 1.12:- Find the Thevenin equivalent circuit of the circuit shown in Fig. 1.33, to the left of the terminals a *and* b. Then find the current through $R_L = 6, 16, and\ 36\ \Omega$.

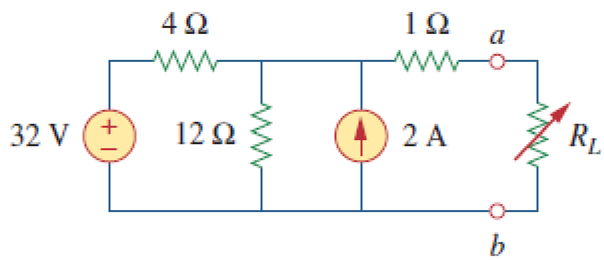


Fig 1.33 : For Example 1.12.

Sol/

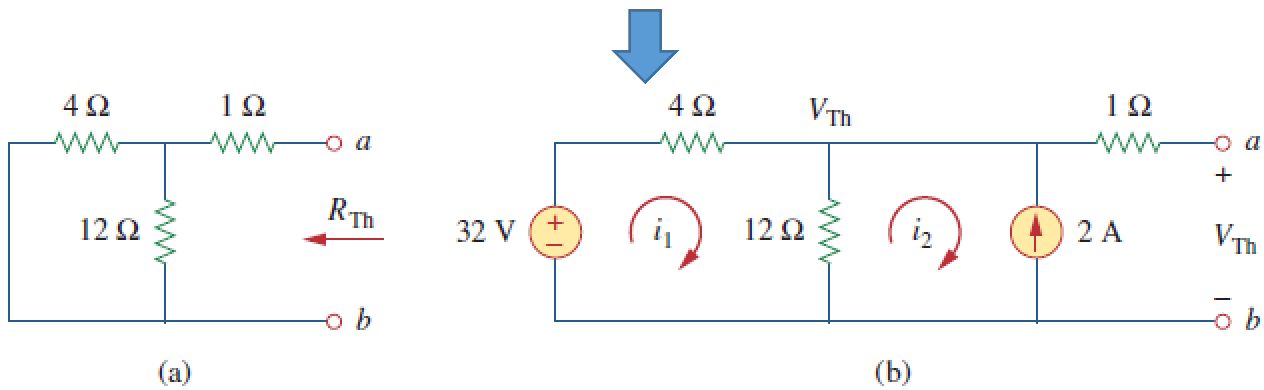


Fig 1.34 : For Example 1.12 (a) finding R_{Th}, (b) finding V_{Th}.

Example 1.12:- Find the Thevenin equivalent of the circuit in Fig. 1.35 at terminals a-b.

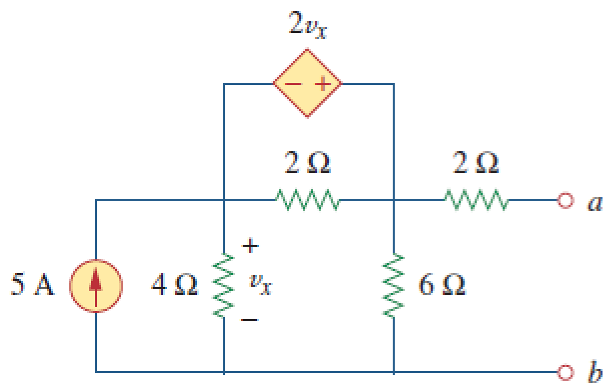


Fig 1.33 : For Example 1.12.

