The concepts of probability

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LEARNING OBJECTIVES

At the end of the lecture the student should:

1. Be able to understand the basic concepts of probability.
2. Know the characteristics of the normal distribution.
4. Know the Standard Normal Distribution curve (z distribution) and understand what is meant by a z score.
5. be able to perform normal calculations, finding areas under the curve

Definitions

Probability is a numerical measure of the likelihood that an event will occur.

An experiment is any process that generates well-defined outcomes.

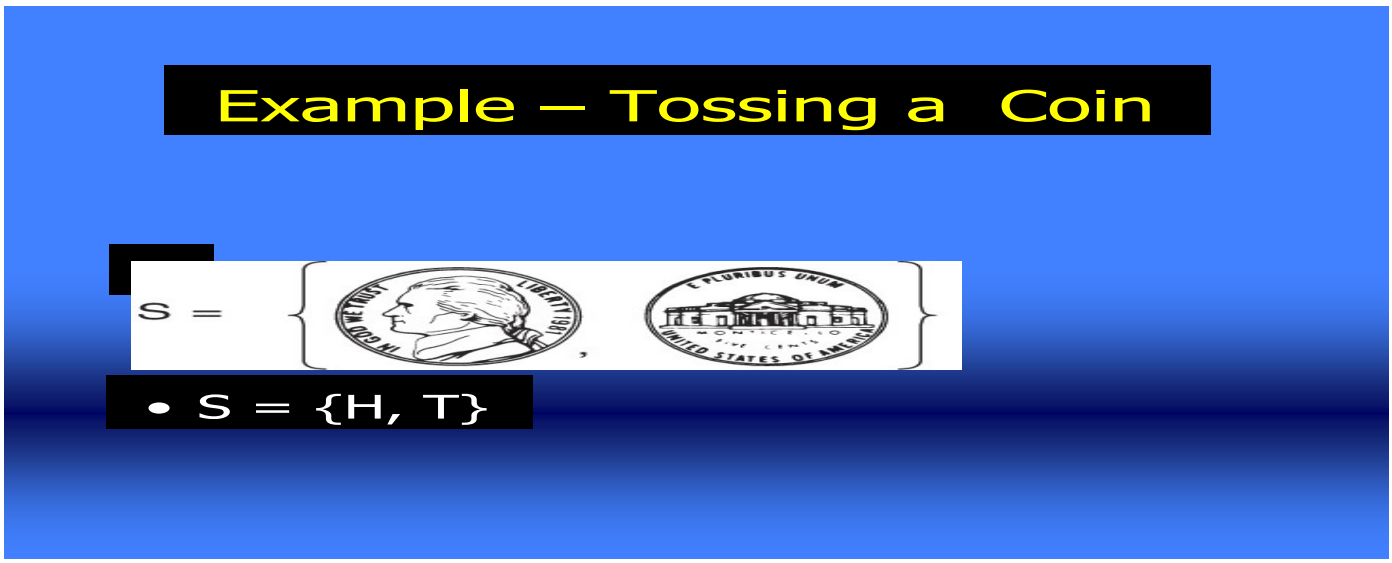
Sample space, *S*, is the set of all possible outcomes of an experiment.

An *event*, *A*, is an outcome or set of outcomes that are of interest to the

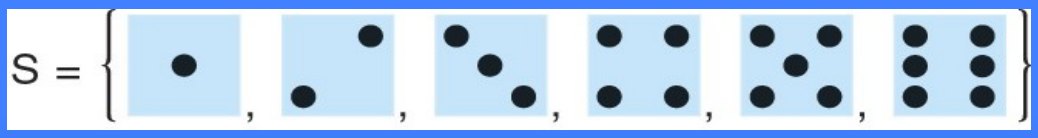
. Experimenter (An event A is a subset of the sample space.

The **probability of an event A, P (A),** is a measure of the likelihood that

. an event *A* will occur.



Example – Tossing a Dice



- S = {1, 2, 3, 4, 5, 6}
- A = {roll an even number}
- A = {2, 4, 6}

Calculation of Probability:-

- In classical probability each outcome is equally likely.
- In these cases, if there are *N* outcomes in S then the probability of any one outcome is 1/*N*.
- If A is any event and n_A is the number of outcomes in A, then

