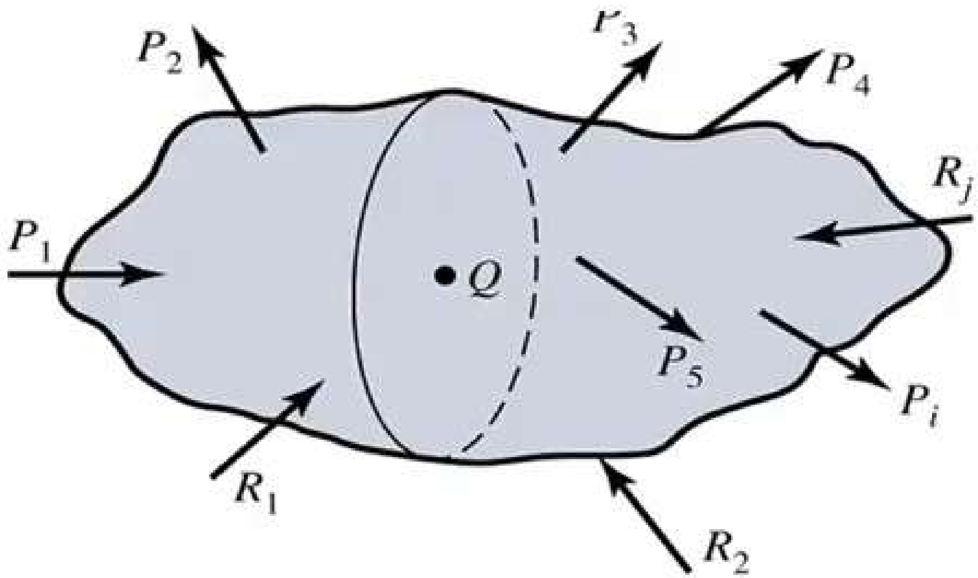
3.4 Stress



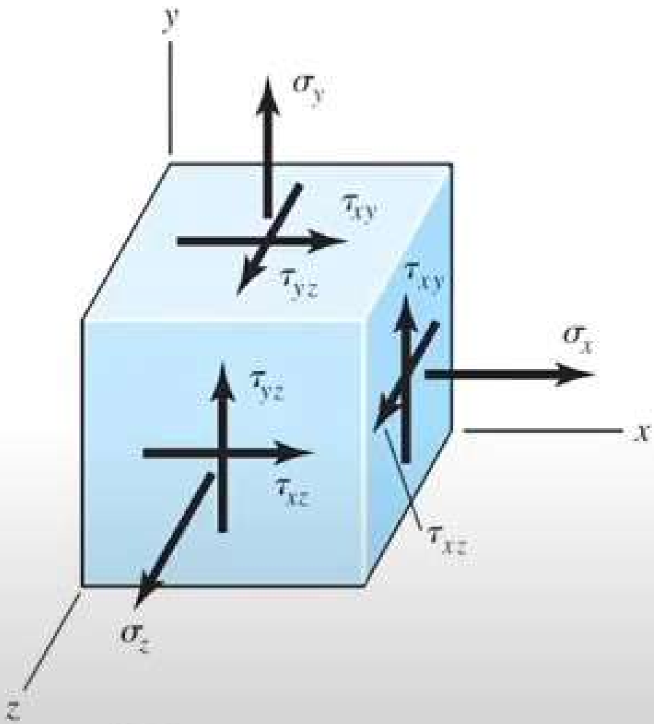
At any point Q in the body, the general stress tensor is

