Shear Design Procedure and Examples

(One Way Shear)

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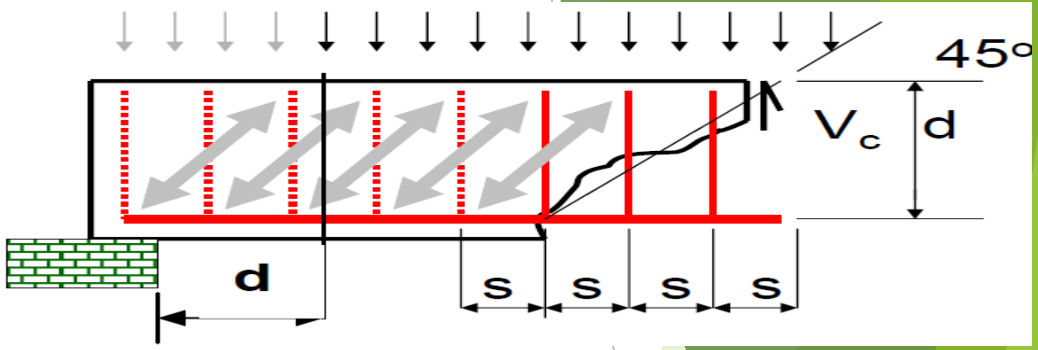
ACI Provisions for Shear



$$V_n \geq \frac{V_u}{\emptyset}$$

$$V_u = 1.2V_D + 1.6V_L \quad \text{and } \phi = 0.75$$

$$V_n = V_c + V_s$$

$$V_c = 0.17\sqrt{f_c'}\, b_w\, d$$

$$V_s = n\, A_v\, f_{yt}$$

$$n \times s = d$$

n ⟷ s

$$V_s = A_v\, f_{yt}\, \frac{d}{s}$$

$$s = \frac{A_v\, f_{yt}\, d}{V_s}$$

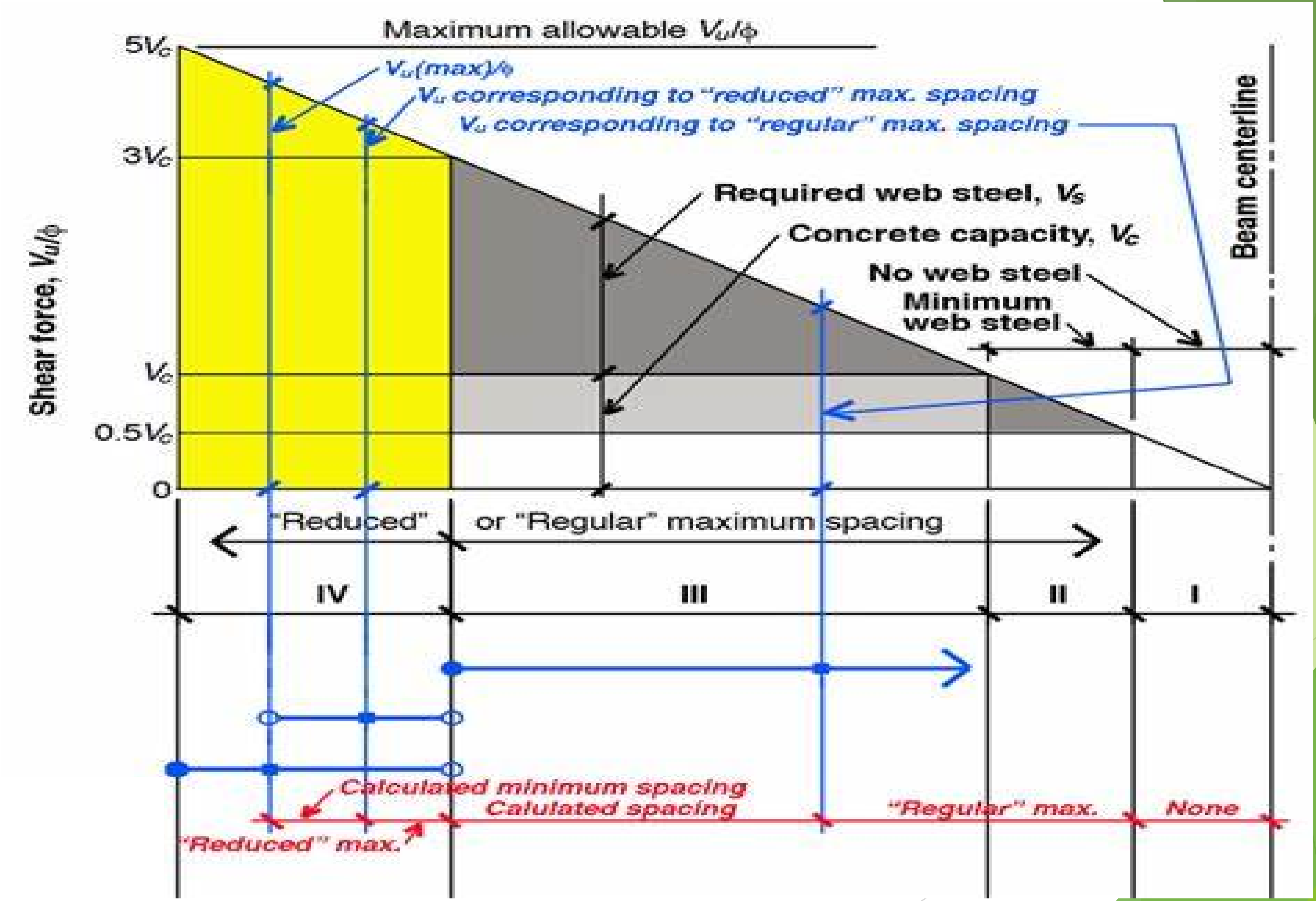
n: number of stirrups intersect a crack of 45° (no. of stirrups of a distance d)