$$P(A) = \frac{n_A}{N}$$

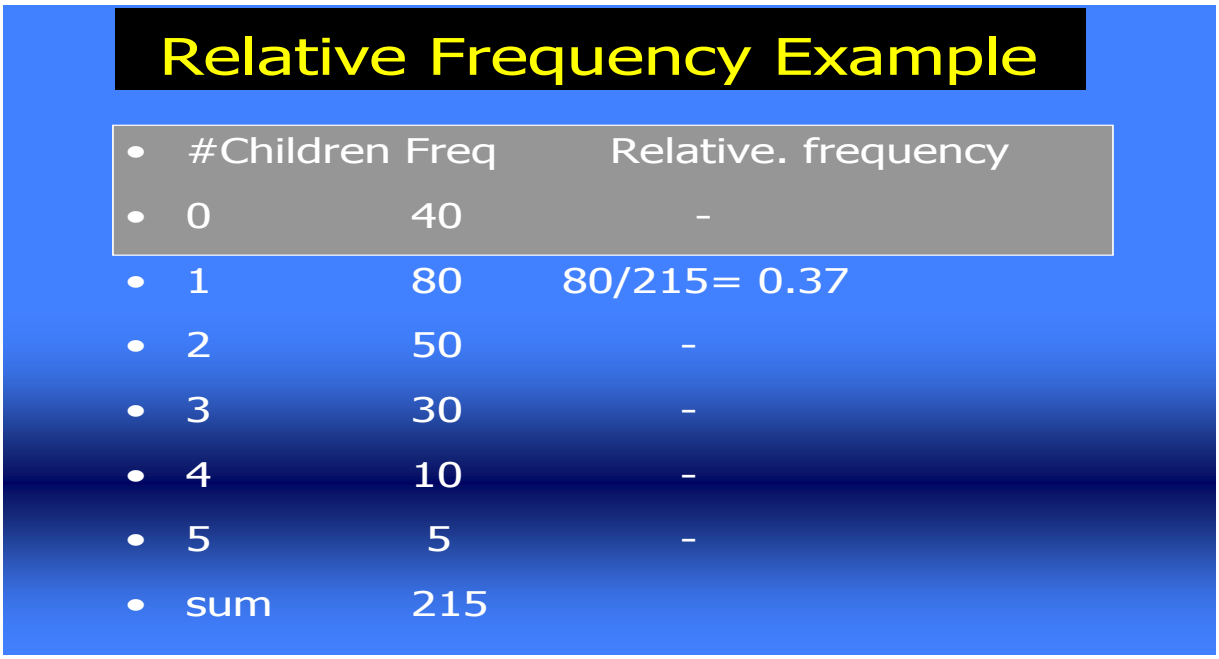
Example – Tossing a Dice

- S = {1, 2, 3, 4, 5, 6}.
- The 6 outcomes are equally likely, i.e., P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = 1/6.
- A = {roll an even number} = {2, 4, 6}.

$$P(A) = \frac{3}{6} = 0.5$$

Empirical probability

- It is simply the relative frequency that some event is observed to happen (or fail to happen).
- Number of times an event occurred divided by the number of trials.



Basic Concepts of Probability:-

- Probability values are always assigned on a scale from 0 to 1.
- A probability near 0 indicates an event is very unlikely to occur.
- A probability near 1 indicates an event is almost certain to occur.
- A probability of 0.5 indicates the occurrence of the event is just as likely as it is unlikely.
- The sum of the probabilities of all outcomes is 1.

Probability Distribution

Probability Distribution is defined as the distribution of all possible values (outcomes) that a random variable can take within a given range.

- **Examples of probability distributions are:**

-The normal distribution
-The binomial distribution
-other distributions such as the t, F, chi-square, etc. that are used to make statistical inferences.

The normal distribution:

The characteristics of the Normal distribution curve

1. The normal curve is bell-shaped and has a single peak at the exact center of the distribution.

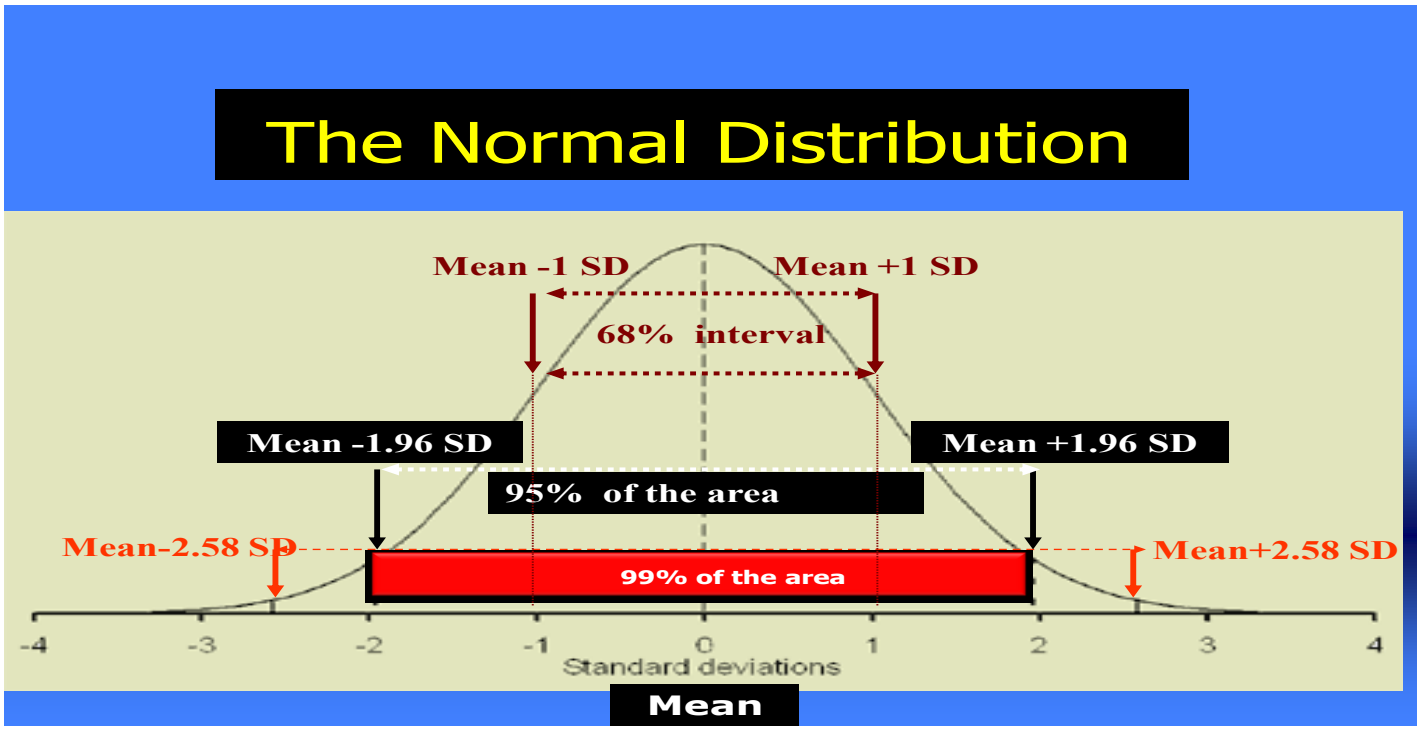
2. The arithmetic mean, median, and mode of the distribution are equal and located in the middle.

