Working Stress Design

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Working Stress Method

Historically the Design method of (Working stress, Allowable Stress, or Service loads) is the first method used in design of Steel and Reinforced Concrete Structures.

The Method is Using (Working or Service Loads) & (Working or Allowable Stress) for design.

Working or Service Loads mean: The ordinary daily loads that expected to be applied on specific structures (without any factors of increasing the loads or factor of safety)

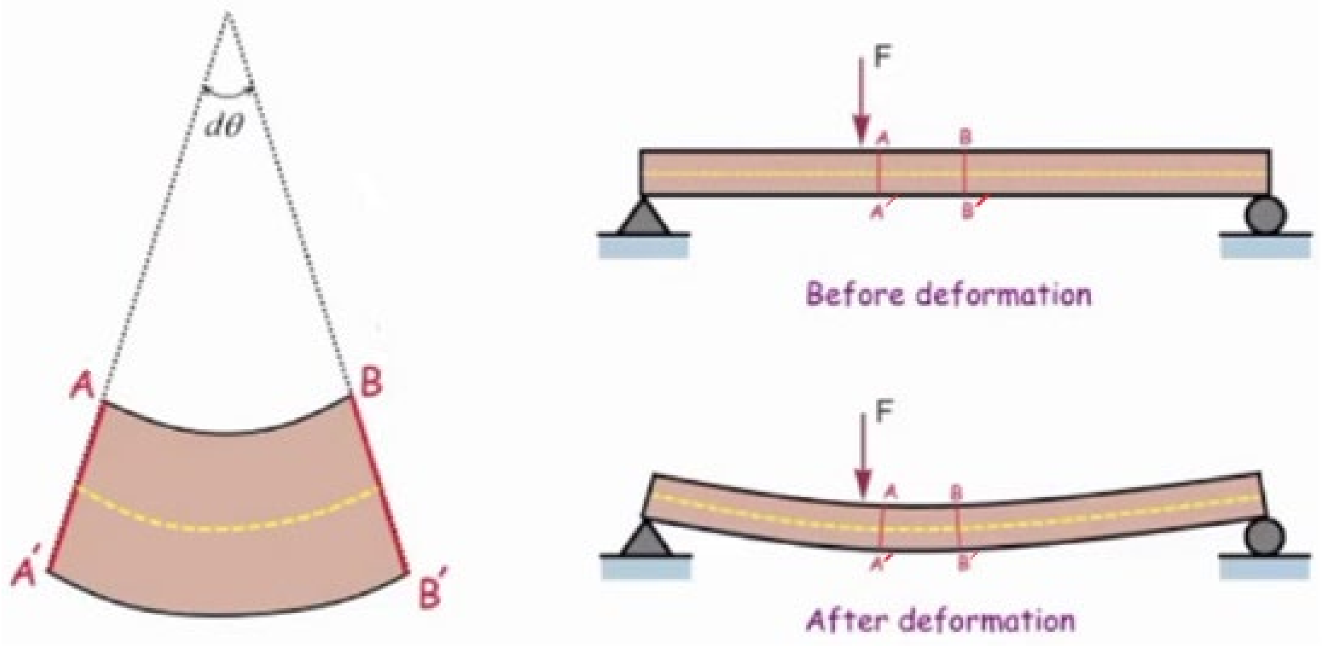
Working or Allowable Stress means: The level of stresses that would be Accepted by the “Designer” or more accurately by the “Code” in the specific member of the structure, that give some Factor of Safety against the failure or against un acceptable deformation or deflection.

Although the recent codes are using another methods for design (The Ultimate Strength Method), the method of Working Stress is still important for the following reasons:

1. The method still being used in Steel Structures, Prestress Concrete and some other applications.
2. It is necessary to calculate the serviceability of the concrete structures (Cracks & Deflection).
3. It is used in concrete tanks that used to reserve water or other fluids, specifically to investigate cracks.
4. Calculate deflection at service loads.

The Flexural Members

As we studied in the Strength of Materials when a flexural member is subjected to Bending Moment it will deform or deflect in a curvature form. Some of the fibers of the material (at the top or Bottom) will be exerted compression stresses, and the other fibers (at the other side) will be exerted tension stresses. The plane or the line between these two zones is called “The Neutral Axis” which will not be deformed at all and will not have tension neither compression stresses.



