## Reservoir Engineering $\Pi$

Reservoir fluid properties

By
Assist lect. Farah N. Almohammad

## Introduction

- Reservoir-fluid properties play a key role in the design and optimization of injection/production strategies and surface facilities for efficient reservoir management.
- Inaccurate fluid characterization often leads to high uncertainties in in-place-volume estimates and recovery predictions.



## Properties Of Natural Gases

- the natural gas is a mixture of hydrocarbon and nonhydrocarbon gases.
- The hydrocarbon gases that are normally found in a natural gas are methanes, ethanes, propanes, butanes, pentanes.
- The nonhydrocarbon gases (i.e., impurities) include carbon dioxide, hydrogen sulfide, and nitrogen.


## Properties Of Natural Gases

- Density, $\rho \mathrm{g}$
- Apparent molecular weight, Ma
- Specific volume, v
- Specific gravity, $\gamma \mathrm{g}$
- Compressibility factor, z
- Isothermal gas compressibility coefficient, cg
- Gas formation volume factor, $\mathbf{B g}$
- Gas expansion factor, $\mathbf{E g}$
- Viscosity, $\mu_{\mathrm{g}}$


## Behavior Of Ideal Gases

- The ideal gas is a hypothetical gas, but it serves as a useful tool to explain the more complex real gas behavior. Moreover, the equation for the ideal gas is also the basis of developing other equations for real gases.
- The physical properties of a gas is controlled by three variables and these are

1. Pressure exerted by the gas,
2. Volume occupied by the gas and
3. Temperature of the gas.

## Behavior Of Ideal Gases

- An ideal gas has the following properties:

1. The volume occupied by gas molecules is negligible compared with the volume occupied by the gas;
2. There is no attractive or repulsive forces among the molecules or between the gas molecules and the wall of the container except during collisions;
3. All collisions between the gas molecules are purely elastic, implying no internal energy loss on collision.

## Laws Of Ideal Gas

- Boyle’s Law
- It states that under constant temperature, the absolute pressure of a given mass of an ideal gas varies inversely as its volume.

$$
\begin{gathered}
p \propto \frac{1}{v} \\
p v=\text { Constant }
\end{gathered}
$$

From the above result we can write this equation as


$$
p_{1} v_{1}=p_{2} v_{2}=p_{3} v_{3}=\cdots=\text { Constant }
$$

## Laws Of Ideal Gas

- Charles's Law
- At constant absolute pressure, the volume of a given mass of an ideal gas varies directly to its temperature.



## Laws Of Ideal Gas

- Avogadro’s Law
- Under constant temperature and pressure, equal volumes of all the gases contains equal number of molecules.

$$
\begin{gathered}
V \propto n \\
\frac{V}{n}=k \\
\frac{V_{1}}{n_{1}}=\frac{V_{2}}{n_{2}}
\end{gathered}
$$

- $k$ = constant which is equals to $R T / P$, where $R$ is the universal gas constant


## Laws Of Ideal Gas

- General Gas Equation or Ideal Gas Law
- Ideal gas law or general gas equation is the combined form of boyle's law, Charles's law and Avogadro law. The ideal gas equation is written as:

$$
P V=n R T . \ldots . .(1)
$$

## Laws Of Ideal Gas

- General Gas Equation or Ideal Gas Law
- $\mathbf{P}=$ pressure of the gas, psia
- $V=$ Volume of the gas, ft 3
- $\mathbf{n}=$ Amount of substance of gas (in moles)
- $R=$ Universal gas constant) $=10.730$ psia ft3/lb-mole ${ }^{\circ} R$
- $\mathbf{T}=$ Absolute temperature of the gas, ${ }^{\circ} \mathbf{R}$


## Laws Of Ideal Gas

- At low pressures, most real gases almost behave like an ideal gas.
- For simplifying engineering calculation, a natural gas may be considered as an ideal gas under low pressures (less than $0.4 \mathrm{Mpa} \sim 58 \mathbf{~ p s i}$ ).
- In reservoirs (high pressure), however, a natural gas behaves far from an ideal gas does.


## Apparent Molecular Weight

- There is no fixed molecular formula for any natural gas. Therefore, it is impossible to calculate the molecular weight of a natural gas in terms of its molecular formula like a pure substance.
- It represents the average molecular weight of all molecules in a natural gas.
- Numerically, the molecular weight of a natural gas is estimated in terms of the composition of a gas. Gas composition measured in the laboratory is customarily reported in mole fraction ( $\mathrm{y}_{\mathrm{i}}$ ).


