Department of Civil Engineering
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Prestressed Concrete Design – Fourth Stage

Syllabus

Prestress Concrete

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PRESTRESS CONCRETE

1- Introduction

Structural engineering tends to progress toward more economical structures through gradually improved methods of design and the use of higher-strength materials. This results in a reduction of cross-sectional dimensions and consequent weight savings. However, there are limitations to this development due mainly to the interrelated problems of cracking and deflection at service loads. The efficient use of high-strength steel is limited by the fact that the amount of cracking (width and number of cracks) is proportional to the strain and stress. The use of high-strength materials is further limited by deflection considerations. This is further aggravated by cracking, which reduces the flexural stiffness of members.

These limiting features of ordinary reinforced concrete have been largely overcome by the development of prestressed concrete. *A prestressed concrete member can be defined as* ***concrete that has prestressed so that the induced internal actions counteract the external loading to the desired degree.***

2- Effects of Prestressing

a. Prestressing applies a pre-compression to the member that reduces or eliminates undesirable tensile stresses that would otherwise be present.
b. Cracking under service loads can be *minimized or even avoided entirely*.
c. Deflections may be limited to *an acceptable value*, and in some cases, *members can be designed to have zero deflection* under the combined effects of service load and prestress force, see Figure (1).
d. Deflection and crack control, achieved through prestressing, permit the engineer to make use of efficient and economical high-strength steels in the form of strands, wires, or bars, in conjunction with high-strength concretes.

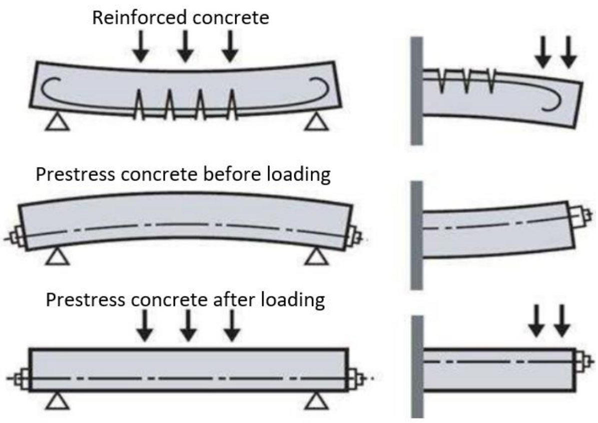


Figure 1: Effects of Prestressing

3- Advantages and Disadvantages

a- Advantages of prestressing:

1. Section remains un-cracked under service loads:
   - Reduction of steel corrosion
   - Full section is utilized
   - Higher moment of inertia (higher stiffness)
   - Less deformations (improved serviceability).
   - Increase in shear capacity.
2. High span-to-depth ratios: Larger spans possible with prestressing (bridges, buildings with large column free spaces).
3. Suitable for precast construction:
   - Rapid construction and better quality control.
   - Reduced maintenance.
   - Suitable for repetitive construction.

b- Disadvantages of prestressing:

1. Closer quality control is required in manufacture.
2. Higher cost of labor and materials.
3. Losses in the initial prestressing forces. When the compressive forces from prestressing are applied to the concrete, it will shorten somewhat, partially relaxing the cables. The result is some reduction in cable tension with a resulting loss in prestressing forces.
4. End anchorages and bearing plates are usually required.

4- Materials of Pretress Concrete

1- Concrete:

High strength concrete is used ($f_c{}'$ > 40 MPa) for the following reasons:

a. High-strength concrete normally has a higher modulus of elasticity. This means a reduction in initial elastic strain under the application of prestressing force and a reduction in creep strain, which is approximately proportional to elastic strain. **This results to a reduction in loss of prestress.**

b. In Prestressed concrete, the entire members are kept in compression, and thus all the concrete is effective in resisting forces. **Hence, it is reasonable to pay for a more expensive but stronger concrete if all of it is going to be used.**

c. Most prestressed work is of the ***precast***, pretensioned type done at the prestress yard, where the work can be carefully controlled. **Consequently, dependable higher-strength concrete can readily be obtained.**

d. For pretensioned work, the higher-strength concretes permit the use of higher bond stresses between the cables and the concrete.

The strain characteristics of concrete under short-term and sustained loads assume even greater importance in prestressed structures than in reinforced concrete structures because of the influence of strain on the loss of prestressing force. Strains due to stress, together with volume changes due to shrinkage and temperature changes, may have considerable influence on prestressed structures.

Table 1: Allowable permissible extreme concrete fiber stresses ACI 318-19

<table>
<tr><th colspan="3">Immediately after prestress transfer
(before losses)</th><th colspan="4">At service load
(after losses)</th></tr>
<tr><td>End of simply-supported members in compression</td><td>$0.7\ f_{ci}'$</td><td>Table 24.5.3.1</td><td>Prestress plus sustained load</td><td colspan="2">$0.45\ f_c'$</td><td>Table 24.5.4.1</td></tr>
<tr><td>All other locations in compression</td><td>$0.6 f_{ci}'$</td><td>Table 24.5.3.1</td><td>Prestress plus total load</td><td colspan="2">$0.6\ f_c'$</td><td>Table 24.5.1.1</td></tr>
<tr><td>Ends of simply-supported members in tension</td><td>$0.5\ \sqrt{f_{ci}'}$</td><td>Table 24.5.3.2</td><td rowspan="2">Extreme fiber stress in tension f_t in precompressed tensile zone under service load
(A.H. Nilson)</td><td>Uncracked</td><td>Transition</td><td>cracked</td></tr>
<tr><td>All other locations in tension</td><td>$0.25\ \sqrt{f_{ci}'}$</td><td>Table 24.5.3.2</td><td>$\leq 0.62\ \sqrt{f_c'}$</td><td>$>0.62\ \sqrt{f_c'}$ and $\leq\ \sqrt{f_c'}$</td><td>-----</td></tr>
</table>

Where:

f_{ci}'= compressive strength of concrete at the time of initial prestress.

f_c' = specified compressive strength of concrete.

In parts $0.45\ f_c'$ and $0.6\ f_c'$ of Table 1**(at service load (after losses)),** *sustained load* is any part of the service load that will be sustained for a sufficient period of time to cause significant time-dependent deflections, whereas ***total load*** refers to the total service load, a part of which may be transient or temporary live load. Thus, sustained load would include dead load and **may or may not** include service live load, depending on its duration. If the live load duration is short or intermittent, the higher limit of $0.6\ f_c'$ is permitted.

2- Steel:

Prestressing is practical only when **High Tensile Steel** (H.T.S.) are used, see Figure (2).

E is the same for all steels.

A- Ordinary steel:

$$fy\ =\ 275\ N/mm^2,\ Es\ =\ 2\,x10^5\ N/mm^2$$

$$\varepsilon_s = \frac{\Delta L}{L} = \frac{fs}{Es} = \frac{275}{2x10^5} = 1.375x10^{-3}$$

B- H.T.S.:

(Prestress bar) $fy\ =\ 880\ N/mm^2\ \ minimum\ \ yield\ stress$

$$\varepsilon_s = \frac{880}{2x10^5} = 4.4x10^{-3}$$

Assume long term strain in concrete due to shrinkage and creep alone = $0.8\ x10^{-3}$

Net strain in steel after losses:

1- $(1.375\text{-}0.8)\ x10^{-3} = 0.575\ x10^{-3}$

2- $(4.4 - 0.8)\ x10^{-3} = 3.6\ x10^{-3}$

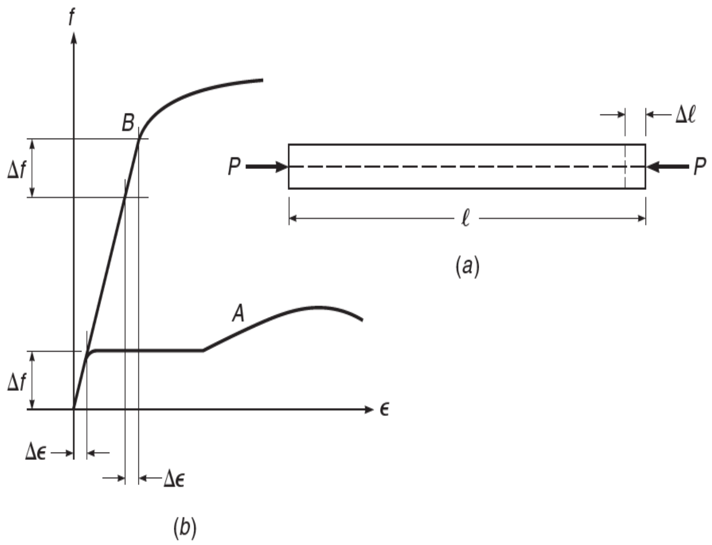


Figure 2: Typical stress strain curve for ordinary and high tensile steel

The corresponding stress after losses:

1- $fs = E_s . \varepsilon_s = 0.575 \, x10^{-3} \, x \, 2 \, x10^5$ = 115 N/mm^2

2- $fs = E_s . \varepsilon_s = 3.6 \, x10^{-3} \, x \, 2 \, x10^5$ = 720 N/mm^2

therefore, the percentage stress loss:

1- $\frac{275-115}{275} = 58 \, \%$

2- $\frac{880-720}{880} = 18 \, \%$

Forms of steel:

1- Individual wires, single, parallel bundles, or cables having diameter 5 to 7 mm, see Figure (3).
2- Standard cable generally made of 7 wires, having diameter 2 to 4 mm.
3- Alloy steel bars of diameter 10 to 32 mm.

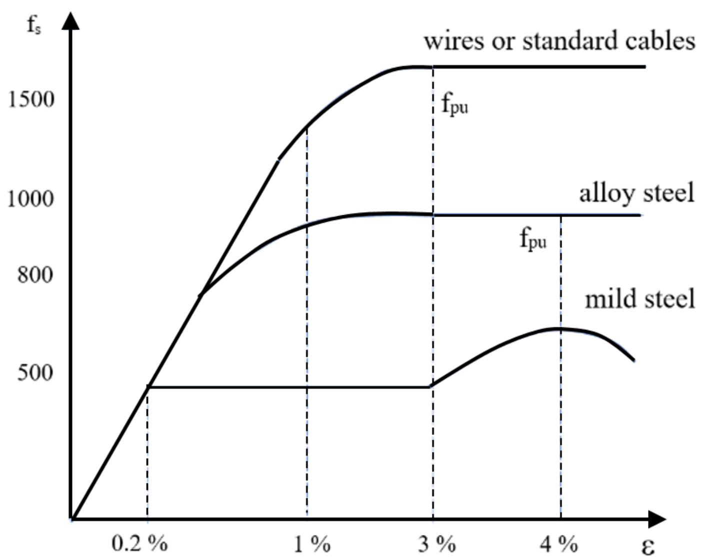


Figure 3: Forms of steel

Table 2: (20.3.2.5.1) – Maximum permissibe tensile stresses in prestressed reinforcement

stage	location	Maximum tensile stress	
During stressing	At jacking end	Least of:	$0.95\ f_{py}$
			$0.80\ f_{pu}$
			Maximum jacking force recommended by the supplier of anchorage device
Immediately after force transfer	At post-tensioning anchorage devices and couplers	$0.70\ f_{pu}$	

Where:
fp_y= yield strength of prestressing steel
fp_u= ultimate strength of prestressing steel

Table 3: Standarized prestessing steel

Product	Specification	Grade	Min. yield strength N/mm^2	Min. tensile strength N/mm^2
Prestress wire	A421		1296-1330	1620-1725
strands	A416	250	1465	1725
		270	1580	1860
Alloy steel bars	A722	Type I	880	1034
		Type II	827	1034

Some types of prestressing steel:

a. Tendons: Hard drawn high tensile steel wire of diameter ranging from 1.5mm to 8 mm.
b. Strands: A few wires are spun together in a helical shape to form a prestressing strand. Two, three or seven wire strand are used, in seven wire strand, six wires are spun around a central wire, the central wire is larger than the other wires. The diameter of strand varies from 7mm to 17mm.
c. Round bars: High tensile alloy steel bars are used in pre-stressing systems. It is available in 10 mm to 32 mm diameter.


Tendons


Strands


Bars

5- Prestressing Equipment

a. Tensioning equipment – hydraulic jacks.

b. Temporary gripping device – Double cone, wedge.

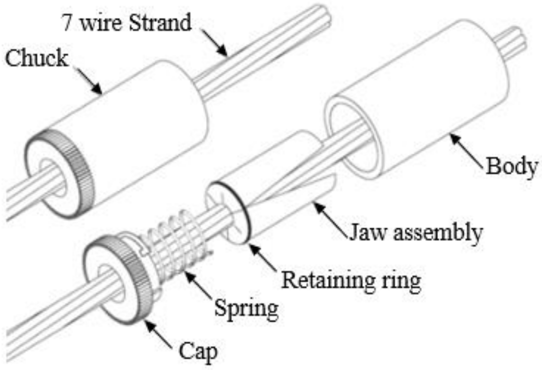
c. Releasing device – for gradual and uniform release.

d. End anchorage – strong enough to hold stress.

a

b

c



d

6- Prestressing Systems

Depending on the design, a prestressed member may be pretensioned or posttensioned. In general, precast beams to be simply supported are pretensioned, although large single-span box beams for bridge construction are often posttensioned. Continuous multi-span beams are posttensioned without exception.

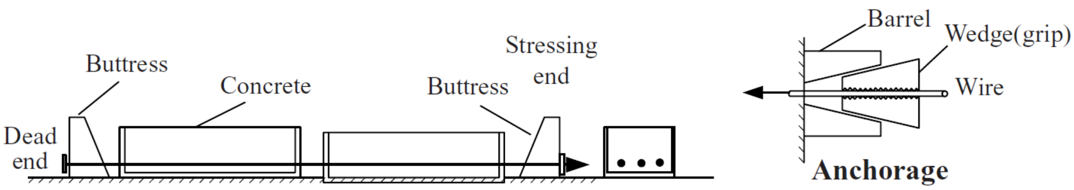
A- Pretensioning:

Steps for pretensioning beams in a precast, Figure (4-a) are as follows:

1. Pull the tendons fixed at the dead-end, one at a time, at predetermined locations from the stressed (live) end. Each tendon is anchored at the dead-end buttress using a barrel-and-wedge anchorage device and each is stressed at the live end with an appropriate prestressing jack using another anchorage device, see Figure (4-b). After the required tension is attained,

the anchorage device now butting against the live-end buttress, also will serve to maintain the tension in the tendon when and after the jack pulling the tendon is deactivated. *Depending on the length of the prestress bed more than one beam can be manufactured in a single operation*.

