Examples for Analysis of Single Reinforced Rectangular Beam Sections

(Analysis of SRRS)

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The Minimum Reinforcement

ACI Code 9.6.1.2: $As_{,min}$ shall be the larger of (a) and (b)

$$a) \quad As_{min} = \frac{1.4}{f_y} b_w d$$

$$b) \quad As_{min} = \frac{0.25\sqrt{f_c'}}{f_y} b_w d$$

OR

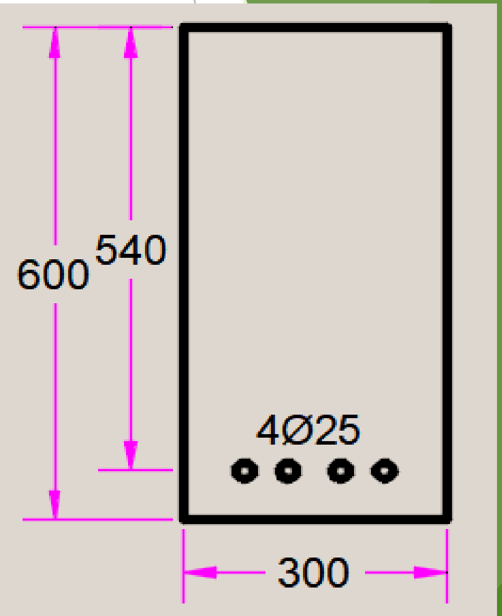
$$As_{min} = \frac{0.25\sqrt{f_c'}}{f_y} b_w d \geq \frac{1.4}{f_y} b_w d$$

OR

$$As_{min} = \begin{cases} \frac{1.4}{f_y} b_w d & for \ f_c' \leq 31.0\ MPa \\ \frac{0.25\sqrt{f_c'}}{f_y} b_w d & for \ f_c' > 31.0\ MPa \end{cases}$$

Ex1: Determine the design moment strength and the position of the neutral axis of the rectangular section shown below, if the reinforcement used is 4Ø25, given f'c=28MPa, fy=420MPa.

Solution:



$$A\text{Ø}25 = \frac{\pi}{4}(25)^2 = 490\ mm^2 \quad \Rightarrow \quad As = 4 \times 490 = 1960\ mm^2$$

Check As Minimum:

$$As_{min} = \begin{cases} (a)\ \dfrac{1.4}{f_y} b_w d & for\ \ f_c' \leq 31.0\ MPa \\ (b)\ \dfrac{0.25\sqrt{f_c'}}{f_y} b_w d & for\ \ f_c' > 31.0\ MPa \end{cases}$$

$$As_{min} = \frac{1.4}{f_y} b_w d = \frac{1.4}{420} \times 300 \times 540 = 540\ mm^2 < 1960\ mm^2 \quad (OK)$$

Equilibrium Equation:

$$0.85\ f_c'\ a\ b = A_s\ f_y \Rightarrow a = \frac{A_s\ f_y}{0.85\ f_c'\ b} \Rightarrow a = \frac{1960 \times 420}{0.85 \times 28 \times 300} = 115.294\ mm$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) \Rightarrow M_n = 1960 \times 420 \left(540 - \frac{115.294}{2}\right) = 397.0729\ kN.m$$

$$c = \frac{a}{\beta_1} = \frac{115.294}{0.85} = 135.64\ mm$$

Check for Ductility:

$$\varepsilon_t = 0.003 \left(\frac{d_t - c}{c}\right) \Rightarrow \varepsilon_t = 0.003 \left(\frac{540}{135.64} - 1\right) = 0.00894 > 0.005 \quad (tension\ controlled\ ok)$$

So, the design moment strength is:

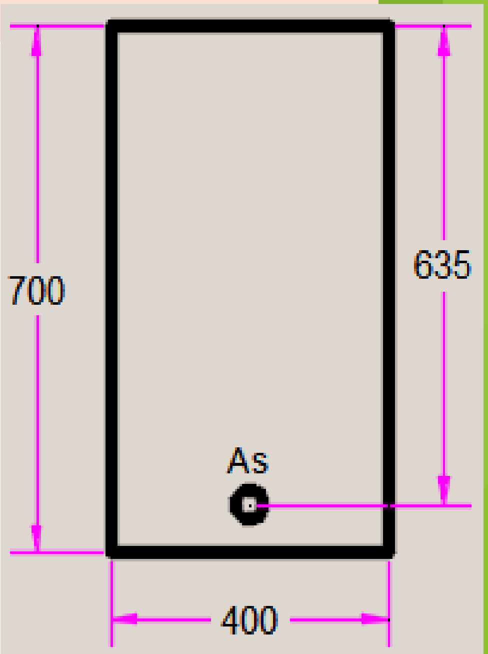
$$\emptyset M_n = 0.9 \times 397.0729 = 357.365\ kN.m$$

Ex2: For the section shown below with f'_c= 28 MPa and fy = 420 MPa, calculate

a. The balanced steel reinforcement

b. The maximum reinforcement area allowed by the ACI Code for a tension-controlled section.

c. The position of the neutral axis and the depth of the equivalent compressive stress block for the tension-controlled section in b.