Three normal stresses

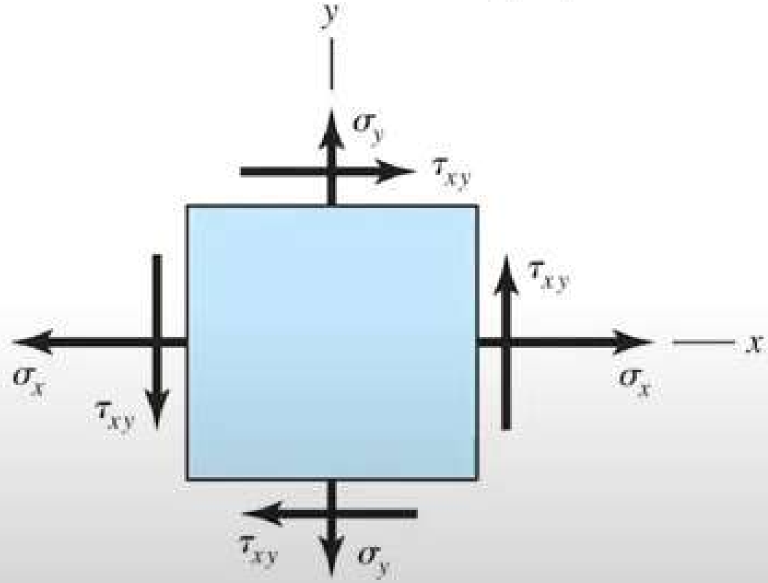
$\sigma_x, \sigma_y, \sigma_z$

Three shear stresses

$\tau_{xy}, \tau_{xz}, \tau_{yz}$

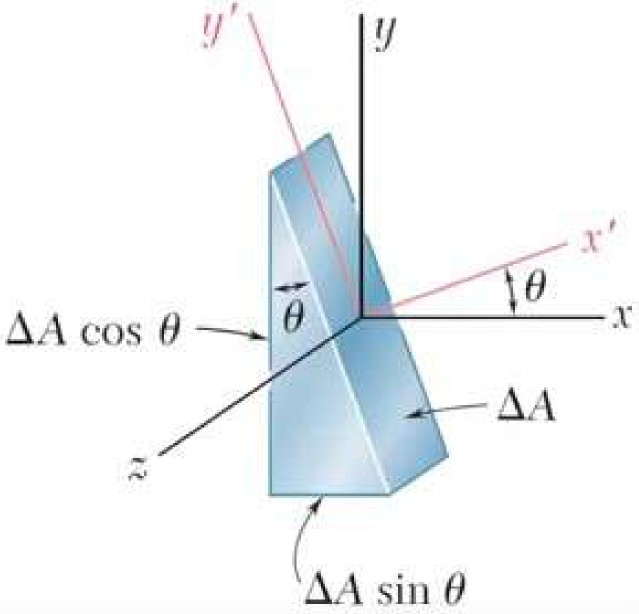


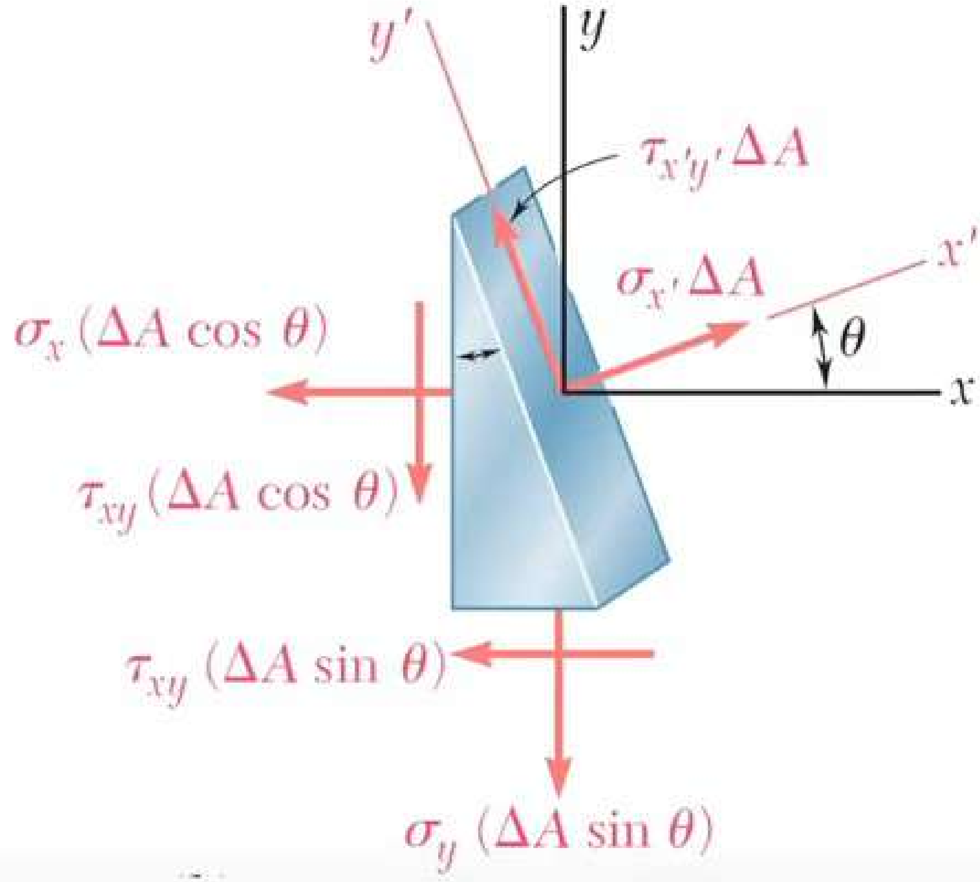
3D stresses



2D stresses (plane stress)

3.6 Mohr's Circle for Plane Stress





$$\sum F_{x'} = 0: \ \sigma_{x'}\Delta A - \sigma_x(\Delta A\cos\theta)\cos\theta - \tau_{xy}(\Delta A\cos\theta)\sin\theta$$
$$-\sigma_y(\Delta A\sin\theta)\sin\theta - \tau_{xy}(\Delta A\sin\theta)\cos\theta = 0$$

$$\sigma_{x'} = \sigma_x\cos^2\theta + \sigma_y\sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta \qquad (1)$$

In same manner we find

$\sigma_{y'}$ and $\tau_{x'y'}$

3.6 Mohr's Circle for Plane Stress cont.

$$\sigma_{x'} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy}\sin\theta\cos\theta \qquad (1)$$

$$\sigma_{y'} = \sigma_x \sin^2\theta + \sigma_y \cos^2\theta - 2\tau_{xy}\sin\theta\cos\theta \qquad (2)$$

$$\tau_{x'y'} = -(\sigma_x - \sigma_y)\sin\theta\cos\theta + \tau_{xy}(\cos^2\theta - \sin^2\theta) \qquad (3)$$

The equations can also be rewritten as:

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta \qquad (1)$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta \qquad (2)$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta \qquad (3)$$

The importance of such equations is that at a specific angle θ you have principal stresses and maximum shear.

3.6 Mohr's Circle for Plane Stress cont.

$$\sigma_{ave} = \frac{\sigma_x + \sigma_y}{2}$$

$$R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \left(\tau_{xy}\right)^2}$$

center $= (\sigma_{ave}, 0)$

current state of stress

$X(\sigma_x, -\tau_{xy})$

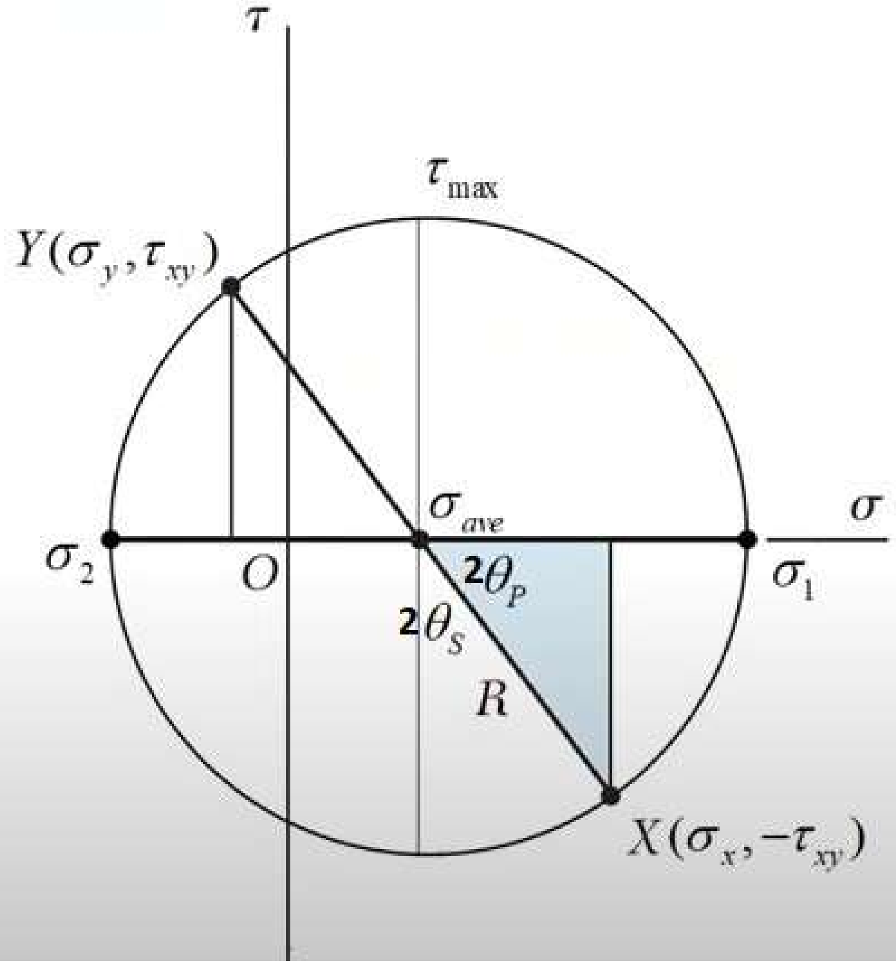
$Y(\sigma_y, \tau_{xy})$

Principal stresses

$\sigma_{1,2} = \sigma_{ave} \pm R$

Max. shear stress (2D)