Sol/

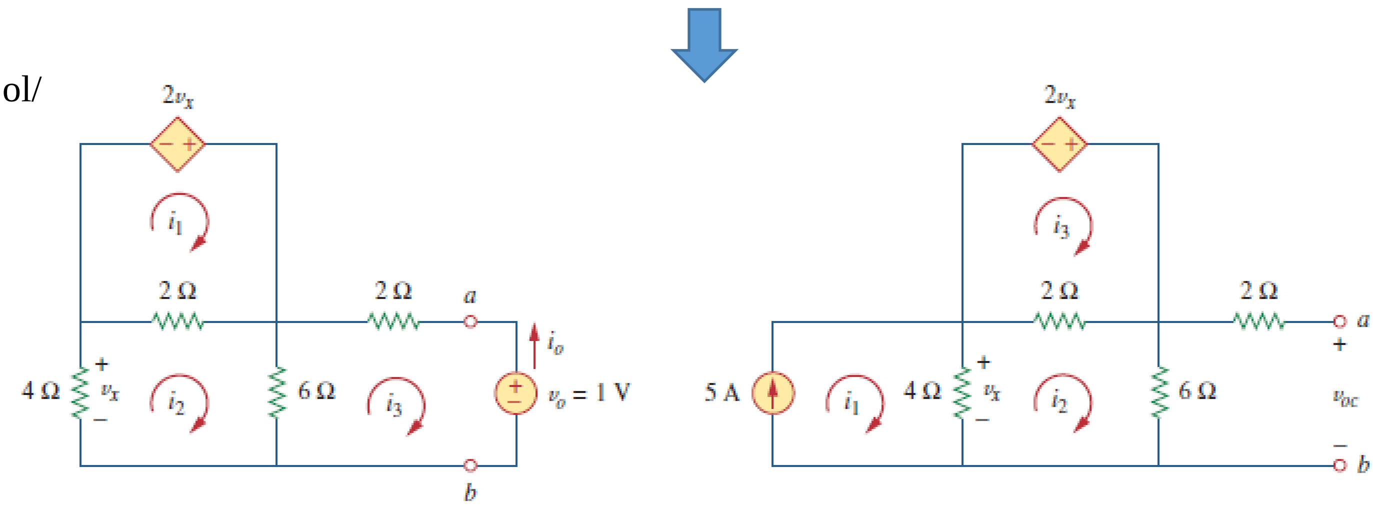


Fig 1.33 : For Example 1.12, finding R_{Th} & V_{Th}.

1.7 Norton's Theorem

Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off. Thus, the circuit in Fig. 1.34(a) can be replaced by the one in Fig. 1.34(b).

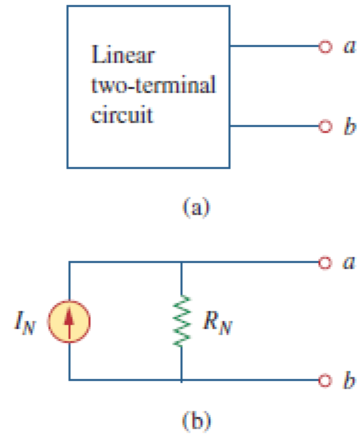


Fig 1.34 : (a) Original circuit, (b) Norton equivalent circuit..

Thevenin and Norton resistances are equal; that is,

$$R_N = R_{Th}$$

To find the Norton current I_N we determine the short-circuit current flowing from terminal a to b in both circuits in Fig. 1.34 (a), and as shown in Fig. 1.35.

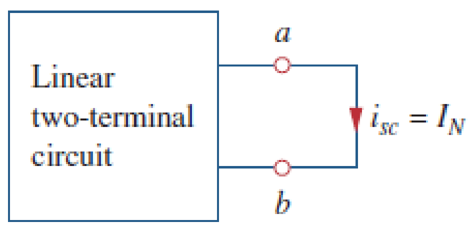


Fig 1.35 : Finding Norton current I_N.

Thus;

$$I_N - i_{sc}$$

As $R_N = R_{Th}$,so the relationship between V_{Th} *and* I_N is;

$$\boxed{I_N = \frac{V_{Th}}{R_{Th}}}$$

Example 1.13:- Find the Norton equivalent circuit of the circuit in Fig. 1.36 at terminals a-b.

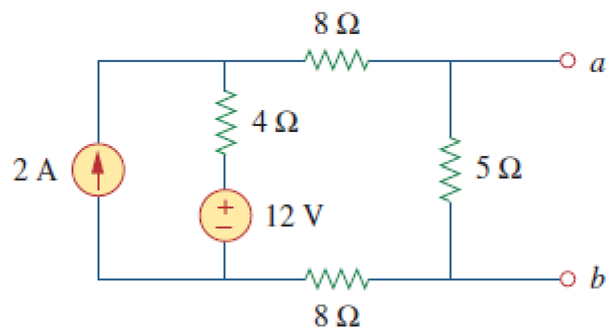


Fig 1.36 : For Example 1.13.

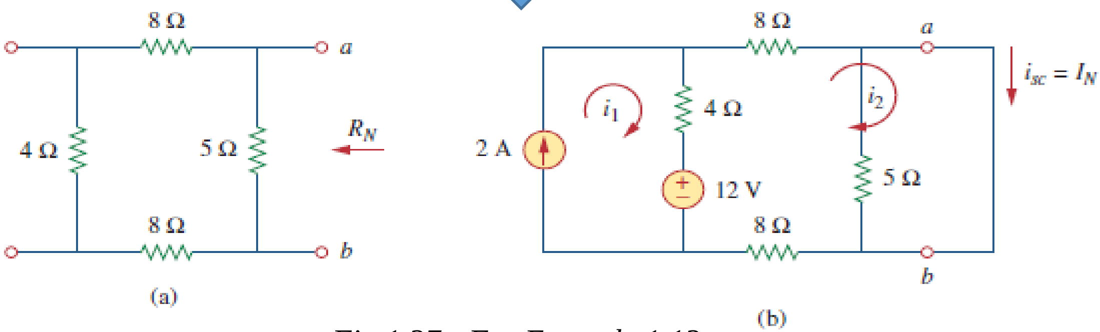


Fig 1.37 : For Example 1.13.

