Short Column Design

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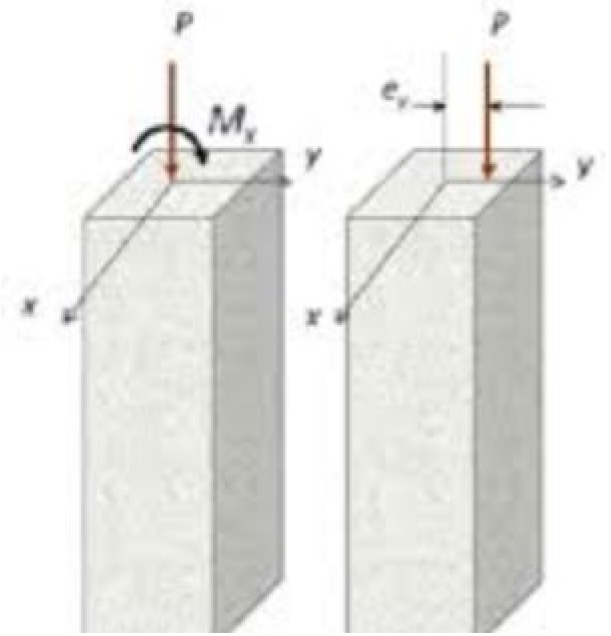
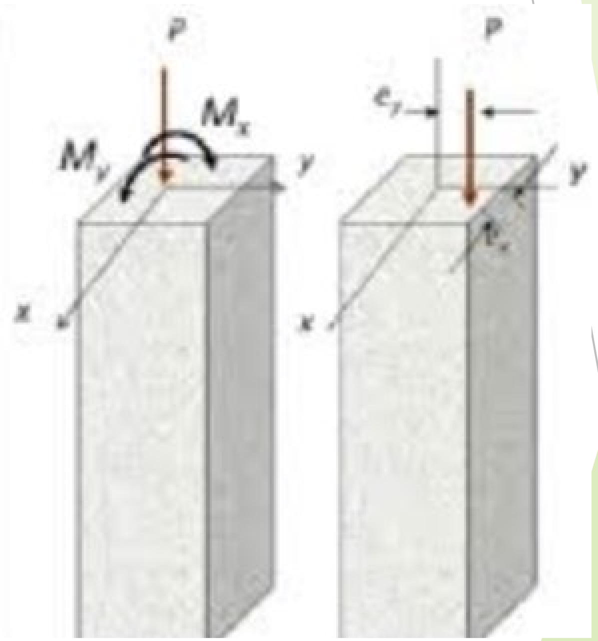
CASE II: Columns subjected to Concentrated load and Uniaxial Moment

The analysis of a structure may shows that the column is subjected to both Compression Force and Bending Moment about one or two of the column cross section main axes (i.e. about x-x or y-y axis or both) such as the case of columns at the edge or at the corner of the building respectively as shown below:



Compression and Uniaxial Moment

$$e = \frac{M}{P}$$

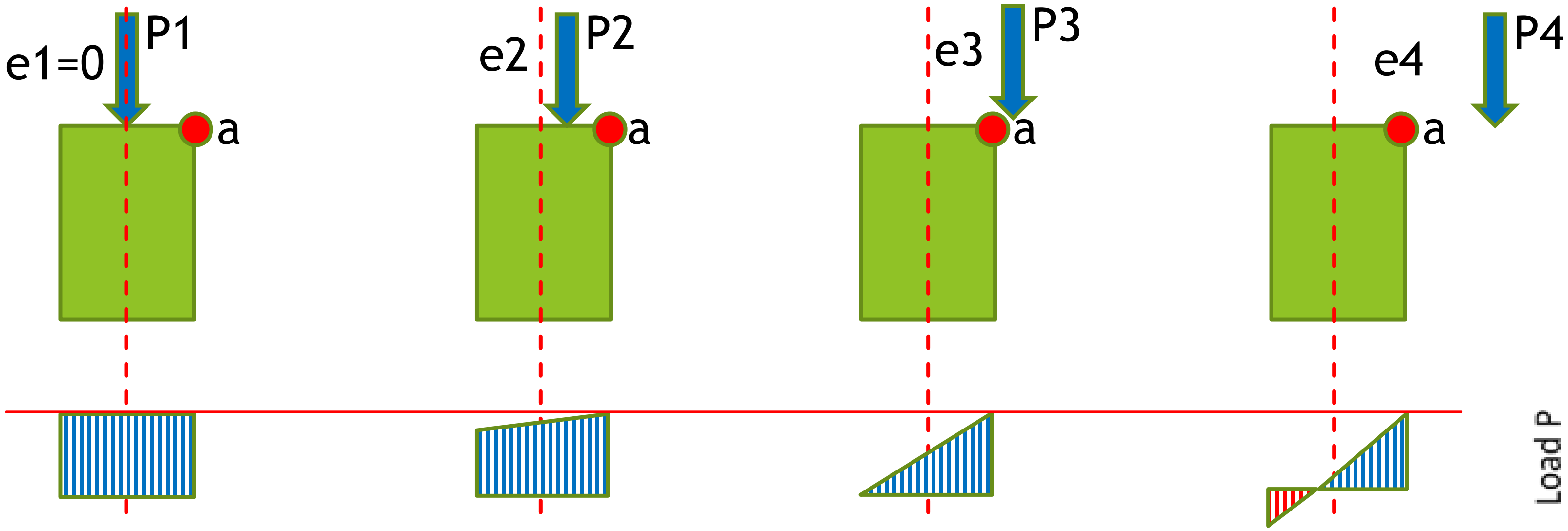


Compression and Biaxial Moment

Although the source of the Axial Force (P) may be different from the source of the Bending Moment (Mx or My), We can transform the system of these two actions into a statically equivalent system of (P) acted at a distance (ey or ex) from the Geometrical Center of the Column using the above equation (e=M/P).