Av: area of all vertical legs of one stirrup (depends on the stirrup shape)

fyt: yield strength of stirrups steel should be ≤ 420 MPa

The ACI Five Regions of Vn

1- $V_n < \frac{1}{2} V_c$	$No\ need\ for\ shear\ reinforcement$		I
2- $\frac{1}{2} V_c < V_n < V_c$	$V_{s(min)} = 0.062 \sqrt{f_c'}\, b_w d \geq 0.35\, b_w\, d$ S ≤ (d/2 or 600mm)	$s = \frac{A_v f_{yt} d}{V_s}$	II
3- $V_c < V_n < 3.0\ V_c$	$V_s = V_n - V_c$ $V_{s(min)} = 0.062 \sqrt{f_c'}\, b_w d \geq 0.35\, b_w\, d$ S ≤ (d/2 or 600mm)	$s = \frac{A_v f_{yt} d}{V_s}$	III
4- $3.0\ V_c < V_n < 5.0\ V_c$	$V_s = V_n - V_c$ S ≤ (d/4 or 300mm)	$s = \frac{A_v f_{yt} d}{V_s}$	IV
5- $V_n > 5.0\ V_c$	$Not\ permitted\ - change\ beam\ section$		V



Maximum allowable V_u/ϕ
$5V_c$
$3V_c$
V_c
$0.5V_c$
0
Shear force, V_u/ϕ
$V_u(max)/\phi$
V_u corresponding to "reduced" max. spacing
V_u corresponding to "regular" max. spacing
Beam centerline
Required web steel, V_s
Concrete capacity, V_c
No web steel
Minimum web steel
"Reduced" or "Regular" maximum spacing
IV
III
II
I
Calculated minimum spacing
Calulated spacing
"Regular" max.
None
"Reduced" max.