$\tau = \pm R$



3.7 General Three-Dimensional Stress

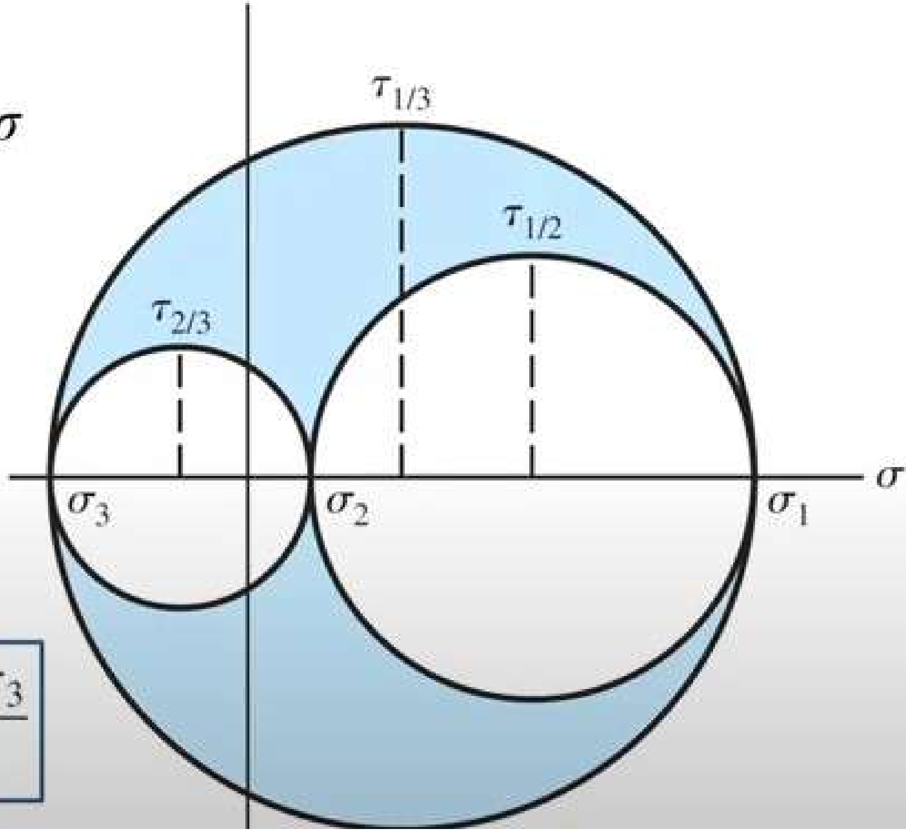
$$\sigma^3 - (\sigma_x + \sigma_y + \sigma_z)\sigma^2 + (\sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{zx}^2)\sigma$$
$$- (\sigma_x\sigma_y\sigma_z + 2\tau_{xy}\tau_{yz}\tau_{zx} - \sigma_x\tau_{yz}^2 - \sigma_y\tau_{zx}^2 - \sigma_z\tau_{xy}^2) = 0$$

Solve the third-degree polynomial to find three values for σ

(principal stresses)

Arrange, such that

$$\sigma_1 \geq \sigma_2 \geq \sigma_3$$



The principal shear stresses are

$$\tau_{1/2} = \frac{\sigma_1 - \sigma_2}{2} \qquad \tau_{2/3} = \frac{\sigma_2 - \sigma_3}{2} \qquad \tau_{1/3} = \frac{\sigma_1 - \sigma_3}{2}$$

τ_{max}

3.8 Elastic Strain cont.

Hook's law

$\sigma = E\varepsilon$ normal stresses

$\tau = G\gamma$ shear stresses

$E = 2G(1+\nu)$

Possion's ratio

$$\nu = -\frac{\text{Lateral strain}}{\text{Axial strain}}$$

Multi-axial loading

$$\epsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

$$\epsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$$

$$\epsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$$

Recall that

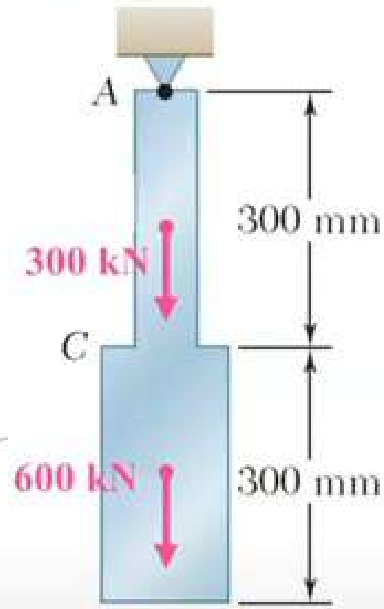
$$\delta = \frac{\mathbf{P}L}{AE}$$

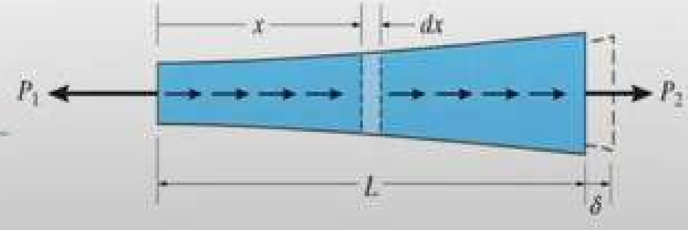
For multi – sections

$$\delta = \sum_i \frac{\mathbf{P_i}L_i}{A_i E_i}$$

For variable cross – section

$$\delta = \int_0^L \frac{\mathbf{P}(x)}{A(x)E}dx$$



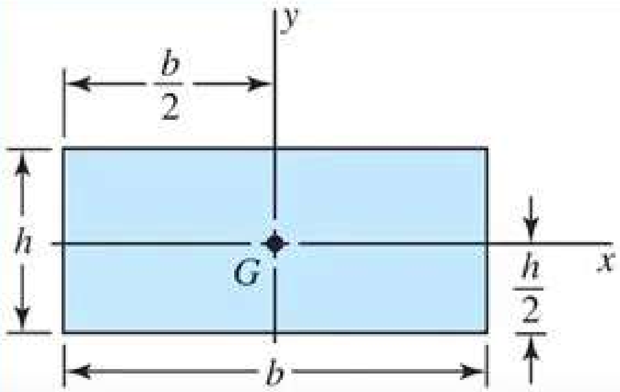


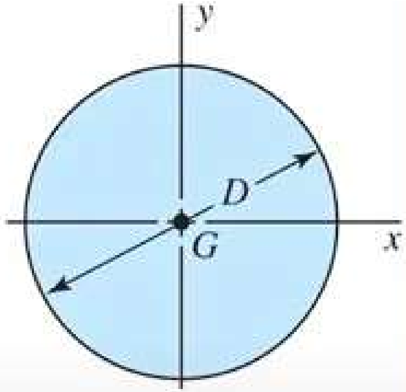
3.10 Normal Stresses for Beams in Bending

$$\sigma = \frac{\mathbf{M}c}{I} = \frac{\mathbf{M}}{I/c} = \frac{\mathbf{M}}{Z}$$

