Analysis of
Double Reinforced
Rectangular Beam Sections

(Analysis of DRRS)

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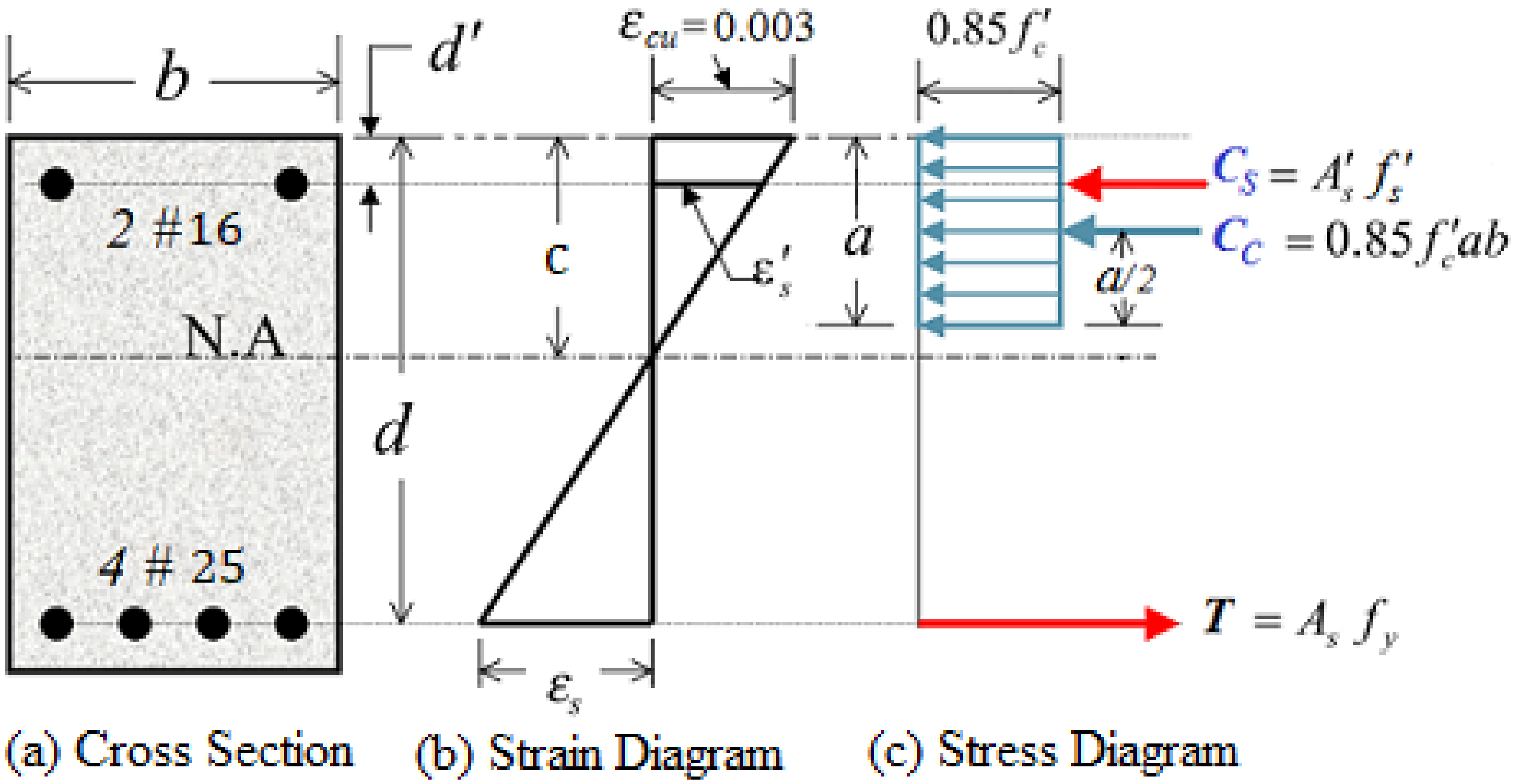


$$A_s\, f_y = 0.85\, f_c'\, \beta_1\, c\, b + A_s'\, f_s'$$

$$A_s\, f_y = 0.85\, f_c'\, \beta_1\, c\, b + A_s'\, f_y$$

$$c = \frac{(A_s - A_s') f_y}{0.85\, \beta_1\, f_c'\, b}$$

$$f_s' = 0.003\, E_s \left(\frac{c - \bar{d}}{c}\right) \quad \ldots ? \ldots \; f_y$$

$$a = \beta_1 c$$

$$M_n = 0.85\, f_c'\, a\, b \left(d - \frac{a}{2}\right) + A_s'\, f_y\, (d - d')$$

$$\varepsilon_t = 0.003 \left(\frac{d_t - c}{c}\right) \quad \geq 0.005$$

$$A_s\, f_y = 0.85\, f_c'\, \beta_1\, c\, b + A_s' \times \left[0.003\left(1 - \frac{\bar{d}}{c}\right)\right] E_s$$

$$Quadratic\ Formla: \quad x = \frac{-b \mp \sqrt{b^2 - 4ac}}{2a}$$

$$f_s' = 0.003\, E_s \left(\frac{c - \bar{d}}{c}\right)$$

$$M_n = 0.85\, f_c'\, a\, b\, \left(d - \frac{a}{2}\right) + A_s'\, f_s'\, (d - d')$$

$$\varepsilon_t = 0.003\left(\frac{d_t - c}{c}\right) \geq 0.005$$

Ex1: Determine the design moment strength of the section shown below. Use f'c=35Mpa, fy=420MPa. As=6ϕ32, A's=3ϕ25