V_n ? $V_c = 0.17\sqrt{f'_c}\, b_w\, d$

Region	$0.5\ V_c$ (I)	⇦▭▭▭▭▭▭▭▭⇨ (II)	V_c	⇦▭▭▭▭▭▭▭▭⇨ (III)	$3V_c$	⇦▭▭▭▭▭▭▭▭⇨ (IV)	$5\ V_c$ (V)
Vs	Not Required	$V_s = 0.062\sqrt{f'_c}\, b_w d \geq 0.35\ b_w d$		$V_s = 0.062\sqrt{f'_c}\, b_w d \geq 0.35\ b_w d$ $V_s = V_n - V_c$		$V_s = V_n - V_c$	Not Permitted
S (Calculated)		$S = \frac{A_v\, f_{yt}\, d}{V_s}$		$S = \frac{A_v\, f_{yt}\, d}{V_s}$		$S = \frac{A_v\, f_{yt}\, d}{V_s}$	
S (Maximum)		$\frac{d}{2}\ or\ 600\ mm$		$\frac{d}{2}\ or\ 600\ mm$		$\frac{d}{4}\ or\ 300\ mm$	

Other limitations:

1- The maximum shear force Vu to be taken at a distance d from the support rather than at the edge of the support (if this area is subjected to compression and there is no concentrated load within this distance)

2-The first stirrup used to be at a distance S/2 from the edge of the support

3-It is preferrable to used s distance as a whole number of centimeters .

Shear at Midspan of Uniformly Loaded Beams:

The beams in normal buildings are designed for uniformly distributed dead and live load. The maximum shear at the face of the support is the shear due to the total UDL of both dead & live load along the whole span of the beam. However, the shear at the mid span section for this UDL will be zero. As the live load is variable, so the maximum shear at the midspan of the beam occur when the live load is uniformly distributed on only one half of the beam. Which equal to (wl×Ln/8), so the design of shear should depend on the envelope of these two cases:

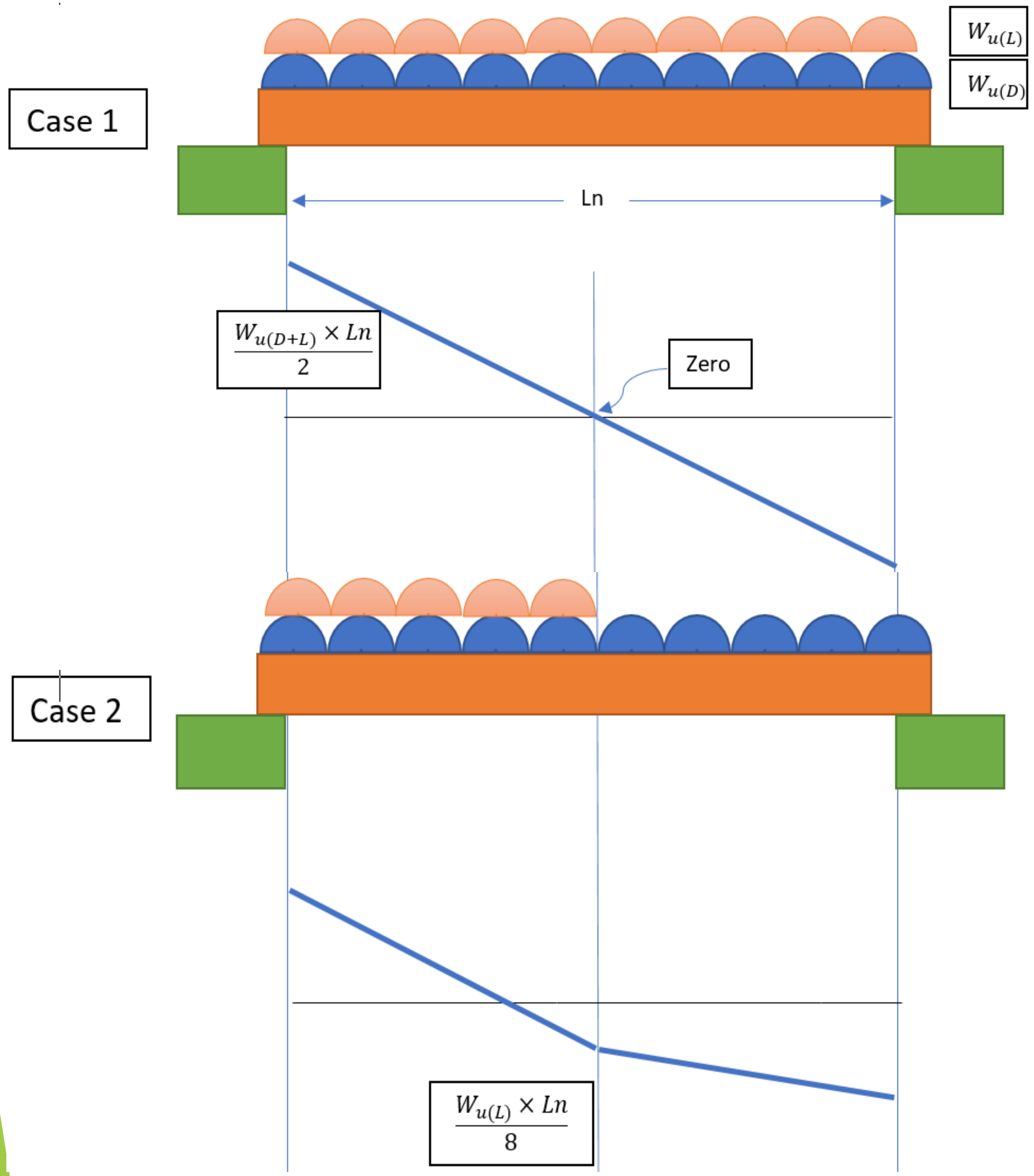
$$V_1(\text{edge}) = \frac{Wu_{(L+D)} \times L_n}{2}$$