2. Cast concrete into the formwork following the proper compaction procedure.
3. After the concrete has hardened to the required strength, sever the tendons (in between the beam ends, and between the beam ends and the buttresses) to 'transfer' the tension in the tendons to the concrete.



(a) Prestressing bed (b) Anchorage system

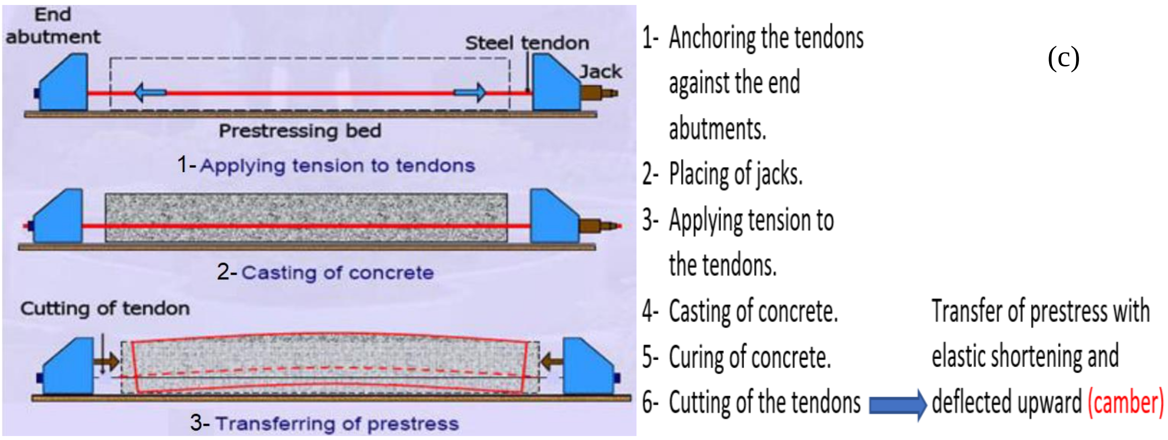
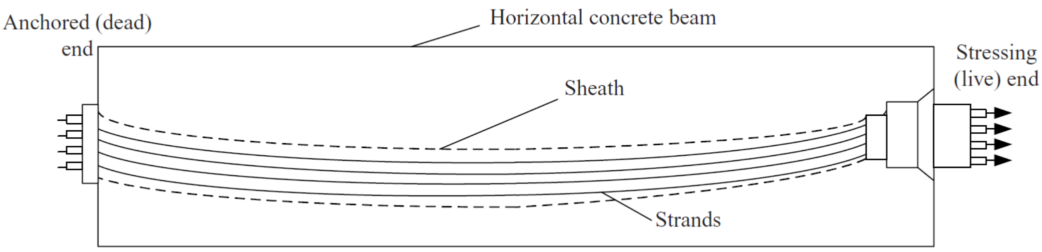


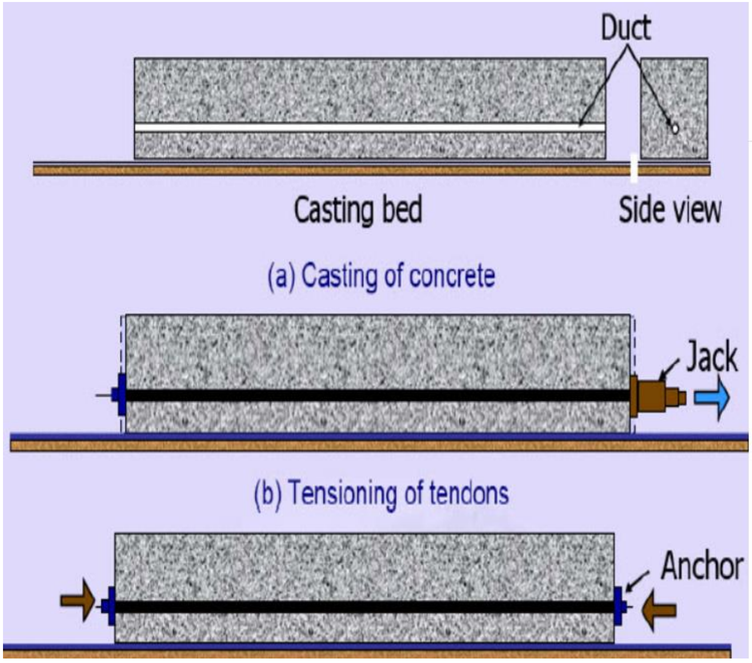
Figure 4: Pretensioned prestressing system (a) prestressing bed (b) anchorage system, and (c) Pretension process

B-Posttensioning

The process for manufacturing a posttensioned beam follows the steps below and is shown schematically in Figure (5).

1. Locate the required strands of tendons or prestress cables, in the desired or predetermined profile. The cables are enclosed in a water-tight sheath; the wedge-barrel anchorage systems at the live and dead ends are proprietary products in most cases.
2. Cast and properly compact the concrete in the formwork with the cables encased in the water-tight sheath unstressed.
3. When the required concrete strength is attained, commence the stressing or posttensioning process to produce the required tension in the group of cables.





1- Constructing the formwork.
2- Placement of the reinforced steel cage.
3- Putting the hollow ducts (sheath).
4- Installing the tendons (unstressed) in the ducts.
5- Casting the concrete
6- Tensioning the tendons after the concrete has reached its required strength.
7- The tendons are then anchored at each stressing end.
8- Filling the ducts with grout under pressure (to ensure an efficient bond between the concrete and tendon)
9- Cutting the tendon.

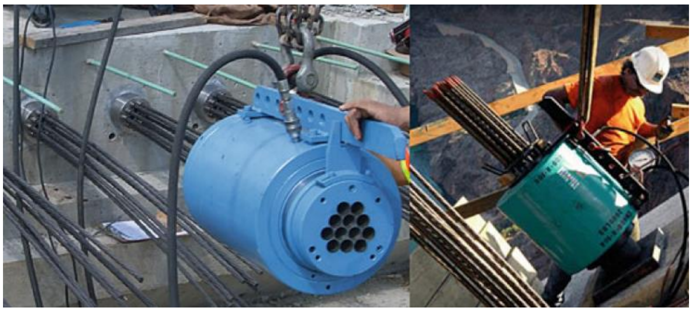


Figure 5: Details of a posttensioned beam

Difference Between Pre-tensioning and Post-tensioning

	Pre-tensioning	**Post-tensioning**
Concept	The prestressing cables also called as strands are tensioned before casting the concrete. Latter concrete is cast enclosing the tensioned cables, so in this method, we can say first tensioning and then casting.	The strands are enclosed within a duct and then concrete is cast. The process of tensioning the strands is carried out after the concrete attains its sufficient strength so in this method we can say first casting and then tensioning
Where it can be done?	Mostly done in factories hence it is suitable for precast construction works.	Can be done in factories as well as on-site.
Size limitations	The size of the sections is limited due to transportation restrictions and also due to the availability of cranes capable of lifting the sections into place.	The size of the member is not restricted long-span bridges can be constructed with the help of this method.
Members	In case of pre-tensioning similar prestressed members are prepared in a factory if the size variation happens then we have to manufacture a separate mould of that size.	Whereas post-tensioning can be done at site hence products are changed according to a structure.
Suitability	Is suitable for small structural elements which are easy to transport.	Is preferred when the structural element is heavy.
Loss in pre-stress	not less than 18 %	not more than 15 %
Cost	Is cheaper because the cost of sheathing is not involved in this method.	Is costlier because of the use of sheathing.

7- Concrete Stress Control by Prestressing

Consider unreinforced (plain) concrete beam of rectangular section, simply supported over a span $\boldsymbol{L}$, and carrying a uniform load ($w = w_L + w_D$), as shown in Figure (6-a). When the tensile strength of concrete is reached in the bottom fibre at mid-span, cracking and a sudden brittle failure occur. If it is assumed that the concrete possesses zero tensile strength, then no load can be carried (at failure $\boldsymbol{w} = \mathbf{0}$). If an axial compressive force $\boldsymbol{P}$ applied to the beam, as shown in Figure (6-b), it will induces a uniform compressive stress of intensity $^P/_A$

- At failure the **mid-span** tensile stress at extreme fibre = $^P/_A$ …… (Concentric tendon).
- The moment caused by the external load w when the linear-elastic occur is given by:

$\frac{M}{Z} = \frac{wL^2}{8Z} = \frac{P}{A}$ … (simple beam theory) and hence,

$w = \frac{8\,Z}{L^2}\ \frac{P}{A}$…. Z = section modulus $= \frac{I}{y} = \frac{(moment\ of\ inertia)}{(distance\ from\ NA\ to\ top\ or\ bottom\ fibers)}$

- If the prestressing force $\boldsymbol{P}$ is applied at an eccentricity $\boldsymbol{e}$ (Eccentric tendon), as shown in Figure (6-c), the compressive stress caused by P in the bottom fibre at mid-span is:

$\frac{P}{A} + \frac{P.e}{Z}$

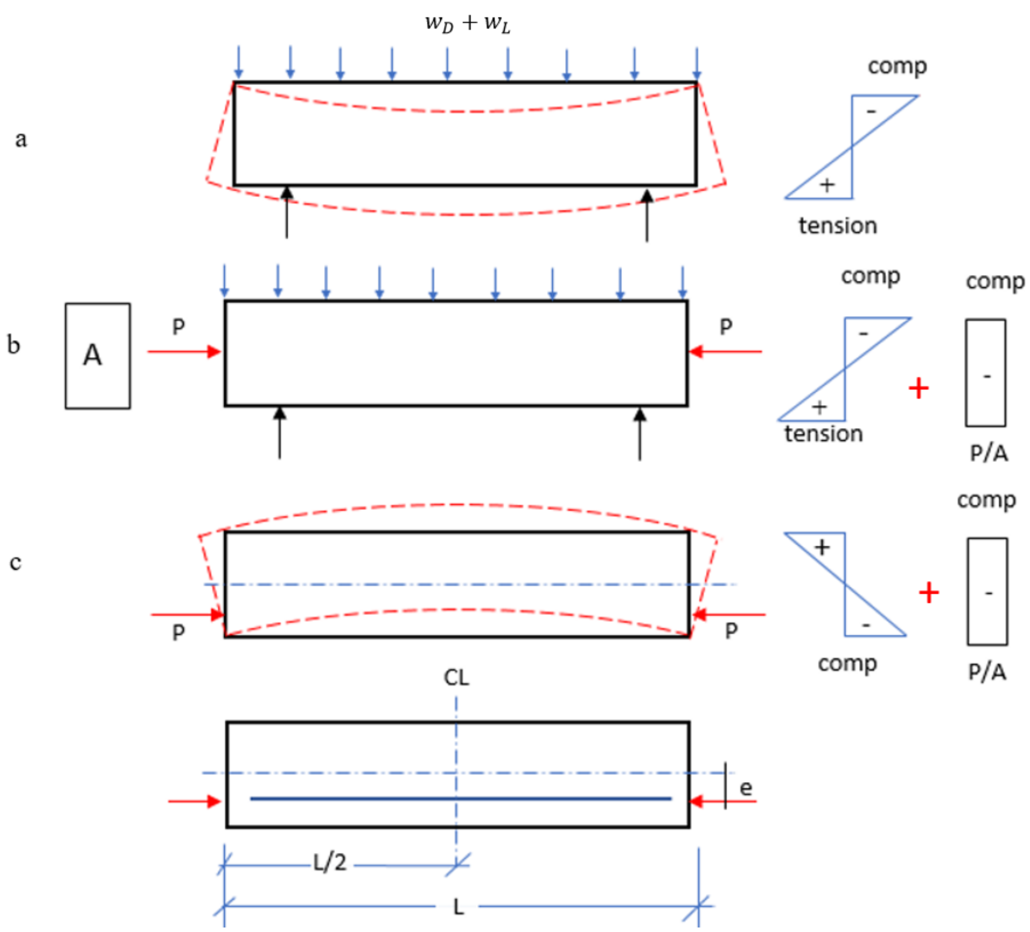


Figure 6: Stresses schemes of prestressing rectangular concrete beam

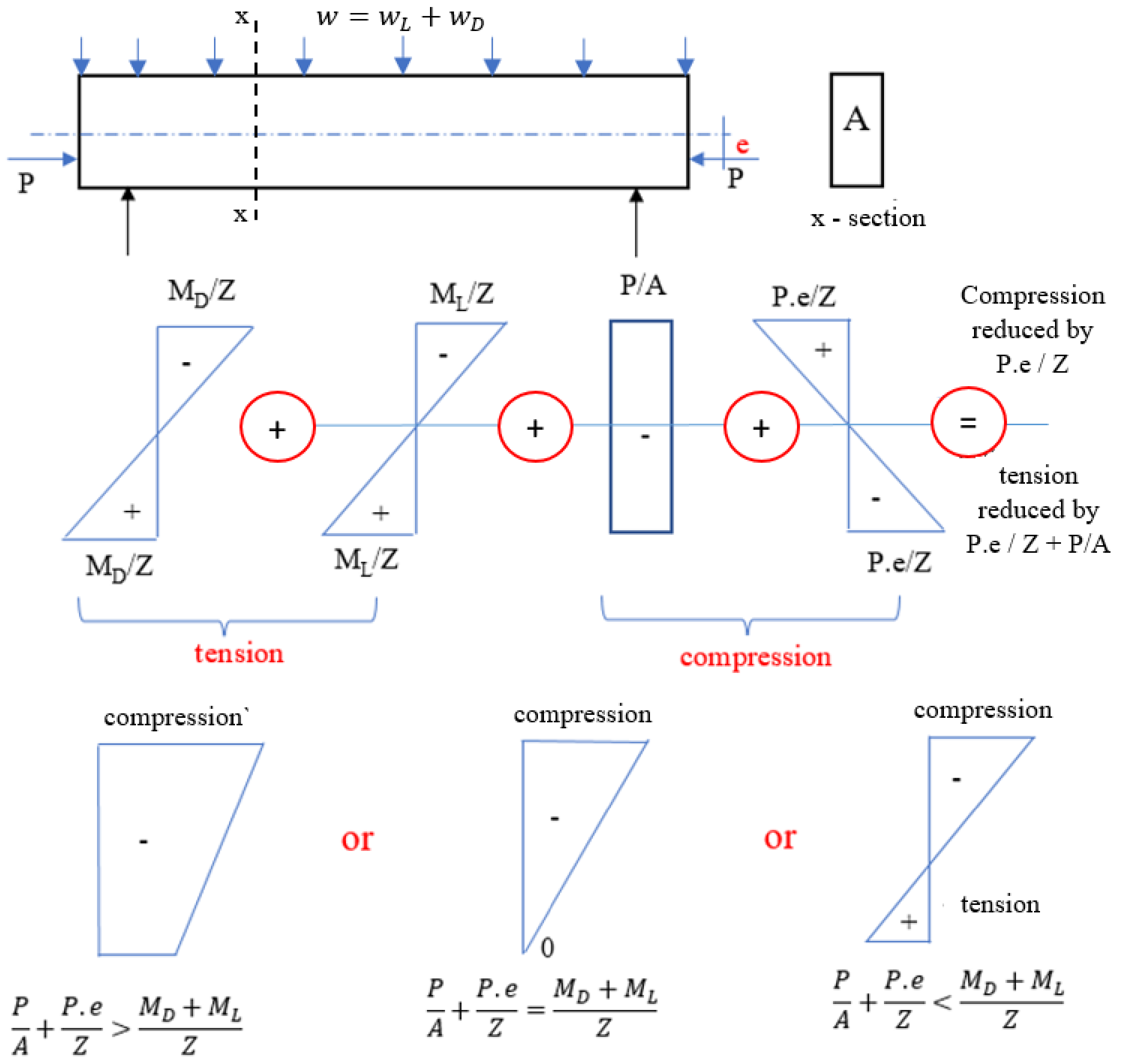


Figure 7: Stress resultant due to prestressing force

Example (7-1):

A simply supported prestressed concrete beam of rectangular section 100 x 250 mm^2 and 9 m span. Prestressed by a straight cable with an effective force 250 kN and an eccentricity e = 40 mm below the centroid of the section. Live load is 1.25 kN/m.