Solution:

$$A_s = 6 \times 804 = 4824\ mm^2$$

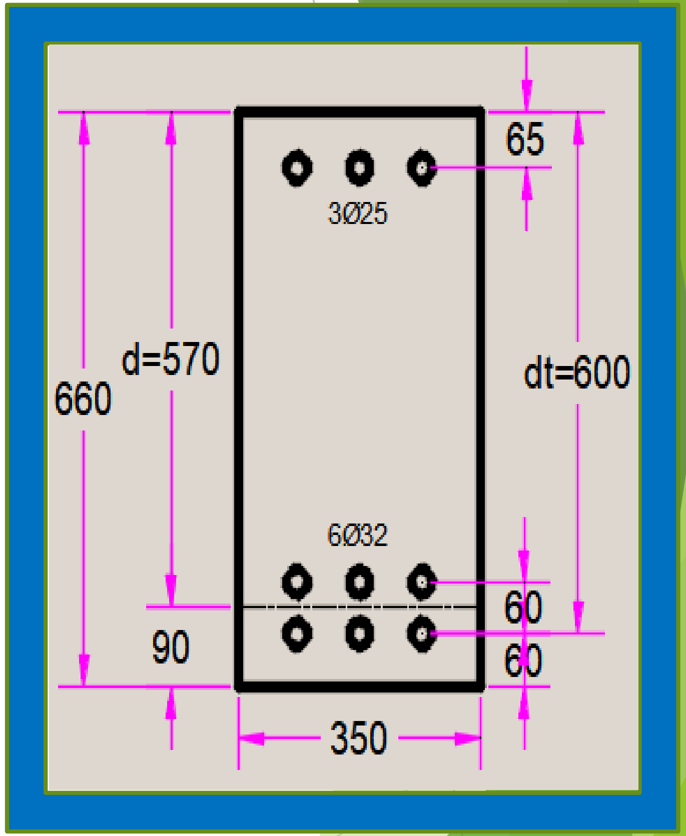
$$A_s' = 3 \times 490 = 1470\ mm^2$$

$$\beta_1 = 0.85 - \frac{0.05\ (f_c' - 28)}{7} \geq 0.65$$

$$\beta_1 = 0.85 - \frac{0.05\ (35 - 28)}{7} = 0.80$$

$$As_{min} = \frac{0.25\sqrt{f_c'}}{f_y}\ b_w\ d$$

$$As_{min} = \frac{0.25 \times \sqrt{35}}{420} \times 350 \times 570 = 702.53 mm^2 < 4824\ (ok)$$



$$A_s\, f_y = 0.85\, f_c'\, \beta_1\, c\, b + A_s'\, f_y$$

$$c = \frac{(A_s - A_s')f_y}{0.85\, \beta_1\, f_c'\, b} = \frac{(4824 - 1470) \times 420}{0.85 \times 0.80 \times 35 \times 350} = 169.11mm$$

$$f_s' = 0.003\, E_s \left(1 - \frac{\bar{d}}{c}\right) = 600 \left(1 - \frac{65}{169.11}\right) = 369.38MPa < 420\ (Not\ Yield)$$

$$A_s\, f_y = 0.85\, f_c'\, \beta_1\, c\, b + A_s' \times\ 0.003\, E_s \left(1 - \frac{\bar{d}}{c}\right)$$

$$4824 \times 420 = 0.85 \times 35 \times 0.80 \times\ c\ \times 350 + 1470 \times 600 \left(1 - \frac{65}{c}\right)$$

$$2026080 = 8330\, c + 882000 - \frac{57330000}{c}$$

$$c^2 - 137.344\, c - 6882.353 = 0$$

$$c = (176.367, -39.023)$$

$$c = 176.367\ mm$$

$$f_s' = 600\left(1 - \frac{65}{176.367}\right) = 378.87\ MPa$$

$$a = \beta_1\ c = 0.8 \times 176.367 = 141.09$$

$$M_n = 0.85\ f_c'\ a\ b\ \left(d - \frac{a}{2}\right) + A_s'\ f_s'\ (d - d')$$

$$M_n = 0.85 \times 35 \times 141.09 \times 350\left(570 - \frac{141.09}{2}\right) + 1470 \times 378.87\ (570 - 65)$$

$$M_n = 733.746 + 281.254 = 1015.0\ kN.m$$

Check for ductility:

$$\varepsilon_s = 0.003\left(\frac{dt}{c} - 1\right) = 0.003\left(\frac{600}{176.367} - 1\right) = 0.0072 > 0.005$$

The section is Tension controlled and ϕ=0.9

$$\emptyset M_n = 0.9 \times 1015.0 = 913.5\ kN.m$$

Thank you...