Interaction Diagram of Column with Uniaxial B.M

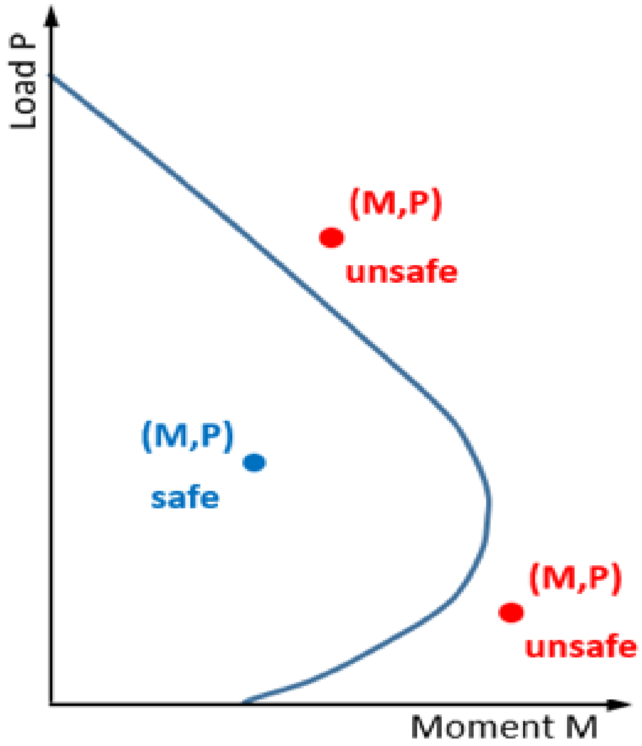
To imagine and construct the Interaction Diagram, We need to focus on one edge of the column (the compression edge at point ‘a’)



e4 > e3 > e2> e1

We should notice for the above Column that the force P that needed to cause a failure (or to make point ‘a’ reaches to a stress of 0.85f’c) will be such that:

P4 < P3 < P2 < P1



Because point ‘a’ will subjected to (two components of Compression) one from the Axial action of Force P and the second from the B.M action, and the B.M will be greater when (e) is greater.

Use of Interaction Diagram

For each column with (Specific Dimensions) and (Number & Distribution of Main Reinforcement) and (Material Properties fy & f'c), there will be (one specific I.D) that represent the capacity of that column for any combination of (P & M). If the point (P,M) is located inside the closed curve the state of the column is Safe, otherwise the column will be Unsafe.

Property of the Interaction Diagram

All points of the I.D has one common property, that is, at the compression edge the concrete compression strain is 0.003 and the Intensity of compression stress block is 0.85f'c. On the other hand, if we looked at the (Steel) at the other edge, the strain and stress of steel reinforcement will be variable (they may be at Compression or Zero or even in Tension state), this depends on the value of both P&M.

Constructing the Interaction Diagram

Practically, we don't need all the points to construct the I.D for a specific column, instead, we will choose 5 special points.

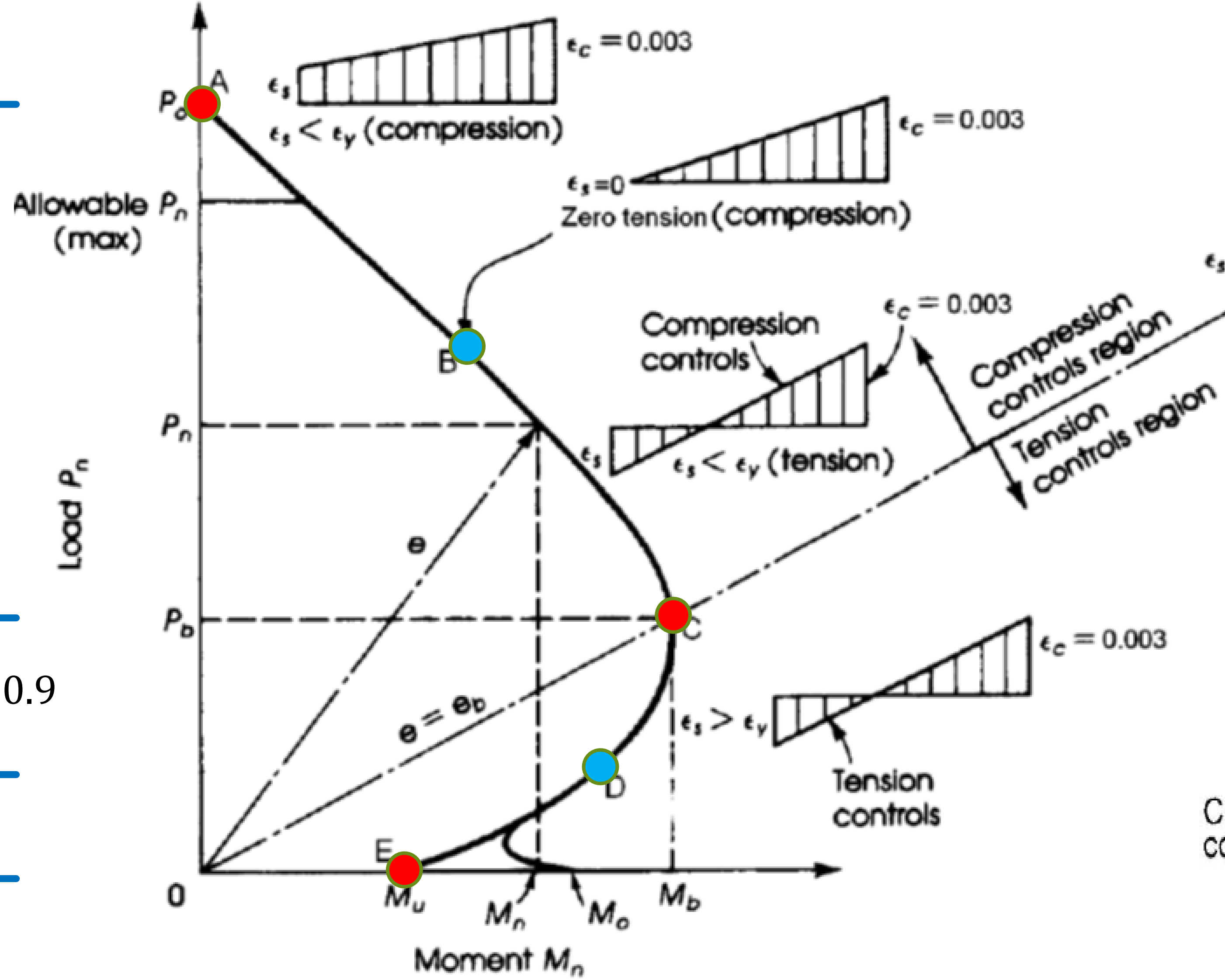
- The First point is the point of Pure Compression (Concentrated P and M=0) this point located on Y-axis
- The Fifth point is the point of Pure B.M (B.M value and P=0) this point is located on X-axis.
- The Third point is splitting the curve into two zones (The upper will be The compression Zone) and (The lower will be The Tension Zone) This point is (the balance point), where the stress in tension steel =fy
- The Second and Forth points are one at the compression zone where (steel strain =0) and the second one at the Tension zone where (steel strain =0.005)(start of tension control)