1- Find the resultant stresses distribution at mid-span.
2- Find the force to give zero stress in the bottom fibres at mid-span.

Solution:

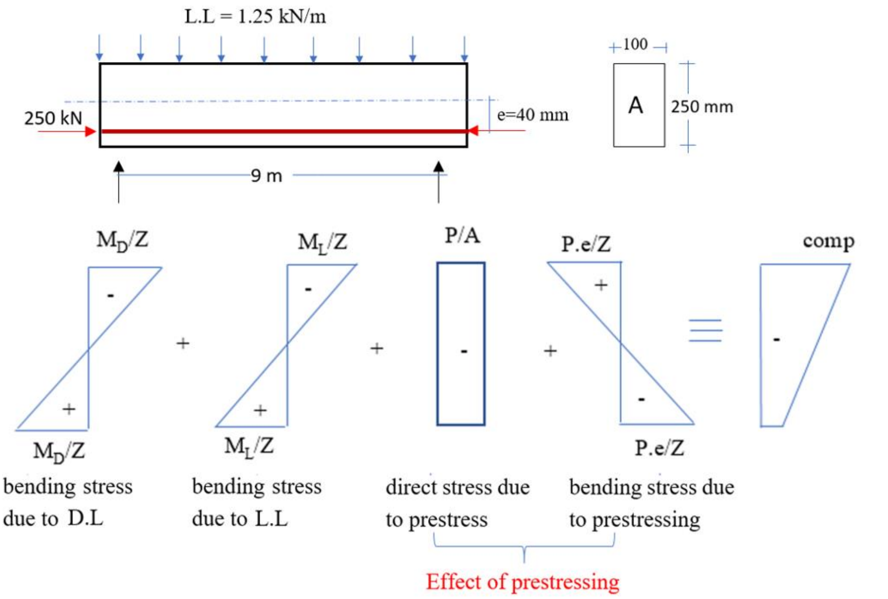
P= 250 kN, e = 40 mm, A = 250 x 100 = 25 x 10^3 mm^2

$I = \frac{1}{12} x 100 x 250^3 = 130.2x10^6\ mm^4$ (moment of inertia)

$y = \frac{250}{2} = 125\ mm$ and $Z = \frac{I}{y} = \frac{130.2x10^6}{125} = 1.04x10^6\ mm^3$

w_D = 0.1 x 0.25 x 24 = 0.6 kN/m

$M_D = \frac{w_D\ x\ L^2}{8} = \frac{0.6\ x\ 9^2}{8} = 6.1\ kN.m$ and $M_L = \frac{w_L\ x\ L^2}{8} = \frac{1.25\ x\ 9^2}{8} = 12.7\ kN.m$



1- Resultant stresses distribution

- At top fiber:

$$= -\frac{P}{A} + \frac{P.e}{Z} - \frac{M_D}{Z} - \frac{M_L}{Z} = -\frac{250x10^3}{25x10^3} + \frac{250x10^3x40}{1.04x10^6} - \frac{6.1x10^6}{1.04x10^6} - \frac{12.7x10^6}{1.04x10^6}$$

$$= -10 + 9.62 - 5.87 - 12.21 = -18.46\ N/mm^2$$ (compression)

- At bottom fiber:

$$= -\frac{P}{A} - \frac{P.e}{Z} + \frac{M_D}{Z} + \frac{M_L}{Z}$$

$$= -10 - 9.62 + 5.87 + 12.21 = -1.54\ N/mm^2$$ (compression)

2- Force at zero stress in the bottom

P = ? e = 40 mm

Balanced sterss i.e. (tension = copmression)

Stresses at bottom fiber

$$-\frac{P}{A} - \frac{P.e}{Z} + \frac{M_D}{Z} + \frac{M_L}{Z} = 0$$

$$-\frac{Px10^3}{25x10^3} - \frac{Px10^3x40}{1.04x10^6} + \frac{6.1x10^6}{1.04x10^6} + \frac{12.6x10^6}{1.04x10^6} = 0$$

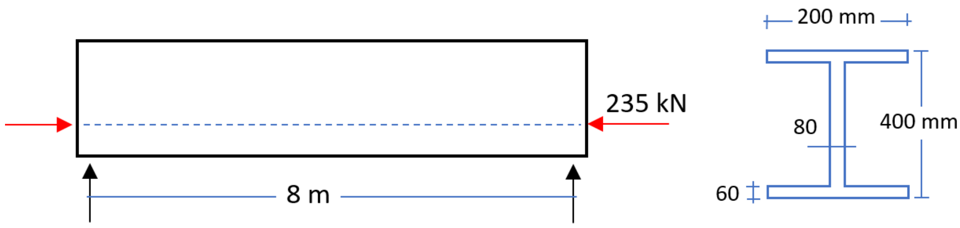
$$-0.04\ P - 0.038P + 5.87 + 12.21 = 0$$

$$\therefore P = \frac{18.1}{0.078} = 232\ kN$$

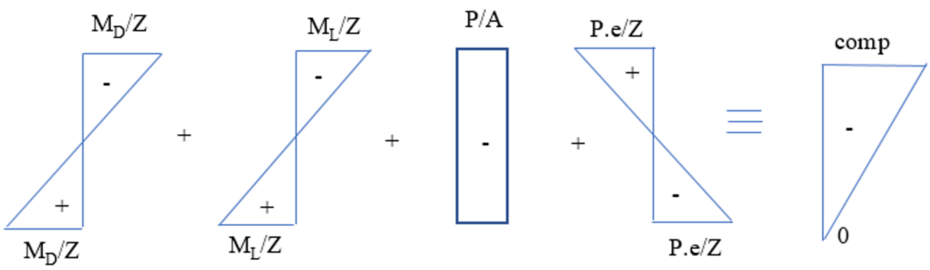
Example (7-2):

A prestressed simply supported beam 8 m span with an I-section and L.L = 4 kN/m. The effective (prestress force) is 235 kN.

1- Find the value of the eccenticricity (e=?) that makes the resultant stress in the bottom fibers at mid-span equal to zero.
2- **If the tendon passes through the centroid of the section**, find the prestress force that makes the resultant stress at the bottom of the section at mid-span equal to zero.



Solution:



A = 200 x 60 x 2 + 80 x (400-2 x 60) = 46.4 x 10^3 mm^2

$$I = \frac{1}{12}x200x60^3x2 + 200x60x170^2x2 + \frac{1}{12}x80x(400-2x60)^3$$

$$= 847.15x10^6\ mm^4$$

$$y = \frac{400}{2} = 200\ mm$$

$$Z = \frac{I}{y} = \frac{847x10^6}{200} = 4.24\ x10^6\ mm^3$$

$$w_D = \frac{46.4x10^3}{10^6}x\ 24 = 1.11\ kN/m$$

$$M_D = \frac{1.11x8^2}{8} = 8.88\ kN.m \quad \text{and } M_L = \frac{4x8^2}{8} = 32\ kN.m$$

1. $0 = -\frac{235x10^3}{46.4x10^3} - \frac{235x10^3x\ e}{4.24x10^6} + \frac{8.88x10^6}{4.24x10^6} + \frac{32x10^6}{4.24x10^6}$ ……………… e = ?

0= – 5.06 – 0.055 e + 2.1 + 7.55

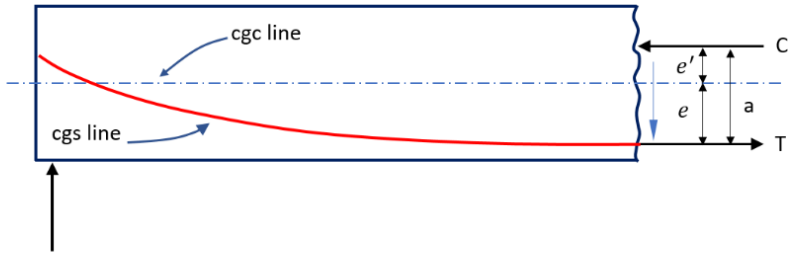
e ≈ 84 mm

2. $0 = -\frac{Px10^3}{46.4x10^3} - 0 + 2.1 + 7.55$ ……………… e = zero

$\Rightarrow P \approx 448\ kN$

Hint:

CGC: *Center of gravity of concrete section*
CGS: *Center of gravity of prestressing tendon*



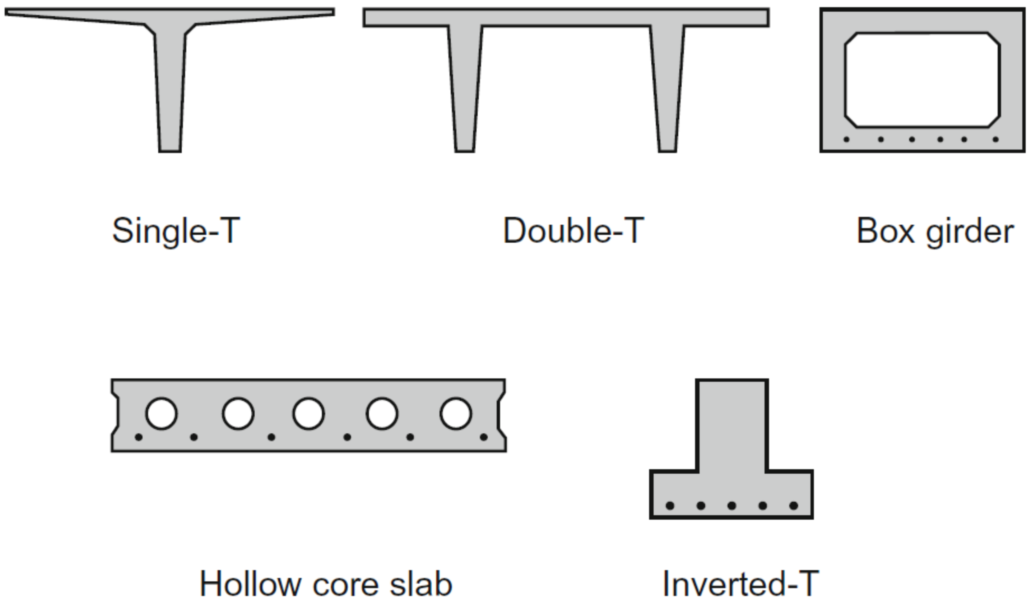


Figure 8: Typical shape of prestress concrete

8- Loss of Prestress Force

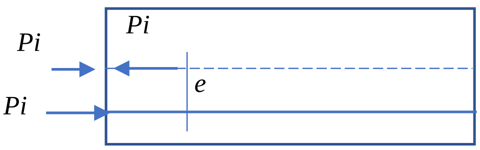
Several factors cause the force in the prestressing tendons to fall from the initial value imparted by the jacking system. Some of these losses are immediate, affecting the prestress force as soon as it has transferred to the concrete member. Other losses occur gradually with time. These short and long-term losses have summarized in Table 4.

Table 4: Short and long term prestress losses

	Losses type	Posttensioned	Pretensioned
Short-term	Elastic shortening	No*	Yes
	Anchorage draw-in (slip)	Yes	No
	Friction	Yes	No
Long-term	Concrete shrinkage	Yes	Yes
	Concrete creep	Yes	Yes
	Steel relaxation	Yes	Yes

* *No loss due to elastic deformation if all wires are simultaneously tensioned. If the wires are successively tensioned, there will be losses of prestress due to elastic deformation equal to one-half of the total initial prestress force.*

1- Elastic shortening of the concrete:



$\Delta fs_{\ loss}(elastic) = Es.\frac{fc}{Ec} = n.fc$

Where *n (modular ratio) = Es / Ec*

fs = stress in steel.

fc = stress in concrete at the level of prestressing wire.

Ec= modules of elasticity of concrete

Es= modules of elasticity of steel

$$fc = -\frac{P_i}{A} - \left(\frac{P_i.e}{I/e}\right) + \frac{M_o}{I/e} = -\frac{P_i}{A}\left(1 + \frac{e^2}{r^2}\right) + \frac{M_o.e}{I}$$

$$-\frac{P_i}{A}\left(1 + \frac{A\,.\,e^2}{I}\right) = -\frac{P_i}{A}\left(1 + \frac{e^2}{I/A}\right)$$

$$= -\frac{P_i}{A}\left(1 + \frac{e^2}{r^2}\right)$$

M_o =full moment due to self-weight

r = radius of gyration = $\sqrt{I/A}$

***P_i* = initial prestressing force, is taken approximately to <u>(10 %)</u> less than the jacking force P_J.**

2- Anchorage slip:

It is caused due to the slipping of wedges and also due to the deformation of anchorage.

$\Delta fs = \frac{\Delta L}{L} Es$

$\frac{\Delta L}{L} = \varepsilon,\ \Delta L$ = shortening of the wire, L = original length of the wire

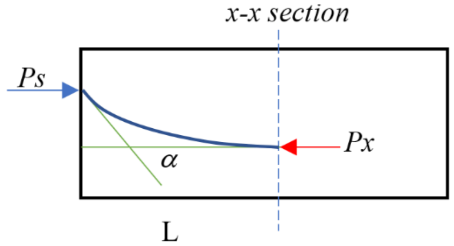
3- Loss of stress due to friction:

The magnitude of the loss of stress due to friction is of the following types:

a. Loss due to curvature effect (intentional friction), depends upon the tendon form or alignment, which generally follows a curved profile along the length of the beam.

b. Loss of stress due to the wobble effect (unintentional friction), which depends upon the local deviations in the alignment of the cable. The wobble or wave effect is the result of accidental or unavoidable misalignment since ducts or sheaths cannot be perfectly located to follow a predetermined profile throughout the length of the beam.

$$Ps = Px.e^{K.L+\mu.\alpha}$$



Where:

Ps = force at the jacking end of the tendons.