The Flexural Formulae

$$\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$$

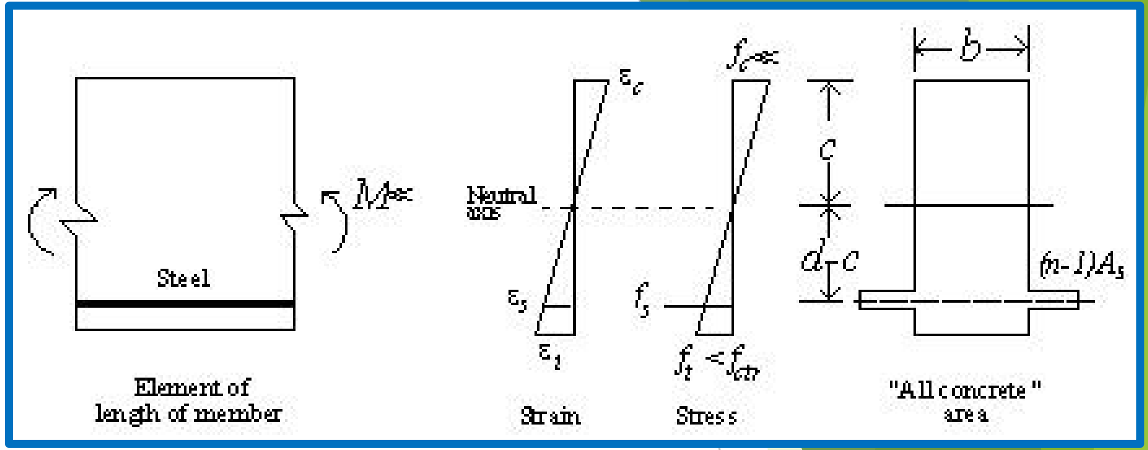
$$\frac{M}{I} = \frac{\sigma}{y}$$

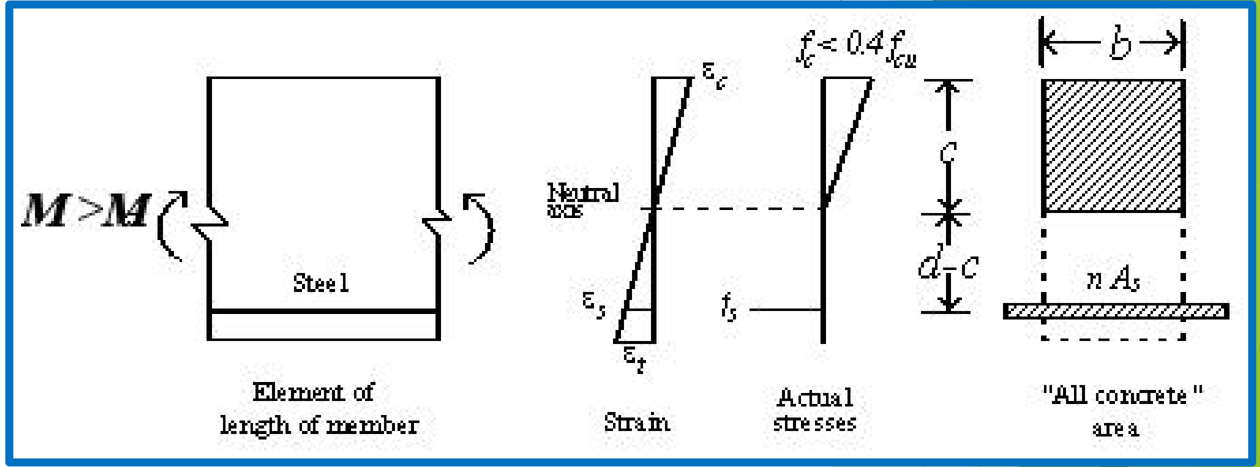
$$\sigma = \frac{M}{I}\, y$$

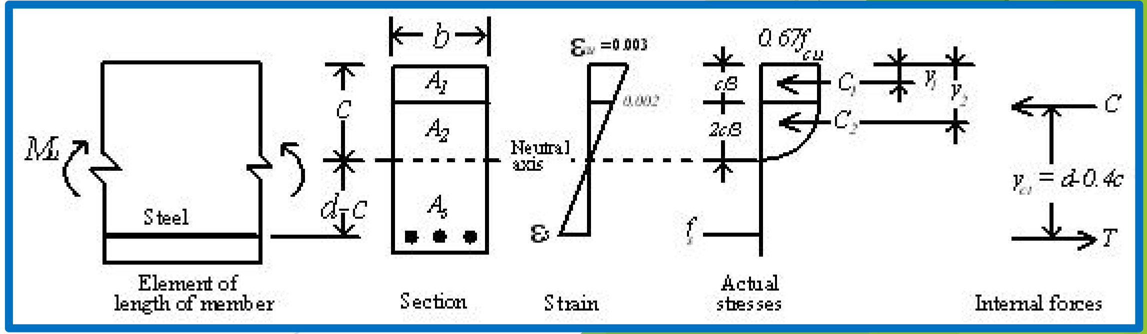
$$\frac{I}{y_{max}} = Z$$

$$\sigma_{max} = \frac{M}{Z}$$

$$n = \frac{E_s}{E_c}$$







The flexural formulae is working directly for the homogenous sections (Steel, Wood Members), However for Composite sections like (Reinforced Concrete), the section need to be transformed to an (Equivalent Section)