3. The normal distribution curve is symmetrical about its mean (half the observations are above the mean, & half below the mean.

4. The random variable has an infinite theoretical range (Tails don't touch X axis).

5. The total area under the curve is equal to 1

6. It is determined by two quantities the mean & the standard deviation.
7. 68% of the area under the curve is between the mean ±1 SD.
8. 95% of the area under the curve is between mean±1.96 SD.
9. 99% of the area under the curve is between mean±2.58 SD.

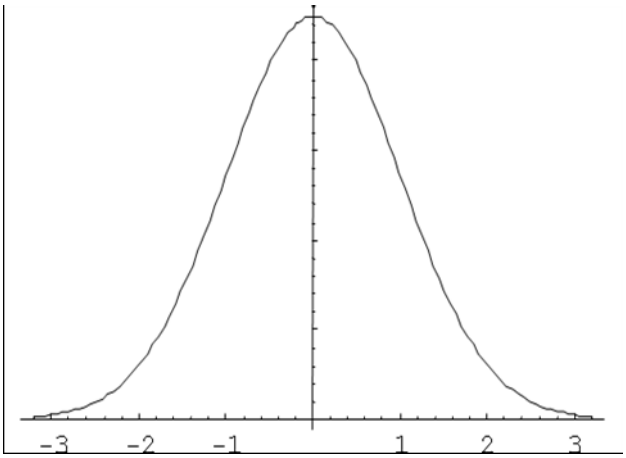


Why the Normal Distribution is Important

- Many types of data that are of interest have a normal distributions (height , weight)
- Means of any distribution has a Normal distribution (Central Limit Theorem). Sampling distribution of means becomes normal as N increases, regardless of shape of original distribution
- Binomial becomes normal as N increases.

Standard Normal Distribution curve (z distribution):

A normal distribution with a mean of 0 and a standard deviation of 1 is called **the standard normal distribution.**



- Any normal distribution can be converted to the standard normal distribution using the ***Z statistic (by calculating what is called a Z score)***
- .
- **Z value (score):** The distance between a selected value, designated X, and the mean, divided by the standard deviation.

$$z = \frac{x_i - \bar{x}}{sd}$$

Z-Scores

- **A z score** is often called **the standardized value or Standard Normal Deviate (SND) .**It represents the

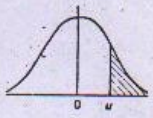
number of standard deviation a data value x is away from the mean and in which direction.

- A data value less than the sample mean will have a z-score less than zero (has a minus sign).
- A data value greater than the sample mean will have a z-score greater than zero (has a positive sign).
- A data value equal to the sample mean will have a z-score of zero.
- By calculating the Z score, a "raw" score (x) is changed to a standardized score (Z).
- The Z-score can be used to determine an area under the curve (probability) by using the standard distribution table

Table 4 Areas of the Standardised Normal Distribution

The function tabulated is $\frac{1}{\sqrt{2\pi}} \int_u^\infty e^{-x^2/2}\, dx$;

the probability that $U > u$, where $U \sim N(0,1)$.