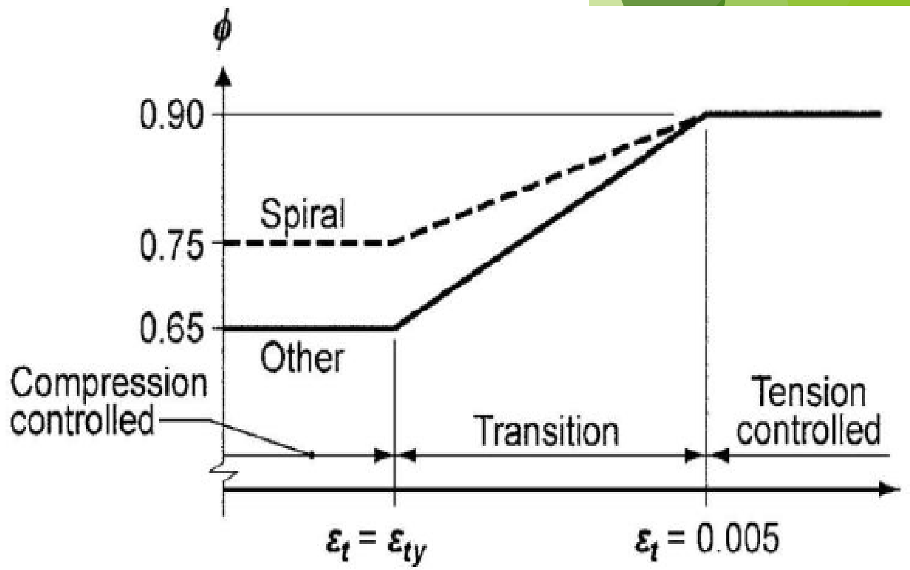
If we ask: how could the column fail by tension? Although it is a compression member? The answer is because at tension side of the column the concrete will not work at all, but at the compression side both the concrete and steel will resist the compression. We can imagine this case at the upper floor of the building where there is no big (P) and there is a big value of (M)

$\emptyset = 0.65$

$\emptyset = 0.65 \rightarrow 0.9$

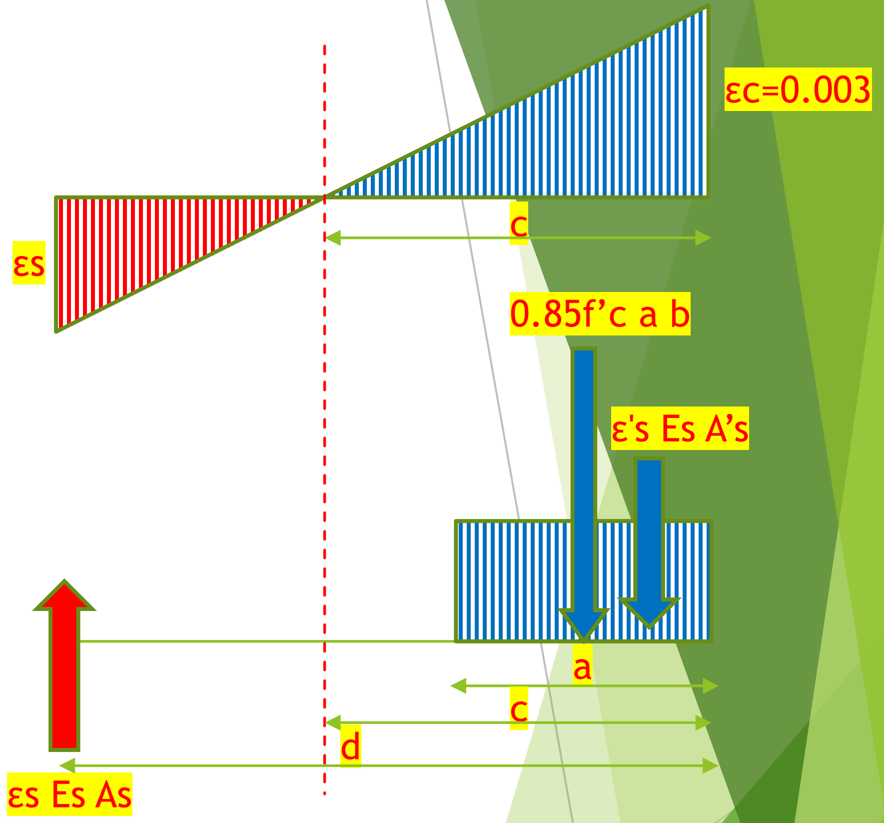
$\emptyset = 0.9$





The procedure to draw the I.D (by calculating the above mentioned 5 points) is depend on the following:

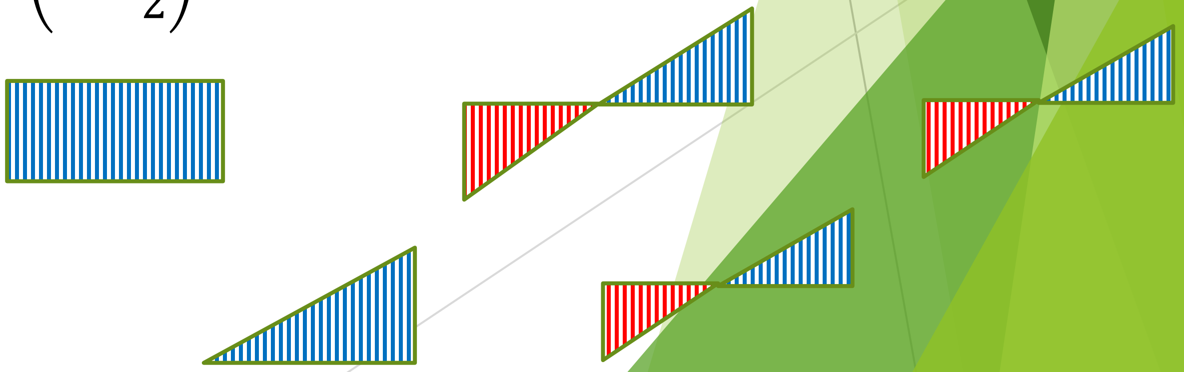
- The summation of forces in y-direction is not equal to Zero like beams, but mostly the compression will be more than tension and the resultant will equal (P)
- Finding the value of c by assuming value of εs
- Take the moment about geometric center of the column which will give the value of (M)



The two governing equations (considering compression positive):

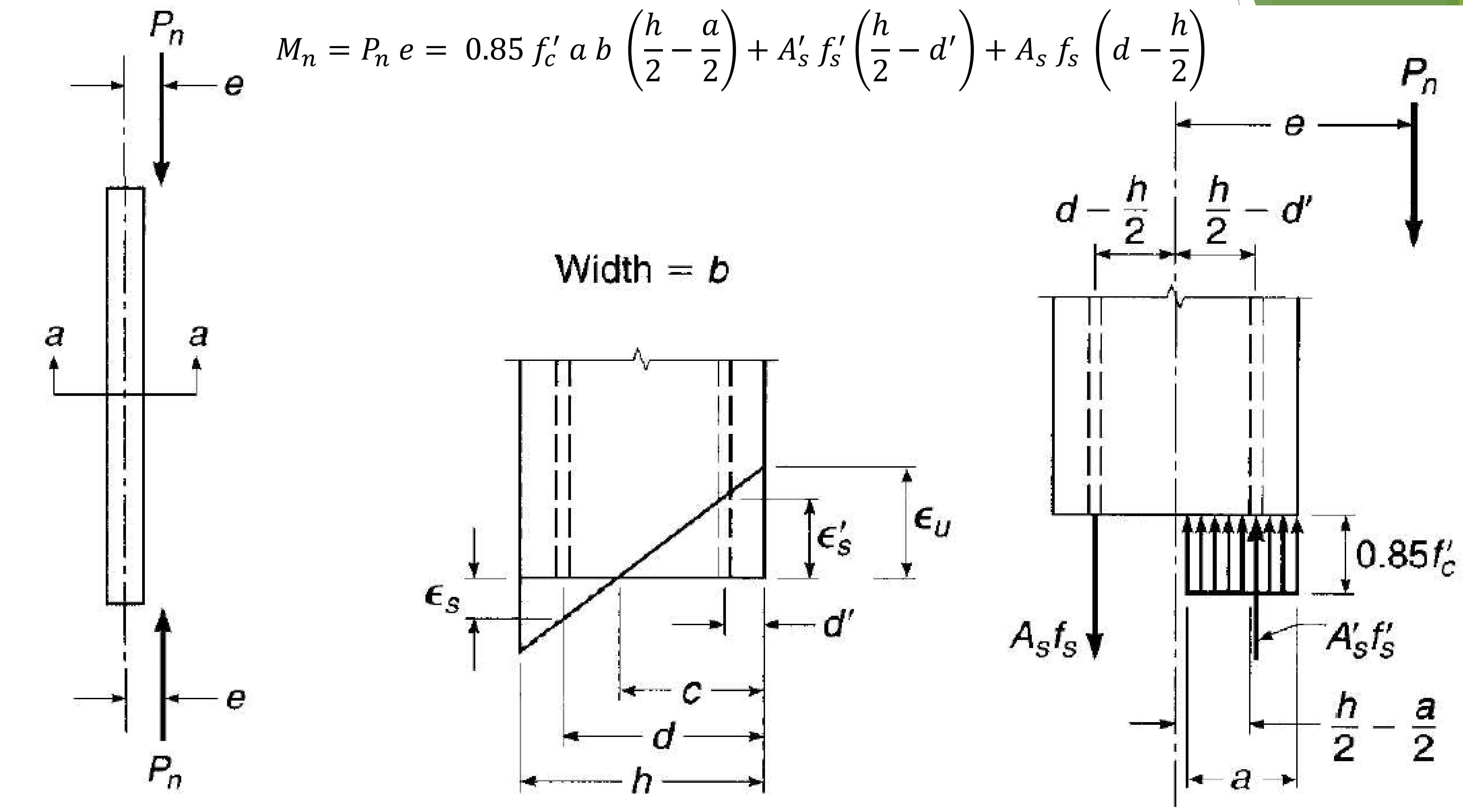
$$P_n = 0.85\, f_c' a\, b + A_s'\, f_s' - A_s\, f_s$$

$$M_n = P_n\, e = \; 0.85\, f_c'\, a\, b\left(\frac{h}{2} - \frac{a}{2}\right) + A_s'\, f_s'\left(\frac{h}{2} - d'\right) + A_s\, f_s\left(d - \frac{h}{2}\right)$$



$$P_n = 0.85\, f_c' a\, b + A_s'\, f_s' - A_s\, f_s$$

$$M_n = P_n\, e = 0.85\, f_c'\, a\, b \left(\frac{h}{2} - \frac{a}{2}\right) + A_s'\, f_s' \left(\frac{h}{2} - d'\right) + A_s\, f_s \left(d - \frac{h}{2}\right)$$



(a) (b) (c)