## Apparent Molecular Weight

- The apparent molecular weight of the gas can be estimated using a mixing rule such as

$$
M_{a}=\sum_{i=1}^{n} y i M i \ldots \ldots(5)
$$

where $\mathrm{Ma}=$ apparent molecular weight of a gas mixture $\mathrm{Mi}=$ molecular weight of the $i$ th component in the mixture $y i=$ mole fraction of component $i$ in the mixture

- The density of gas mixture is

$$
\rho_{\mathrm{g}}=\frac{\mathbf{m}}{\mathrm{V}}=\frac{\mathrm{P} M_{a}}{\mathrm{RT}} \ldots \ldots(6)
$$

## Apparent Molecular Weight

Example:- Calculate the molecular weight of a natural gas according to its mole composition given in Table (1-3) .

## Solution

According to the mole composition of the gas, the values of $y_{i} M_{i}$ for each component are first calculated, which are listed in the last column of Table The molecular weight of the gas is then determined by Eq.(5) :
Table (1-3) Mole compositions of a natural gas

| Components | Mole fraction, $y_{i}$ | Molecular weight, $M_{i}$ | $y_{i} M_{i}$ |
| :--- | :--- | :--- | :--- |
| Methane | 0.85 | 16.0 | 13.60 |
| Ethane | 0.09 | 30.1 | 7.71 |
| Propane | 0.04 | 44.1 | 1.76 |
| Butane | 0.02 | 58.1 | 1.16 |
| Sum | 1.00 |  | $M=19.23$ |

$$
M_{\mathrm{a}}=\sum_{i=1}^{N} y_{i} M_{i}=13.6+7.71+1.76+1.16=19.23
$$

## Gas Density

- The density is defined as the mass per unit volume of the substance

$$
n=\frac{m}{M} \ldots .(2)
$$

where $m=$ weight of gas, lb
$M$ = molecular weight, lb/lb-mol

$$
\begin{gather*}
P V=\frac{m}{M} R T \\
\rho_{\mathrm{g}}=\frac{\mathrm{m}}{\mathrm{~V}}=\frac{\mathrm{PM}}{\mathrm{RT}} \tag{4}
\end{gather*}
$$

where $\rho_{g}=$ density of the gas, $\mathbf{l b} / \mathbf{f} \mathbf{t}^{3}$

## Gas Density

## Example

Three pounds of n-butane are placed in a vessel at $120^{\circ} \mathrm{F}$ and 60 psia. Calculate the volume of the gas assuming an ideal gas behavior.

## Solution

Step 1. Determine the molecular weight of n-butane from Table 1-1 to give:

$$
\mathrm{M}=58.123
$$

Step 2. Solve Equation : 3 for the volume of gas:

$$
\begin{aligned}
V & =\left(\frac{m}{M}\right) \frac{R T}{p} \\
V & =\left(\frac{3}{58.123}\right) \frac{(10.73)(120+460)}{60}=5.35 \mathrm{ft}^{3}
\end{aligned}
$$

- $\mathrm{T}(\mathrm{R})=\mathrm{T}(\mathrm{F})+460$


## Gas Density

Table 1-1 Physical properties of hydrocarbon gases

| Gases | Molecular <br> formula | Molecular <br> weight | Boiling <br> point <br> $(0.1 \mathrm{MPa})$ <br> $\left({ }^{\circ} \mathrm{C}\right)$ | Critical <br> pressure, $p_{\mathrm{ci}}$ <br> $(\mathrm{MPa})$ | Critical <br> temperature, <br> $T_{\mathrm{c} i}(\mathrm{~K})$ | Acentric <br> factor |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Methane | $\mathrm{CH}_{4}$ | 16.043 | -161.50 | 4.6408 | 190.67 | 0.0115 |
| Ethane | $\mathrm{C}_{2} \mathrm{H}_{6}$ | 30.070 | -88.61 | 4.8835 | 303.50 | 0.098 |
| Propane | $\mathrm{C}_{3} \mathrm{H}_{8}$ | 44.097 | -42.06 | 4.2568 | 370.00 | 0.1454 |
| Isobutane | $i \mathrm{C}_{4} \mathrm{H}_{8}$ | 58.124 | -11.72 | 3.6480 | 408.11 | 0.1756 |
| Normalbutane | $n \mathrm{C}_{4} \mathrm{H}_{8}$ | 58.124 | -0.50 | 3.7928 | 425.39 | 0.1928 |
| Isobutane | $i \mathrm{C}_{5} \mathrm{H}_{12}$ | 72.151 | 27.83 | 3.3336 | 460.89 | 0.2273 |
| Normalpentane | $n \mathrm{C}_{5} \mathrm{H}_{12}$ | 72.151 | 36.06 | 3.3770 | 470.11 | 0.2510 |