u	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.50000	0.49601	0.49202	0.48803	0.48405	0.48006	0.47608	0.47210	0.46812	0.46414
0.1	0.46017	0.45620	0.45224	0.44828	0.44433	0.44038	0.43644	0.43250	0.42858	0.42465
0.2	0.42074	0.41683	0.41294	0.40905	0.40517	0.40129	0.39743	0.39358	0.38974	0.38591
0.3	0.38209	0.37828	0.37448	0.37070	0.36693	0.36317	0.35942	0.35569	0.35197	0.34827
0.4	0.34458	0.34090	0.33724	0.33360	0.32997	0.32636	0.32276	0.31918	0.31561	0.31207
0.5	0.30854	0.30503	0.30153	0.29806	0.29460	0.29116	0.28774	0.28434	0.28096	0.27760
0.6	0.27425	0.27093	0.26763	0.26435	0.26109	0.25785	0.25463	0.25143	0.24825	0.24510
0.7	0.24196	0.23885	0.23576	0.23269	0.22965	0.22663	0.22363	0.22065	0.21770	0.21476
0.8	0.21186	0.20897	0.20611	0.20327	0.20045	0.19766	0.19489	0.19215	0.18943	0.18673
0.9	0.18406	0.18141	0.17879	0.17619	0.17361	0.17106	0.16853	0.16602	0.16354	0.16109
1.0	0.15866	0.15625	0.15386	0.15150	0.14917	0.14686	0.14457	0.14231	0.14007	0.13786
1.1	0.13567	0.13350	0.13136	0.12924	0.12714	0.12507	0.12302	0.12100	0.11900	0.11702
1.2	0.11507	0.11314	0.11123	0.10935	0.10749	0.10565	0.10383	0.10204	0.10027	0.09853
1.3	0.09680	0.09510	0.09342	0.09176	0.09012	0.08851	0.08692	0.08534	0.08379	0.08226
1.4	0.08076	0.07927	0.07780	0.07636	0.07493	0.07353	0.07215	0.07078	0.06944	0.06811
1.5	0.06681	0.06552	0.06426	0.06301	0.06178	0.06057	0.05938	0.05821	0.05705	0.05592
1.6	0.05480	0.05370	0.05262	0.05155	0.05050	0.04947	0.04846	0.04746	0.04648	0.04551
1.7	0.04457	0.04363	0.04272	0.04182	0.04093	0.04006	0.03920	0.03836	0.03754	0.03673
1.8	0.03593	0.03515	0.03438	0.03362	0.03288	0.03216	0.03144	0.03074	0.03005	0.02938
1.9	0.02872	0.02807	0.02743	0.02680	0.02619	0.02559	0.02500	0.02442	0.02385	0.02330
2.0	0.02275	0.02222	0.02169	0.02118	0.02068	0.02018	0.01970	0.01923	0.01876	0.01831
2.1	0.01786	0.01743	0.01700	0.01659	0.01618	0.01578	0.01539	0.01500	0.01463	0.01426
2.2	0.01390	0.01355	0.01321	0.01287	0.01255	0.01222	0.01191	0.01160	0.01130	0.01101
2.3	0.01072	0.01044	0.01017	0.00990	0.00964	0.00939	0.00914	0.00889	0.00866	0.00842
2.4	0.00820	0.00798	0.00776	0.00755	0.00734	0.00714	0.00695	0.00676	0.00657	0.00639
2.5	0.00621	0.00604	0.00587	0.00570	0.00554	0.00539	0.00523	0.00508	0.00494	0.00480
2.6	0.00466	0.00453	0.00440	0.00427	0.00415	0.00402	0.00391	0.00379	0.00368	0.00357
2.7	0.00347	0.00336	0.00326	0.00317	0.00307	0.00298	0.00289	0.00280	0.00272	0.00264
2.8	0.00256	0.00248	0.00240	0.00233	0.00226	0.00219	0.00212	0.00205	0.00199	0.00193
2.9	0.00187	0.00181	0.00175	0.00169	0.00164	0.00159	0.00154	0.00149	0.00144	0.00139
3.0	0.00135	0.00131	0.00126	0.00122	0.00118	0.00114	0.00111	0.00107	0.00104	0.00100
3.1	0.00097	0.00094	0.00090	0.00087	0.00084	0.00082	0.00079	0.00076	0.00074	0.00071
3.2	0.00069	0.00066	0.00064	0.00062	0.00060	0.00058	0.00056	0.00054	0.00052	0.00050
3.3	0.00048	0.00047	0.00045	0.00043	0.00042	0.00040	0.00039	0.00038	0.00036	0.00035
3.4	0.00034	0.00032	0.00031	0.00030	0.00029	0.00028	0.00027	0.00026	0.00025	0.00024
3.5	0.00023	0.00022	0.00022	0.00021	0.00020	0.00019	0.00019	0.00018	0.00017	0.00017
3.6	0.00016	0.00015	0.00015	0.00014	0.00014	0.00013	0.00013	0.00012	0.00012	0.00011
3.7	0.00011	0.00010	0.00010	0.00010	0.00009	0.00009	0.00008	0.00008	0.00008	0.00008
3.8	0.00007	0.00007	0.00007	0.00006	0.00006	0.00006	0.00006	0.00005	0.00005	0.00005
3.9	0.00005	0.00005	0.00004	0.00004	0.00004	0.00004	0.00004	0.00004	0.00003	0.00003

Table 1 AREAS OF THE STANDARDISED NORMAL DISTRIBUTION

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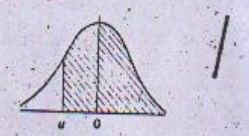
the probability that $U > u$, where $U \sim N(0,1)$.