Strain Compatibility Equations

Tension Steel

$$\varepsilon_s = 0.003\,\frac{d-c}{c}$$

$$f_s = 600\,\frac{d-c}{c} \le fy$$

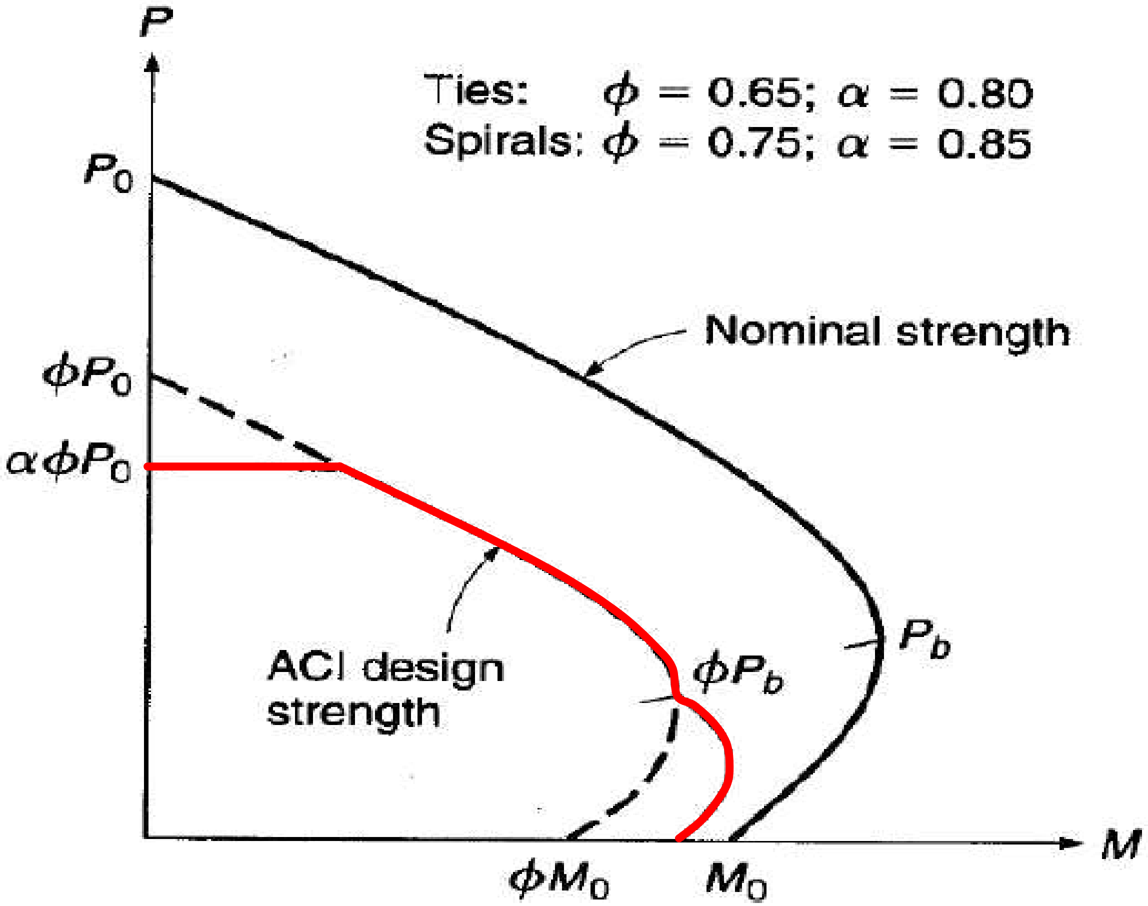
$$c = \frac{0.003}{0.003+\varepsilon_s}\,d$$

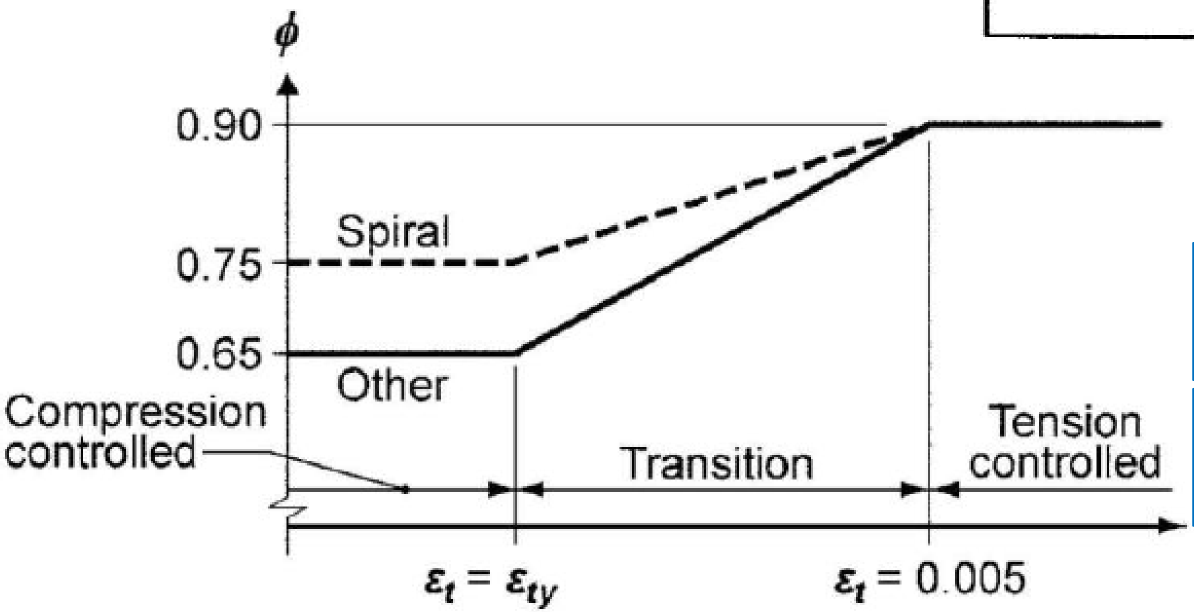
$$a = \beta_1\,c \le h$$

Compression Steel

$$\varepsilon_s' = 0.003\,\frac{c-d'}{c}$$

$$f_s' = 600\,\frac{c-d'}{c} \le fy$$





Ø for Transition Zone

$$\emptyset = 0.65 + 0.25\,\frac{\varepsilon_s - \varepsilon_y}{0.005 - \varepsilon_y} \quad Ties$$

$$\emptyset = 0.75 + 0.15\,\frac{\varepsilon_s - \varepsilon_y}{0.005 - \varepsilon_y} \quad Spiral$$