## Ggs Jensity

Table 1-2 Physical properties of nonhydrocarbon gases

| Gases | Molecular <br> formula | Molecular <br> weight | Boiling point <br> $(0.1 \mathrm{MPa})$ <br> $\left({ }^{\circ} \mathrm{C}\right)$ | Critical <br> pressure, <br> $p_{\mathrm{ci}}(\mathrm{MPa})$ | Critical <br> temperature, <br> $T_{\mathrm{ci}}(\mathrm{K})$ | Acentric <br> factor, $\omega$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Carbon dioxide | $\mathrm{CO}_{2}$ | 44.010 | -78.51 | 7.3787 | 304.17 | 0.2250 |
| Helium | He | 4.003 | -268.93 | 0.2289 | 5.278 |  |
| Hydrogen | $\mathrm{H}_{2}$ | 2.016 | -252.87 | 1.3031 | 33.22 | -0.2234 |
| Hydrogensulfide | $\mathrm{H}_{2} \mathrm{~S}$ | 34.076 | -60.31 | 9.0080 | 373.56 | 0.0949 |
| Nitrogen | $\mathrm{N}_{2}$ | 28.013 | -195.80 | 3.3936 | 126.11 | 0.0355 |
| Oxygen | $\mathrm{O}_{2}$ | 31.999 | -182.96 | 5.0807 | 154.78 | 0.0196 |
| Aqueous vapor | $\mathrm{H}_{2} \mathrm{O}$ | 18.015 | 100 | 22.1286 | 647.33 | 0.3210 |

## Standard Volume

- The standard volume is defined as the volume of gas occupied by $1 \mathrm{lb}-\mathrm{mol}$ of gas at standard conditions.
- The standard conditions are usually 14.7 psia and $60^{\circ} \mathrm{F}$.
- $V_{s c}=\frac{(1) R T_{s c}}{P_{s c}}=\frac{1 \times 10.73 \times 520}{14.7}$.
- Vsc $=379.4$ scf/lb-mol
where Vsc = standard volume, scf/lb-mol scf = standard cubic feet
$\mathrm{T}_{\mathrm{sc}}=$ standard temperature, ${ }^{\circ} \mathbf{R}$
$\mathbf{P}_{\mathrm{sc}}=$ standard pressure, psia


## Specific Volume

- The specific volume is defined as the volume occupied by a unit mass of the gas.
- $v=\frac{V}{m}=\frac{R T}{P M}=\frac{1}{\rho_{g}}$
where $v=$ specific volume, ft $3 / \mathrm{lb}$
$\rho_{\mathrm{g}}=$ gas density, lb/ft3


## Specific Gravity

- The specific gravity is defined as the ratio of the gas density to that of the air.
- Both densities are measured or expressed at the same pressure and temperature.
- Commonly, the standard pressure psc and standard temperature $\mathrm{T}_{\text {sc }}$ are used in defining the gas specific gravity.


## Specific Gravity

$$
\begin{gather*}
\gamma_{g}=\frac{\rho_{g}}{\rho_{\text {air }}} \ldots  \tag{8}\\
\gamma_{g}=\frac{\frac{P_{s c} M_{a}}{\mathrm{RT} T_{s c}}}{\frac{P_{s c} M_{a i r}}{\mathrm{RT} T_{s c}}}  \tag{9}\\
\gamma_{g}=\frac{M_{a}}{M_{a i r}} \tag{10}
\end{gather*}
$$

Where $\gamma_{g}=$ gas specific gravity
$\rho$ air $=$ density of the air
$M_{\text {air }}=$ apparent molecular weight of the air $=28.96$
$\mathrm{Ma}=$ apparent molecular weight of the gas
$\mathbf{P s c}_{\mathrm{sc}}=$ standard pressure, psia
$\mathrm{T}_{\mathrm{sc}}=$ standard temperature, ${ }^{\circ} \mathbf{R}$

## Gas Mixture Density

## Example

A gas well is producing a natural gas with the following composition:
Assuming an ideal gas behavior, calculate:
a. Apparent molecular weight
b. Specific gravity
c. Gas density at $2,000 \mathrm{psia}$ and $150^{\circ} \mathrm{F}$
d. Specific volume at $2,000 \mathrm{psia}$ and $150^{\circ} \mathrm{F}$

Solution

| Component | $\mathbf{y}_{\mathbf{i}}$ | $\mathbf{M}_{\mathbf{i}}$ | $\mathbf{y}_{\mathbf{i}} \cdot \mathbf{M}_{\mathbf{i}}$ |
| :---: | :---: | :---: | :---: |
| $\mathrm{CO}_{2}$ | 0.05 | 44.01 | 2.200 |
| $\mathrm{C}_{1}$ | 0.90 | 16.04 | 14.436 |
| $\mathrm{C}_{2}$ | 0.03 | 30.07 | 0.902 |
| $\mathrm{C}_{3}$ | 0.02 | 44.11 | 0.882 |