Z is the section modulus

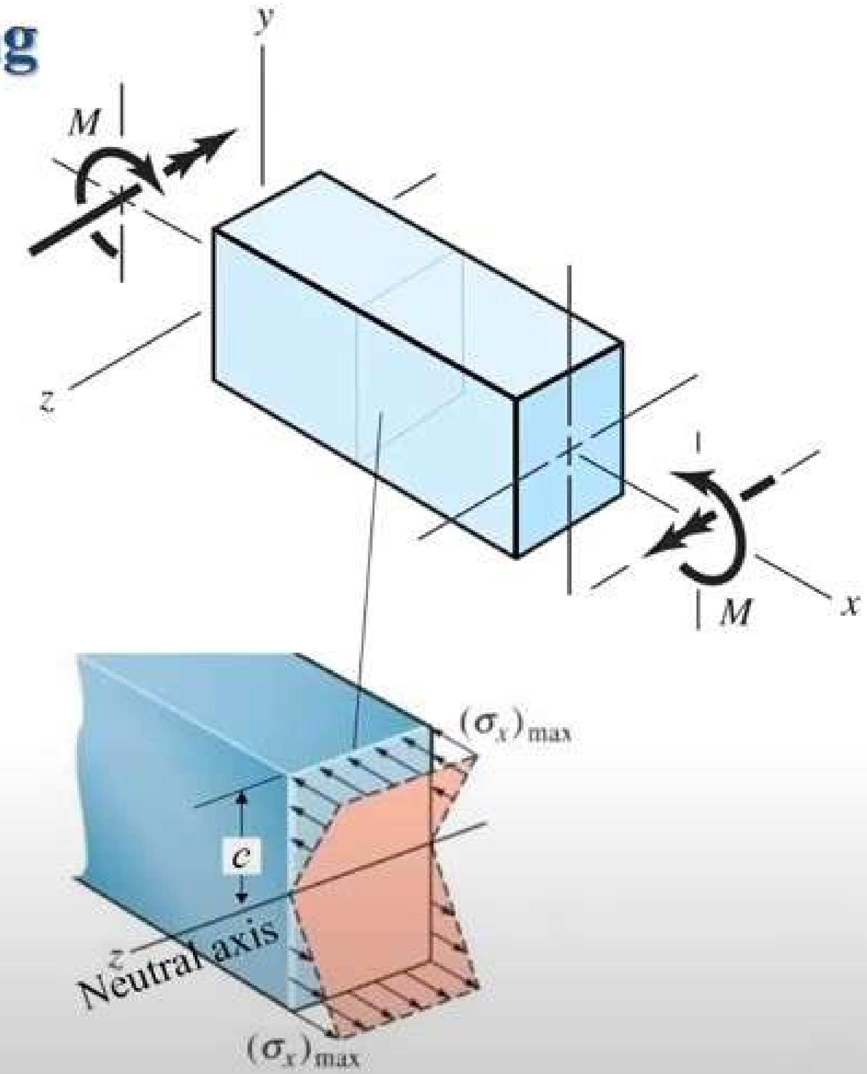
Second moment of area





$$I_x = \frac{bh^3}{12} \qquad I_y = \frac{b^3h}{12}$$

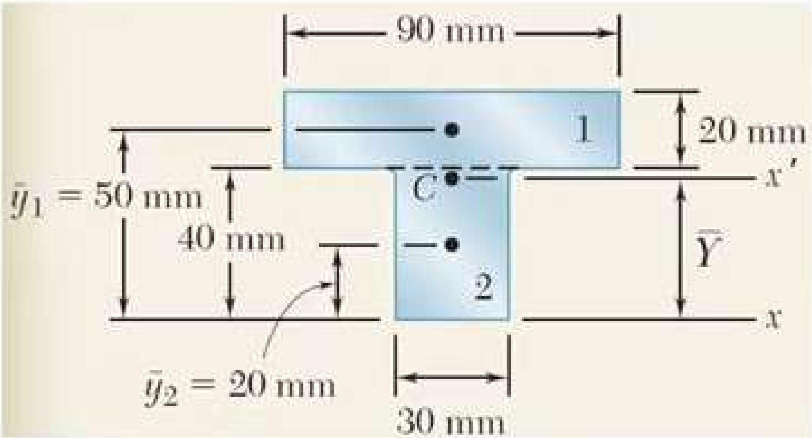
$$I_x = I_y = \frac{\pi D^4}{64}$$

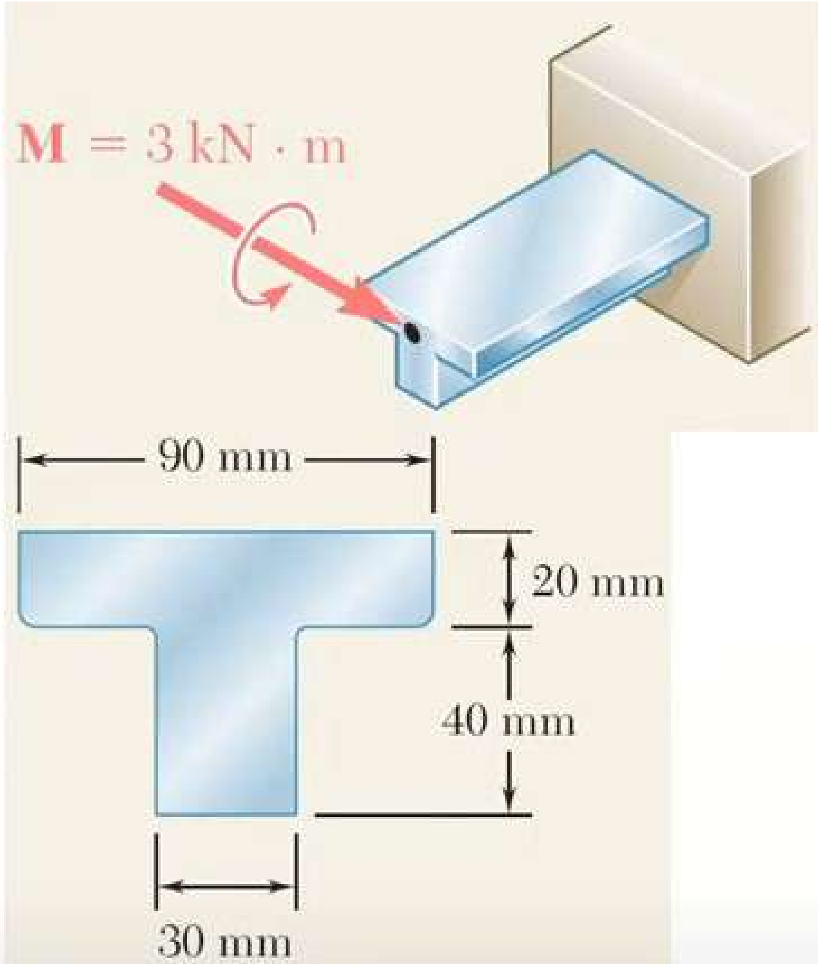


Example: Find maximum tensile and compressive stresses.

