Design of
Single Reinforced
Rectangular Beam Sections

(Design of SRRS)

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Before getting into design it is good to introduce the following new three terms or definitions:
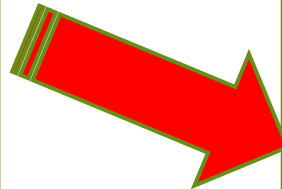
$$\rho = \frac{A_s}{bd}$$

$$m = \frac{f_y}{0.85\, f_c'}$$

$$R_n = \frac{M_n}{bd^2}$$

$$A_s\, f_y = 0.85\, f_c'\, a\, b \qquad (1)$$

$$M_n = A_s\, f_y \left(d - \frac{a}{2}\right) \qquad (2)$$

$$\varepsilon_t = 0.003 \left(\frac{d_t - c}{c}\right) \qquad (3)$$

$$a = \rho\, m\, d \qquad (1')$$

$$R_n = \frac{M_n}{bd^2} = \rho\, f_y \left(1 - \frac{1}{2}\rho m\right) \qquad (2')$$

$$\rho = \left(\frac{0.003}{0.003 + \varepsilon_s}\right) \frac{\beta_1}{m} \left(\frac{d_t}{d}\right) \qquad (3')$$

$$\rho = \frac{1}{m} \left(1 - \sqrt{1 - \frac{2m\, R_n}{f_y}}\right) \qquad (4')$$

As an example of usefulness of the above (dash equations) we can calculate ρb and ρmax for (fy=420MPa) by substituting εs=0.002 and εs=0.005 in eq.(3') respectively:

$$\rho_b = \frac{3}{5}\frac{\beta_1}{m}\left(\frac{d_t}{d}\right) \qquad for\ fy = 420\ MPa\ because\ \varepsilon y = 0.002$$

$$\rho_{max} = \frac{3}{8}\frac{\beta_1}{m}\left(\frac{d_t}{d}\right) \qquad for\ all\ Cases$$

Or in design problems we can start estimating the As by assuming εs=0.007 to stay in Tension Control Zone so:

$$\rho_{start} = \frac{3}{10}\frac{\beta_1}{m}\left(\frac{d_t}{d}\right)$$

Design problems will mainly depend on eq.(2')

$$R_n = \frac{M_n}{bd^2} = \rho\, f_y\left(1 - \frac{1}{2}\rho m\right) \quad (2')$$
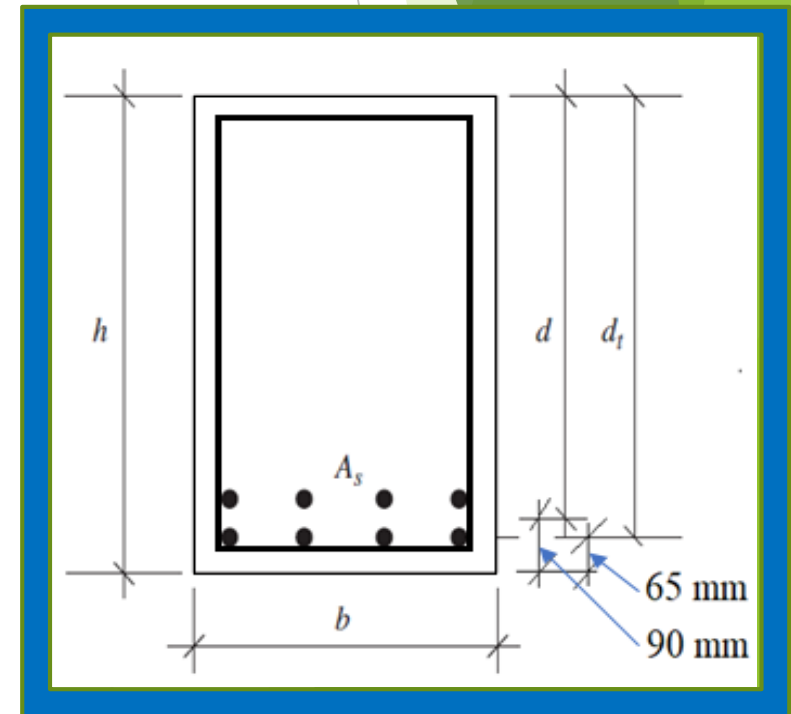
Table 9.3.1.1—Minimum depth of nonprestressed beams

Support condition	Minimum h [1]
Simply supported	$\ell/16$
One end continuous	$\ell/18.5$
Both ends continuous	$\ell/21$
Cantilever	$\ell/8$

$$h = [1.5 \ to \ 2] \times b$$

Minimum concrete cover for beams=40mm

ACI 25.2.1 the minimum clear bar spacing:

$$S_{min} \geq \begin{cases} 25mm \\ d_b \\ \left(\dfrac{4}{3}\right) d_{agg} \end{cases}$$

Ex1: Find the necessary reinforcement for a given section that has a width of 250mm and total depth of 500mm, if it is subjected to a factored moment of 180kN.m. Given f'c=21Mpa, fy=375MPa.