Px = force at any section (x) along the tendon.

K = wobble friction coeifficent.

L = tendon length from jacking end to the section (x).

μ = curvature friction coefficent.

α = angular change of tendon from jacking end to section x.

Hint:

When $K.L + \mu.\alpha \ngtr 0.3$, the formula will be:

$Ps = Px.(1 + K.L + \mu.\alpha)$……………………….. ACI Code simplified form

Table 5: Friction coefficients for post-tensioned tendons (A. H. Nilson)

Type of Tendon	Wobble Coefficient K, per m	Curvature Coefficient μ
Grouted tendons in metal sheathing		
Wire tendons	0.0033 - 0.0049	0.15 - 0.25
High-strength bars	0.0003 - 0.0020	0.08 – 0.30
Seven-wire strand	0.0016 - 0.0066	0.15 – 0.25
Unbonded tendons		
Mastic-coated wire tendons	0.0033 - 0.0066	0.05 - 0.15
Mastic-coated seven-wire strand	0.0033 - 0.0066	0.05 - 0.15
Pre-greased wire tendons	0.0010 - 0.0066	0.05 - 0.15
Pre-greased seven-wire strand	0.0010 - 0.0066	0.05 - 0.15

4- Creep of concrete:

The sustained prestress in the concrete of a prestress member results in the creep of concrete, which is effectively reduces the stress in high tensile steel. The loss of steel stresses due to the creep of concrete can be estimated if the magnitude of ultimate creep strain or creep-coefficient is known.

$\Delta fs_{creep} = C_c . n . fc$

C_c= creep coefficient, from (1.4 for $f_c{}' = 83\ MPa$) to (3.1 $f_c{}' = 21\ MPa$)

$n = \frac{Es}{Ec}$

fc = stress in concrete at the level of steel centroid, when the eccentric prestress force plus all sustained loads are acting.

Hint: Equation for calculating *fc* in elastic shortening can be used except that M_o should be replaced by moment due to all dead loads plus that due to any position of the live load that may be considered sustained. To account for this, it is taken 10 % less than (*Pi*).

5- Shrinkage of concrete:

$\Delta fs_{shrink} = \varepsilon_{sh} . Es$

ε_{sh} = 0.0004 - 0.0008 and the typical value = 0.0006

6- Relaxation of steel:

Elongation of steel under sustained loads is called relaxation.

$\frac{f_p}{f_{pi}} = 1 - \frac{\log t}{10} (\frac{f_{pi}}{f_{py}} - 0.55)$, $\frac{f_{pi}}{f_{py}}$ **not less than 0.55, below that value no relaxation occurs.**

f_p: finl stress after (t) hours, *log t* is to the base 10.

f_{pi} : initial stress

f_{py}: nominal yield stress

7-Lump-Sum Losses

Following the transfer of the prestressing force from the jack to the concrete member, a continuous loss in the prestressing force occurs; the total loss of prestress is the reduction in the prestressing force during the life span of the structure.

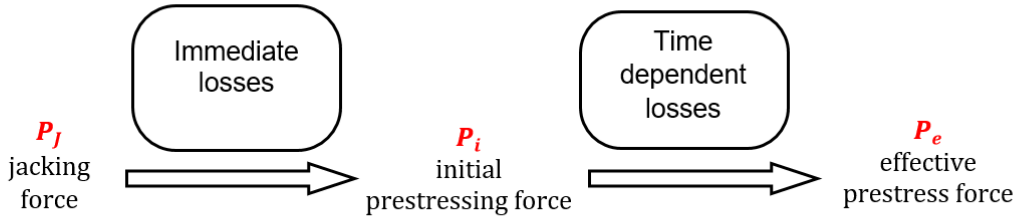
For most normal-weight concrete structures constructed by standard methods, the tendon stress loss due to elastic shortening, shrinkage, creep, and relaxation of steel (friction and anchorage slip are not included) is about:

- *pretensioned members* - 241 MPa
- *posttensioned members* - 172 MPa

Prestressing Forces Reduced with Losses (How?):

1	P_J = jacking force		
2	Immediate losses P_i = initial prestressing force (after):	• Elastic shortening loss • Frictional loss • Slip of anckorage loss	Loss in P_J
3	Time dependent losses P_e = effective prestress force (after):	• Shrankage loss • creep loss • relaxation loss	Loss in P_i

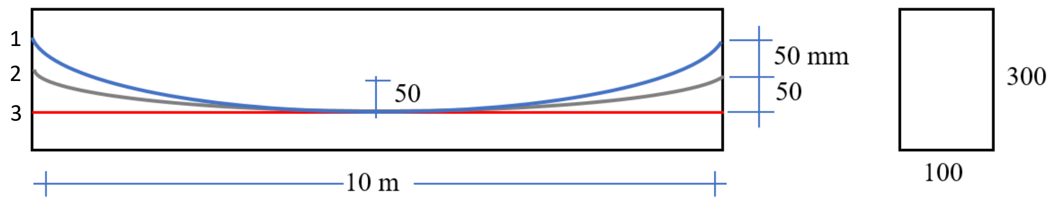
$$\textbf{Effective ratio } (\boldsymbol{R}) = \frac{\boldsymbol{P_e}}{\boldsymbol{P_i}}$$

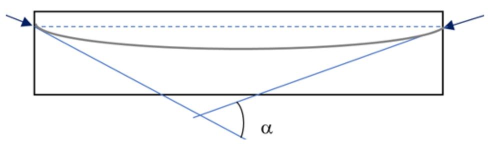


Example (8-1):

A posttentioned concrete beam (100 mm x 300 mm) is tentioned by three cables and anchoraged. The cross-sectional area of each cable is 200 mm^2 and the initial stress in the cable is 1200 N/mm^2. *Estemate the percentage friction loss in each cable.*

Note that (μ Curvature Coefficient =0.25, K wobble Coefficient = 0.0015), cable profile $y = \frac{4.e.x}{L^2}(L - x)$.





Solution:

$y = \frac{4.e.}{L^2}(L.x - x^2)$ $\Rightarrow$ $\frac{dy}{dx} = \frac{4.e}{L^2}(L - 2.x)$,

$$Ps = Px.e^{K.L+\mu.\alpha}$$

when x = 0, then $\frac{dy}{dx})_0 = \frac{4.e}{L}$

Cable 1	Cable 2	Cable 3
Slope at the end = $\frac{4x100}{10x1000}$ = 0.04 rad, Comulative angle between tangents $\alpha = 2\ x\ 0.04 = 0.08\ rad$	Slope at the end = $\frac{4x50}{10x1000}$ = 0.02 rad, Comulative angle between tangents $\alpha = 2\ x\ 0.02 = 0.04\ rad$	Slope at the end =0, Comulative angle between tangents $\alpha = 2\ x\ 0 = 0$

Iinitial prestressing force in each cable at jacking end:

Ps = cable area *x* initial stress = $200 \times 1200 = 240 \times 10^3$ *N* = *240 kN*

If *Px* is the prestresssing force at the far end, then

$K.L + \mu.\alpha = 0.0015 \times 10 + 0.25 \times 0.08 = 0.035 < 0.3$

Hance $Ps = Px\,(1 + K.L + \mu.\alpha) \Rightarrow Px = \frac{Ps}{1+K.L+\mu.\alpha}$

Losses of prestress $\Rightarrow$ $Ps = Px + Px\,(K.L + \mu.\alpha) \Rightarrow Ps - Px = Px\,(K.L + \mu.\alpha)$

Cable 1 $Px = \frac{Ps}{1+K.L+\mu.\alpha} = \frac{240x10^3}{1+0.035} = \frac{240x10^3}{1.035} = 231.88x10^3\ N = 231.88\ kN$

Losses of prestress = $231.88\ x\ 0.035 = 8.12\ kN$ *or = 240 – 231.88 = 8.12 kN*

Cable 2 $Px = \frac{Ps}{1+(0.0015x10+0.25\ x\ 0.04)} = \frac{240x10^3}{1.025} = 234.15x10^3\ N = 234.15\ kN$

Losses of prestress =$234.15\ x\ 0.025 = 5.85\ kN$ or =$240 - 234.15 = 5.85\ kN$

Cable 3 $Px = \frac{Ps}{1+(0.0015x10+0.25\ x\ 0)} = \frac{240x10^3}{1.015} = 236.45x10^3 N = 236.45\ kN$

Losses of prestress =$236.45\ x\ 0.015 = 3.55\ kN$ or =$240 - 236.45 = 3.55\ kN$

	Cable1	Cable 2	Cable 3
Losses % =	$\frac{240-231.88}{240}\ x100$= 3.38 %	$\frac{240-234.15}{240}x100$ = 2.44 %	$\frac{240-236.45}{240}x100$ = 1.48 %

Example (8-2):

A simply supported concrete beam with cross section (100 x 300 mm) is posttentioned by three (striaght) cables, **neglect beam self weight**. Estimate the percentage loss due to elastic shortenting of the concrete when:

1- The whole cables are tentioned simulatonsouly.
2- The three clables are tentioned successively.

e = 50 mm, initial stress = 1200 N/mm^2*, cable cross section = 50 mm*^{2}*,* $f_c{}'$ *= 35.5 N/ mm*^{2}

$Ec = 4700\sqrt{f_c{}'} = 4700\sqrt{35.5} = 28000\ N/mm^2$ (ACI 318-19)

$Es = 2\ x10^5\ N/mm^2$

$n\ (modular\ ratio) = \frac{Es}{Ec} = \frac{2x10^5}{28x10^3} = 7.14 \approx 7$

Solution:

Case I: No losses (No loss due to elastic deformation if all wires are simultaneously tensioned)

Case II:

$P = 50 \times 1200 = 60 \times 10^3\ N = 60\ kN$

A = 30 $x10^3$ mm^2, I = 225 $x10^6$ mm^4, e = y = 50 mm, Z = I/e

Stress in concrete at level of steel;

$$fc = -\frac{P}{A} - \frac{P.e}{I/e} = -\frac{60x10^3}{30x10^3} - \frac{60\ x\ 10^3\ x\ 50\ x\ 50}{225x10^6} = -2 - 0.67 = -2.67\ N/mm^2$$

Cable 1: tenstioned and anchorage = No losses

Cable 2: tenstioned and anchorage, losses in cable (1)

Losses (cable 1) = $n\ x\ fc = 7\ x\ 2.67 = 18.69\ N/mm^2$

Cable 3: tenstioned and anchorage, losses in cables (1) and (2)

Losses (cable 1) = $n\ x\ fc = 7\ x\ 2.67 = 18.69\ N/mm^2$

Losses (cable 2) = $n\ x\ fc = 7\ x\ 2.67 = 18.69\ N/mm^2$

Therefore the whole losses of stress due to elastic shortening of concrete is:

Cable 1 = $18.69 + 18.69$ = 37.38 N/mm^2

Cable 2 = 18.69= 18.69 N/mm^2

Cable 3 = zero

The avarage losses in the whole cables = (37.38+18.69) / 3 = 18.69 N/mm^2

Perecntage losses = $\frac{18.69}{1200}\ x100 = 1.56\ \%$

9- Elastic Flexural Analysis

Figure (9 *a*) shows a prestressed simple beam span with curved tendons. The portion of the beam to the left of a vertical cutting plane *x*-*x* is taken as a free body, with forces acting as shown in Figure (9 *b*). The force ***P*** at the left end is exerted on the concrete through the tendon anchorage, while the force ***P*** at the cutting plane *x*-*x* results from combined shear and normal stresses acting at the concrete surface at that location. The direction of ***P*** is tangent to the curve of the tendon at each location.

Note the presence of the force ***N***, acting on the concrete from the tendon, due to tendon curvature. This force will be distributed in some manner along the length of the tendon, ***the exact distribution depending upon the tendon profile***. Its resultant and the direction in which the resultant acts can be found from the force diagram of Figure (9 *c*).

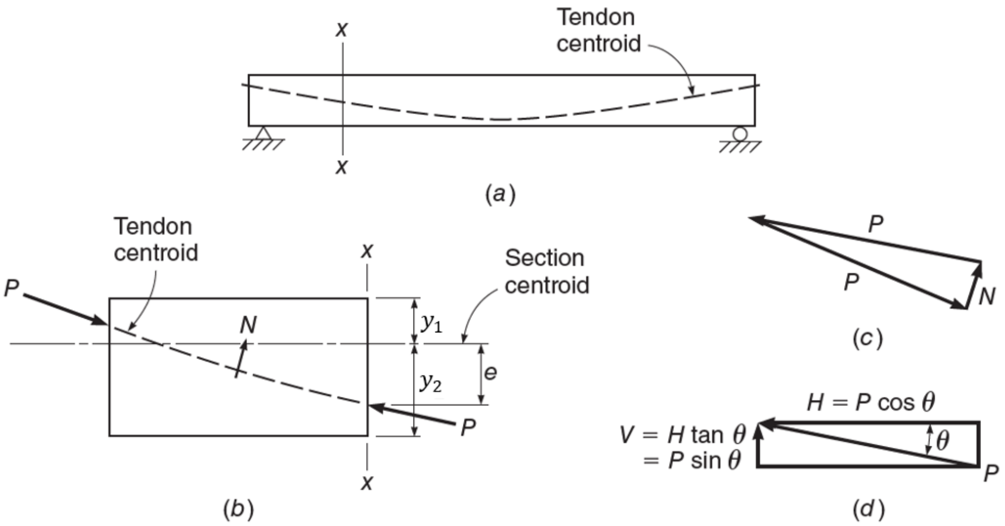


Figure 9: Prestressing forces acting on concrete.

It is convenient to divide the prestressing force ***P*** into its components in the horizontal (H = $\boldsymbol{P\cos\theta}$) and vertical directions ($\boldsymbol{V = H\tan\theta = P\sin\theta}$), Figure (8 *d*). Since the slope angle is normally quite small, ($\boldsymbol{\cos\theta \approx 1}$), and therefore $\boldsymbol{H = P}$. The magnitude of the prestress force is not constant. The *jacking force* $\boldsymbol{P_j}$ is immediately reduced to the *initial prestress force* $\boldsymbol{P_i}$ because of:

- Elastic shortening of the concrete upon transfer,
- Slip of the tendon as the force is transferred from the jacks to the beam ends, and
- Loss due to friction between the tendon and the concrete (posttensioning) or between the tendon and the strand alignment devices (pretensioning).