## Gas Mixture Density

a. Apply Equation ( 5 ) to calculate the apparent molecular weight:

$$
\mathrm{M}_{\mathrm{a}}=18.42
$$

b. Calculate the specific gravity

$$
\gamma_{\mathrm{g}}=\mathrm{M}_{\mathrm{a}} / 28.96=18.42 / 28.96=0.636
$$

c. Solve for the density

$$
\rho_{\mathrm{g}}=\frac{\mathrm{PM}_{\mathrm{a}}}{\mathrm{RT}}=\frac{(2000)(18.42)}{(10.73)(610)}=5.628 \mathrm{lb} / \mathrm{ft}^{3}
$$

d. Determine the specific volume

$$
\mathrm{v}=\frac{1}{\rho}=\frac{1}{5.628}=0.178 \mathrm{ft}^{3} / \mathrm{lb} \underset{\substack{\text { Gas Properties } \\ \text { Farah N. Almohammad }}}{ }
$$

## Behavior Of Real Gases

- In most cases, the deviation from ideal behavior of real gases can be modified by a correction factor; and the results can satisfy the accuracy in engineering application.
- The correction factor called the gas compressibility factor, gas deviation factor, or simply the z-factor.
- The compressibility equation of state for real gases is developed as

$$
P V=z n R T . . . . .(11)
$$

## Behavior Of Real Gases

- General Gas Equation of Real Gas
- $P=$ pressure of the gas, psia
- $V=$ Volume of the gas, ft 3
- $\mathbf{n}=$ Amount of substance of gas (in moles)
- $R=$ Universal gas constant) $=10.730$ psia ft3/lb-mole ${ }^{\circ} R$
- $\mathbf{T}=$ Absolute temperature of the gas, ${ }^{\circ} \mathbf{R}$
- $\mathrm{z}=$ The gas compressibility factor, dimensionless


## Gas Compressibility Factor

- The gas compressibility factor $z$ is a dimensionless quantity.
- It is defined as the ratio of the actual volume of $\mathbf{n}$-moles of gas at $T$ and $p$ to the ideal volume of the same number of moles at the same $T$ and $p$.

$$
\begin{equation*}
Z=\frac{V_{\text {actual }}}{V_{\text {ideal }}}=\frac{V}{(n R T) / P} \tag{1}
\end{equation*}
$$

## Gas Compressibility Factor Calculation

- pseudo-critical properties, i.e., Ppc and $\mathbf{T p c}_{\mathrm{p}}$, calculation.

$$
\begin{gathered}
\boldsymbol{P}_{\boldsymbol{p c}}=\sum_{i=1}^{n} y_{i} P_{c i} \ldots . .(13) \\
\boldsymbol{T}_{\boldsymbol{p c}}=\sum_{i=1}^{n} y_{i} T_{c i} \ldots . .(14)
\end{gathered}
$$

- Dimensionless terms, Pseudo-reduced pressure and Pseudo-reduce temperature calculation .

$$
\begin{aligned}
P_{p r} & =\frac{P}{P_{p c}} . . . .(15) \\
T_{p r} & =\frac{T}{T_{p c}} \ldots . .(16)
\end{aligned}
$$

## Gas Compressibility Factor Calculation

where $\mathbf{p}=$ system pressure, psia
$\mathbf{P}_{\mathrm{pr}}=$ pseudo-reduced pressure, dimensionless
T = system temperature, ${ }^{\circ} \mathbf{R}$
$\mathbf{T p r}=$ pseudo-reduced temperature, dimensionless
$\mathbf{P}_{\mathrm{pc}}, \mathbf{T}_{\mathbf{p c}}=$ pseudo-critical pressure and temperature, respectively

## Gas Compressibility Factor



[^0]
## Standing And Katz Compressibility Factor Chart



## Gas Compressibility Factor

## Example

A gas reservoir has the following gas composition: the initial reservoir pressure and temperature are $3,000 \mathrm{psia}$ and $180^{\circ} \mathrm{F}$, respectively.

| Component | $\boldsymbol{y}_{\mathbf{i}}$ |
| :--- | :---: |
| $\mathrm{CO}_{2}$ | 0.02 |
| $\mathrm{~N}_{2}$ | 0.01 |
| $\mathrm{C}_{1}$ | 0.85 |
| $\mathrm{C}_{2}$ | 0.04 |
| $\mathrm{C}_{3}$ | 0.03 |
| $\mathrm{i}-\mathrm{C}_{4}$ | 0.03 |
| $\mathrm{n}-\mathrm{C}_{4}$ | 0.02 |

Calculate the gas compressibility factor under initial reservoir conditions.