Solution :

$$\bar{y} = \frac{A_1\bar{y}_1 + A_2\bar{y}_2}{A_1 + A_2} = 38 \text{ mm}$$





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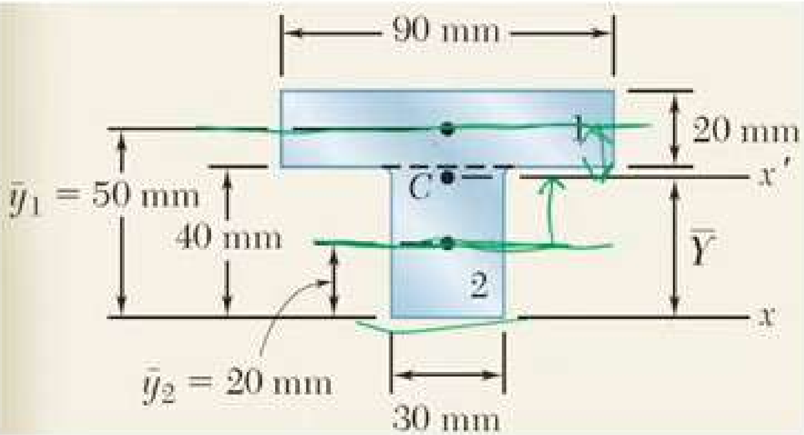
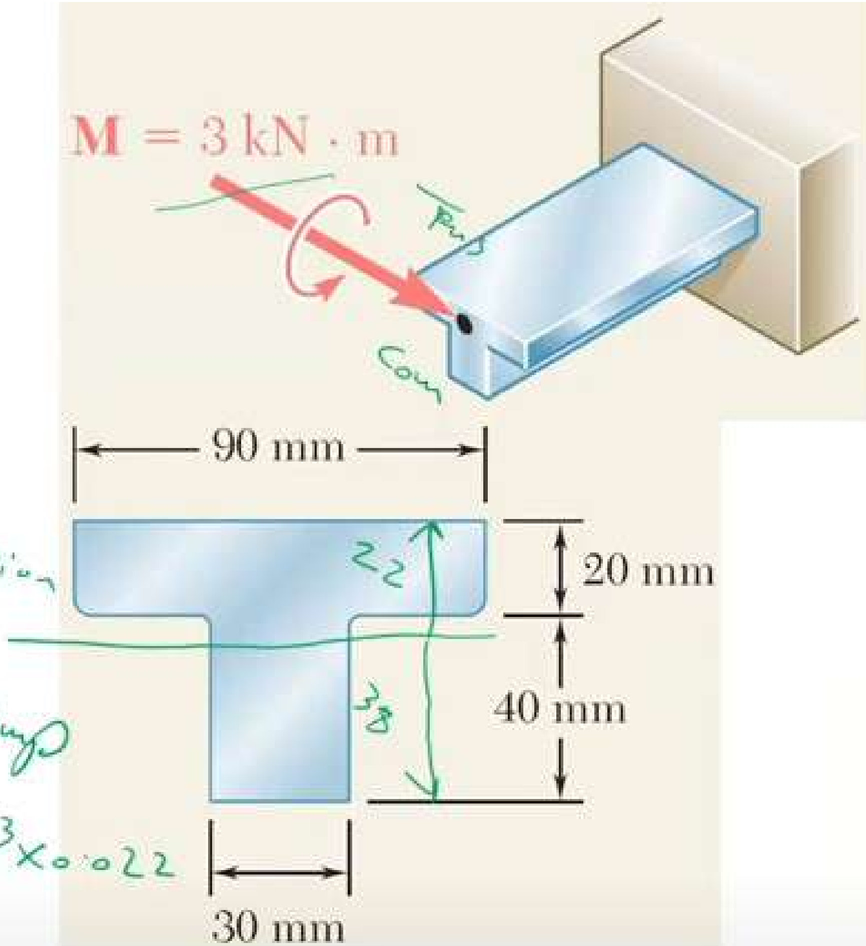


$$I_1 = \frac{1}{12}\times 90\times(20)^3 + 90\times 20\times(12)^2 \text{ mm}^4$$

$$I_2 = \frac{1}{12}\times 30\times(40)^3 + 30\times 40\times(18)^2 \text{ mm}^4$$

$$I = I_1 + I_2 = 868\times 10^{-9} \text{ m}^4$$



$$(\sigma_t)_{max} = \frac{3\times 10^3\times 22\times 10^{-3}}{868\times 10^{-9}} = 76 \text{ MPa}$$

$$(\sigma_c)_{max} = \frac{3\times 10^3\times 38\times 10^{-3}}{868\times 10^{-9}} = 131.3 \text{ MPa}$$

Example: Find $\sigma_A, \sigma_B, \sigma_C, \sigma_D$

$t = 12$ mm, $A = 4.224\times10^{-3}$ m^2

$$I_y = \frac{1}{12}\times0.125\times(0.075)^3 - \frac{1}{12}\times0.101\times(0.051)^3 = 3.278\times10^{-6}$$

$$I_z = \frac{1}{12}\times0.075\times(0.125)^3 - \frac{1}{12}\times0.051\times(0.101)^3 = 7.828\times10^{-6}$$

$$\sigma_0 = \frac{\mathbf{F}}{A} = \frac{70\times10^3}{4.224\times10^{-3}} = 16.57 \text{ MPa}$$

$$\sigma_1 = \frac{\mathbf{M_y}\cdot z_{max}}{I_y} = \frac{0.525\times10^3\times0.0375}{3.278\times10^{-6}} = 6 \text{ MPa}$$

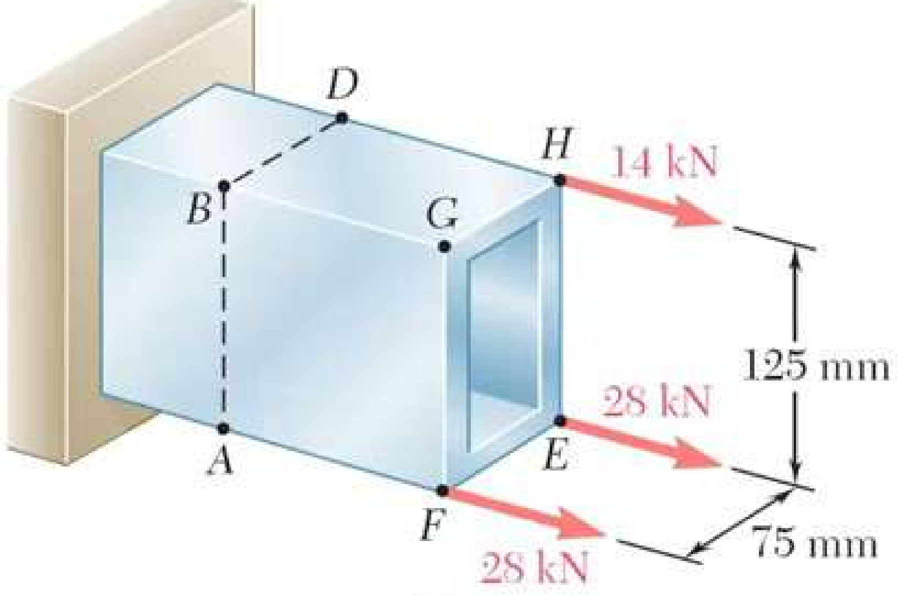
$$\sigma_2 = \frac{\mathbf{M_z}\cdot y_{max}}{I_z} = \frac{2.625\times10^3\times0.0625}{7.828\times10^{-6}} = 20.96 \text{ MPa}$$