There is a further reduction of force from $\boldsymbol{P_i}$ to the *effective prestress* $\boldsymbol{P_e}$, occurring over a long period of time at a gradually decreasing rate, because of:

- Concrete creep under the sustained prestress force,
- Concrete shrinkage, and
- Relaxation of stress in the steel.

In developing elastic equations for flexural stress, the effects of prestressing force, self-weight moment, and dead and live load moments **are calculated separately**, and the separate stresses are superimposed.

When the initial prestress force $\boldsymbol{P_i}$ is applied with an eccentricity $\boldsymbol{e}$ below the centroid of the cross section with area $\boldsymbol{Ac}$ and top and bottom fiber distances y_1 and y_2, respectively, it causes the compressive stress $-\frac{Pi}{Ac}$ and the bending stresses $\frac{P_i.e.y_1}{Ic}$ and $-\frac{P_i.e.y_2}{Ic}$ in the top and bottom fibers, respectively (compressive stresses are designated as negative, tensile stresses as positive), as shown in Figure (9*a*), the stress is:

Top fiber stress	$f^t = -\frac{P_i}{Ac} + \frac{P_i.e.y_1}{Ic} = -\frac{P_i}{Ac}(1 - \frac{e.y_1}{r^2})$
Bottom fiber stress	$f_b = -\frac{P_i}{Ac} - \frac{P_i.e.y_2}{Ic} = -\frac{P_i}{Ac}(1 + \frac{e.y_2}{r^2})$

Where r is the radius of gyration of the concrete section. Normally, as the eccentric prestress force is applied, the beam deflects upward. The beam self-weight $\boldsymbol{w_o}$ then causes additional moment $\boldsymbol{M_o}$ to act, and stresses become as shown in Figure (*b*).

Top fiber stress	$f^t = -\frac{P_i}{Ac}\left(1 - \frac{e.y_1}{r^2}\right) - \frac{M_o.y_1}{Ic}$
Bottom fiber stress	$f_b = -\frac{P_i}{Ac}(1 + \frac{e.y_2}{r^2}) + \frac{M_o.y_2}{Ic}$

At this stage, time-dependent losses due to shrinkage, creep, and relaxation commence, and the prestressing force gradually decreases from $\boldsymbol{P_i}$ to $\boldsymbol{P_e}$. It is usually acceptable to assume that all such losses occur prior to the application of service loads, since the concrete stresses at service loads will be critical after losses, not before. Accordingly, the stresses in the top and bottom fiber, with $\boldsymbol{P_e}$ and beam load acting, become

Top fiber stress	$f^t = -\frac{P_e}{Ac}\left(1 - \frac{e.y_1}{r^2}\right) - \frac{M_o.y_1}{Ic}$
Bottom fiber stress	$f_b = -\frac{P_e}{Ac}(1 + \frac{e.y_2}{r^2}) + \frac{M_o.y_2}{Ic}$

When the full service loads (dead load and self-weight of the beam + service live load) are applied, see Figure (9*c*), the stress are:

Top fiber stress	$f^t = -\frac{P_e}{Ac}\left(1 - \frac{e.y_1}{r^2}\right) - \frac{(M_o + M_D + M_L).y_1}{Ic}$
Bottom fiber stress	$f_b = -\frac{P_e}{Ac}\left(1 + \frac{e.y_2}{r^2}\right) + \frac{(M_o + M_D + M_L).y_2}{Ic}$

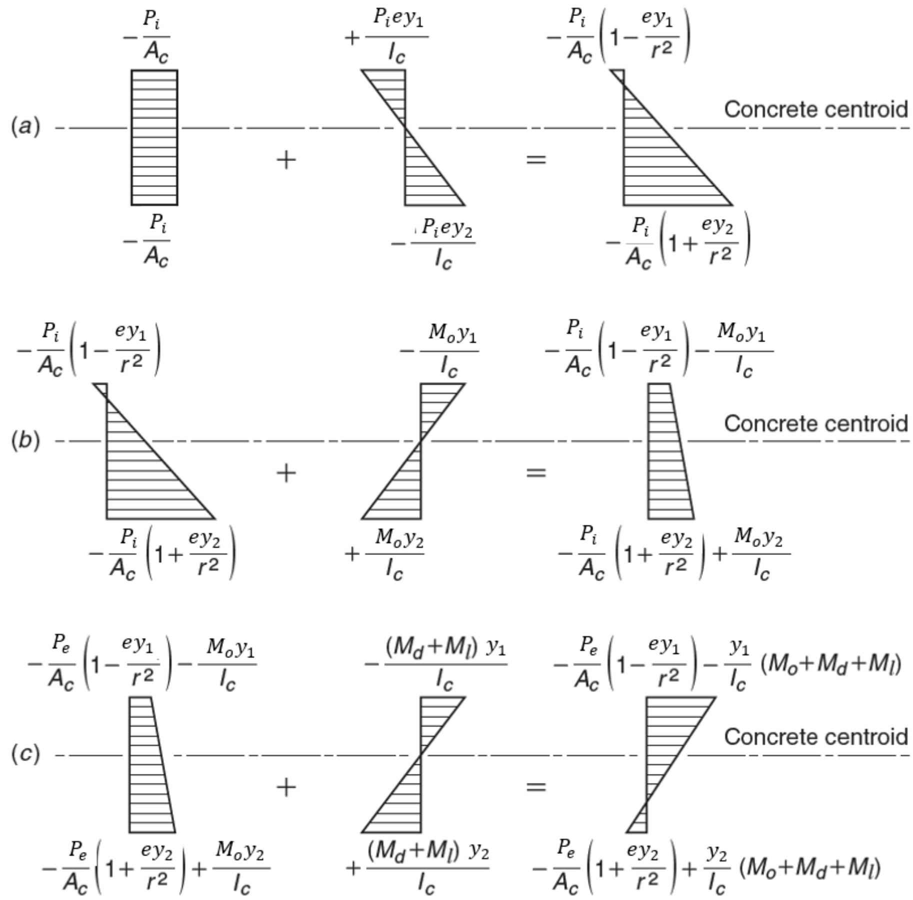
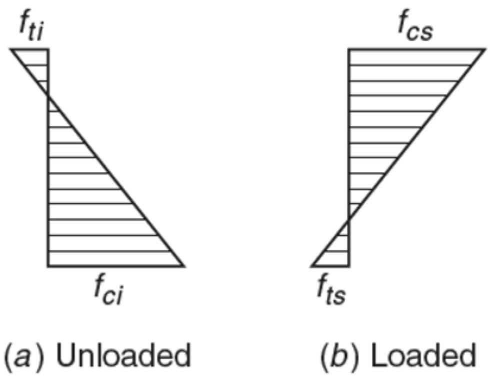


Figure 10: Concrete stress distributions in beams: (a) effect of prestress; (b) effect of prestress plus self-weight of beam; and (*c*) effect of prestress, self-weight, and external dead and live service loads.

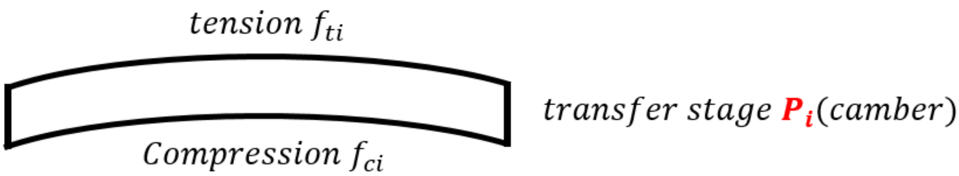
It is necessary, in reviewing the adequacy of a beam (or in designing a beam on the basis of permissible stresses, **see Table (1)**), that the stresses in the extreme fibers remain within specified limits under any combination of loadings that can occur. Normally, the stresses at the section of maximum moment, in a properly designed beam, must stay within the limit states defined by the distributions shown in Figure (10) as the beam passes from the unloaded stage ($\boldsymbol{P_i}$ plus self-weight) to the loaded stage ($\boldsymbol{P_e}$ plus full service loads).

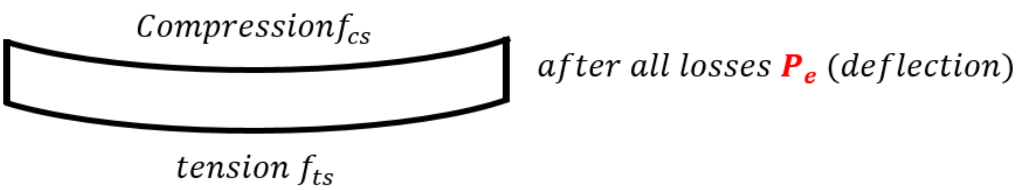


f_{ci} and f_{ti} are the permissible compressive and tensile stresses, respectively, in the concrete immediately after transfer.
f_{cs} and f_{ts} are the permissible compressive and tensile stresses at service loads.

Figure 11: Stress limits: (*a*) unloaded beam with initial prestress plus self-weight, and (b) loaded beam, with effective prestress, self-weight, and full service load.

Stress at Transfer	$f^t = -\frac{P_i}{Ac}\left(1-\frac{e.y_1}{r^2}\right) - \frac{M_o.y_1}{Ic}$	$\leq f_{ti}$
	$f_b = -\frac{P_i}{Ac}(1+\frac{e.y_2}{r^2}) + \frac{M_o.y_2}{Ic}$	$\leq f_{ci}$
Effective stresses after losses	$f^t = -\frac{P_e}{Ac}\left(1-\frac{e.y_1}{r^2}\right) - \frac{M_o.y_1}{Ic}$	$\leq f_{ts}$
	$f_b = -\frac{P_e}{Ac}(1+\frac{e.y_2}{r^2}) + \frac{M_o.y_2}{Ic}$	$\leq f_{cs}$
Service load (Final stress)	$f^t = -\frac{P_e}{Ac}\left(1-\frac{e.y_1}{r^2}\right) - \frac{(M_o+M_D+M_L).y_1}{Ic}$	$\leq f_{cs}$
	$f_b = -\frac{P_e}{Ac}\left(1+\frac{e.y_2}{r^2}\right) + \frac{(M_o+M_D+M_L).y_2}{Ic}$	$\leq f_{ts}$





Example (9-1):

A posttentioned simply supported concrete beam of 12 m span, tensioned by single tendon of parabolic profile, beam cross section (b=280 mm, h=710 mm), e_{max} = 200 mm, e_{min}=0, initial prestresing force (Pi) =1500 kN. Effective ratio R = 0.84, L.L. = 15 kN/mm^2, D.L.=4.4 kN/mm^2, $f_c{}'$ = 34 N/mm^2 , $f_{ci}{}'$ (compressive strength of concrete at the time of initial prestress)= 28 N/mm^2.

Find bending stress distribtion at mid-span sction for the following cases:

1- At initial condition before applying imposed load.
2- At full service load.

Check reslts with the ACI-Code requirements

Solution:

Allowable stress (ACI-Code), see Table 1

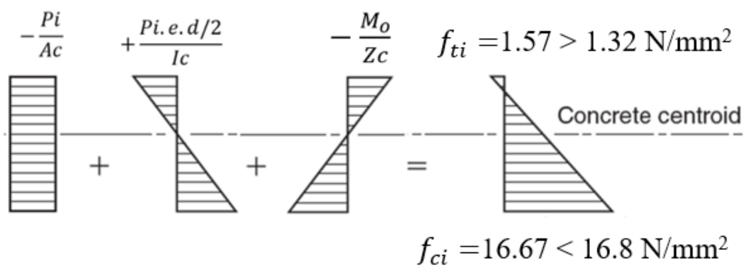
1- At transfare ($f_{ci}{}'$)
   a- Compression fci = 0.6 $f_{ci}{}'$ = 0.6 x 28 = 16.8 N/mm^2
   b- Tension fti = 0.25 $\sqrt{f_{ci}{}'}$ = 0.25 $\sqrt{28}$ = 1.32 N/mm^2
2- At service load ($f_c{}'$)
   a- Compression fcs = 0.45 $f_c{}'$ = 0.45 x 34 = 15.3 N/mm^2
   b- Tension fts = 0.6 $\sqrt{f_c{}'}$ = 0.6 $\sqrt{34}$ = 3.5 N/mm^2

Case 1:

Initial condition = prestress + self-weight

A = 280 x 710 = 198.8 x 10^3 mm^2, I = $\frac{280}{12}\ x\ 710^3$ = 8351 x 10^6 mm^4

Z = $\frac{I}{y} = \frac{8351x10^6}{710/2}$= 23.5 $x\ 10^6$ mm^3



w$_O$ = 0.28 x 0.71 x 24 = $4.77\ kN/m$

M$_O$= $\frac{w.L^2}{8} = \frac{4.77x\ 12^2}{8} = 85.86\ kN.m$

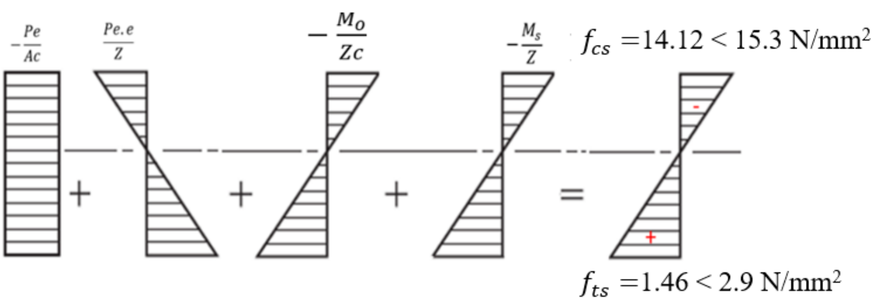
Stress at top fiber (f_{ti}) = $-\frac{P_i}{Ac} + \frac{P_i.e}{Zc} - \frac{M_O}{Zc} = -\frac{1500x\ 10^3}{198.8\ x\ 10^3} + \frac{1500x\ 10^3x200}{23.5\ x\ 10^6} - \frac{85.86x\ 10^6}{23.5\ x\ 10^6}$

= -7.55 + 12.77 – 3.65 = + 1.57 N/mm^2 (tension) > 1.32 N/mm^2 Not ok

Try new section 300 x 710, is f_{ti} check?