## Gas Compressibility Factor

## Solution

| Component | yi | $\mathrm{T}_{\text {ciiver }}{ }^{\text {a }} \mathrm{R}$ | $\mathrm{yi}_{\mathrm{i}} \mathrm{T}_{\text {cii }}$ | Pci | Yi Pci |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{CO}_{2}$ | 0.02 | 547.91 | 10.96 | 1071 | 21.42 |
| $\mathrm{N}_{2}$ | 0.01 | 227.49 | 2.27 | 493.1 | 4.93 |
| $\mathrm{C}_{1}$ | 0.85 | 343.33 | 291.83 | 666.4 | 566.44 |
| $C_{2}$ | 0.04 | 549.92 | 22.00 | 706.5 | 28.26 |
| $\mathrm{C}_{3}$ | 0.03 | 666.06 | 19.98 | 616.4 | 18.48 |
| $\mathrm{i}-\mathrm{C}_{4}$ | 0.03 | 734.46 | 22.03 | 527.9 | 15.84 |
| $\mathrm{n}-\mathrm{C}_{4}$ | 0.02 | 765.62 | 15.31 | 550.6 | 11.01 |

Step 1. Determine the pseudo-critical pressure from Equation 13

$$
P_{p c}=666.18
$$

Step 2. Calculate the pseudo-critical temperature from Equation

$$
T_{p \mathrm{pc}}=383.38
$$

Step 3. Calculate the pseudo-reduced pressure and temperature by applying Equations 15 and 16 , respectively:

$$
\begin{aligned}
& P_{p r}=\frac{3000}{666.38}=4.50 \\
& T_{p r}=\frac{640}{383.38}=1.67
\end{aligned}
$$

## Standing And Katz Compressibility Factor Chart



Gas Properties
Farah N. Almohammad

## Gas Compressibility Factor

Step 4. Determine the z -factor

$$
\mathrm{z}=0.85
$$

Equation 11 can be written in terms of the apparent molecular weight $\mathrm{M}_{\mathrm{a}}$ and the weight of the gas m :

$$
\mathrm{pV}=\mathrm{z}\left(\frac{\mathrm{~m}}{\mathrm{M}_{\mathrm{a}}}\right) \mathrm{RT}
$$

Solving the above relationship for the gas specific volume and density, give:

$$
\begin{aligned}
& \mathrm{v}=\frac{\mathrm{V}}{\mathrm{~m}}=\frac{\mathrm{zRT}}{\mathrm{pM}_{\mathrm{a}}} \\
& \rho_{\mathrm{g}}=\frac{1}{\mathrm{v}}=\frac{\mathrm{pM}_{\mathrm{a}}}{\mathrm{zRT}}
\end{aligned}
$$

where $\mathrm{v}=$ specific volume, $\mathrm{ft}^{3} / \mathrm{lb}$

$$
\rho_{\mathrm{g}}=\text { density }, \mathrm{lb} / \mathrm{ft}^{3}
$$

## Gas Compressibility Factor

- In cases where the composition of a natural gas is not available, the pseudo-critical properties, i.e., ppc and $T_{p c}$, can be predicted solely from the specific gravity of the gas.
- This can be done either by Pseudo-critical properties of natural gases chart or by using the following equations.
- Case : Natural Gas Systems

$$
\begin{align*}
& T_{\mathrm{pc}}=168+325 \gamma \mathrm{~g}-12.5 \gamma \mathrm{~g}^{2}  \tag{17}\\
& \mathrm{Ppc}=677+15.0 \gamma \mathrm{~g}-37.5 \gamma \mathrm{~g}^{2} \tag{18}
\end{align*}
$$

## Gas Compressibility Factor

## Pseudo-Critical Properties of Natural Gases



## Adjustment of The Z-factor

- Higher contents of nonhydrocarbons in natural gases result in large deviations in the pseudoreduced properties of natural gases by Kay's rule. Therefore, the correction to the Z-factor from a chart is necessary.
- Besides the hydrogen sulfide and carbon dioxide, nitrogen is common in natural gases. Generally, the presence of nitrogen does not greatly affect the accuracy of a Z-factor from a generalized chart; Z-factor increases by about $1 \%$ for each $5 \%$ of nitrogen in the gas.
- However, the presence of hydrogen sulfide and carbon dioxide in natural gases may cause large errors in Z-factors from a generalized chart.