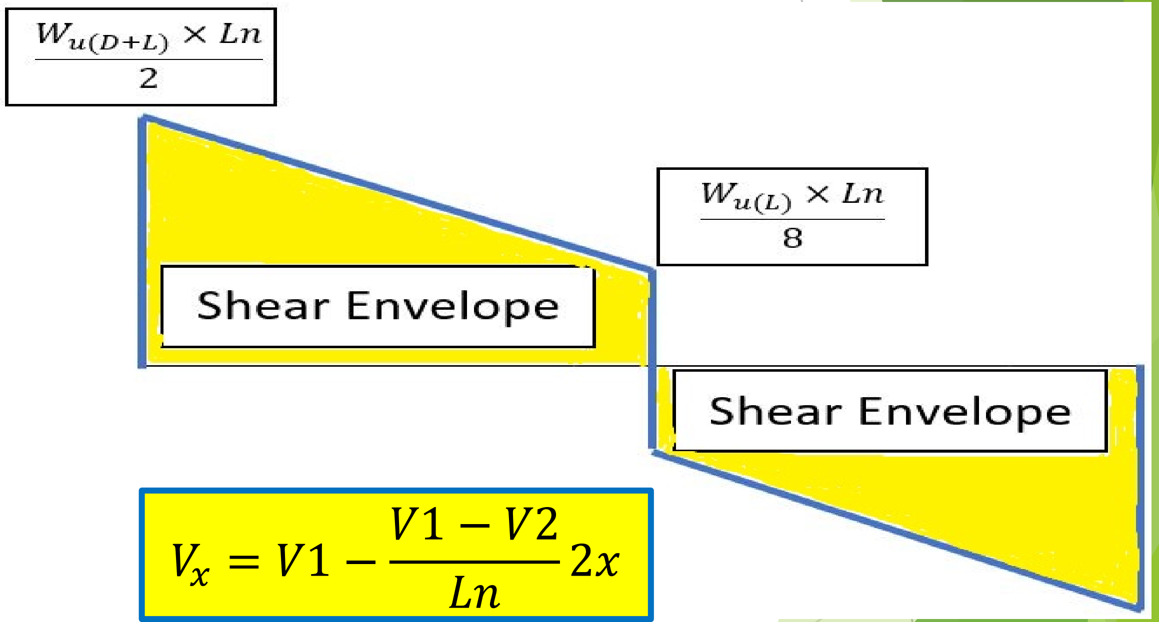
$$V_2(\text{mid}) = \frac{Wu_{(L)} \times L_n}{8}$$

The shear force of the remaining points on the beam takes the linear variation between these to points.

$$V_x = V_1 - \left(\frac{V_1 - V_2}{Ln/2}\right)x$$

Shear Envelope





$$V_x = V1 - \frac{V1 - V2}{Ln} 2x$$

Ex1: A simply supported beam section shown below, f'c=28Mpa, fyt=420MPa. As=4ϕ25

Check the section adequacy for each of the following factored shear forces:

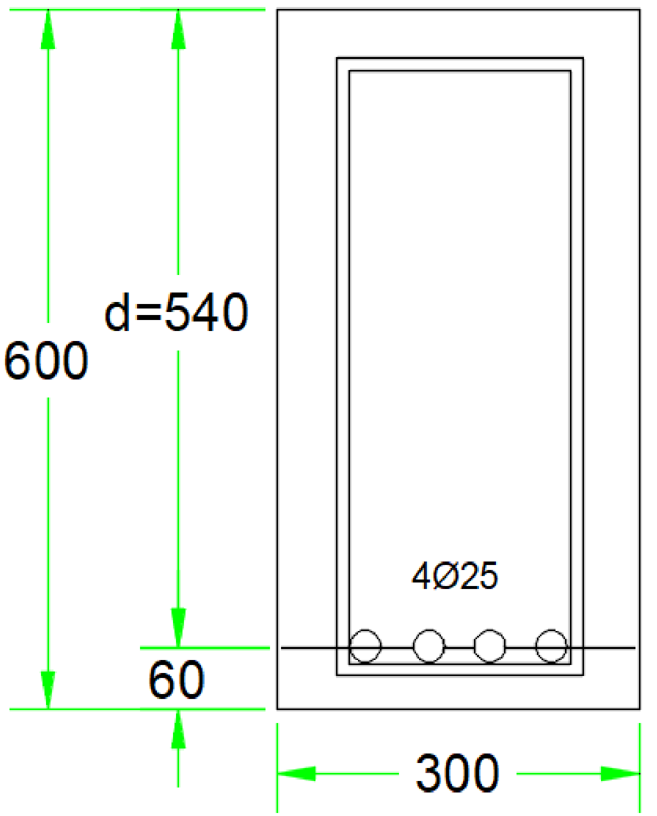
a) Vu=52kN b)Vu=104kN c)Vu=243kN d)Vu=337kN e)Vu=560kN

Solution:

1) Our calculation will depend on Vn as Vn=Vu/Ø , Ø=0.75 so:

a) Vn=69.33kN b) Vn=138.66kN c) Vn=324kN d) Vn=449.33kN e) Vn=746.66kN

2) Calculate the 4 limits (5 Zones) to Compare with Required Vn



$$V_c = 0.17\sqrt{f_c'}\, b_w\, d = 0.17\sqrt{28} \times 300 \times 540 = 145.72\ kN$$

$$0.5V_c = 0.5 \times 145.72 = 72.86\ kN$$

$$V_c = 145.72\ kN$$

$$3V_c = 3 \times 145.72 = 437.16\ kN$$

$$5V_c = 5 \times 145.72 = 728.6\ kN$$

For Av calculation use two legs Ø10 stirrups:

$A_b = 78.5\ mm^2$ ⟹ $A_v = 2 \times 78.5 = 157 mm^2$

a) Vn=69.33kN

69.33≤ 72.86 (0.5Vc) (Region I) No shear reinforcement is required

b) Vn=138.66kN

72.86<Vn=138.66kN≤145.72 (Vc) (Region II)

$$V_{s(\min)} = 0.062\sqrt{f_c'}\, b_w d \geq 0.35\, b_w d$$

$$V_{s(\min)} = 0.062\sqrt{28}\ \times 300 \times 540 \geq 0.35 \times 300 \times 540$$

$$V_{s(\min)} = 53.147 \geq 56.7\ kN$$ ⟹ Vs(min)=56.7kN

$$S = \frac{A_v\, f_{yt}\, d}{V_s} = \frac{157 \times 420 \times 540}{56.7 \times 1000} = 628mm$$

$$Smax = \left(\frac{d}{2}, 600mm\right) ⟹ \left(\frac{540}{2}, 600mm\right) ⟹ (270mm, 600mm)$$