Stress at bottom fiber (f_{ci}) = -7.55 - 12.77 + 3.65 = - 16.67 N/mm^2 (comp.) < 16.8 N/mm^2 ok

Case 2:



Ms= $\frac{(w_D+w_L)x\ L^2}{8} = \frac{(4.4+15)x\ 12^2}{8} = 349.2\ kN.m$

R= $\frac{Pe}{Pi}$, P$_e$ = R x P$_i$ = 0.84 x 1500 = 1260 kN

Stress at top fiber (f_{cs}) = $-\frac{P_e}{Ac} + \frac{P_e.e}{Zc} - \frac{M_O}{Zc} - \frac{M_S}{Zc} = -\frac{Pe}{Ac} + \frac{P_e.e}{Zc} - \frac{M_O+M_S}{Zc}$

$= -\frac{1260x\ 10^3}{198.8\ x\ 10^3} + \frac{1260x\ 10^3x200}{23.5\ x\ 10^6} - \frac{(85.86+349.2)x\ 10^6}{23.5\ x\ 10^6}$

= -6.33+10.72 – 18.51 = -14.12 N/mm^2 (comp) < 15.3 N/mm^2 ok

f_{ts}= - 6.33 - 10.72 +18.51 = 1.46 N/mm^2 (tension) < 3.5 N/mm^2 ok

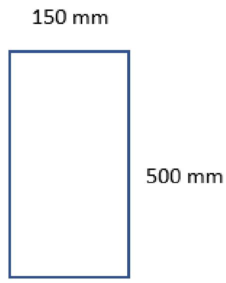
Example (9-2):

A posttentioned simply supported concrete beam of 10 m span with cross section of (b=150 mm, h=500 mm), at transfer $f_c' = 27$ N/mm^2 , $f_{ci}' = 21$ N/mm^2, Effective ratio R = 0.8

Find the following:

1. Value of (Pi) and (e) for maximum bending moment that can be applied due to service loading ($Ms = M_D + M_L$).
2. Maximum UDL which the beam can carry.

Solution:



Allowable stress (ACI-Code)

a- $fcs = -0.45\ f_c' = -0.45 \times 27 = -12.15$ N/mm^2

b- $fts = 0.6\sqrt{f_c'} = 0.6 \times \sqrt{27} = +3.12$ N/mm^2

c- $fci = -0.6\ f_{ci}' = -0.6 \times 21 = -12.6$ N/mm^2

d- $fti = 0.25\sqrt{f_{ci}'} = 0.25 \times \sqrt{21} = +1.14$ N/mm^2

$Ac = 150 \text{ x } 500 = 75 \text{ x} 10^3$ mm^2, $w_o = 0.15 \text{ x } 0.5 \text{ x } 24 = 1.8$ kN/m

$I = \frac{1}{12}\ x\ 150\ x\ 500^3 = 1.5625 \text{ x} 10^9$ mm^4, $Z = \frac{I}{y} = \frac{1.5625\ x 10^9}{250} = 6.25\ \ x 10^6$ mm^3

$$fcs = -\frac{P_e}{Ac} + \frac{P_e.e}{Z} - \frac{M_o}{Z} - \frac{Ms}{Z} \quad -1$$

$$fts = -\frac{P_e}{Ac} - \frac{P_e.e}{Z} + \frac{M_o}{Z} + \frac{Ms}{Z} \quad -2$$

$$fcs + fts = -2\ \frac{P_e}{Ac} \quad \text{Adding eqs (1+2)}$$

$$\Rightarrow -12.15 + 3.12 = -2\ \frac{P_e}{Ac} \quad -3$$

$$P_e = \frac{9.03\ x\ 75\ x\ 10^3}{2\ x\ 10^3} = 338.6 \text{ kN}$$

$$R = 0.8 = \frac{Pe}{Pi} \Rightarrow P_i = \frac{338.6}{0.8} = 423.3\ kN$$

$$M_o = \frac{1.8\ x 10^2}{8} = 22.5\ kN.m$$

To find e:

- From $fti = -\frac{P_i}{Ac} + \frac{P_i.e}{Z} - \frac{M_o}{Z} \Rightarrow 1.14 = -\frac{423.3\ \ x\ 10^3}{75\ x\ 10^3} + \frac{423.3\ \ x\ 10^3.e}{6.25\ \ x\ 10^6} - \frac{22.5\ \ x\ 10^6}{6.25\ \ x\ 10^6}$

  $1.14 = -5.64 + 0.068\ e - 3.6 \Rightarrow e = 152.6\ mm$

- From $fci = -\frac{P_i}{Ac} - \frac{P_i.e}{Z} + \frac{M_o}{Z} \Rightarrow -12.6 = -\frac{423.3\ \ x\ 10^3}{75\ x\ 10^3} - \frac{423.3\ \ x\ 10^3.e}{6.25\ \ x\ 10^6} + \frac{22.5\ \ x\ 10^6}{6.25\ \ x\ 10^6}$

  $-12.6 = -5.64 - 0.068\ e + 3.6 \Rightarrow e = 155.3\ mm$ Hance $e_{max} = 152$ mm…… why?

$Ms = ?$

$$fcs = -\frac{P_e}{Ac} + \frac{P_e.e}{Z} - \frac{M_o}{Z} - \frac{Ms}{Z} \Rightarrow$$

$$-12.15 = -\frac{338.6 \ x\ 10^3}{75\ x\ 10^3} + \frac{338.6\ x\ 10^3 x 152}{6.25\ \ x\ 10^6} - \frac{22.5\ \ x\ 10^6}{6.25\ \ x\ 10^6} - \frac{Ms}{6.25\ \ x\ 10^6}$$

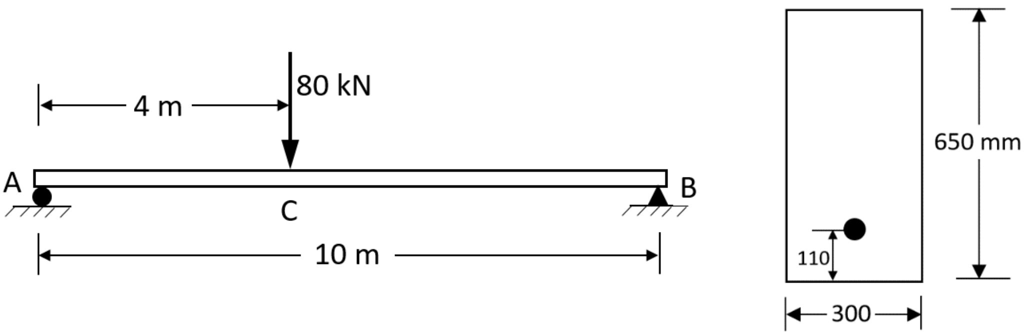
$$-12.15 = -4.51 + 8.23 - 3.6 - \frac{Ms}{6.25\ \ x\ 10^6} \Rightarrow Ms = 76.69 \text{ kN.m}$$

$M_s = M_D + M_L = 76.69$ kN.m

$w_s = \frac{Ms\ x\ 8}{10^2} = \frac{76.69\ x\ 8}{10^2} = 6.14$ kN/m

Example (9-3):

A prestressed concrete simply supported beam of 10 m span with a rctangular cross section, is loaded by a concentrated (live) load at 4 m from support A **(under point C)**, in addition to its own weight, as shown in figure (Q1). If the initial prestressed force is 980 kN, check the top and bottom stresses at **point (C)** for both transfer and service stages, and compare with the ACI code permissible stresses. $f_c{}' = 36$ N/mm^2 , $f_{ci}{}' = 23$ N/mm^2, Effective ratio R = 0.8.



Solution:

Allowable stress (ACI-Code)

$fci = -0.6\ f_{ci}{}' = -0.6 \times 23 = -13.8$ N/mm^2

$fti = 0.25\sqrt{f_{ci}{}'} = 0.25 \times \sqrt{23} = +1.2$ N/mm^2

$fcs = -0.45\ f_c{}' = -0.45 \times 36 = -16.2$ N/mm^2

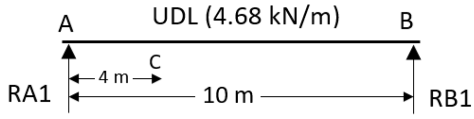
$fts = 0.6\sqrt{f_c{}'} = 0.6 \times \sqrt{36} = +3.6$ N/mm^2

$Ac = 300 \text{ x } 650 = 195 \text{ x}10^3$ mm^2,

$w_o = 0.65 \times 0.3 \times 24 = 4.68$ kN/m

$I = \frac{1}{12}\ x\ 300\ x\ 650^3 = 6.866 \text{ x}10^9$ mm^4, ………… y1=y2 = 650/2 = 325 mm

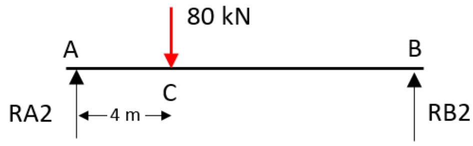
$Z = \frac{I}{y} = \frac{6.866\ x10^9}{325} = 21.126\ x10^6$ mm^3



$\sum fy = 0$ RA1=RB1 = 10 x 4.68/2 = 23.4 kN

$\sum Mo(c) = 23.4\ x\ 4 - 4.68\ x \frac{4x4}{2} = 56.16\ kN.m$

$\sum MB = RA2x10 - 80x6 = 0$



RA2 = 48 kN

$\sum ML(c) = 48\ x\ 4 = 192\ kN.m$

At transfer:

$Top\ stresses = -\frac{P_i}{Ac} + \frac{P_i.e}{Z} - \frac{M_o}{Z}$

$= -\frac{980\ x\ 10^3}{195x\ 10^3} + \frac{980\ x\ 10^3 x215}{21.126\ x\ 10^6} - \frac{56.16\ x\ 10^6}{21.126\ x\ 10^6}$

= - 5.03 + 9.97 – 2.66 = 2.28 MPa > 1.2 MPa ……. Not good

$bottom\ stresses = -\frac{P_i}{Ac} - \frac{P_i.e}{Z} + \frac{M_o}{Z}$

= - 5.03 - 9.97 + 2.66 = - 12.34 MPa < 13.8 MPa ……. OK

At service:

$Top = -\frac{P_e}{Ac} + \frac{P_e.e}{Z} - \frac{M_o}{Z} - \frac{Ms}{Z}$ …….. Pe = 0.8 x 980 = 784 kN

$= -\frac{784\ x\ 10^3}{195x\ 10^3} + \frac{784\ x\ 10^3 x215}{21.126\ x\ 10^6} - \frac{(56.16+192)\ x\ 10^6}{21.126\ x\ 10^6}$

= -4.02 + 7.98 – 11.74 = - 7.78 MPa < 16.2 MPa OK

Bottom = -4.02 - 7.98 + 11.74 = - 0.26 MPa < 3.6 MPa OK

Example (9-4):

Design a posttentioned simply supported concrete beam having the follwing data:

Beam length = 12 m, L.L. = 14.6 kN/m, D.L.= 7.3 kN/m, $f_c{}'$ = 40 N/mm^2,

$f_{ci}{}'$ = 29 N/mm^2, Effective ratio R = 0.85, e = variable eccentricity.

Specified tensile strength of prestressing reinforcement f_{pu}= 1034 N/mm^2 (alloy steel),

Solution:

Permissible stresses according to ACI Code:

- Comprssion at transfer fci = - 0.6 $f_{ci}{}'$ = - 0.6 × 29 = - 17.4 N/mm^2
- Tension at transfer fti = 0.25 $\sqrt{f_{ci}{}'}$ = 0.25 × $\sqrt{29}$ = + 1.34 N/ mm^2
- Comprssion at service fcs = - 0.45 $f_c{}'$ = - 0.45 × 40 = - 18 N/mm^2
- Tension at service fts = 0.6 $\sqrt{f_c{}'}$ = 0. 6× $\sqrt{40}$ = + 3.8 N/mm^2

Assume beam self-weight = 6 kN/m, therefore $M_o = \frac{6\ x\ 12^2}{8} = 108\ kN.m$

Ms = $M_D + M_L = \frac{(7.3+14.6)\,x\,12^2}{8} = 394.2\ kN.m$

At transfer

Top $fti = -\frac{P_i}{Ac} + \frac{P_i.e}{Z1} - \frac{M_o}{Z1}$ - 1

Bottom $fci = -\frac{P_i}{Ac} - \frac{P_i.e}{Z2} + \frac{M_o}{Z2}$ - 2

After all losses

Bottom $fts = -\frac{P_e}{Ac} - \frac{P_e.e}{Z2} + \frac{M_o}{Z2} + \frac{Ms}{Z2}$ - 3

Top $fcs = -\frac{P_e}{Ac} + \frac{P_e.e}{Z1} - \frac{M_o}{Z1} - \frac{Ms}{Z1}$ - 4

Consider Eqs. 1 & 4............hence R = 0.85 = $\frac{P_e}{P_i} \Longrightarrow P_e = R.P_i$

$1.34 = -\frac{P_i}{Ac} + \frac{P_i.e}{Z1} - \frac{108\,x\,10^6}{Z1}$ - 1

$-18 = -R\frac{P_i}{Ac} + R\frac{P_i.e}{Z1} - \frac{108\,x\,10^6}{Z1} - \frac{394.2\,x\,10^6}{Z1}$ - 4

Multiply Eq. (1) by R and subtract Eq. (4) from Eq. (1), gives

$1.34\,x\,R = -R\frac{P_i}{Ac} + R\frac{P_i.e}{Z1} - R\frac{108\,x\,10^6}{Z1}$

$-18 = -R\frac{P_i}{Ac} + R\frac{P_i.e}{Z1} - \frac{108\,x\,10^6}{Z1} - \frac{394.2\,x\,10^6}{Z1}$ .... subtracting

(+) (+) (-) (+) (+)

$18 + 0.85\,x\,1.34 = \frac{108\,x\,10^6}{Z1}(1\text{-}0.85) + \frac{394.2\,x\,10^6}{Z1} \Rightarrow Z1 = 21.44\,x\,10^6\ mm^3$

Again use eqs. (2) & (3) with mmultiplying Eq. (2) by R

$-17.4\,x\,R = -R\frac{P_i}{Ac} - R\frac{P_i.e}{Z2} + R\frac{108\,x\,10^6}{Z2}$ - 2

$3.8 = -R\frac{P_i}{Ac} - R\frac{P_i.e}{Z2} + \frac{108\,x\,10^6}{Z2} + \frac{394.2\,x\,10^6}{Z2}$ - 3

Subtracting

$-18.59 = -\frac{108\,x\,10^6}{Z2}(1\text{-}0.85) - \frac{394.2\,x\,10^6}{Z2} \Rightarrow Z2 = 22.08\,x\,10^6\ mm^3$

Take the largest value of Z which is equal to $Z = 22.08\,x\,10^6\ mm^3$ .... why?