## Adjustment of The Z-factor

The equations used for the adjustment by The Wichert-Aziz Correction Method are:

$$
\begin{align*}
& \mathrm{T}_{\mathrm{pc}}^{\prime}=\mathrm{T}_{\mathrm{pc}}-\varepsilon  \tag{19}\\
& \mathrm{p}_{\mathrm{pc}}^{\prime}=\frac{\mathrm{p}_{\mathrm{pc}} \mathrm{~T}_{\mathrm{pc}}^{\prime}}{\mathrm{T}_{\mathrm{pc}}+\mathrm{B}(1-\mathrm{B}) \varepsilon}
\end{align*}
$$

where $\mathrm{T}_{\mathrm{pc}}=$ pseudo-critical temperature, ${ }^{\circ} \mathrm{R}$
$\mathrm{p}_{\mathrm{pc}}=$ pseudo-critical pressure, psia
$\mathrm{T}_{\mathrm{pc}}^{\prime}=$ corrected pseudo-critical temperature, ${ }^{\circ} \mathrm{R}$
$\mathrm{p}_{\mathrm{pc}}^{\prime}=$ corrected pseudo-critical pressure, psia
$\mathrm{B}=$ mole fraction of $\mathrm{H}_{2} \mathrm{~S}$ in the gas mixture
$\varepsilon=$ pseudo-critical temperature adjustment factor and is defined mathematically by the following expression

$$
\begin{equation*}
\varepsilon=120\left[\mathrm{~A}^{0.9}-\mathrm{A}^{1.6}\right]+15\left(\mathrm{~B}^{0.5}-\mathrm{B}^{4.0}\right) \tag{21}
\end{equation*}
$$

where the coefficient A is the sum of the mole fraction $\mathrm{H}_{2} \mathrm{~S}$ and $\mathrm{CO}_{2}$ in the gas mixture,

## Adjustment of The Z-factor

## Example

A sour natural gas has a specific gravity of 0.7. The compositional analysis of the gas shows that it contains $5 \% \mathrm{CO}_{2}$ and $10 \% \mathrm{H}_{2} \mathrm{~S}$. Calculate the density of the gas at 3,500 psia and $160^{\circ} \mathrm{F}$.

## Solution

Step 1. Calculate the uncorrected pseudo-critical properties of the gas from Equations 17 and 18

$$
\begin{aligned}
& \mathrm{T}_{\mathrm{pc}}=168+325(0.7)-12.5(0.7)^{2}=389.38^{\circ} \mathrm{R} \\
& \mathrm{p}_{\mathrm{pc}}=677+15(0.7)-37.5(0.7)^{2}=669.1 \mathrm{psia}
\end{aligned}
$$

Step 2. Calculate the pseudo-critical temperature adjustment factor from Equation 21 :
$\varepsilon=120\left(0.15^{0.9}-0.15^{1.6}\right)+15\left(0.1^{0.5}-0.1^{4}\right)=20.735$
Step 3. Calculate the corrected pseudo-critical temperature by applying Equation 19

$$
T_{p \mathrm{c}}^{\prime}=389.38-20.735=368.64
$$

## Adjustment of The Z-factor

Step 4. Adjust the pseudo-critical pressure $\mathrm{p}_{\mathrm{pc}}$ by applying Equation 20

$$
\mathrm{P}_{\mathrm{pc}}^{\prime}=\frac{(669.1)(368.64)}{389.38+0.1(1-0.1)(20.635)}
$$

Step 5. Calculate $\mathrm{p}_{\mathrm{pr}}$ and $\mathrm{T}_{\mathrm{pr}}$ :

$$
\begin{aligned}
& \mathrm{p}_{\mathrm{pr}}=\frac{3500}{630.44}=5.55 \\
& \mathrm{~T}_{\mathrm{pr}}=\frac{160+460}{368.64}=1.68
\end{aligned}
$$

Step 6. Determine the z-factor

$$
\mathrm{z}=0.89
$$

Step 7. Calculate the apparent molecular weight of the gas from Equation 10

$$
\mathrm{M}_{\mathrm{a}}=(28.96)(0.7)=20.27
$$

Step 8. Solve for gas density:

$$
\rho_{\mathrm{g}}=\frac{(3500)(20.27)}{(0.89)(10.73)(620)}=11.98 \mathrm{lb} / \mathrm{ft}^{3}
$$

## Compressibility Of Natural Gases

- Gas isothermal compressibility is a measure of a change in the volume of a gas with respect to a change in reservoir pressure. Since reservoir temperature is always constant during gas production, the isothermal gas compressibility is extensively used in determining the compressible properties of gas reservoirs.
- In general, the compressibility of natural gas is significantly higher than that of crude oil and interstitial water in reservoirs, typically by two orders of magnitude. The gas compressibility is about in the order of $720 \times 10^{-4} \mathrm{MPa}^{-1}$, whereas typical water and oil compressibilities are about $6 \times 10^{-4}$ and $15 \times 10^{-4} \mathrm{MPa}^{-1}$, respectively

$$
c_{g}=-\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_{T}
$$

where $\mathbf{c g}=$ isothermal gas compressibility, $1 / \mathrm{psi}$.