−0.09	−0.08	−0.07	−0.06	−0.05	−0.04	−0.03	−0.02	−0.01	−0.00	u
0.99997	0.99997	0.99996	0.99996	0.99996	0.99996	0.99996	0.99996	0.99995	0.99995	−3.9
0.99995	0.99995	0.99995	0.99994	0.99994	0.99994	0.99994	0.99993	0.99993	0.99993	−3.8
0.99992	0.99992	0.99992	0.99992	0.99991	0.99991	0.99990	0.99990	0.99990	0.99989	−3.7
0.99989	0.99988	0.99988	0.99987	0.99987	0.99986	0.99986	0.99985	0.99985	0.99984	−3.6
0.99983	0.99983	0.99982	0.99981	0.99981	0.99980	0.99979	0.99978	0.99978	0.99977	−3.5
0.99976	0.99975	0.99974	0.99973	0.99972	0.99971	0.99970	0.99969	0.99968	0.99966	−3.4
0.99965	0.99964	0.99962	0.99961	0.99960	0.99958	0.99957	0.99955	0.99953	0.99952	−3.3
0.99950	0.99948	0.99946	0.99944	0.99942	0.99940	0.99938	0.99936	0.99934	0.99931	−3.2
0.99929	0.99926	0.99924	0.99921	0.99918	0.99916	0.99913	0.99910	0.99906	0.99903	−3.1
0.99900	0.99896	0.99893	0.99889	0.99886	0.99882	0.99878	0.99874	0.99869	0.99865	−3.0
0.99861	0.99856	0.99851	0.99846	0.99841	0.99836	0.99831	0.99825	0.99819	0.99813	−2.9
0.99807	0.99801	0.99795	0.99788	0.99781	0.99774	0.99767	0.99760	0.99752	0.99744	−2.8
0.99736	0.99728	0.99720	0.99711	0.99702	0.99693	0.99683	0.99674	0.99664	0.99653	−2.7
0.99643	0.99632	0.99621	0.99609	0.99598	0.99585	0.99573	0.99560	0.99547	0.99534	−2.6
0.99520	0.99506	0.99492	0.99477	0.99461	0.99446	0.99430	0.99413	0.99396	0.99379	−2.5
0.99361	0.99343	0.99324	0.99305	0.99286	0.99266	0.99245	0.99224	0.99202	0.99180	−2.4
0.99158	0.99134	0.99111	0.99086	0.99061	0.99036	0.99010	0.98983	0.98956	0.98928	−2.3
0.98899	0.98870	0.98840	0.98809	0.98778	0.98745	0.98713	0.98679	0.98645	0.98610	−2.2
0.98574	0.98537	0.98500	0.98461	0.98422	0.98382	0.98341	0.98300	0.98257	0.98214	−2.1
0.98169	0.98124	0.98077	0.98030	0.97982	0.97932	0.97882	0.97831	0.97778	0.97725	−2.0
0.97670	0.97615	0.97558	0.97500	0.97441	0.97381	0.97320	0.97257	0.97193	0.97128	−1.9
0.97062	0.96995	0.96926	0.96856	0.96784	0.96712	0.96638	0.96562	0.96485	0.96407	−1.8
0.96327	0.96246	0.96164	0.96080	0.95994	0.95907	0.95818	0.95728	0.95637	0.95543	−1.7
0.95449	0.95352	0.95254	0.95154	0.95053	0.94950	0.94845	0.94738	0.94630	0.94520	−1.6
0.94408	0.94295	0.94179	0.94062	0.93943	0.93822	0.93699	0.93574	0.93448	0.93319	−1.5
0.93189	0.93056	0.92922	0.92785	0.92647	0.92507	0.92364	0.92220	0.92073	0.91924	−1.4
0.91774	0.91621	0.91466	0.91308	0.91149	0.90988	0.90824	0.90658	0.90490	0.90320	−1.3
0.90147	0.89973	0.89796	0.89617	0.89435	0.89251	0.89065	0.88877	0.88686	0.88493	−1.2
0.88298	0.88100	0.87900	0.87698	0.87493	0.87286	0.87076	0.86864	0.86650	0.86433	−1.1
0.86214	0.85993	0.85769	0.85543	0.85314	0.85083	0.84850	0.84614	0.84375	0.84134	−1.0
0.83891	0.83646	0.83398	0.83147	0.82894	0.82639	0.82381	0.82121	0.81859	0.81594	−0.9
0.81327	0.81057	0.80785	0.80511	0.80234	0.79955	0.79673	0.79389	0.79103	0.78814	−0.8
0.78524	0.78230	0.77935	0.77637	0.77337	0.77035	0.76731	0.76424	0.76115	0.75804	−0.7
0.75490	0.75175	0.74857	0.74537	0.74215	0.73891	0.73565	0.73237	0.72907	0.72575	−0.6
0.72240	0.71904	0.71566	0.71226	0.70884	0.70540	0.70194	0.69847	0.69497	0.69146	−0.5
0.68793	0.68439	0.68082	0.67724	0.67364	0.67003	0.66640	0.66276	0.65910	0.65542	−0.4
0.65173	0.64803	0.64431	0.64058	0.63683	0.63307	0.62930	0.62552	0.62172	0.61791	−0.3
0.61409	0.61026	0.60642	0.60257	0.59871	0.59483	0.59095	0.58706	0.58317	0.57926	−0.2
0.57535	0.57142	0.56750	0.56356	0.55962	0.55567	0.55172	0.54776	0.54380	0.53983	−0.1
0.53586	0.53188	0.52790	0.52392	0.51994	0.51595	0.51197	0.50798	0.50399	0.50000	−0.0