Procedure of drawing the 5 points of I.D

1) Point 1

$$P_o = 0.85\, f_c' \left(A_g - A_{st}\right) + f_y\, A_{st}$$

$$M = 0$$

$$P_n = 0.80\, P_o \quad (Tied)$$

$$\emptyset P_n = 0.65\, P_n \quad (Tied)$$

$$\boxed{\emptyset P_n = 0.65 \times 0.8 \times P_o \quad (Tied)}$$

$$\boxed{\emptyset M_n = 0}$$

2) Point 2 (tension steel strain=0)

$$\varepsilon_s = 0 \;\rightarrow c = d$$

$$P_n = 0.85\, f_c' a\, b + A_s' f_s' \cancel{- f_s\, A_s}$$

$$f_s = 600\,\frac{d-c}{c} = 0 \le fy$$

$$f_s' = 600\,\frac{c-d'}{c} = 600\,\frac{d-d'}{d} \le fy$$

$$M_n = 0.85\, f_c' a\, b \left(\frac{h}{2} - \frac{a}{2}\right) + A_s' f_s' \left(\frac{h}{2} - d'\right) \cancel{+ f_s\, A_s \left(d - \frac{h}{2}\right)}$$

$$\emptyset P_n = 0.65\, P_n \quad (Tied)$$

$$\emptyset M_n = 0.65\, M_n \quad (Tied)$$

$$\boxed{\emptyset P_n = 0.65\, P_n \quad (Tied)}$$

$$\boxed{\emptyset M_n = 0.65\, M_n \quad (Tied)}$$

3) Point 3 (balance condition εs=εy, fs=fy)

$$c_b = \frac{600}{600 + f_y}\ d$$

$$P_n = 0.85\ f_c' a\ b + A_s' f_s' - f_s\ A_s$$

$$f_s' = 600\frac{c_b - d'}{c_b} \le fy$$

$$M_n = 0.85\ f_c' a\ b\left(\frac{h}{2} - \frac{a}{2}\right) + A_s' f_s'\left(\frac{h}{2} - d'\right) + f_s\ A_s\left(d - \frac{h}{2}\right)$$

$$\emptyset P_n = 0.65\ P_n \qquad (Tied)$$

$$\emptyset M_n = 0.65\ M_n \qquad (Tied)$$

4) Point 4 (tension control margin εs=0.005)

$$c = \frac{0.003}{0.003 + \varepsilon_s}\ d = \frac{0.003}{0.003 + 0.005} d = \frac{3}{8} d$$

$$P_n = 0.85\ f_c' a\ b + A_s' f_s' - f_s . A_s$$

$$f_s = f_y$$

$$f_s' = 600\frac{c - d'}{c} \le fy$$

$$M_n = 0.85\ f_c' a\ b\left(\frac{h}{2} - \frac{a}{2}\right) + A_s' f_s'\left(\frac{h}{2} - d'\right) + f_s\ A_s\left(d - \frac{h}{2}\right)$$

$$\emptyset P_n = 0.9\ P_n$$

$$\emptyset M_n = 0.9\ M_n$$

5) Point 5 (pure bending Pn=0)

$$P_n = 0.85\, f_c' a\, b + A_s' f_s' - f_s\, A_s = 0$$

$$f_s = f_y$$

$$f_s' = 600 \frac{c - d'}{c} \leq fy$$

$$f_y\, A_s = 0.85\, f_c' a\, b + A_s' f_s'$$

Note: For symmetrical columns and symmetrical reinforcement, the fs' should be less than fy, otherwise the P≠0

Substituting and Solve for c:

$$0.85\, f_c'\, b\, \beta_1 c^2 + \left(600 A_s' - f_y A_s\right) c - 600\, A_s' d' = 0$$

$$M_n = 0.85\, f_c' a\, b \left(\frac{h}{2} - \frac{a}{2}\right) + A_s' f_s' \left(\frac{h}{2} - d'\right) + f_y\, A_s \left(d - \frac{h}{2}\right)$$

$$\emptyset M_n = 0.9\, M_n$$

$$\emptyset P_n = 0$$

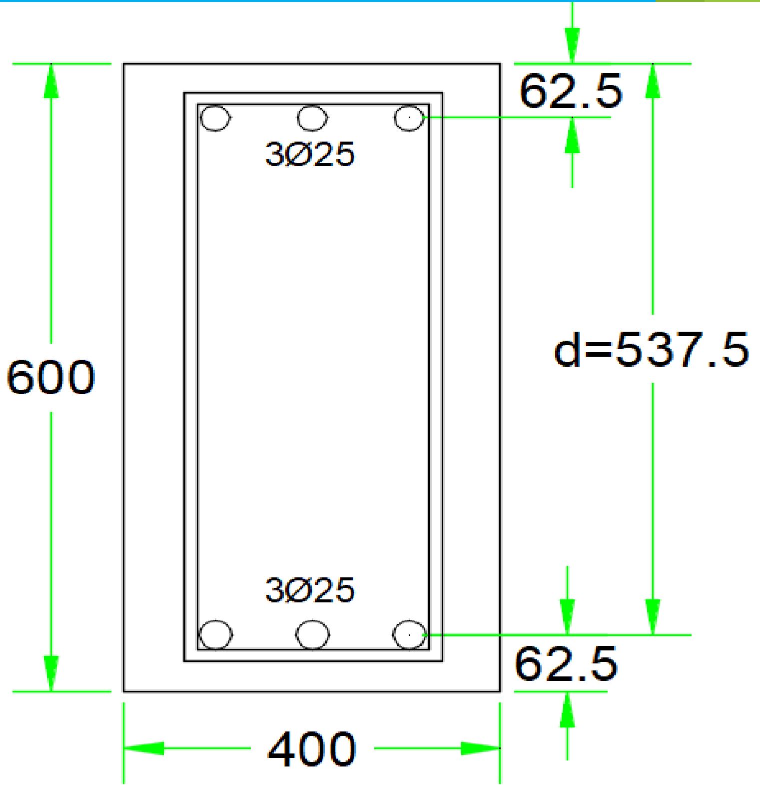
$$\emptyset M_n = 0.9\, M_n$$

Ex: Draw the Interaction Diagram of the short tied column with cross section shown below in the (h direction) if f'c=20MPa & fy=380MPa