## Compressibility Of Natural Gases

- Care should be taken so that one is not confused with the two concepts: $C g$ and $Z$ because they both use the word "compressibility", and both are related to the effect of pressure on gas volume.
- Cg (gas compressibility) means the ability of the gas being compressed at constant temperature; whereas $\mathbf{Z}$ (compressibility factor or Z-factor) denotes the deviation from ideality of a gas.
- Cg is used for the calculation of the elastic energy of a gas reservoir, but Z-factor is used for the calculation of gas behavior in the equation of states.


## Compressibility Of Natural Gases

From the real gas equation-of-state:

$$
V=\frac{\mathbf{n R T Z}}{\mathbb{P}}
$$

Differentiating the above equation with respect to pressure at constant temperature $T$ gives:

$$
\left(\frac{\partial V}{\partial p}\right)_{T}=n \mathbb{R} T\left[\frac{1}{p}\left(\frac{\partial z}{\partial p}\right)-\frac{z}{p^{2}}\right]
$$

Substituting into Equation $2-44$ produces the following generalized relationship:

$$
\begin{equation*}
c_{\mathrm{B}}=\frac{1}{\mathrm{P}}-\frac{1}{z}\left(\frac{\partial z}{\partial \mathrm{P}}\right)_{\mathrm{T}} \tag{22}
\end{equation*}
$$

For an ideal gas, $z=1$ and $(\partial z / \partial p)_{T}=0$, therefore:

$$
\begin{equation*}
c_{\mathrm{E}}=\frac{1}{\mathrm{D}} \tag{23}
\end{equation*}
$$

Equation (22) can be conveniently expressed in terms of the pseudoreduced pressure and temperature by simply replacing $p$ with ( $P_{p o}$ Ppr), or:

$$
c_{\mathrm{B}}=\frac{1}{\mathrm{P}_{\mathrm{pr}} \mathrm{P}_{\mathrm{pc}}}-\frac{1}{z}\left[\frac{\partial z}{\partial\left(\mathrm{P}_{\mathrm{Pr}} \mathrm{P}_{\mathrm{Es}}\right)}\right]_{\mathrm{T}_{\mathrm{Pr}}}
$$

Multiplying the above equation by $P_{p o}$ yields:

$$
\begin{equation*}
c_{\mathrm{B}} \mathbb{P}_{\mathrm{pcc}}=c_{\mathrm{Pr}}=\frac{1}{P_{r m}}-\frac{1}{z}\left\lceil\frac{\partial z}{\partial P_{r u r}}\right\rceil \tag{24}
\end{equation*}
$$

The term $c_{p r}$ is called the isothermal pseudo-reduced compressibility and is defined by the relationship

$$
\begin{equation*}
c_{p r}=c_{\mathrm{g}} \mathrm{P}_{\mathrm{pc}} \tag{25}
\end{equation*}
$$

Farah N. Almohammad

## Compressibility Of Natural Gases

where $\mathbf{c p r}_{\mathrm{pr}}=$ isothermal pseudo-reduced compressibility
$\mathrm{cg}=$ isothermal gas compressibility, $\mathbf{p s i}-1$
$\mathbf{p p c}_{\mathrm{p}}=$ pseudo-reduced pressure, $\mathbf{p s i}$

- Values of $\left(\partial z / \partial p_{p r}\right) T_{p r}$ can be calculated from the slope of the Tpr isotherm on the Standing and Katz z-factor chart


## Compressibility Of Natural Gases

## Example

A hydrocarbon gas mixture has a specific gravity of 0.72. Calculate the isothermal gas compressibility coefficient at 2,000 psia and $140^{\circ} \mathrm{F}$ by assuming:
a. An ideal gas behavior
b. A real gas behavior

## Solution

a. Assuming an ideal gas behavior, determine $c_{g}$ by applying

$$
c_{\mathrm{g}}=\frac{1}{2000}=500 \times 10^{-6} \mathrm{psi}^{-1}
$$

b. Assuming a real gas behavior

Step 1. Calculate $T_{p c}$ and $p_{p c}$ by applying Equations 17 and 18

$$
\mathrm{T}_{\mathrm{pc}}=168+325(0.72)-12.5(0.72)^{2}=395.5^{\circ} \mathbf{R}
$$

$$
P_{P c}=677+15(0.72)-37.5(0.72)^{2}=668.4 \text { psia }
$$

Step 2. Compute $P_{p r}$ and $T_{p r}$ from Equations 15 and 16

$$
\begin{aligned}
& p_{p r}=\frac{2000}{668.4}=2.99 \\
& T_{\mathrm{pr}}=\frac{600}{395.5}=1.52
\end{aligned}
$$

## Compressibility Of Natural Gases

Calculated $\mathbf{P p r}=2.99$ at $\mathbf{T p r}=1.52$
For these pseudo conditions $\mathrm{z}=0.78$

To find $\left(\frac{\partial z}{\partial P_{P r}}\right)_{T_{p r}}$ :