Assume b = $\frac{h}{2}$, and rembmer that $Z = \frac{1}{6}b.h^2 \Rightarrow Z = 22.08\,x\,10^6 = \frac{1}{6}\frac{h}{2}.h^2$

therefore, $h^3/12 = 22.08\,x\,10^6 \Rightarrow h = 642.3\ mm$, ...*say* $h = 650\ mm$, *and* $b = 325\ mm$

$Z = \frac{1}{6}325\,x\,650^2 = 22.88\,x\,10^6\ mm^3 > 22.08\,x\,10^6$ ok

To find P_i use eqs 1 and 2

$1.34 = -\frac{P_i}{Ac} + \frac{P_i.e}{Z} - \frac{M_o}{Z}$

$-17.4 = -\frac{P_i}{Ac} - \frac{P_i.e}{Z} + \frac{M_o}{Z}$ adding

$1.34\text{-}17.4 = -2\,\frac{P_i}{Ac} \;\Rightarrow\; P_i = \frac{16.06}{2}\,x\,Ac = \frac{16.06}{2}\,x\,325\,x\,650 = 1696\,kN$

From Eq. (1)

$$1.34 = -\frac{1696\,x\,10^3}{325\,x\,650} + \frac{1696\,x\,10^3.e}{22.88\,x\,10^6} - \frac{108\,x\,10^6}{22.88\,x\,10^6}$$

$$\Rightarrow 1.34 = -8.03 + 0.074\,e - 4.737 \Rightarrow \; e = 190\,mm$$

For alloy steel bars, f_{pu}= 1034 N/mm^2

Permissible stress immediately after transfer:

f_s = 0.7 x f_{pu}= 0.7 x 1034 = 723.8 N/mm^2 see table 2

$$As = \frac{P_i}{f_s} = \frac{1696\,x\,10^3}{723.8} \approx 2344\,mm^2$$

Provide 5 ϕ25 mm, $A_{st} \approx 2455\,mm^2$

- Check the design:

Check for self-weight

w_o = 0.325 x 0.65 x 24 = 5.07 kN/m < 6.0 kN/m …… o.k

$$M_o = \frac{5.07\,x\,12^2}{8} = 91.26\,kN.m$$

Check for stress

$fti = -\frac{1696\,x\,10^3}{325\,x\,650} + \frac{1696\,x\,10^3 x190}{22.88\,x\,10^6} - \frac{91.26\,x\,10^6}{22.88\,x\,10^6}$ = -8.03+14.08 - 3.99 = 2.06 N/mm^2 >1.34 Not o.k

Use a new value of **e** which must be less than 190, try **e = 180 mm**

$fti = -8.03 + 14.08\,x\frac{180}{190} - 3.99$ = 1.32 N/mm^2 < 1.34 o.k

$fci = -8.03 - 13.34 + 3.99$ = -17.38 N/mm^2 < -17.4 o.k

$fcs = -8.03\,x\,0.85 + 13.34\,x\,0.85 - \frac{(91.26+394.2)\,x\,10^6}{22.88\,x\,10^6}$ Check is ok

$fcs = -6.83 + 11.34 - 21.22$ = - 16.71 N/mm^2 < - 18 o.k

$fts = -6.83 - 11.34 + 21.22$ = 3.05 N/mm^2 < 3.8 o.k

Example (9-5):

Design the beam in the privous example, using cables with constant eccentricity.

Hint:

1-The allwoble concrete tensile stress in the ends of simply supported members is twice of what it is eleswhere in the beam, $(0.5\sqrt{f_{ci}'})$ ....see table 1.

2- For beams with a constant steel eccentricity, the critical sections at transfer are the beam ends, where the self-weight moment (M_o) does not counter the prestressing moment.

Solution:

1- The critical section is at the supports at transfer:

$$fti = -\frac{P_i}{Ac} + \frac{P_i.e}{Z1} \qquad -1$$

$$fci = -\frac{P_i}{Ac} - \frac{P_i.e}{Z2} \qquad -2$$

2- The critical section is at mid-span at service load:

$$fts = -\frac{P_e}{Ac} - \frac{P_e.e}{Z2} + \frac{M_o}{Z2} + \frac{Ms}{Z2} \qquad -3$$

$$fcs = -\frac{P_e}{Ac} + \frac{P_e.e}{Z1} - \frac{M_o}{Z1} - \frac{Ms}{Z1} \qquad -4$$

From eqs. 1 and 4 (multiplying Eq.1 by R, and replacing $P_e = R.P_i$ in Eq.4)

$$R.fti = -R\,\frac{P_i}{Ac} + R\,\frac{P_i.e}{Z1}$$

$$fcs = -R\,\frac{P_i}{Ac} + R\,\frac{P_i.e}{Z1} - \frac{M_o}{Z1} - \frac{Ms}{Z1}$$

Subtracting

$$0.85\,(2 \text{ x } 1.34) + 18 = \frac{(108+394.2)}{Z1}\, x\, 10^6 \qquad Z1 = 24.77\, x\, 10^6 \;\; mm^3$$

From eqs. 2 and 3 (multiplying Eq.2 by R, and replacing $P_e = R.P_i$ in Eq.3)

$$R.fci = -R\,\frac{P_i}{Ac} - R\,\frac{P_i.e}{Z2}$$

$$fts = -R\,\frac{P_i}{Ac} - R\,\frac{P_i.e}{Z2} + \frac{M_o + Ms}{Z2}$$

Subtracting

$$0.85 \text{ x } (-17.4) - 3.8 = \frac{(108+394.2)}{Z2}\, x\, 10^6 \qquad Z2 = 27.01\, x\, 10^6 \;\; mm^3$$

Take the largest value of Z which is equal to $=27.01\, x\, 10^6 \;\; mm^3$

Assume $b = \frac{h}{2}$, $Z = \frac{1}{6} b.h^2 \Rightarrow Z = 27.01\, x\, 10^6 = \frac{1}{6}\frac{h}{2}.h^2$

therefore, $h^3/12 = 27.99\, x\, 10^6 \Rightarrow h = 686.9\ mm$ ... ... say $h = 700\ mm$, and $b = 350\ mm$

$Z = \frac{1}{6}\, 350 \text{ x } 700^2 = 28.58\, x\, 10^6$ mm^3

To find P_i

$fti = -\frac{P_i}{Ac} + \frac{P_i.e}{Z}$

$- fci = -\frac{P_i}{Ac} - \frac{P_i.e}{Z}$ adding

2 x 1.34 - 17.4 = $-2\ \frac{Pi}{Ac} \Rightarrow P_i = \frac{14.74}{2} x\ Ac = \frac{14.72}{2}\ x\ 350\ x\ 700 = 1803.2\ kN$

Subtracting Eq 2 from Eq 1

2.68 + 17.4 = 2 $x\ \frac{1803.2\ x\ e\ x\ 10^3}{28.58\ x\ 10^6} \Rightarrow\ e = 159\ mm$

$As = \frac{P_i}{f_s} = \frac{1803.2\ x\ 10^3}{0.7\ x\ 1034} = 2491\ mm^2$

Provide 8 ϕ20 mm, $Ast = 2512\ mm^2$

Check stress at service load $P_e = 0.85\ xP_i = 1803.2\ x\ 0.85 = 1532.7\ kN$

$fcs = -\frac{P_e}{Ac} + \frac{P_e.e}{Z} - \frac{M_o+Ms}{Z} = -\frac{1532.7x\ 10^3}{350\ x\ 700} + \frac{1532.7\ x\ 10^3 x159}{28.58\ x\ 10^6} - \frac{(108+394.2)\ x\ 10^6}{28.58\ x\ 10^6}$

= -6.26 + 8.53 – 17.57 = -15.3 N/mm^2 < - 18 o.k

$fts = -6.26 - 8.48 + 17.57$= 2.78 N/mm^2 < 3.16 o.k

10- Flexural Strength

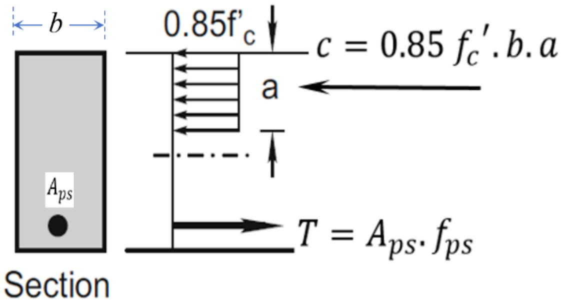
Bonded tendon:

When there is adequate bond between the prestressing tendon and concrete, it is called a bonded tendon. Pretensioned and grouted posttensioned tendons are bonded tendons.

Unbonded tendon:

When there is no bond between the prestressing tendon and concrete, it is called an unbonded tendon. When the grout is not applied after posttensioning, the tendon is unbonded.

1- Rectangular Sections:



1- If $\rho_p \frac{f_{ps}}{f_c{'}} < 0.3$, (under reinforced)

the strength of prestressed beams can be found from the follwing expersion:

$M_u = \emptyset.T\,(d_p - \frac{a}{2})$ ….. $\emptyset = 0.90$

$M_u = \emptyset.A_{ps}.f_{ps}\,(d_p - \frac{a}{2})$

ρ_p: tensile reinforcement ratio

f_{ps}: Stress in prestressing reinforcement at nominal flexural strength

A_{ps}: cross-sectional area of the prestressing steel = $\rho_p.b.d_p$

Where $a = \frac{A_{ps}.f_{ps}}{0.85\, f_c{'}.b}$, then M_u becomes:

$M_u = \emptyset.A_{ps}.f_{ps}.d_p\,(1 - \frac{0.59\,.\rho_p.f_{ps}}{f_c{'}})$, but $A_{ps} = \rho_p.b.d_p$

$M_u = \emptyset.\rho_p.f_{ps}.b.d_p{}^2\,(1 - \frac{0.59\,.\rho_p.f_{ps}}{f_c{'}})$

The values of steel stress f_{ps} can be calclated from:

a. For bonded members $f_{ps} = f_{pu}\,(1 - \frac{\gamma_p}{\beta_1}\frac{\rho_p.f_{pu}}{f_c{'}})$

Where $\beta_1 = 0.85$

$\gamma_p =$	0.55	For $f_{py}/f_{pu} \geq 0.8$ (typical high strenght bars)
	0.40	For $f_{py}/f_{pu} \geq 0.85$ (typical ordinary strands)
	0.28	For $f_{py}/f_{pu} \geq 0.9$ (typical low-relaxtion strands)

For simplicity take $\frac{\gamma_p}{\beta_1} \approx 0.5$ (typical high strenght bars)

b. For unbonded members $f_{ps} = f_{pe} + 70 + \frac{f_c{'}}{100\,.\rho_p}$, *but not more than* f_{py}

and not more than $f_{pe} + 420$ MPa

Where:

f_{pu} : tensile strength of the prestressing steel.

f_{py} : yield stress of the prestressing reinforcement.

f_{pe}: effective stress in prestressing steel after losses (not less than 0.5 f_{pu})

2- If $\rho_p \frac{f_{ps}}{f_c'} > 0.3$, (over reinforced)

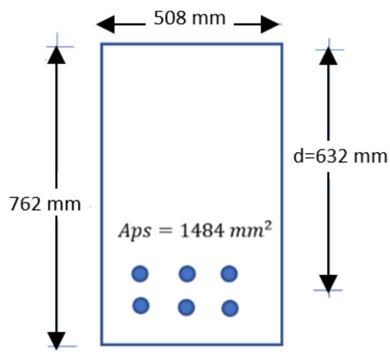
the ultimate moment can be taken as:

$M_u = \emptyset.(0.25\ f_c'\ b.d_p^{\ 2})$

Example (10-1):

A posttentioned bonded prestressed concrete beam with a cross section shown in the figure below, what is the design strength of the beam?

$f_c' = 34\ MPa, f_{pu} = 1725\ MPa$



Solution:

Tensile steel ratio $\rho_p = \frac{A_{ps}}{b.d_p} = \frac{1484}{508\ x\ 632} = 4.6\ x\ 10^{-3}$

Steel stress at the design load $f_{ps} = f_{pu}\ (1 - \frac{0.5\ .\rho_p.f_{pu}}{f_c'})$

$f_{ps} = 1725\ x(1 - \frac{0.5\ x\ 4.6\ x\ 10^{-3} x\ 1725}{34}) \approx 1524$ N/mm^2

check ratio $\rho_p \frac{f_{ps}}{f_c'} = 4.6\ x\ 10^{-3}\ x\ \frac{1524}{34} = 0.206 < 0.3$ (under reinforced section)

$M_u = \emptyset.A_{ps}\ .f_{ps}\ .d_p\ \left(1 - \frac{0.59\ .\rho_p.f_{ps}}{f_c'}\right)$

$M_u = 0.9\ x\ 1484\ x\ 1524\ x\ 632\ x\ (1 - 0.59\ x\ 0.233) = 1109.6$ kN.m

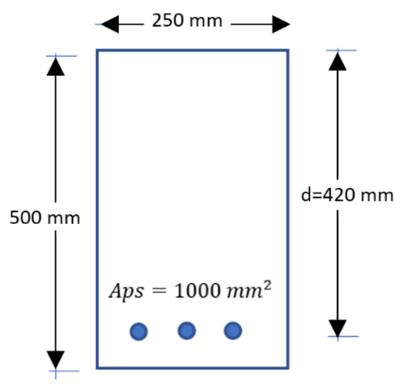
Example (10-2):

A posttentioned prestressed concrete beam with a cross section shown in the figure below.

$f_c' = 35\ MPa, f_{pu} = 1350\ MPa$, $M_D = 80$ kN.m, $M_L = 90$ kN.m, initial stress $f_{pi} = 930$ N/mm^2, total losses = 170 MPa

Find the ultimate flexural strength for the following cases:

1- Well grouted, 2- Grouting is omitted.