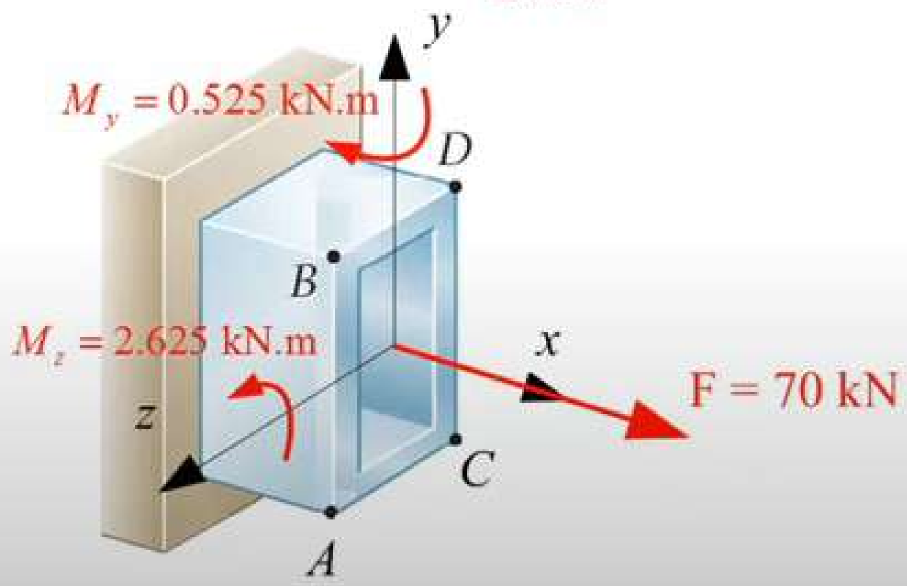
$\sigma_A = 16.57 - 6 + 20.96 = 31.53$ MPa

$\sigma_B = 16.57 - 6 - 20.96 = -10.39$ MPa

$\sigma_C = 16.57 + 6 + 20.96 = 43.53$ MPa

$\sigma_D = 16.57 + 6 - 20.96 = 1.61$ MPa

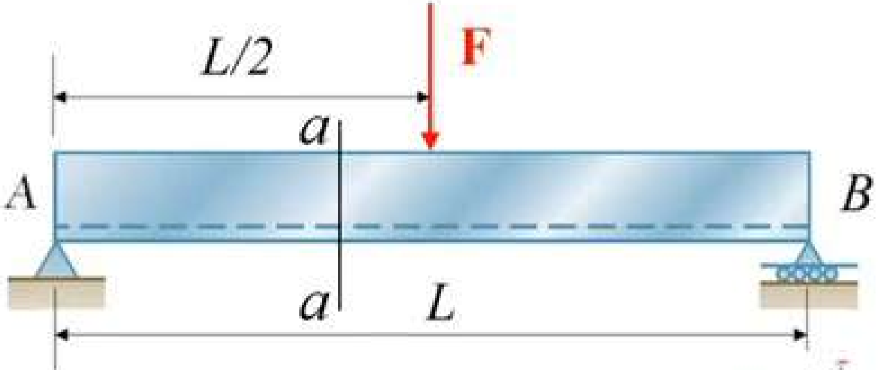




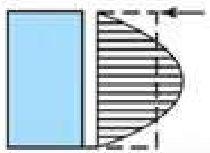
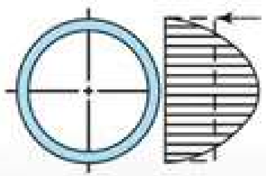
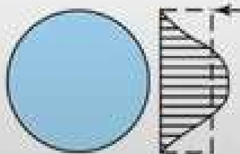
3.11 Shear Stresses for Beams in Bending (Transverse shear)

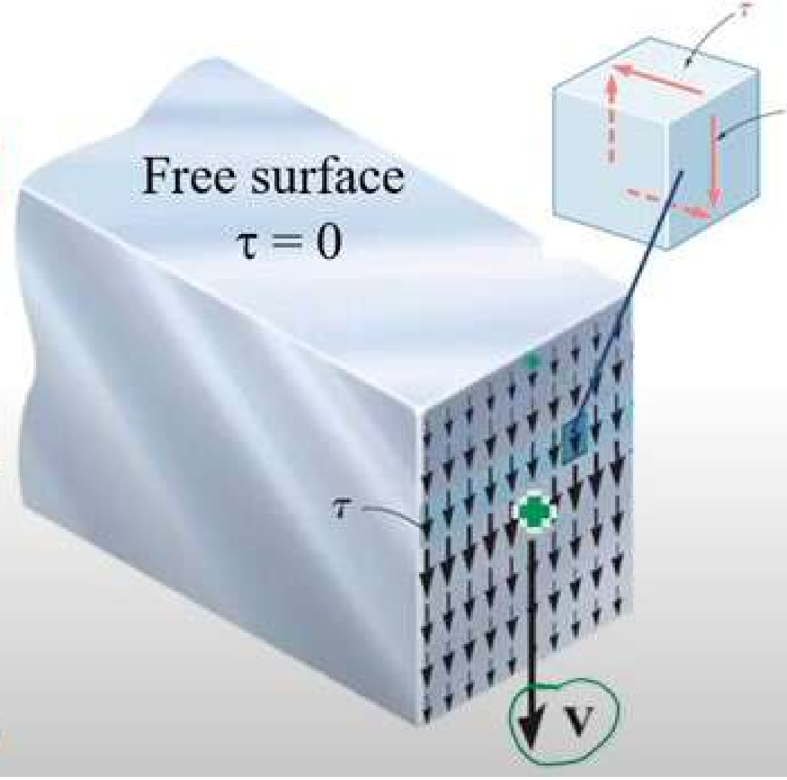
$$\tau = \frac{VQ}{It}$$

V → shear
Q → first moment of line
t → thickness



Shortcut

Beam Shape	Formula	Beam Shape	Formula
 $\tau_{avc} = \frac{V}{A}$ Rectangular	$\tau_{max} = \frac{3V}{2A}$	 $\tau_{avc} = \frac{V}{A}$ Hollow, thin-walled round	$\tau_{max} = \frac{2V}{A}$
 $\tau_{avc} = \frac{V}{A}$ Circular	$\tau_{max} = \frac{4V}{3A}$	 A_{web} Structural I beam (thin-walled)	$\tau_{max} \approx \frac{V}{A_{web}}$