Solution:

a)

$$\varepsilon_s = 0.003\left(\frac{d_t - c}{c}\right) \rightarrow 0.0021 = 0.003\left(\frac{635 - c}{c}\right)$$

$$\frac{0.0021}{0.003} c = 635 - c \rightarrow 1.7c=635 \rightarrow c=373.53mm$$

$$a = \beta_1\, c = 0.85 \times 373.53 = 317.5mm$$

$$0.85\, f'_c\, a\, b = A_s\, f_y$$

$$A_s = \frac{0.85\, f'_c\, a\, b}{f_y} = \frac{0.85 \times 28 \times 317.5 \times 400}{420} = 7196.66\, mm^2$$

b)

$$\varepsilon_s = 0.003\left(\frac{d_t - c}{c}\right) \qquad 0.005 = 0.003\left(\frac{635 - c}{c}\right) \qquad \frac{0.005}{0.003}c = 635 - c$$

2.666c=635 c=238.18 mm

$$a = \beta_1\, c = 0.85 \times 238.18 = 202.45\, mm$$

$$0.85\, f_c'\, a\, b = A_s\, f_y$$

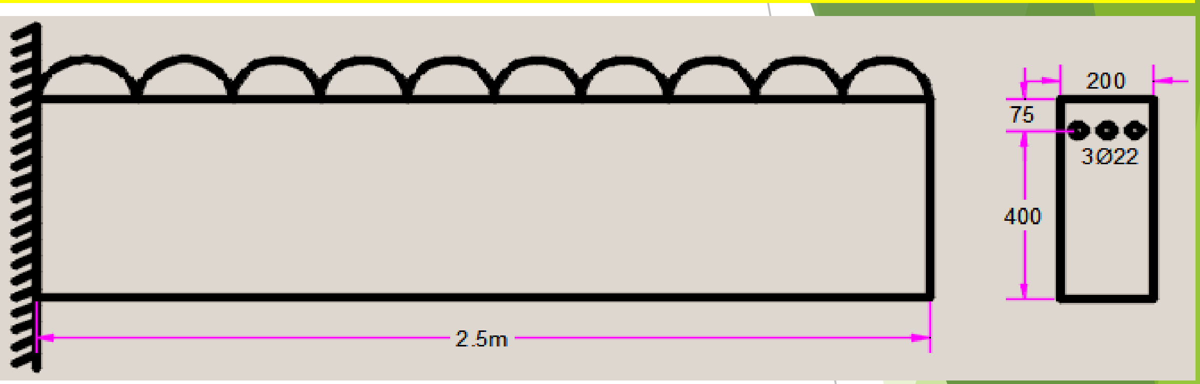
$$A_s = \frac{0.85\, f_c'\, a\, b}{f_y} = \frac{0.85 \times 28 \times 202.45 \times 400}{420} = 4589.02\, mm^2$$

C)

c=238.18 mm

a=202.45 mm

EX3: A 2.5m span cantilever beam has a rectangular section and reinforcement as shown in the figure. The beam carries a dead load including its own weight of 22 kN/m and a live load of 13 kN/m, using f'c=28 MPa, and fy=420 MPa, check if the beam is safe to carry the above loads.



$$A\emptyset 22 = \frac{\pi}{4}(22)^2 = 380\ mm^2$$

$$As = 3 \times 380 = 1140\ mm^2$$

$$As_{min} = \begin{cases} \dfrac{1.4}{f_y} b_w d & for\ \ f_c' \le 31.0\ MPa \\ \dfrac{0.25\sqrt{f_c'}}{f_y} b_w d & for\ \ f_c' > 31.0\ MPa \end{cases}$$

$$As_{min} = \frac{1.4}{f_y} b_w d = \frac{1.4}{420} \times 200 \times 400 = 266.66\ mm^2 < 1140\ mm^2 \quad (OK)$$

$$w_u = 1.2\ D + 1.6\ L \quad \Rightarrow \quad w_u = 1.2 \times 22 + 1.6 \times 13 = 47.2\ kN/m$$

$$M_{u,max} = \frac{w_u \ell^2}{2} = \frac{47.2 \times 2.5^2}{2} = 147.5\ kN.m$$

$$a = \frac{A_s f_y}{0.85 f_c' b} \quad \Rightarrow \quad a = \frac{1140 \times 420}{0.85 \times 28 \times 200} = 100\ mm$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right)$$

$$M_n = 1140 \times 420 \left(400 - \frac{100}{2}\right) = 167.58\ kN.m$$

Check the Ductility:

$$c = \frac{a}{\beta_1} = \frac{100}{0.85} = 117.64\ mm$$

$$\varepsilon_s = 0.003 \left(\frac{d_t - c}{c}\right) = 0.003 \left(\frac{400 - 117.64}{117.64}\right) = 0.0072 > 0.005\ (OK)$$

$$\emptyset M_n = 0.9 \times 167.58 = 150.822\ kN.m > 147.5 \qquad (OK)$$

Thank you…