The flexural formulae needs also the material to be in the range of (Linear Stress-Strain Relationship) and does not work for the range of nonlinearity.

$f_c \leq 0.45\, f_c'$ $\quad$ $(Allowable\ concrete\ compression\ stress)$

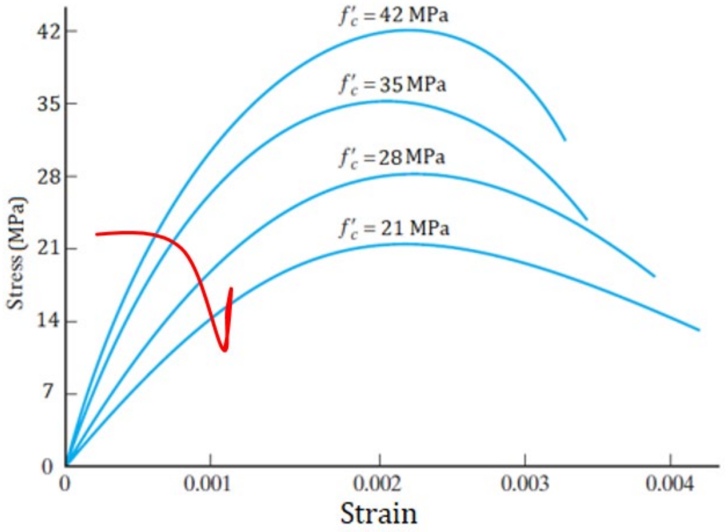
$f_s \leq 0.5\, f_y$ $\quad$ (Allowable steel stress)

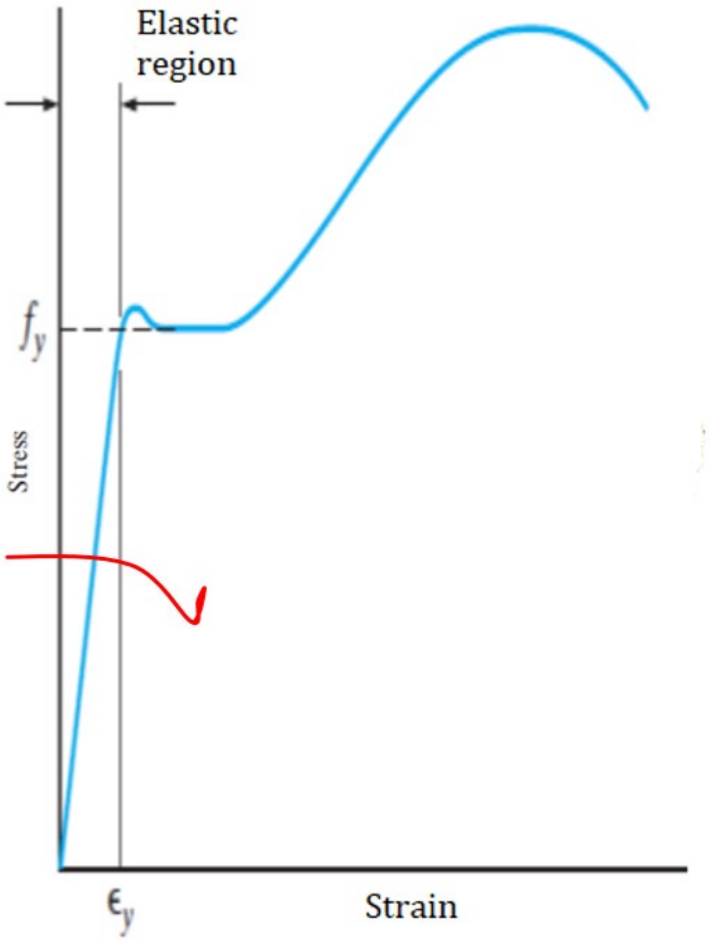
$E_s = 200{,}000\ MPa$

$E_c = 4730\sqrt{f_c'}\ MPa$ $\quad$ (Code)