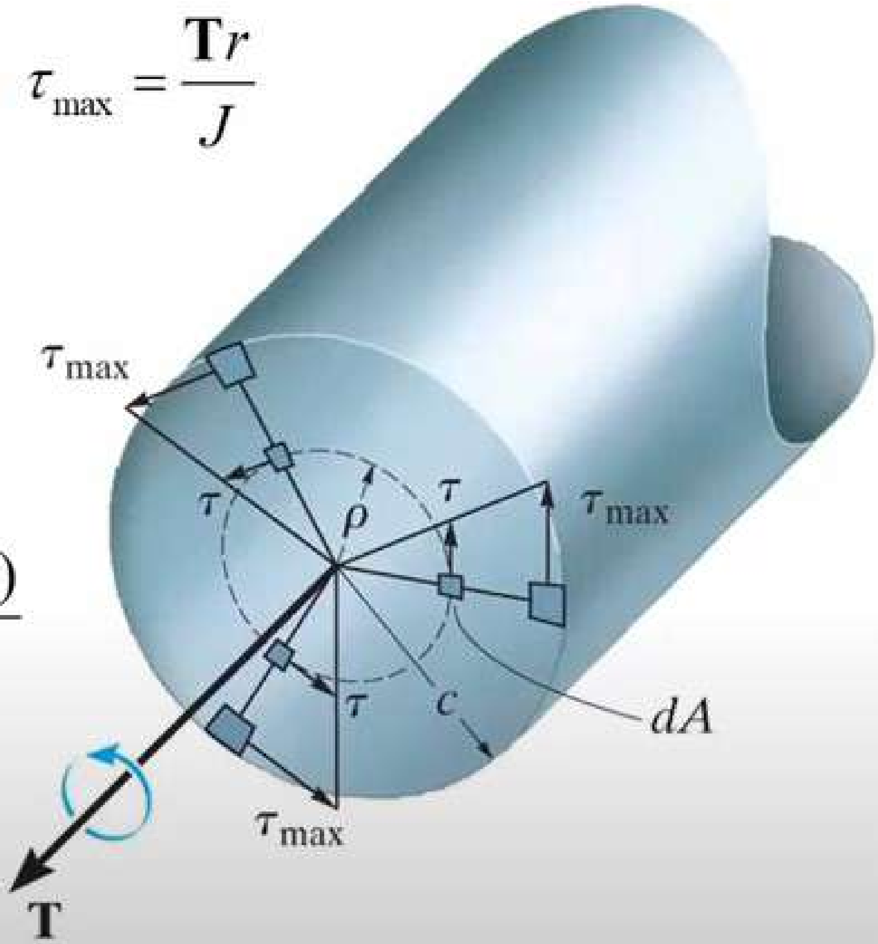


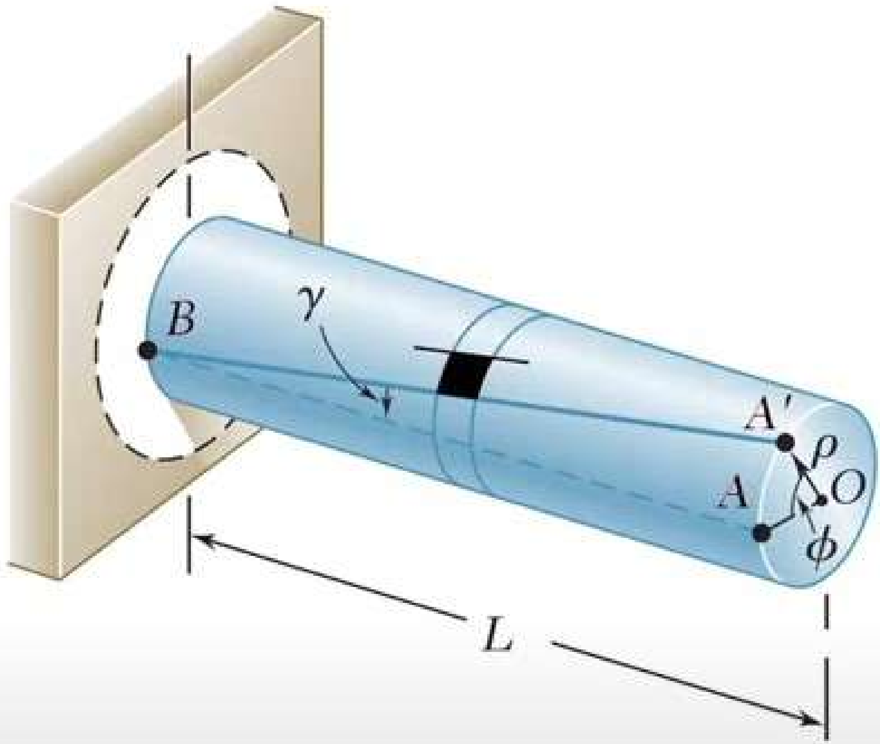
3.12 Torsion

$$\tau = \frac{\mathbf{T}\rho}{J} \Rightarrow \tau_{\max} = \frac{\mathbf{T}r}{J}$$

$$J = \frac{\pi d^4}{32}$$

$$J = \frac{\pi(d_o^4 - d_i^4)}{32}$$

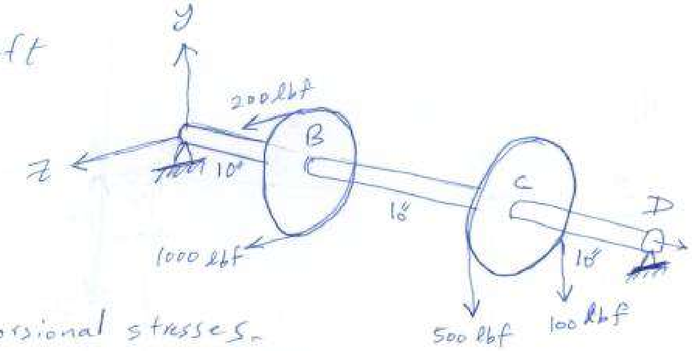




$$\gamma L = \rho\phi$$

$$\tau = G\gamma \Rightarrow \phi = \frac{TL}{GJ}$$

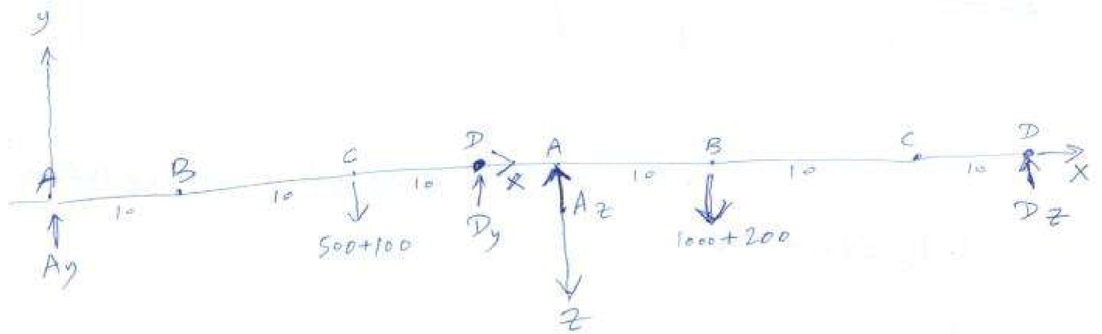
1.5" diameter solid shaft
Steel.
Simply supported -
Pulley B of 4" dia.
Pulley C of 8" dia.



considering bending & torsional stresses.

Determine the locations & magnitudes
of the greatest tensile & compressive, shear stresses.