Use Ø10 stirrups @270mm c/c

c) Vn=324kN

145.72 (Vc)<Vn=324kN≤437.16 (34Vc) (Region III)

$$V_{s(\min)} = 56.7\ kN$$

$$V_s = V_n - V_c$$

$$V_s = 324 - 145.72 = 178.28\ kN$$ ⇨ Vs=178.28 kN

$$S = \frac{A_v\ f_{yt}\ d}{V_s} = \frac{157 \times 420 \times 540}{178.28 \times 1000} = 199.72mm$$

$$Smax = \left(\frac{d}{2}, 600mm\right) \Rightarrow \left(\frac{540}{2}, 600mm\right) \Rightarrow (270mm, 600mm)$$

Use Ø10 stirrups @190mm c/c

d) Vn=449.33kN

437.16(3Vc)<Vn=449.33kN≤728.6 (5Vc) (Region IV)

$$V_s = V_n - V_c$$

$$V_s = 449.33 - 145.72 = 303.61\ kN$$

$$S = \frac{A_v\ f_{yt}\ d}{V_s} = \frac{157 \times 420 \times 540}{303.61 \times 1000} = 117.28mm$$

$$Smax = \left(\frac{d}{4}, 300mm\right) \Rightarrow \left(\frac{540}{4}, 300mm\right) \Rightarrow (135mm, 300mm)$$

Use Ø10 stirrups @110mm c/c

e) Vn=746.66kN

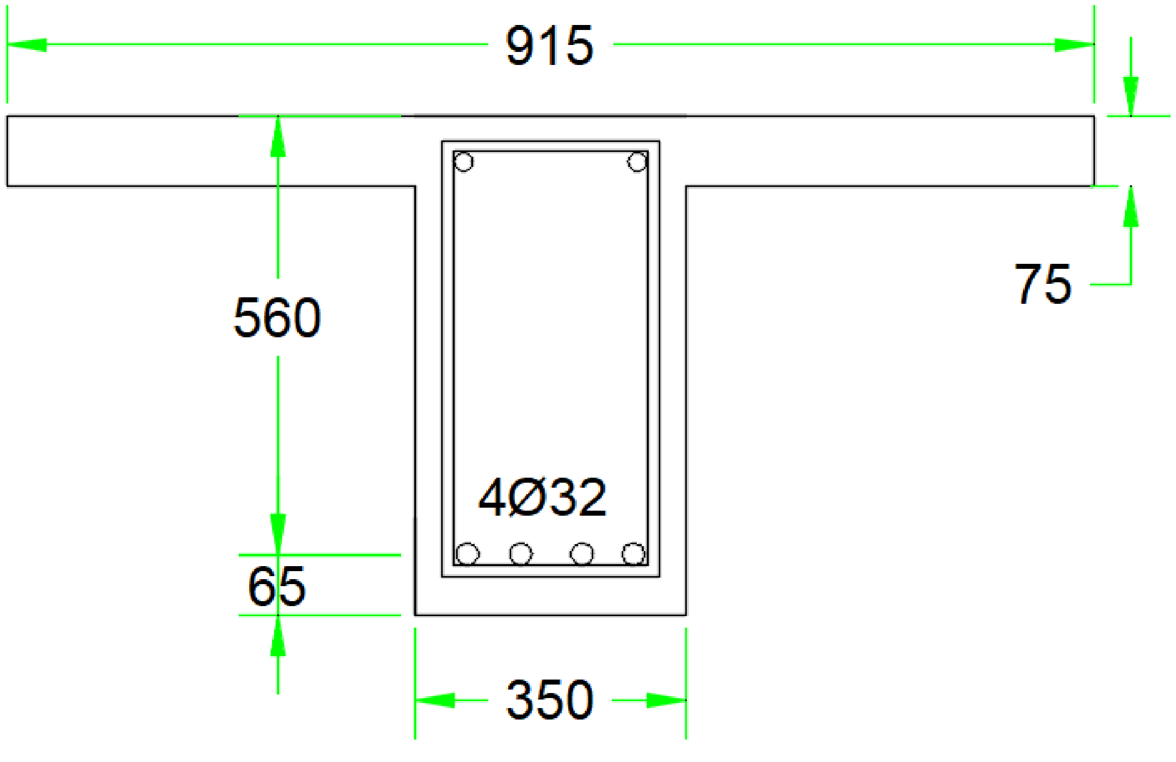
Vn=746.66kN>728.6 (5Vc) (Region V)

The section is not adequate for the applied Shear force……Change section dimensions.