$$A_g = 400 \times 600 = 240000\ mm^2$$

$$A_{st} = 6 \times 490 = 2940\ mm^2$$

$$A_s = A_s' = 1470\ mm^2$$



$h = 600mm$ $\quad d = 537.5mm$ $\quad d' = 62.5mm$ $\quad b = 400mm$

1)) Point 1

$$P_o = 0.85\, f_c'\,(A_g - A_{st}) + f_y\, A_{st}$$

$$P_o = 0.85 \times 20\,(240000 - 2940) + 380 \times 2940$$

$$P_o = 5147220N$$

$$P_n = 0.8\, P_o = 4117.776\ kN$$

$$\emptyset P_n = 0.65 \times P_n = 2676.554\ kN$$

2)) Point 2

$$\varepsilon_s = 0 \quad \& \quad f_s = 0$$

$$c = d$$

$$f_s' = 600\frac{c - d'}{c} = 600\frac{d - d'}{d} \le fy$$

$$f_s' = 600\frac{537.5 - 62.5}{537.5} = 530 > f_y \Rightarrow f_s' = 380MPa$$

$$a = \beta_1 c = 0.85 \times 537.5 = 456.875mm$$

$$P_n = 0.85\ f_c' a\ b + A_s' f_s' - f_s\ A_s$$

$$P_n = 0.85\ \times 20 \times 456.875 \times 400 + 1470 \times 380 - 0.0$$

$$P_n = 3665350\ N$$

$$M_n = 0.85\ f_c' a\ b\left(\frac{h}{2} - \frac{a}{2}\right) + A_s' f_s' \left(\frac{h}{2} - d'\right) + f_s\ A_s\left(d - \frac{h}{2}\right)$$

$$M_n = 0.85 \times 20 \times 456.875 \times 400\left(\frac{600}{2} - \frac{456.875}{2}\right) + 1470 \times 380\left(\frac{600}{2} - 62.5\right) + 0.0$$

$$M_n = 354994296.9\ N.mm$$

$$\emptyset P_n = 0.65 \times 3665.350 = 2382.477\ kN$$

$$\emptyset M_n = 0.65 \times 354.994 = 230.746\ kN.m$$

3)) Point 3

$$c_b = \frac{600}{600 + f_y}\ d = \frac{600}{600 + 380} \times 537.5 = 329.08\ mm$$

$$f_s = fy = 380 \quad \text{(by definition)}$$

$$f_s' = 600\frac{c_b - d'}{c_b} \leq fy$$

$$f_s' = 600\frac{329.08 - 62.5}{329.08} = 486.04 > fy \Longrightarrow f_s' = 380MPa$$

$$P_n = 0.85\, f_c' a\, b + A_s' f_s' - f_s\, A_s$$

$$P_n = 0.85 \times 20 \times 0.85 \times 329.08 \times 400 + \cancel{1470} \times 380 - \cancel{1470} \times 380$$

$$P_n = 1902082.4\ N$$

$$M_n = 0.85\, f_c' a\, b \left(\frac{h}{2} - \frac{a}{2}\right) + A_s' f_s' \left(\frac{h}{2} - d'\right) + f_s . A_s \left(d - \frac{h}{2}\right)$$

$$M_n = 0.85 \times 20 \times 0.85 \times 329.08 \times 400 \left(\frac{600}{2} - \frac{0.85 \times 329.08}{2}\right) + 1470 \times 380 \left(\frac{600}{2} - 62.5\right) + 1470 \times 380 \left(537.5 - \frac{600}{2}\right)$$

$$M_n = 304601377.6 + 132667500 + 132667500 = 569936377.6\ N.mm$$

$$\emptyset P_n = 0.65 \times 1902.082 = 1236.353\ kN$$

$$\emptyset M_n = 0.65 \times 569.936 = 370.458\ kN.m$$

4)) Point 4

$$\varepsilon_s = 0.005$$

$$c = \frac{0.003}{0.003 + \varepsilon_s}\, d = \frac{3}{8} d = \frac{3}{8} \times 537.5 = 201.562mm$$

$$f_s = fy = 380 \quad \text{(by definition)}$$

$$f_s' = 600 \frac{c - d'}{c} \le fy$$

$$f_s' = 600 \frac{201.562 - 62.5}{201.562} = 414 > f_y \Rightarrow f_s' = 380MPa$$

$$P_n = 0.85\, f_c' a\, b + A_s' f_s' - f_s\, A_s$$

$$P_n = 0.85 \times 20 \times 0.85 \times 201.562 \times 400 + \cancel{1470 \times 380} - \cancel{1470 \times 380}$$

$$P_n = 1165028.36\, N$$

$$M_n = 0.85\, f_c' a\, b \left(\frac{h}{2} - \frac{a}{2}\right) + A_s' f_s' \left(\frac{h}{2} - d'\right) + f_s . A_s \left(d - \frac{h}{2}\right)$$

$$M_n = 1165028.36 \left(\frac{600}{2} - \frac{0.85 \times 201.562}{2}\right) + 1470 \times 380 \left(\frac{600}{2} - 62.5\right) + 1470 \times 380 \left(537.5 - \frac{600}{2}\right)$$

$$M_n = 249707693.3 + 132667500 + 132667500 = 515042693.3\, N.mm$$

$$\emptyset P_n = 0.9 \times 1165.028 = 1048.525\, kN$$

$$\emptyset M_n = 0.9 \times 515.042 = 463.538\, kN.m$$

5)) Point 5

$$P_n = 0.85\, f_c' a\, b + A_s' f_s' - f_s\, A_s = 0$$

$$f_s = f_y \quad \text{(by definition)}$$

Note: For symmetrical columns and symmetrical reinforcement (for pure bending case), the fs' should be less than fy, otherwise the P≠0