E =



$\Sigma M_z = 0,$

$A_y * 30 - 600 * 10 = 0 \Rightarrow$ $\boxed{A_y = 200}$

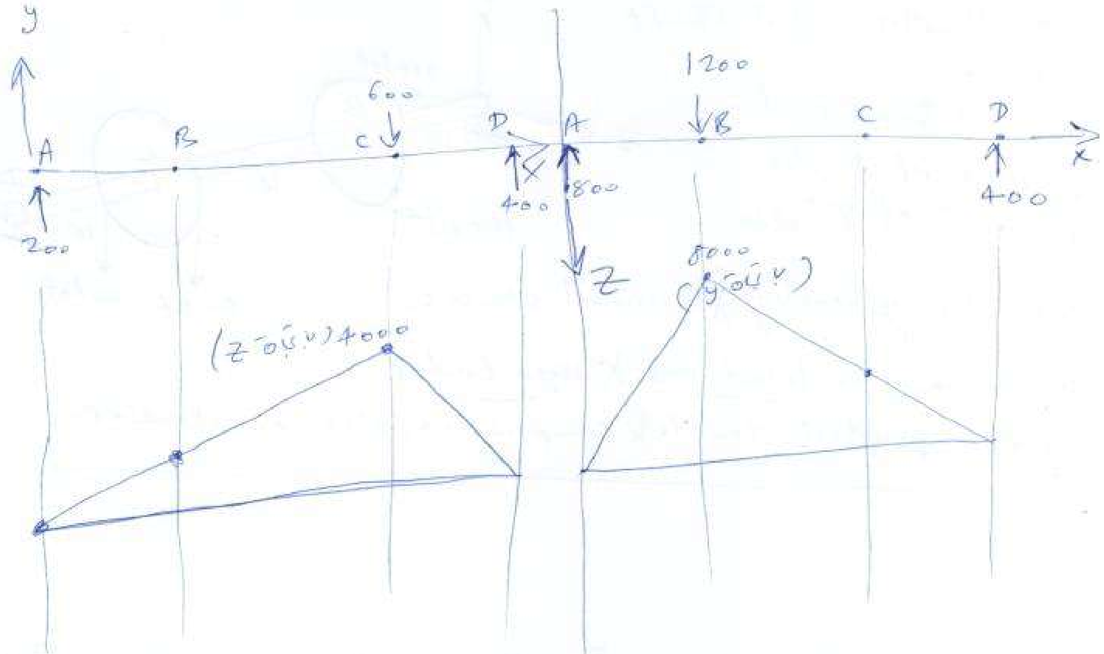
$\Sigma f_y = 0, \Longrightarrow$ $\boxed{D_y = 400}$

$\Sigma M_y = 0,$

$A_z * 30 - 1200 * 20 = 0$

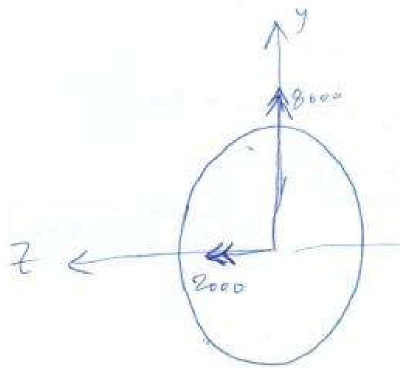
$\boxed{A_z = 800}$

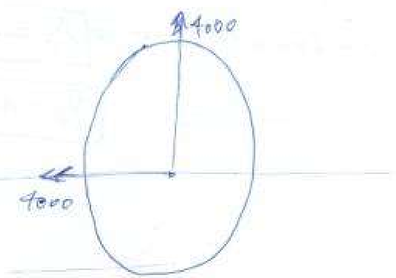
$\Sigma f_z = 0 \Longrightarrow$ $\boxed{D_z = 400}$



BM at B = 2000 lbf-in

BM at C = 4000 lbf-in





point B

point C

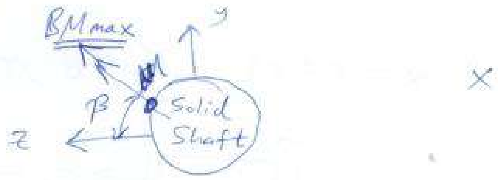
$M_{res} = \sqrt{8000^2 + 2000^2} =$

8246.2

$M_{res} = \sqrt{4000^2 + 4000^2} =$

5656.8

Max. BM at Point B y-z plane



$\beta = 76^\circ$

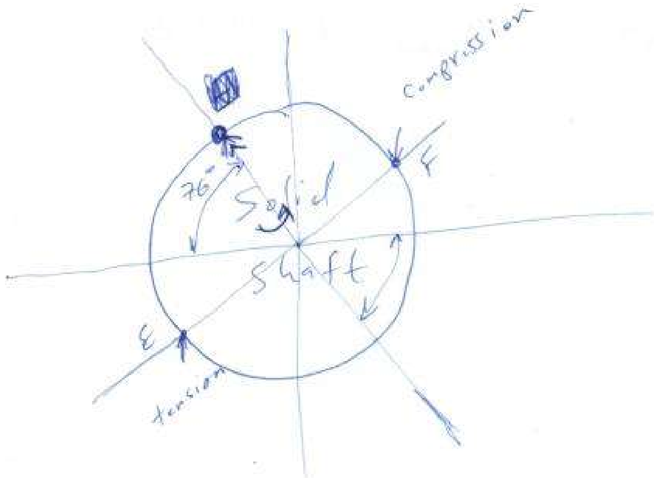
Shear stress $\frac{T}{J} = \frac{\tau}{R} \Rightarrow T_B = (1000-200) \times 2$

$= 1600$ lbf-in

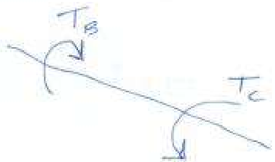
$= T_C$ (opposite)

$$\tau = \frac{16\,(1600)}{\pi\,(1.5)^3} = 2.414 \text{ Kpsi}$$

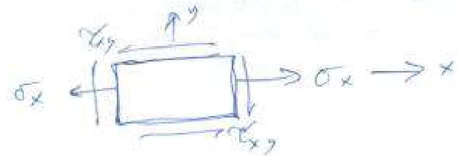
Bending stress $\sigma = \frac{32M}{\pi d^3} = \frac{32 \times 8246}{\pi\,(1.5)^3} = 24.89$ Kpsi



Tension (point E)



$\sigma_x = 24.89$, $\sigma_y = 0$, $\tau_{xy} = 2.414$

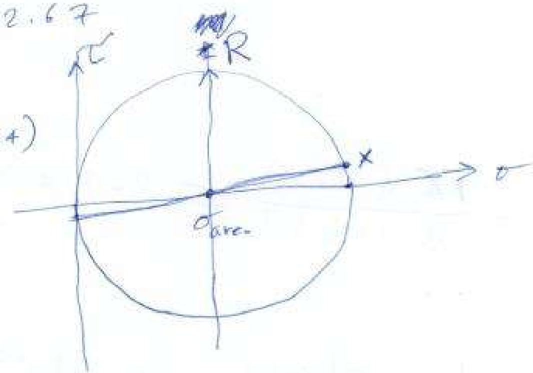


$\sigma_x = 24.89$ $\sigma_y = 0$, $\tau_{xy} = -2.414$

$\sigma_{ave} = 12.44$, $R = 12.67$

$X(\sigma_x, -\tau_{xy}) \Rightarrow X(24.89, 2.414)$

$Y(\sigma_y, \tau_{xy}) \Rightarrow Y(0, -2.414)$



$\sigma_1 = \sigma_{av.} + R = 25.11$

$\sigma_2 \simeq 0$

σ_3 (NA) $\tau_{max} = R = 12.67$