Ex2: A 5.2m simply supported beam has a clear span of 4.9m, and carries a uniformly distributed dead load (including self weight) and live load of 65.7kN/m , 54.75kN/m, respectively. The dimension of the section and steel reinforcement are shown in the figure. Check the section for shear and design the necessary shear reinforcement (Take Shear Envelope in Consideration). Use: f'c=21Mpa , fyt=420MPa.

Solution:

1) Our calculation will depend on Vn as Vn=Vu/Ø , Ø=0.75 for the shear envelope:



$$V1(\text{edge}) = \frac{Wu_{(L+D)} \times L_n}{2Ø}$$

$$V1(\text{edge}) = \frac{(1.2 \times 65.7 + 1.6 \times 54.75) \times 4.9}{2 \times 0.75} = 543.7kN$$

$$V2(\text{mid}) = \frac{Wu_{(L)} \times L_n}{8Ø} = \frac{1.6 \times 54.75 \times 4.9}{8 \times 0.75} = 71.54kN$$

$$V_x = V1 - \frac{V1 - V2}{Ln} 2x$$

$$V_{max} = V_d = 543.7 - \left(\frac{543.7-71.54}{4.9} \times 2\right) \times 0.56$$

$$V_{max} = V_d = 543.7 - (192.71) \times 0.56 = 435.77\text{kN}$$

2) Calculate the 4 limits (5 Zones) to compare with the required Vn

$$V_c = 0.17\sqrt{f_c'}\, b_w\, d \;= 0.17\sqrt{21}\; \times 350\; \times 560 = 152.69\ kN$$

$$0.5V_c = 0.5 \times 152.69 = 76.34\ kN$$

$$V_c = 152.69\ kN$$

$$3V_c = 3 \times 152.69 = 458.07\ kN$$

$$5V_c = 5 \times 152.69 = 763.45\ kN$$

$*\ The\ distribution\ of\ the\ applied\ shear\ works\ for\ Zones\ I, II\ \&\ III$

$*\ Find\ the\ value\ of\ x\ (from\ the\ ege\ of\ support)\ that\ separate\ these\ regions:$

$$V_x = V1 - \left(\frac{V1 - V2}{Ln} \times 2\right)x \qquad 152.69 = 543.7 - (192.71)x$$

$$x = 2.03\ m$$

$$Zone\ III\ from\ x = 0 \dashrightarrow x = 2.03m$$

$$V_x = V1 - \left(\frac{V1 - V2}{Ln} \times 2\right)x \qquad 76.34 = 543.7 - (192.71)x$$

$$x = 2.425\ m$$

$$Zone\ II\ from\ x = 2.03 \dashrightarrow x = 2.425m$$

$Zone\ I\ from\ x = 2.425 \dashrightarrow x = \frac{Ln}{2}$ =2.45m (Mid of Beam) (Will be Neglected)

$Region\ III\ from\ x = 0 \dashrightarrow x = 2.03m$

Vn=Vd=435.77kN

$$V_{s(\min)} = 0.062\sqrt{f_c'}\ b_w d \geq 0.35\ b_w d$$

$$V_{s(\min)} = 0.062\sqrt{21}\ \times 350 \times 560 \geq 0.35 \times 350 \times 560$$

$V_{s(\min)} = 55.68 \geq 68.6\ kN$ □□□□⇨ Vs(min)=68.6kN

$$V_s = V_n - V_c$$

$$V_s = 435.77 - 152.69 = 283.08\ kN$$

$$S = \frac{A_v\ f_{yt}\ d}{V_s} = \frac{157 \times 420 \times 560}{283.08 \times 1000} = 130.44mm$$

$Smax = \left(\frac{d}{2}, 600mm\right)$ □□□□⇨ $\left(\frac{560}{2}, 600mm\right)$ □□□□⇨ $(280mm, 600mm)$