$f_{cr} = 0.625\sqrt{f_c'}\ MPa$ $\quad$ (Code)

$Modulus\ of\ Rupture$

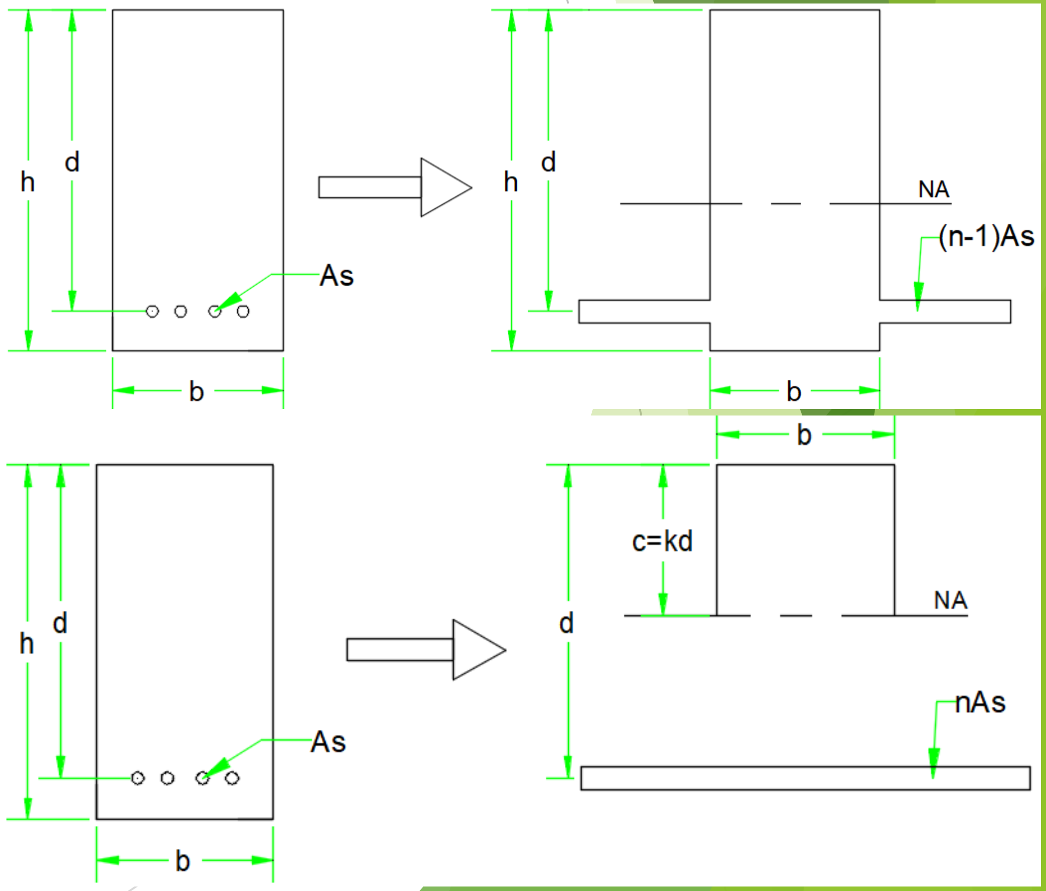




Stress-Strain Curves for Concrete and Steel

General Solution Steps:

- ❖ Transform the original section (two materials) into an equivalent section (one material)
- ❖ Calculate the location of the Neutral Axis (N.A)
- ❖ Calculate second moment of area (the moment of Inertia) of the equivalent section (about N.A)
- ❖ Apply the Flexural Formulae to calculate the stresses everywhere in the section
- ❖ Check the resulting stresses with the Code limitations



