University of Basrah - College of Engineering

Department of Mechanical Engineering

Subject: *Numerical Analysis* **Stage:** *Third*

Lecture: *Dr. Emad A. Khazal*

Syllabus of Numerical Analysis

1. Roots of Equations
2. Solution of Linear Simultaneous Equations
3. Finite Difference, Interpolation and Extrapolation
4. Numerical Integration and Differentiation
5. Curve Fitting
6. Numerical Solution of Ordinary Differential Equations
7. Numerical Solution of Ordinary Partial Equations
8. Introduction to Finite Element Method.

1. Roots of Equations

Years ago, you learned to use the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \tag{1}$$

To solve

$$f(x) = ax^2 + bx + c = 0 \tag{2}$$

The values calculated with eq.(2) are called the "**roots**" of eq.(1). They represent the vakues of x that make eq.(2) equal to zero.

Root of equation: The value of x that makes f(x)=0. The roots are sometimes are called ***zeros*** of equation.

There are many other functions for which the root cannot be determined so easily such as $f(x) = e^{-x} - x$. For these cases the numerical methods efficient means to obtain answer.

Before the advent of digital computers, there were several ways to solve for roots of ***algebraic*** and ***transcendental*** equations. Such as ***Graphical technique method*** is to plot the function and determine where it crosses the x-axis. This method is useful for obtaining rough estimated of roots, this is limited because of their lack precision.

The other method is ***Trial and error***. This method consists of guessing a value of x and evaluating whether f(x) is zero. If not, another guess is made, and f(x) is again evaluated to determine whether the new value provides a better estimate of the root.

Note: **transcendental** function is one that is nonalgebraic. These include trigonometric, exponential, logarithmic, and other, less familiar, functions. Ex.

$$f(x) = \ln x^2 - 1, \quad or \quad f(x) = e^x + \sin(3x)$$

1.1. Iteration Method (Fixed point method) (open method)

We put x=g(x), assume initial value for x_o then calculating next value as follows

$$x_{n+1} = g(x_n)$$

and continue in this iteration. The absolute relative approximate error as

$$|\in_a| = \left|\frac{x_{new} - x_{old}}{x_{new}}\right| \times 100 = \left|\frac{x_{i+1} - x_i}{x_{i+1}}\right| \times 100$$

Ex1. Solve the equation $f(x) = x^2 - 3x + 1 = 0$ using iteration method.

Since we know the solution $x = 1.5 \pm \sqrt{1.25}$ thus 2.618 and 0.3819

Sol. The equation $f(x) = x^2 - 3x + 1 = 0$ may be written

$$x = g_1(x) = \frac{1}{3}\left(x^2 + 1\right), \qquad \Rightarrow \quad x_{n+1} = \frac{1}{3}\left(x_n^2 + 1\right)$$

If Choose $x_0 = 1$

N	x_n	$g_1(x_n)$	$\lvert\in_a\rvert$
0	1	0.667	50%
1	0.667	0.481	39%
2	0.481	0.411	17%
3	0.411	0.39	5.3%
4	0.39	0.38	2.6%
Convergence			

If Choose $x_0 = 3$

N	x_n	$g_1(x_n)$	$\lvert\in_a\rvert$
0	3	3.33	10%
1	3.33	4.037	17.5%
2	4.037	5.766	30%
3	5.766	11.415	50%
4	11.415	Divergence	

**** To Find other roots may be rewritten above equation (divided by x)**

$$f(x) = x^2 - 3x + 1 = 0 \quad \Rightarrow \quad x - 3 + \frac{1}{x} = 0$$

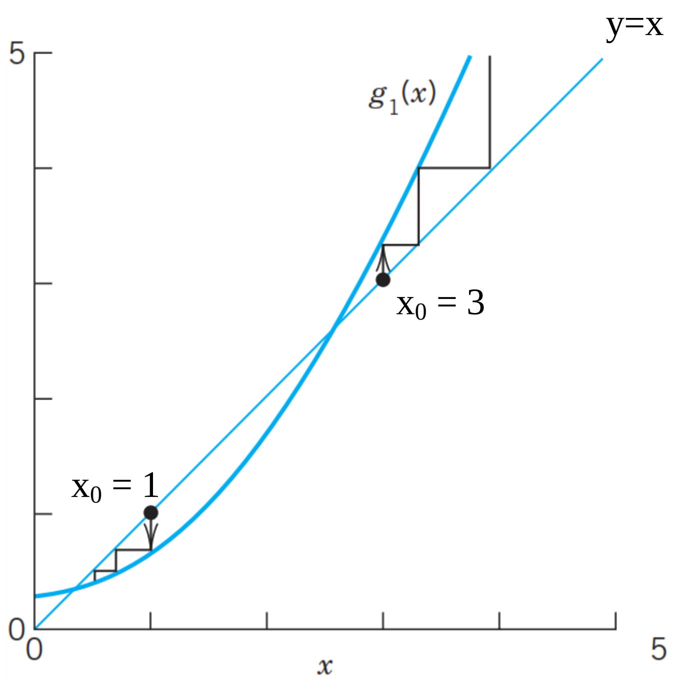
$$x = g_2(x) = 3 - \frac{1}{x}, \qquad \Rightarrow \quad x_{n+1} = 3 - \frac{1}{x_n}$$