Sol/ Set the independent sources equal to zero. This leads to the circuit in Fig. 1.37(a), from which we find Thus,

$$R_N = 5 \,\|\, (8 + 4 + 8) = 5 \,\|\, 20 = \frac{20 \times 5}{25} = 4\ \Omega$$

To find I_N we short-circuit terminals a and b, as shown in Fig. 1.37(b).

$$i_1 = 2\text{ A}, \qquad 20i_2 - 4i_1 - 12 = 0$$

From these equations, we obtain

$$i_2 = 1\text{ A} = i_{sc} = I_N$$

So, the equivalent Norton circuit is shown in Fig. 1.38.

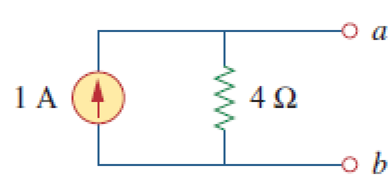


Fig 1.38 : Norton equivalent circut.

Example 1.14:-Using Norton's theorem, find I_N and R_N of the circuit in Fig. 1.39 at terminals a-b.

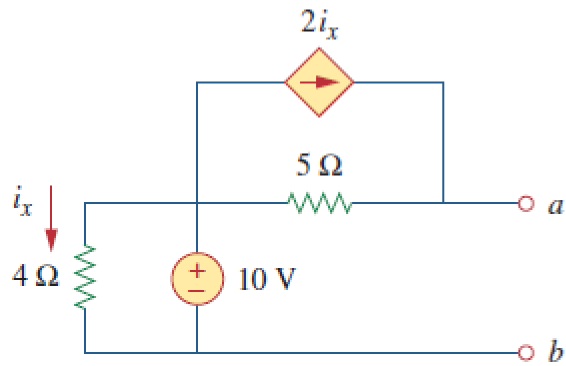


Fig 1.39 : For Example 1.14.

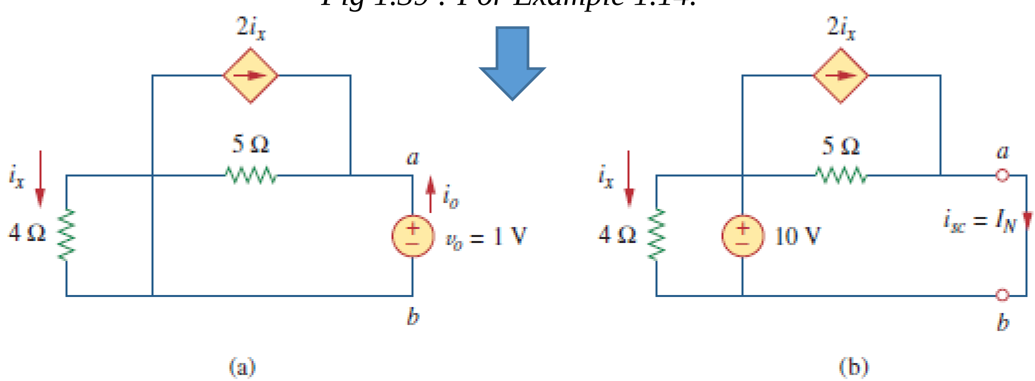


Fig 1.40 : For Example 1.14. (a) finding R_N, (b) finding I_N.

1.8 Maximum Power Transfer

In many practical situations, a circuit is designed to provide power to a load. There are applications in areas such as communications where it is desirable to maximize the power delivered to a load. We now address the problem of delivering the maximum power to a load when given a system with known internal losses. It should be noted that this will result in significant internal losses greater than or equal to the power delivered to the load.

The Thevenin equivalent is useful in finding the maximum power a linear circuit can deliver to a load. We assume that we can adjust the load resistance R_L .If the entire circuit is replaced by its Thevenin equivalent except for the load, as shown in Fig. 1.41, the power delivered to the load is

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L \qquad \ldots (1)$$

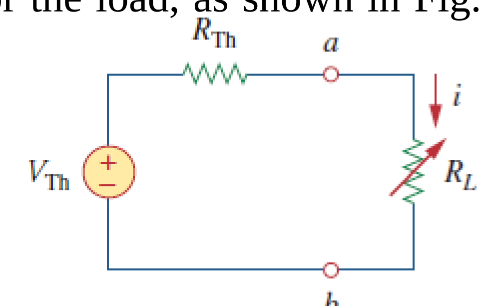


Fig 1.41 : The circuit used for maximum power transfer.