Useful Equations for Cracked Sections

$$\rho = \frac{As}{bd}$$

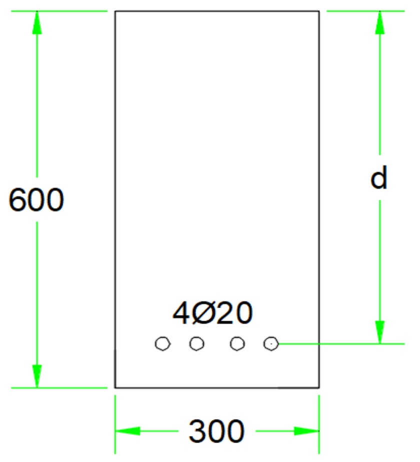
$$k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n$$

$$c = k\,d$$

$$n = \frac{E_s}{E_c}$$

Example 1: Determine the crack moment for the section shown below, and the stress state.

Es=200000 MPa	f'c=28 MPa	fy=414 MPa	As=4ϕ20mm
b=300 mm	h=600 mm	Cover=50 mm	



$$A\emptyset 20 = \frac{\pi\, D^2}{4} = \frac{\pi}{4}(20)^2 = 314\, mm^2$$

$$A_s = 4 \times 314 = 1256\, mm^2$$

$$E_c = 4730\sqrt{f_c'} = 4730\sqrt{28} = 25028.8\, MPa$$

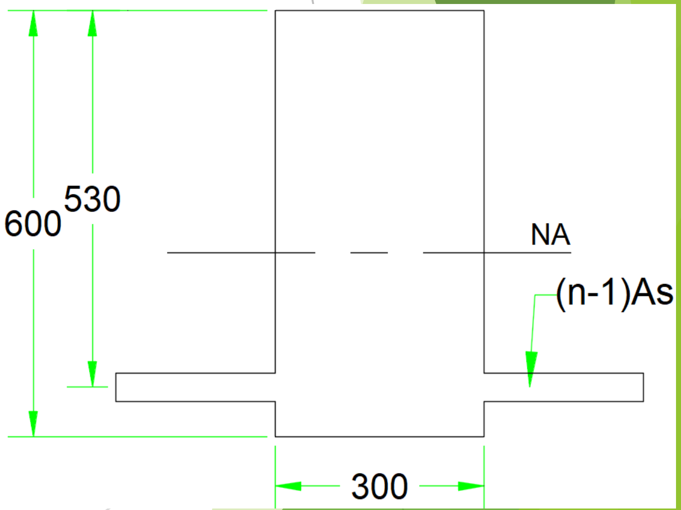
$$f_{c(allowable)} = 0.45 f_c' = 0.45 \times 28 = 12.6\, MPa$$

$$f_{s(allowable)} = 0.5 f_y = 0.5 \times 414 = 207\, MPa$$

$$n = \frac{E_s}{E_c} = \frac{200000}{25028.8} = 7.99 \cong 8$$

$$d = h - Cover - 10 - \frac{\emptyset}{2}$$

$$d = 600 - 50 - 10 - \frac{20}{2} = 530\, mm$$



$$\bar{y} = \frac{b\, h\, \frac{h}{2} + (n-1) A_s\, d}{b\, h + (n-1) A_s}$$ (from top fiber)

$$\bar{y} = \frac{\frac{300 \times 600 \times 600}{2} + (8-1) \times 1256 \times 530}{300 \times 6000 + (8-1) \times 1256} = 310.7\, mm$$

$$I_{gr} = \frac{b\,h^3}{12} + bh\left(\bar{y} - \frac{h}{2}\right)^2 + (n-1)A_s\,(d-\bar{y})^2$$

$$I_{gr} = \frac{300 \times 600^3}{12} + 300 \times 600\left(310.7 - \frac{600}{2}\right)^2 + (8-1) \times 1256 \times (530 - 310.7)^2$$

$$I_{gr} = 5.843 \times 10^9\ mm^4$$

$$y_{top} = \bar{y} = 310.7\ mm$$

$$y_{bot} = h - \bar{y} = 600 - 310.7 = 289.3\ mm$$

$$y_{steel} = y_{bot} - cover - 10 - \frac{\emptyset}{2} = 289.3 - 70 = 219.3\ mm$$