Use Ø10 stirrups @130 mm c/c

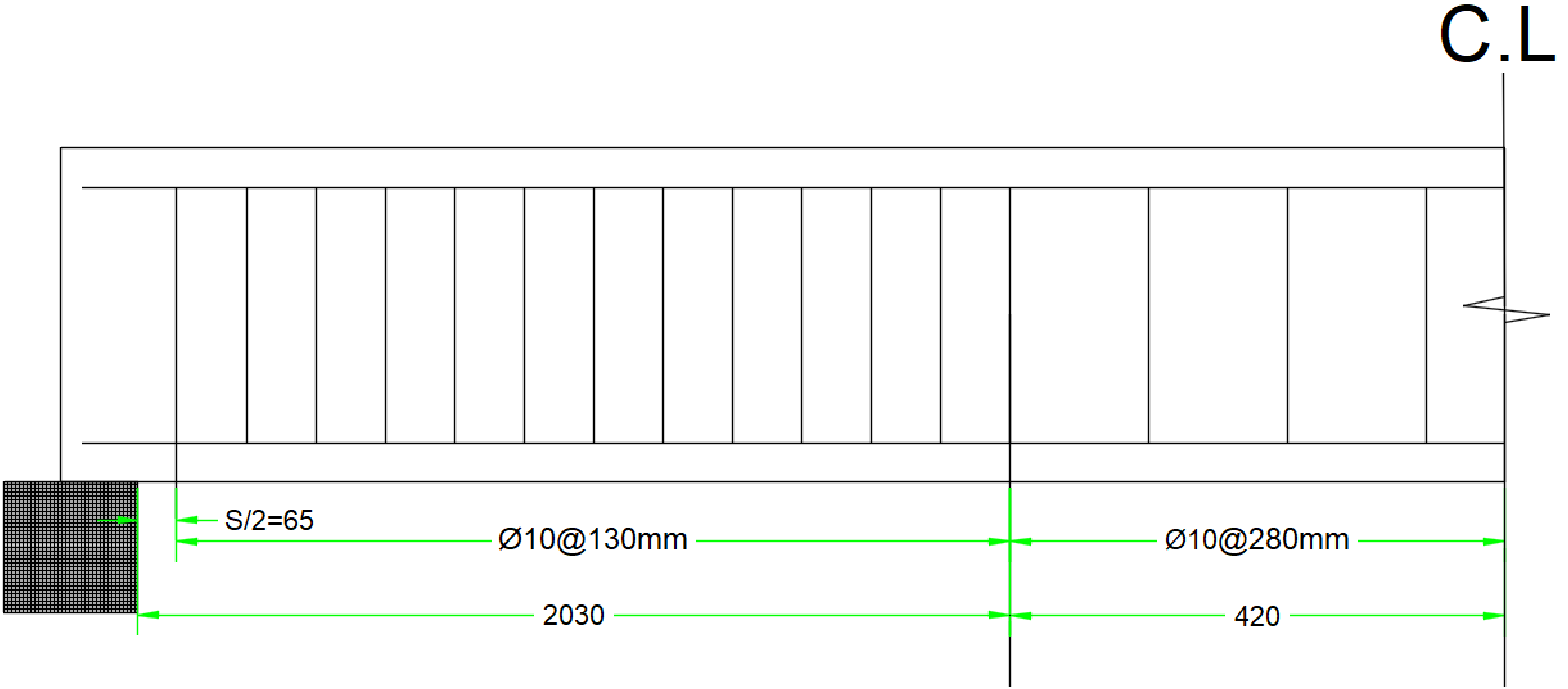
$Region\ II\ from\ x = 2.03 \dashrightarrow x =$2.45m

$V_{s(\min)} =$ 68.6kN

$$S = \frac{A_v\ f_{yt}\ d}{V_s} = \frac{157 \times 420 \times 560}{68.6 \times 1000} = 538.28mm$$

$Smax = \left(\frac{d}{2}, 600mm\right)$ □□□□⇨ $\left(\frac{560}{2}, 600mm\right)$ □□□□⇨ $(280mm, 600mm)$

Use Ø10 stirrups @280mm c/c


C.L
S/2=65
Ø10@130mm
Ø10@280mm
2030
420

Thank you…

Thank you…