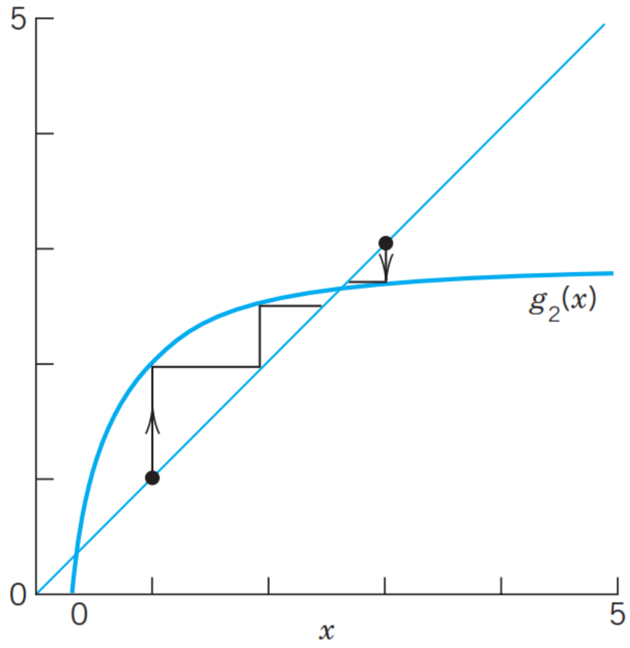
If Choose $x_0 = 1$

N	x_n	$g_1(x_n)$	$\lvert \epsilon_a \rvert$
0	1	2	50%
1	2	2.5	20%
2	2.5	2.6	4%
3	2.6	2.615	0.57%
4	2.615	2.617	0.07%
Convergence			

If Choose $x_0 = 3$

N	x_n	$g_1(x_n)$	$\lvert \epsilon_a \rvert$
0	3	2.667	12.5%
1	2.667	2.625	1.6%
2	2.625	2.619	0.22%
3	2.619	2.618	0.04%
4	2.618	2.618	0%
		Convergence	





*** Convergence of fixed point method.**

Then if $|g'(x)| < 1$ The iteration process converge for any x_0

For above example

$$g_1(x) = \frac{1}{3}(x^2 + 1) \quad \Rightarrow g'(x) = \frac{2}{3}x$$

$$\therefore |g'(x)| < 1 \quad \Rightarrow \quad \left|\frac{2}{3}x\right| < 1 \quad \rightarrow x < \frac{3}{2} \quad \textit{to converges}$$

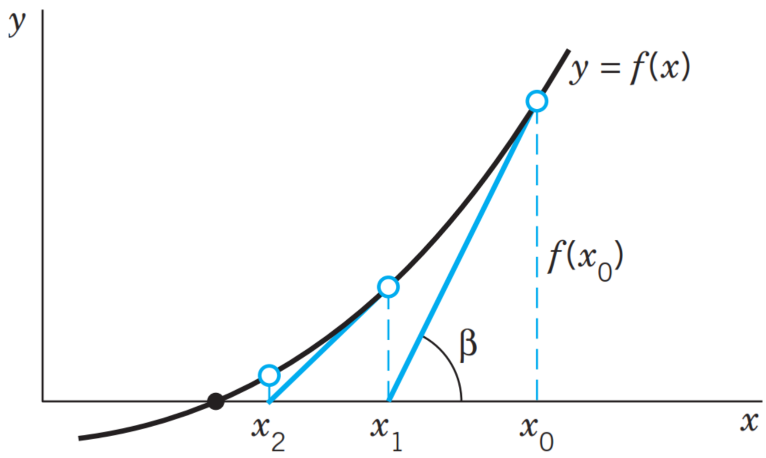
1.2. Newton - Raphson Method

The Newton method is another iteration method for solving equation f(x) = 0, where f(x) is assumed to have a continuous derivative f(x). The method commonly used because of its simplicity and great speed.

$$\tan\beta = f'(x) = \frac{f(x_0)}{x_0 - x_1}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$



Ex1. Solve the equation $f(x) = x^2 - 3x + 1 = 0$ using Newton Raphson method.

$$f(x) = x^2 - 3x + 1 \quad \Rightarrow \quad f'(x) = 2x - 3$$

$$\therefore x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{x_n^2 - 3x_n + 1}{2x_n - 3} = \frac{2x_n^2 - 3x_n - x_n^2 + 3x_n - 1}{2x_n - 3}$$

$$\therefore x_{n+1} = \frac{x_n^2 - 1}{2x_n - 3}$$

If Choose $x_0 = 1$

N	x_n	x_{n+1}	$\lvert\epsilon_a\rvert$
0	1	0	-
1	0	0.333	100%
2	0.333	0.381	12%
3	0.381	0.381	0%
Convergence			

If Choose $x_0 = 3$

N	x_n	x_{n+1}	$\lvert\epsilon_a\rvert$
0	3	2.667	12.5%
1	2.667	2.619	1.8%
2	2.619	2.618	0.04%
3	2.618	2.618	0%
		Convergence	

H.W. Write a computer program to solve the following equation using the above two methods with percent error less than 1%.

$\cos x - 10x + N = 0$ **where N is your number**