Solution:

$$\varepsilon_y = \frac{f_y}{E_s} = \frac{375}{200000} = 0.001875$$

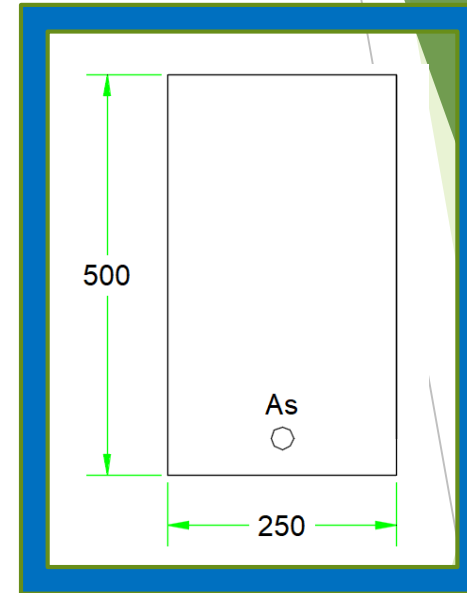$$m = \frac{f_y}{0.85\ f_c'} = \frac{375}{0.85 \times 21} = 21.0$$

$$M_n = \frac{180}{0.9} = 200\ kN.m$$

Assume one layer of reinforcement

$$d = h - 65 = 500 - 65 = 435mm$$

$$R_n = \frac{M_n}{bd^2}$$

$$R_n = \frac{200 \times 10^6}{250 \times 435^2} = 4.227$$

500

As
○

250

$$\rho = \frac{1}{m} \left(1 - \sqrt{1 - \frac{2m\,R_n}{f_y}} \right)$$

$$\rho = \frac{1}{21} \left(1 - \sqrt{1 - \frac{2 \times 21 \times 4.227}{375}} \right) = 0.013$$

$$\rho_{max} = \frac{3}{8} \frac{\beta_1}{m} \left(\frac{d_t}{d} \right)$$

$$\rho_{max} = \frac{3 \times 0.85}{8 \times 21.0} (1) = 0.0151 > 0.013 \ (ok\ Tension\ control)$$

$$A_s = \rho bd = 0.013 \times 250 \times 435 = 1413.75\ mm^2$$

Try Ø25→Ab=490 mm²

$$n = \frac{As}{Ab} = \frac{1413.75}{490} = 2.88$$

Use 3 Ø25

$$As_{provided} = 3 \times 490 = 1470 \ mm^2$$

$$\rho_{provided} = \frac{1470}{250 \times 435} = 0.0135 < 0.0151 \ (ok \ T.C)$$

$$\rho_{min} = \frac{1.4}{f_y} = \frac{1.4}{375} = 0.00373 < 0.0135 \ (ok$$
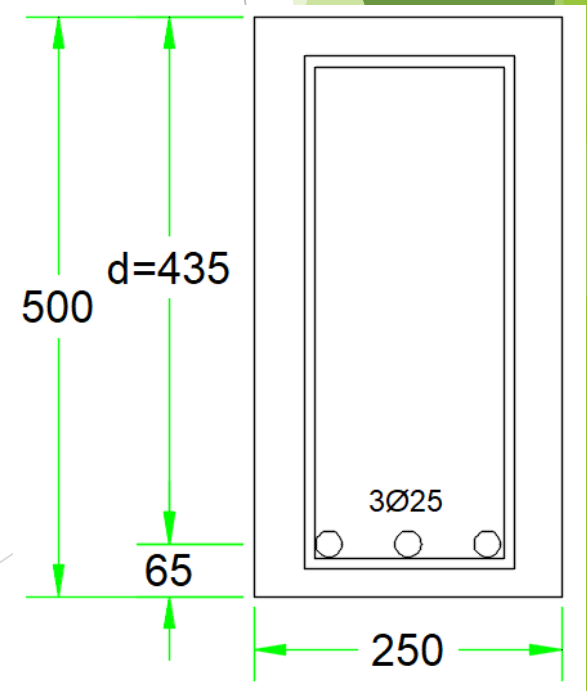
Check for bar spacing:

$$S_{min} = \begin{cases} 25mm \\ \\ d_b = 25 \\ \\ \left(\frac{4}{3}\right)d_{agg} = \frac{4}{3} \times 20 = 26.66 \end{cases}$$

$$s = \frac{250 - 100 - 3 \times 25}{2} = 37.5 > 26.66 \ (ok)$$

$$M_n = \rho \ f_y \ bd^2 \left(1 - \frac{1}{2}\rho m\right)$$

$$M_n = 0.0135 \times 375 \times 250 \times 435^2 \left(1 - \frac{1}{2} \times 0.0135 \times 21.0\right) = 205.54 kN.m$$

$$M_u = \emptyset \ M_n = 0.9 \times 205.54 = 184.98 \ kN.m > 180 \ (OK)$$

d=435

500

65

250

3Ø25

Ex2: Design a rectangular section to resist a factored moment of 300kN.m. Given f'c=28MPa, and fy=420MPa.