To prove the maximum power transfer theorem, we differentiate p in Eq. (1) with respect R_L to and set the result equal to zero. We obtain

$$\frac{dp}{dR_L} = V_{Th}^2 \left[\frac{(R_{Th}+R_L)^2 - 2R_L(R_{Th}+R_L)}{(R_{Th}+R_L)^4}\right] = 0$$

This implies that

$$R_{Th} + R_L - 2R_L = 0 \quad \Rightarrow \quad \boxed{R_L = R_{Th}}$$

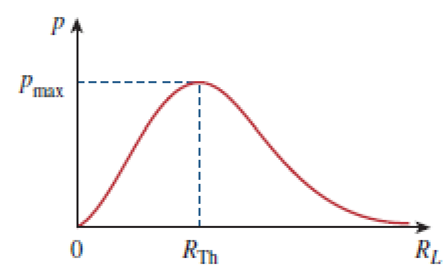


Fig 1.42 : Power delivered to the load as a function of R_L

Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load ($R_L = R_{Th}$) as shown in Fig. 1. 42, so p_{Max} will be

$$p_{Max} = \frac{V_{Th}^2}{4R_{Th}}$$

Example 1.15:- Find the value of for maximum power transfer in the circuit in Fig. 1.43. Find the maximum power.

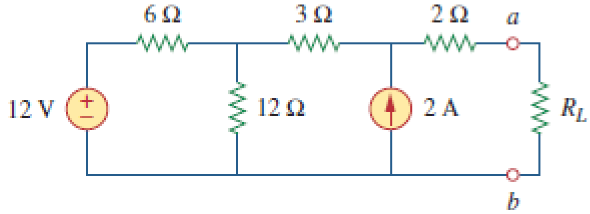


Fig 1.43 : For Example 1.15.

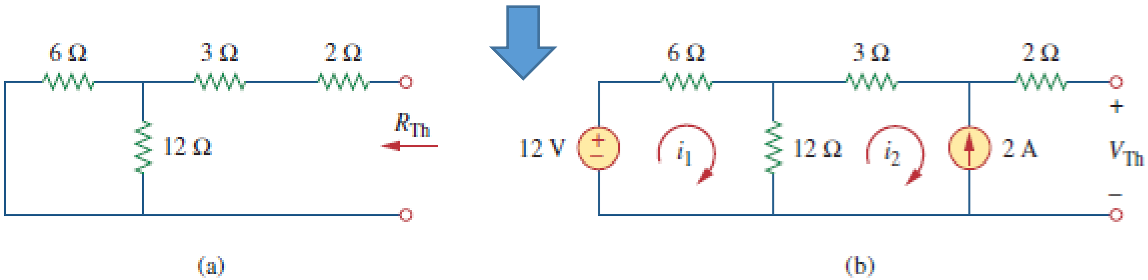


Fig 1.44 : For Example 1.15 (a) finding R_{Th}, (b) finding V_{Th}.

Sol/ To get R_{Th} we use the circuit in Fig. 1.44(a) and obtain

$$R_{Th} = 2 + 3 + (12||6) = 2 + 3 + 4 = 9\,\Omega$$

To get V_{Th} we consider the circuit in Fig. 1.44(b). Applying mesh analysis gives

$$-12 + 18i_1 - 12i_2 = 0, \qquad i_2 = -2\text{ A}$$

Solving for i_1, we get $i_1 = -2/3$. Applying KVL around the outer loop to get V_{Th} across terminals a-b, we obtain

$$-12 + 6i_1 + 3i_2 + 2(0) + V_{Th} = 0 \quad \Rightarrow \quad V_{Th} = 22\text{ V}$$

For maximum power transfer,

$$R_L = R_{Th} = 9\,\Omega$$

and the maximum power is

$$p_{max} = \frac{V_{Th}^2}{4R_L} = \frac{22^2}{4 \times 9} = 13.44\text{ W}$$

Homework:- Determine the value of R_L that will draw the maximum power from the rest of the circuit in Fig. 1.45. Calculate the maximum power.

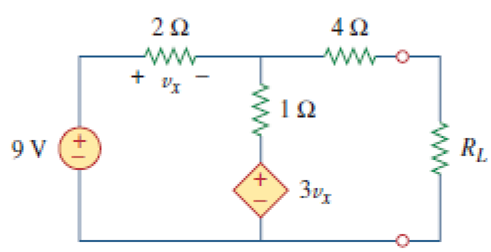


Fig 1.43 : For Homework