Let $\mathrm{Ppr}=2.5$ at constant $\mathrm{T} p r=1.52$
For these pseudo conditions $\mathrm{z}=0.792$
$\left(\frac{\partial z}{\partial P_{P r}}\right)_{T_{p r}}=\frac{z 1-z 2}{P_{P r} 1-P_{P r} 2}$

## Compressibility Of Natural Gases

Step 3. Determine the z-factor

$$
z=0.78
$$

Step 4. Calculate the slope $\left[\partial \mathrm{z} / \partial \mathrm{ppr}_{\mathrm{pr}}\right]_{\mathrm{Tpr}}=1.52$ :

$$
\left[\frac{\partial \mathrm{z}}{\partial \mathrm{p}_{\mathrm{pr}}}\right]_{\mathrm{Tpr}}=-0.022
$$

Step 5. Solve for $\mathrm{c}_{\mathrm{pr}}$ by applying Equation 24

$$
c_{\mathrm{pr}}=\frac{1}{2.99}-\frac{1}{0.78}[-0.022]=0.3627
$$

Step 6. Calculate $\mathrm{c}_{\mathrm{g}}$ from Equation 25

$$
c_{g}=\frac{0.327}{668.4}=543 \times 10^{-6} \mathrm{psi}^{-1}
$$

## Gas Formation Volume Factor

- Gas formation volume factor is defined as the volume of free gas, measured at reservoir conditions, required to produce unit volume of the gas at surface standard conditions. Thus

$$
B_{g}=\frac{V_{R}}{V_{s c}} \ldots \ldots(26)
$$

where $\mathrm{Bg}_{\mathrm{g}}=$ gas formation volume factor, $\mathrm{ft} 3 / \mathrm{scf}$
$\mathrm{V}_{\mathrm{R}}=$ volume of gas at pressure p and temperature, $\mathrm{T}, \mathrm{ft} 3$
$\mathbf{V}_{\mathrm{sc}}=$ volume of gas at standard conditions, scf

- The gas formation volume factor is used to relate the volume of gas, as measured at reservoir conditions, to the volume of the gas as measured at standard conditions, i.e., $60^{\circ} \mathrm{F}$ and 14.7 psia.


## Gas Formation Volume Factor

Applying the real gas equation-of-state,

$$
\mathrm{B}_{\mathrm{g}}=\frac{\frac{\mathrm{zn} \mathrm{RT}}{\mathrm{p}}}{\frac{\mathrm{Z}_{\mathrm{sc}} \mathrm{n} R \mathrm{~T}_{\mathrm{sc}}}{\mathrm{P}_{\mathrm{sc}}}}=\frac{\mathrm{P}_{\mathrm{sc}}}{\mathrm{~T}_{\mathrm{sc}}} \frac{\mathrm{zT}}{\mathrm{P}}
$$

where $\quad z_{s c}=z$-factor at standard conditions $=1.0$
$p_{\mathrm{sc}}, \mathrm{T}_{\mathrm{sc}}=$ standard pressure and temperature
Assuming that the standard conditions are represented by $p_{s c}=14.7$ psia and $T_{s c}=520$, the above expression can be reduced to the following relationship:

$$
\begin{equation*}
\mathrm{B}_{\mathrm{g}}=0.02827 \frac{z T}{\mathrm{p}} \tag{26}
\end{equation*}
$$

In other field units, the gas formation volume factor can be expressed in bbl/scf to give:

$$
\begin{equation*}
B_{\mathrm{z}}=0.005035 \frac{\mathrm{zT}}{\mathrm{p}} \tag{27}
\end{equation*}
$$

## Gas Formation Volume Factor

A typical curve of gas formation volume factor as a function of pressure at reservoir temperature


## Gas Expansion Factor

- The reciprocal of the formation value of the factor sometimes is called gas expansion factor.

$$
\mathrm{E}_{\mathrm{g}}=35.37 \frac{\mathrm{p}}{\mathrm{zT}} ; \quad \mathrm{scf} / \mathrm{ft}^{3}
$$

or in terms of the gas density $\rho_{\mathrm{g}}$ :

$$
\mathrm{E}_{\mathrm{g}}=35.37 \frac{\mathrm{R} \rho_{\mathrm{g}}}{\mathrm{M}_{\mathrm{a}}}=379.52 \frac{\rho_{\mathrm{g}}}{\mathrm{M}_{\mathrm{a}}} ; \quad \mathrm{scf} / \mathrm{ft} \mathrm{t}^{3}
$$

In other units:

$$
\mathrm{E}_{\mathrm{g}}=198.6 \frac{\mathrm{p}}{\mathrm{zT}} ; \quad \mathrm{scf} / \mathrm{bbl}
$$


[^0]:    