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Eample Z Score

- Calculate the Z score for blood pressure of 140 if the sample mean is 110 and the standard deviation is 10
- Z = 140 – 110 / 10 = 3

Using the Normal Curve: Z Scores

Procedure:

■First calculate the Z score (Convert raw score to Z score), taking careful note of the sign of the score.

■Draw a normal curve.

-Indicate where Z score falls.

- Shade area in which you are interested (you are trying to find).

■Use the standard distribution table.

1-Determine the percentage above or below a Z score

Example:

The mean of examination scores for statistics class is 60, with a standard deviation of 10.

Suppose a student's exam score is 70. What percentage of students has a score below his score? Above?

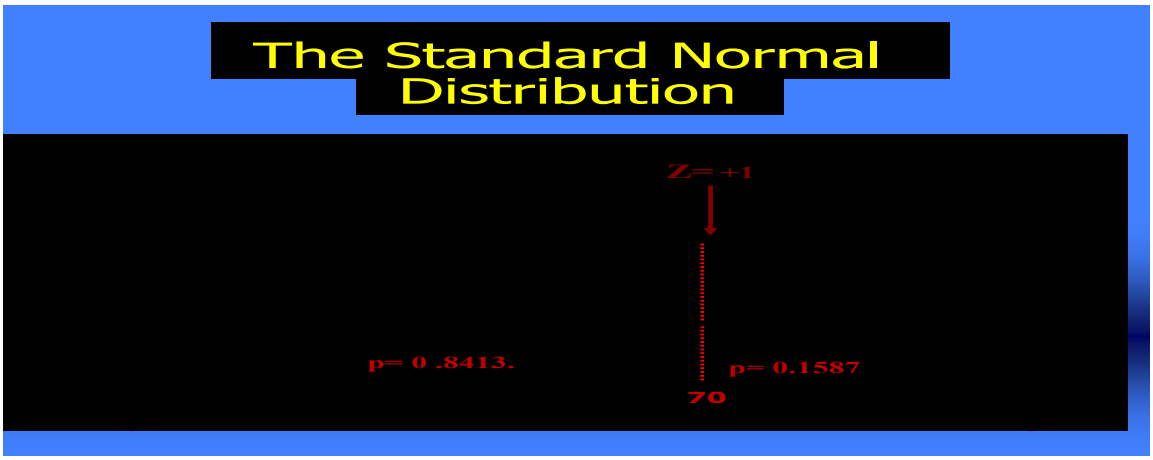
Solution:

- $Z = (X_i - \bar{X}) / Sd$

$= (70 - 60) / 10$

$= +1.0$

- The Z-score of +1.0 is exactly 1 sd above the mean.
- From the table the probability above z +1.0 = 0.1587
- (% of marks above 70= 15.87%)
- Therefore the probability below z (+1.0) = 1-0.1587 =0.8413. (% of marks below 70= 84.13%).