Solution:

$$M_n = \frac{M_u}{\emptyset} = \frac{300}{0.9} = 333.33 \; kN.m$$

$$m = \frac{f_y}{0.85 \; f_c'} = \frac{420}{0.85 \times 28} = 17.647$$

$$R_n = \frac{M_n}{bd^2} = \rho \; f_y \left(1 - \frac{1}{2}\rho m\right)$$

Start with ρ depending on εs=0.007 and assuming one layer of reinforcement:

$$\rho_{start} = \frac{3}{10}\frac{\beta_1}{m}\left(\frac{d_t}{d}\right)$$

$$\rho_{start} = \left(\frac{3}{10}\right)\frac{0.85}{17.647}(1) = 0.0144$$

$$R_n = \rho \; f_y \left(1 - \frac{1}{2}\rho m\right)$$

$$R_n = 0.0144 \times 420 \left(1 - \frac{1}{2} \times 0.0144 \times 17.647\right) = 5.277$$

$$R_n = \frac{M_n}{bd^2}$$

$$5.277 = \frac{333.33 \times 10^6}{bd^2}$$
$$bd^2 = 63166571.92 \ mm^3$$

Assume b to get d:

b	d	d/b
250	502.66	2.01
300	458.86	1.529
350	424.82	1.213

$$A_s = \rho bd = 0.0144 \times 300 \times 460 = 1987.2 \ mm^2$$

Try Ø22→Ab=380

$$n = \frac{1987.2}{380} = 5.23$$

Use 6Ø22 in two layers:

$$As_{provided} = 6 \times 380 = 2280 \ mm^2$$

$$\rho_{provided} = \frac{2280}{300 \times 460} = 0.0165$$

$$\rho_{max} = \frac{3}{8} \frac{\beta_1}{m} \left(\frac{d_t}{d} \right)$$

$$\rho_{max} = \frac{3}{8} \times \frac{0.85}{17.647} \times \left(\frac{489}{460} \right) = 0.0192 > 0.0165 \ (ok \ T.C)$$
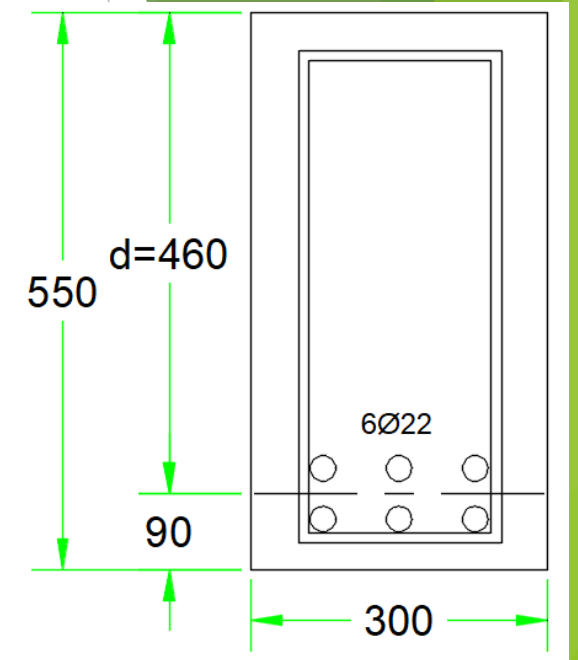
$$\rho_{min} = \frac{1.4}{f_y} = \frac{1.4}{420} = 0.00333 < 0.0165 \ (ok)$$

$$h = d + 90 = 460 + 90 = 550 \ mm$$

$$s = \frac{300 - 100 - 3 \times 22}{2} = 67 > 26.66 \ (ok)$$

$$M_n = 0.0165 \times 420 \times 300 \times 460^2 \left(1 - \frac{1}{2} \times 0.0165 \times 17.647 \right) = 375.87 \ kN.m$$

$$M_u = \emptyset \ M_n = 0.9 \times 375.87 = 338.283 \ kN.m > 300 \ (OK)$$

d=460
550
90
6Ø22
300

Thank you…