$$f_s' = 600 \frac{c - d'}{c} \le fy$$

Substitute and Solve for c:

$$0.85\, f_c'\, b\, \beta_1 c^2 + \left(600 A_s' - f_y A_s\right) c - 600\, A_s' d' = 0$$

$$0.85 \times 20 \times 400 \times 0.85 \times c^2 + (600 \times 1470 - 380 \times 1470)c - 600 \times 1470 \times 62.5 = 0$$

$$5780c^2 + 323400c - 55125000 = 0$$

$$c1 = 73.61 \qquad \& \qquad c2 = -129.56$$

$$\Rightarrow c = 73.61$$

$$f_s' = 600\frac{c - d'}{c} \le fy$$

$$f_s' = 600\frac{73.61 - 62.5}{73.61} = 90.558$$

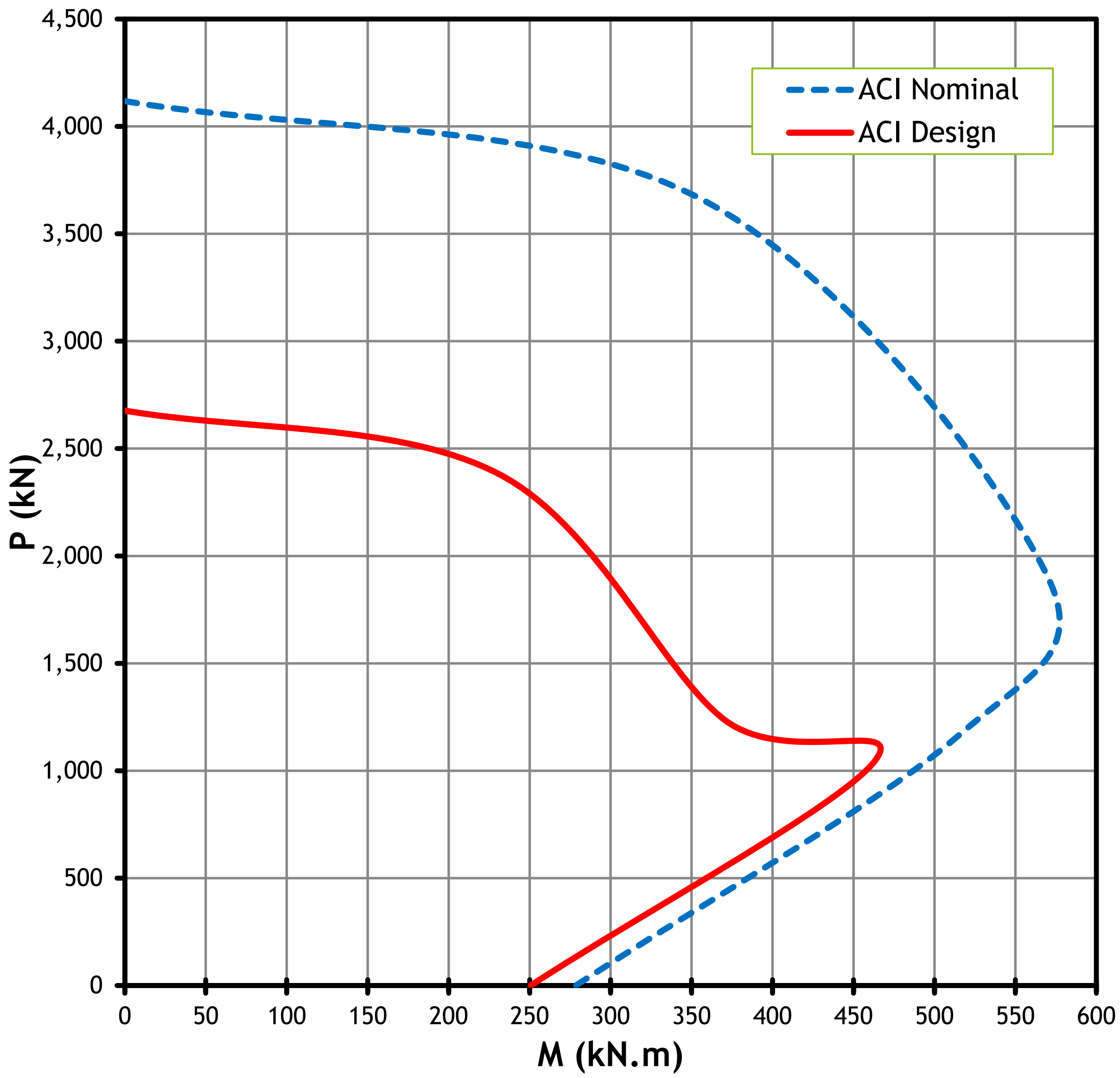
$$M_n = 0.85\, f_c' a\, b\left(\frac{h}{2} - \frac{a}{2}\right) + A_s' f_s'\left(\frac{h}{2} - d'\right) + f_y\, A_s\left(d - \frac{h}{2}\right)$$

$$M_n = 0.85 \times 20 \times 0.85 \times 73.61 \times 400\left(\frac{600}{2} - \frac{0.85 \times 73.61}{2}\right) + 1470 \times 90.558\left(\frac{600}{2} - 62.5\right) + 1470 \times 380\left(537.5 - \frac{600}{2}\right)$$

$$M_n = 114329361.5 + 31616061.75 + 132667500 = 278612923.3\ N.mm$$

$$\emptyset P_n = 0$$

$$\emptyset M_n = 0.9 \times 278.612 = 250.75\ kN.m$$



Summary:

Point	Pn(kN)	Mn(kN.m)		ØPn(kN)	ØMn(kN.m)
1	4117.77	0		2676.55	0
2	3665.35	354.99		2382.47	230.74
3	1902.08	569.93		1236.35	370.45
4	1165.02	515.04		1048.52	463.53
5	0	278.61		0	250.75

Thank you…