$$f_{cr} = 0.625\sqrt{f_c'} = 0.625 \times \sqrt{28} = 3.31\ MPa$$

$$f_b = f_{cr} = \frac{M_{cr}\,y_b}{I_{gr}}$$

$$3.31 = \frac{M_{cr} \times 289.3}{5.843 \times 10^9}$$

$$M_{cr} = 66.852 \times 10^6\ N.mm$$

$$M_{cr} = 66.852\ kN.m$$

$$f_c = \frac{M_{cr}\, y_t}{I_{gr}}$$

$$f_c = \frac{66.852 \times 10^6 \times\ 310.7}{5.843 \times 10^9} = 3.55 \ll 12.6\ MPa$$

$$f_s = nf_c =\ n\frac{M\ (h - \bar{y} - cover - 10 - \emptyset/2)}{I_{gr}}$$

$$f_s = 8 \times \frac{66.852 \times 10^6 \times 219.3}{5.843 \times 10^9} = 20.07 \ll 207\ MPa$$

Example 2: Determine the stress state for the section in the previous example if it is subjected to service moment M=100 kN.m

From the previous example: Mcr=66.85 kN.m

$$M = 100\ kN.m > 66.85\ (the\ section\ will\ crack)$$

$$\rho = \frac{As}{bd} = \frac{1256}{300 \times 530} = 0.0079$$

$$\rho n = 0.0079 \times 8 = 0.0632$$

$$k = \sqrt{(\rho n)^2 + 2\rho n} - \rho n$$

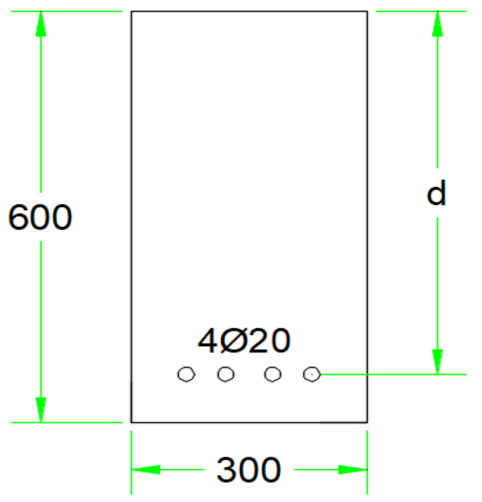
$$k = \sqrt{(0.0632)^2 + 2 \times 0.0632} - 0.0632 = 0.297$$

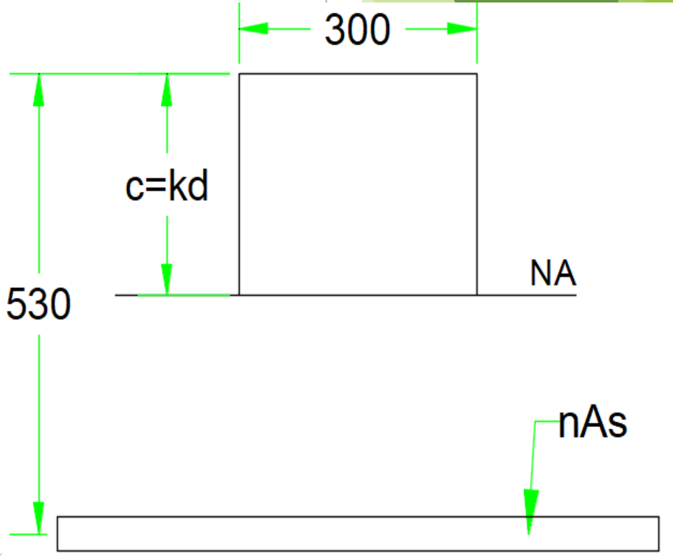
$$c = k\ d = 0.2975 \times 530 = 157.68$$

$$I_{cr} = \frac{b\ c^3}{3} + n\ A_s\ (d - c)^2$$

$$I_{cr} = \frac{300 \times (157.68)^3}{3} + 8 \times\ 1256 \times\ (530 - 157.68)^2$$

$$I_{cr} = \ 392039506 + 1392875689 = 1784915195 = 1.785 \times 10^9$$





$$f_{c(top)} = \frac{M\,y_t}{I_{cr}} = \frac{M\,c}{I_{cr}}$$

$$f_c = \frac{100 \times 10^6 \times \ 157.68}{1.785 \times 10^9} = 8.833 < 12.6\ MPa$$

$$f_s = \ nf_c = n\frac{M\,(d-c)}{I_{cr}}$$

$$f_s = \ 8 \times \frac{100 \times 10^6 \times (530 - 157.68)}{1.785 \times 10^9} = 166.866\ MPa < 207$$

Thank you…