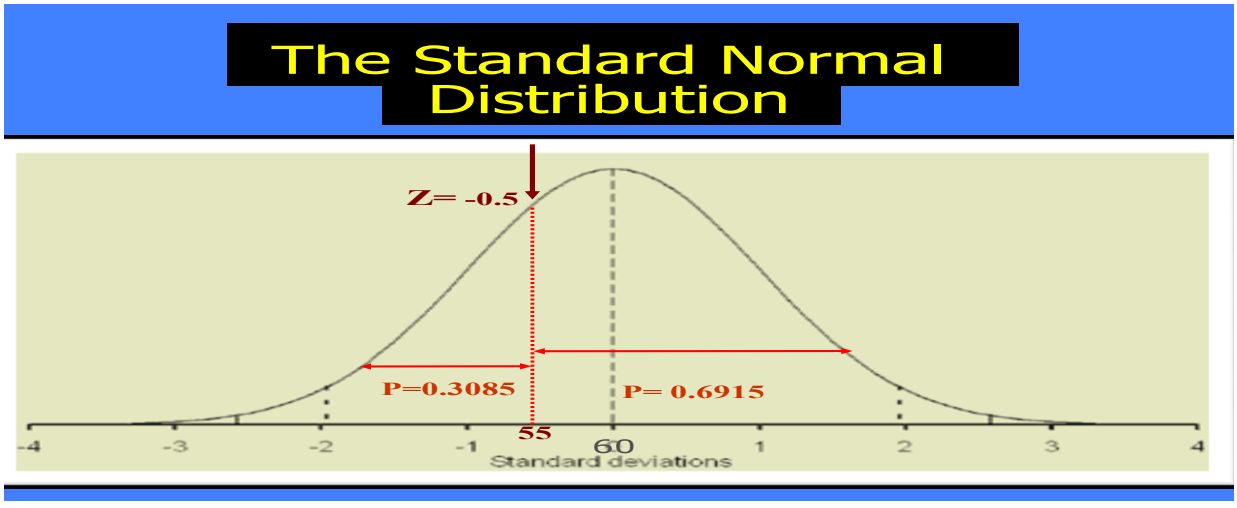


2. Suppose the student's exam score is 55. What percentage of students has a score below his score? Above?

Solution:

Z = (55 – 60) /10 = -0.5

- Area above Z=-0.5 = 0.6915
  (% of marks above 55= 69.15%).
- The area below Z = 1-0.6915=0.3085
  (% of marks below 55=30.85%).



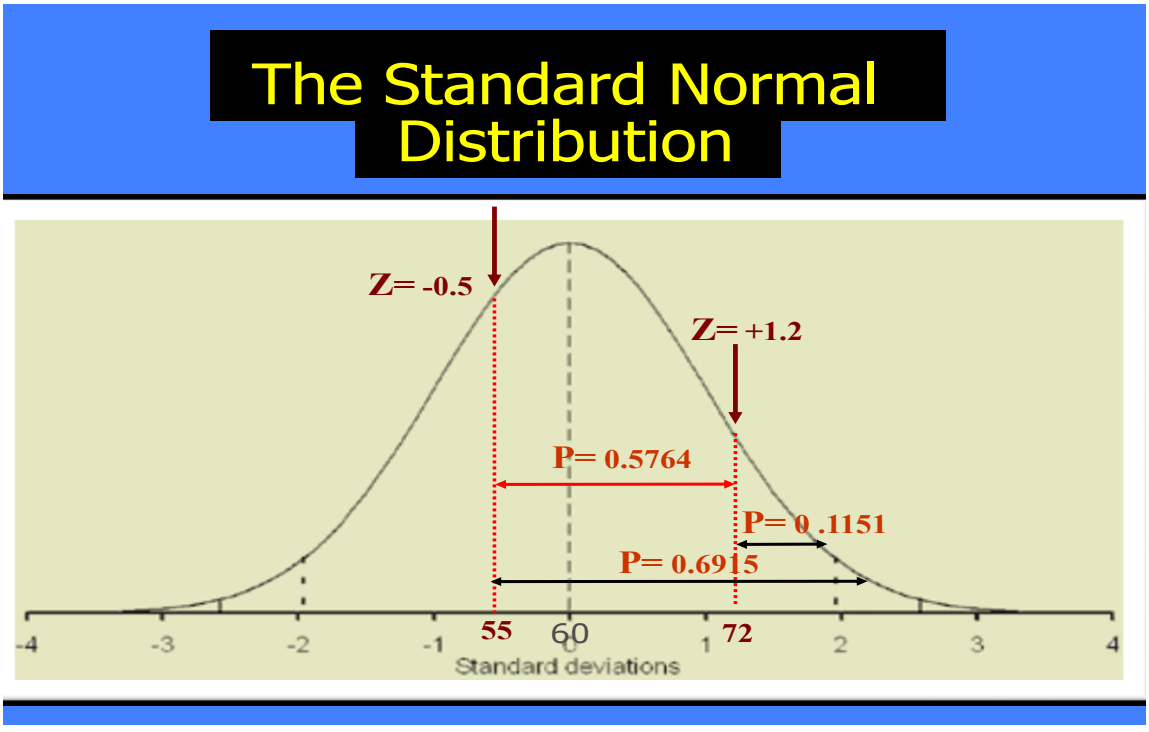
2-Determine the percentage between 2 Z scores

- What is the probability of someone having a mark between 72 and 55?

Solution:

- Z for 72 = (72 – 60) /10 = +1.2
- The area above Z (1.2) =0.1151
- Z for 55% = (55 – 60) /10 = -0.5

  The area above Z(-0.5)= 0.6915
- The area between Z = 1.2 and Z = -0.5 would be 0.6915 -0 .1151 = 0.5764
- The probability of having a mark between 72 and 55 for this distribution is p =0 .5764 or 57.64%



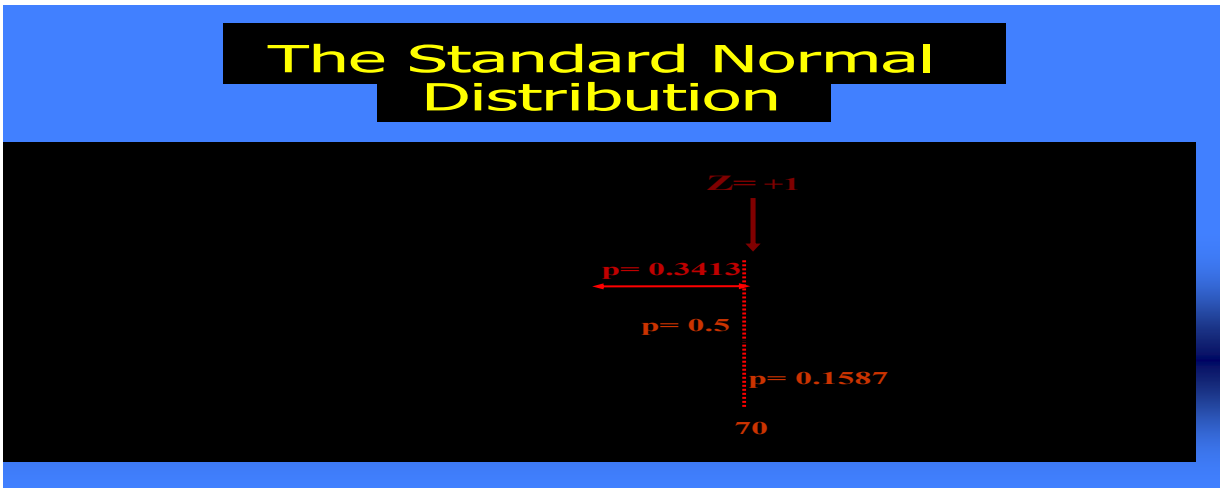
3-Determine the percentage of scores between the mean and particular z score

●What is the probability of having a mark between 60% and 70%?

The area above Z score 1=0.1587

0.5-0.1587=0.3413

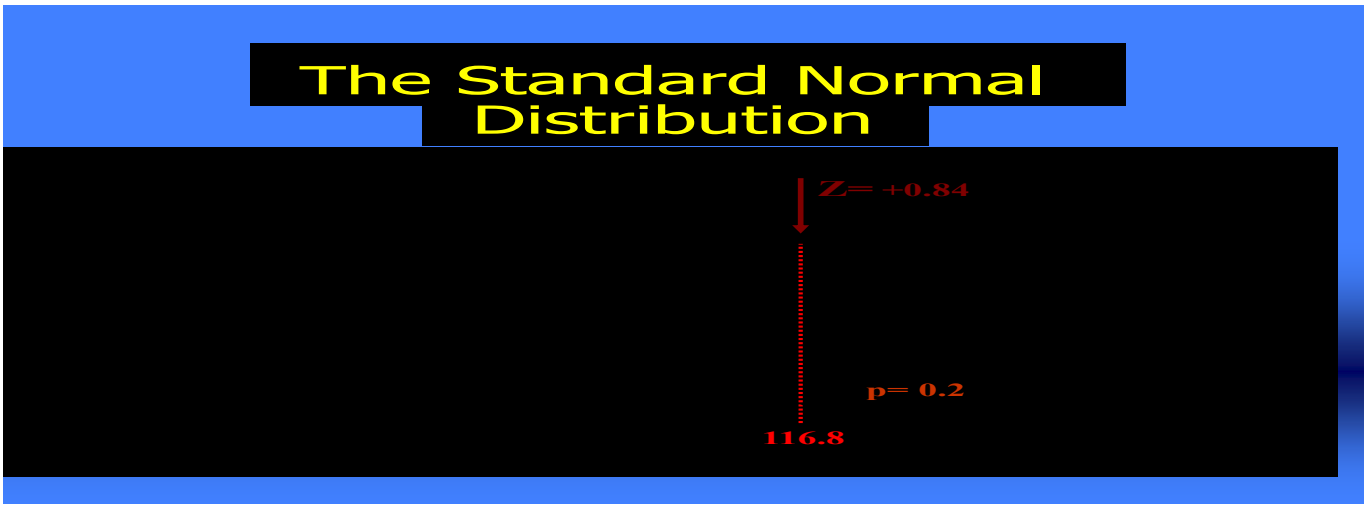
●There is a 0.3413 probability (or 34.13% chance) of having a mark between 60 and 70.



4- Steps for determining a Z score or raw score from percentage (proportion)

●If, instead, we are given a proportion and asked to find the original value of x corresponding to it?

●Draw normal curve, shade approximation area for percentage desired.



- We have to use the Table backwards.
- What proportion are we interested in?
- Look inside the body of the Table for the value closest to this "proportion. That will give us a *z* score. Then the original value of *x* is simply.

$$x = \bar{x} + zsd.$$

Example

- For a normal distribution with mean of 100 and standard deviation of 20, find the score that cuts the upper 20%.
- From the table Z=0.84
- 0.84=x-100/20

X=20*0.84+100=116.8