Solution:

1- Well grouted:

Tensile steel ratio $\rho_p = \frac{A_{ps}}{b.d_p} = \frac{1000}{250\ x\ 420} = 9.5 \times 10^{-3}$

Steel stress at the design load $f_{ps} = f_{pu}\ (1 - \frac{0.5\ .\rho_p.f_{pu}}{f_c'})$

$f_{ps} = 1350\ x\ (1 - \frac{0.5\ x\ 9.5x\ 10^{-3}\ \ x\ 1350}{35}) = 1102$ N/mm^2 (MPa) check ratio $\rho_p \frac{f_{ps}}{f_c'} = \frac{9.5\ x\ 10^{-3}\ x\ 1102}{35}$ = 0.299 < 0.3 (under reinforced section)

$a = \frac{A_{ps}.f_{ps}}{0.85\ f_c'.b} = \frac{1000\ x\ 1102}{0.85\ x\ 35\ x\ 250} = 148.2$ ≈149 mm

$M_u = \emptyset. A_{ps}.f_{ps}\ (d_p - \frac{a}{2})\ = 0.9\ x\ 1000\ x\ 1102\ x\ (420 - 149/2)$ = 342.7 kN.m

Required moment = 1.2 x 80 + 1.6 x 90 = 240 kN.m < 342.7 kN.m

Reserve = $\frac{\mathbf{342.7-240}}{\mathbf{342.7}}\boldsymbol{x100} = \mathbf{30}$ %

2- Grouting is omitted:

Tensile steel ratio $\rho_p = 9.5 \times 10^{-3}$

Steel stress at the design load $f_{ps} = f_{pe} + 70 + \frac{f_c'}{100.\rho_p}$……… $f_{pe} = f_{pi} - total\ losses$

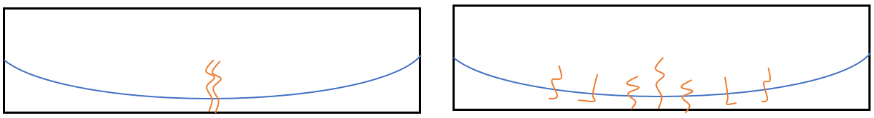
= (930-170) + 70 + $\frac{35}{100x9.5\ x\ 10^{-3}}$ = 867 MPa < (760 + 420) o.k

check ratio $\rho_p \frac{f_{ps}}{f_c'} = 9.5\ x\ 10^{-3}\ x\ \frac{867}{35}$ = 0.235 < 0.3 (under reinforced section)

$a = \frac{1000\ x\ 867}{0.85\ x\ 35\ x\ 250} = 116.6$ ≈117 mm

$M_u = \emptyset. A_{ps}.f_{ps}\ (d_p - \frac{a}{2})\ = 0.9\ x\ 1000\ x\ 867\ x\ (420 - 117/2)$ = 282 kN.m > 265 kN.m

Reserve = $\frac{\mathbf{282-240}}{\mathbf{282}}\boldsymbol{x100} \approx \mathbf{14.9}$ %



ungrouted
One or few wide cracks

grouted
Several spread cracks

2- Flanged Sections:

For flanged sections such as I and T, the flexural strength of the beams can be calculated using the same method which is used in rectangular beams. The total steel area is divided into two parts for computational purposes. The **first part** balances the compression in the overhanging portion of the flange; the **second part** balances the compression in the web. In the meantime, ρ_p is taken as the steel ratio of the tension steel area, which is required to develop the compression in the web.

- If $\rho_p \frac{f_{ps}}{f_c'} < 0.3$ (under reinforced)

$$M_u = \emptyset \left[A_{pw}.f_{ps}.d_p \left(1 - \frac{0.59\ A_{pw}.f_{ps}}{b_w.d.\ f_c'}\right) + 0.85\ f_c'(b - b_w)\ h_f(d_p - \frac{h_f}{2})\right]$$

Where $A_{pw} = A_{ps} - A_{pf}$, and $A_{pf} = 0.85\ \frac{f_c'}{f_{ps}}\ (b - b_w).h_f$

- If $\rho_p \frac{f_{ps}}{f_c'} > 0.3$ (over reinforced)

$$M_u = \emptyset\left[0.25\ f_c'\ b_w\,.d_p{}^2 + 0.85\ f_c'\ (b - b_w)\ h_f\ (d_p - \frac{h_f}{2})\right]$$

How to solve problems of flexural strength?

1. find $\rho_p = \frac{A_{ps}}{b.d_p}$

2. find f_{ps} → Bonded tendon $f_{ps} = f_{pu}\,(1 - \frac{0.5\ \rho_p.f_{pu}}{f_c'})$

→ Unbonded tendon $f_{ps} = f_{pe} + 70 + \frac{f_c'}{100\ .\rho_p} < f_{pe} + 420$ MPa

3. Check whether the section is a rectangular or flanged section (use one of the following methods):

a- Stress block …… find $a = \frac{A_{ps}.f_{ps}}{0.85\ f_c'.b}$ → a < h_f……….. rectangular ection

→ a > h_f……….. flanged section

b- Compute the compresion force indeuced due to the flange (C) and the tension force due to the (ordinady and/or prestressed steel) area (T) → if C > T ……….. rectangular ection

→ if C < T ……….. flanged ection

4. Chech the ratio $\rho_p \frac{f_{ps}}{f_c'}$

If $\rho_p \frac{f_{ps}}{f_c'} < 0.3$ (under reinforced)

$M_u = \emptyset.\rho_p\,.f_{ps}\,.b.d_p{}^2\ (1 - \frac{0.59\ .\rho_p.f_{ps}}{f_c'})$ ………………… rectangular sction

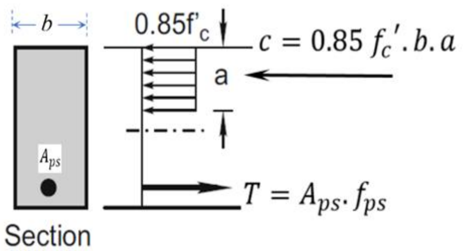
$M_u = \emptyset\left[A_{pw}\,.f_{ps}\,.d_p\ \left(1 - \frac{0.59\ A_{pw}.f_{ps}}{b_w.d.\ f_c'}\right) + 0.85\ f_c'(b - b_w)\ h_f(d_p - \frac{h_f}{2})\right]$ ……. flanged section

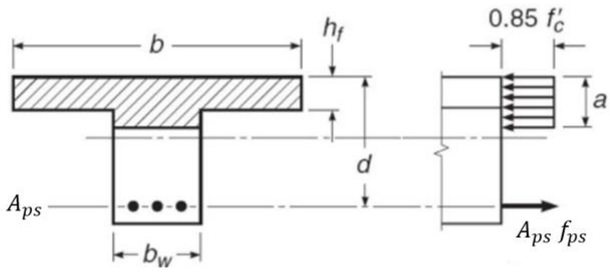
Where $A_{pw} = A_{ps} - A_{pf}$, and $A_{pf} = 0.85\ \frac{f_c'}{f_{ps}}\ (b - b_w).h_f$, $\emptyset = 0.90$

If $\rho_p \frac{f_{ps}}{f_c'} > 0.3$ (over reinforced)

$M_u = \emptyset[0.25\ f_c'\ b\,.d_p{}^2]$ ……. Rectangular section

$M_u = \emptyset\left[0.25\ f_c'\ b_w\,.d_p{}^2 + 0.85\ f_c'\ (b - b_w)\ h_f\ (d_p - \frac{h_f}{2})\right]$ ……. flanged section





Example (10-3):

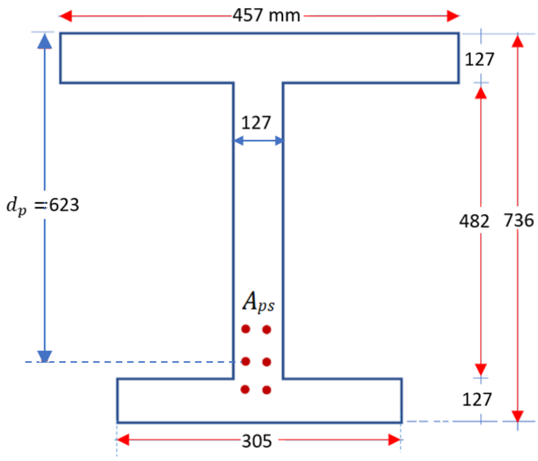
A posttentioned bonded prestressed concrete beam with a cross section shown in the figure, what is the ultimate bending moment that the beam can carry?

$f_c{}' = 34\ MPa,$

$f_{pu} = 1897\ MPa,$

A_{ps} = 1129 mm^2, and

A_c = 157.6 x 10^3 mm^2



Solution: A_{ps}

Tensile steel ratio $\rho_p = \frac{A_{ps}}{b.d_p} = \frac{1129}{457\ x\ 623} = 3.97x\ 10^{-3}$

Steel stress at the design load $f_{ps} = f_{pu}\ (1 - \frac{0.5\ .\rho_p.f_{pu}}{f_c{}'})$

$f_{ps} = 1897x(1 - \frac{0.5\ x3.97x\ 10^{-3}\ \ x\ 1897}{34})$ = 1687 N/mm^2,

Next, it is necessary to check whether the stress block depth is greater or less than the average flange thickness of 127 mm. On the assumption that it is not greater than the flange thickness, the equation of a is used:

$a = \frac{A_{ps}.f_{ps}}{0.85\ f_c{}'.b} = \frac{1129\ x\ 1687}{0.85\ x\ 34x\ 457} = 144.2\ mm\ >$ 127 mm

It is concluded from this trial calculation that a actually exceeds h_f, so the trial calculation is not valid and equations for flanged members must be used. The steel that acts with the overhanging flanges is found from:

$A_{pf} = 0.85\frac{f_c{}'}{f_{ps}}(b - b_w)\ .h_f$ $= 0.85\ x\ \frac{34}{1687}(457 - 127)\ x127$ = 718 mm^2

$A_{pw} = A_{ps} - A_{pf} = 1129 - 718 = 411$ mm^2

check ratio $\rho_p \frac{f_{ps}}{f_c{}'}$ ……. $\rho_p = \frac{A_{pw}}{b_w.d}$, then

$\frac{A_{pw}}{b_w.d}\frac{f_{ps}}{f_c{}'} = \frac{411\ \ x\ 1687}{127x\ 623x34}$ = 0.26 < 0.3 (under reinforced section)

steel ratio balance the compression in the web

$$M_u = \emptyset.\left[A_{pw}\ .f_{ps}\ .d_p\ \left(1 - \frac{0.59\ .A_{pw}.f_{ps}}{b_w x d_p\ x\ f_c{}'}\right) + 0.85\ \ f_c{}'(b - b_w)\ h_f\left(d_p - \frac{h_f}{2}\right)\right]$$

$$M_u = 0.9\ x\left[\begin{array}{c} 411\ x1687\ x\ 623\ \left(1 - \frac{0.59\ x411x1687}{127x623x34}\right) \\ +0.85\ x\ 34\ x\ (457 - 127)\ x\ 127\left(623 - \frac{127}{2}\right)\end{array}\right]$$

$M_u = 0.9x[366.3\ x\ 10^6 + 677.7x10^6] = 939.6$ kN.m

Example (10-4):

A pertensioned prestress beam of recangular section (150 mm width and 350 mm depth) has an effective cover of 50 mm. If $f_c{}'$ = 32 MPa and f_{pu} = 1600 MPa, determine:

a- The maximum area of steel which just ensures failure by excessive elongation of steel followed by crashing of concrete.

b- Flexural strength corresonding to case (a); and

c- The flexural strength of the section if the area of steel in case (a) is doubled.

Solution:

a- To find the maximum area of steel,

$\rho_p \frac{f_{ps}}{f_c{}'} < 0.3$ (under reinforced)

$\rho_p \frac{f_{ps}}{f_c{}'} > 0.3$ (over reinforced)

Take $\rho_p \frac{f_{ps}}{f_c{}'} = 0.3$

Steel stress at the design load

$$f_{ps} = f_{pu}\left(1 - \frac{0.5\,.\rho_p.f_{pu}}{f_c{}'}\right) = 1600x(1 - \frac{0.5\,.\rho_p.1600}{32})$$

substituting f_{ps} in the effective raio

$$\frac{\rho_p}{32}\ x\ 1600\ x\ (1 - \frac{0.5\ \rho_p x 1600}{32})\ = 0.3 \Longrightarrow$$

$$\rho_p{}^2 - 0.04\ \rho_p + 0.24x10^{-3} = 0$$

ρ_p= 0.0074……… d_p = 350 – 50 = 300 mm

A_{ps} = 0.0074 x 150 x 300 = 333 mm^2

b-

$$f_{ps} = f_{pu}\left(1 - \frac{0.5\,.\rho_p.f_{pu}}{f_c{}'}\right) = 1600x\left(1 - \frac{0.5\ x0.0074x1600}{32}\right) = 1304\ \text{N/mm}^2$$

$$M_u = \emptyset.A_{ps}\,.f_{ps}\,.d_p\ \left(1 - \frac{0.59\,.\rho_p.f_{ps}}{f_c{}'}\right)$$

$$M_u = 0.9\ x\ 333\ x\ 1304\ x\ 300\left(1 - \frac{0.59\ x\ 0.0074\ x\ 1304}{32}\right) = 96.3\ kN.m$$

c- if the area is doubled

A_{ps} = 2 x 333 = 666 mm^2

The section is over rinforced and hance

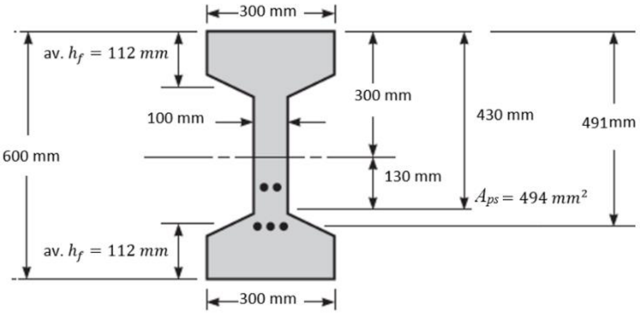
$$M_u = \emptyset\left[0.25\ \ f_c{}'.b\,.d_p{}^2\right]$$

$$= 0.9\ x\ 0.25\ x\ 32\ x\ 150\ x\ 300^2$$

$$= 97.2\ kN.m$$

Example (10-5):

The prestressed I beam shown below is pretensioned using five 12.7 mm diameter strands Grade 1900, carrying effective prestress f_{pe}= 1100 MPa. Concrete strength is $f_c{}'$ = 27 MPa. Calculate the design strength of the beam.



Solution:

The effective prestress in the strands of 1100 MPa is well above 0.50 × 1900 = 950 MPa, confirming that the approximate ACI equations are applicable.

The tensile reinforcement ratio is

$$\rho_p = \frac{494}{300\ x\ 430} = \ 0.0038$$

and the steel stress f_{ps} when the beam fails in flexure is found from:

$$f_{ps} = f_{pu}\left(1 - \frac{0.5\ .\rho_p.f_{pu}}{f_c{}'}\right) = 1900x\left(1 - \frac{0.5\ x0.0038x\ 1900}{27}\right) = 1646\ \text{ N/mm}^2$$

Next, it is necessary to check whether the stress block depth is greater or less than the average flange thickness of 112 mm. On the assumption that it is not greater than the flange thickness, the equation of a is used:

$$a = \frac{A_{ps}.f_{ps}}{0.85\ f_c{}'.b} = \frac{494\ x\ 1646}{0.85\ x\ 27\text{x}\ 300} = 118\ mm\ >\ 112\text{ mm}$$

It is concluded from this trial calculation that a actually exceeds h_f, so the trial calculation is not valid and equations for flanged members must be used. The steel that acts with the overhanging flanges is found from:

$$A_{pf} = 0.85\frac{f_c{}'}{f_{ps}}(b - b_w)\ .\ h_f\ = 0.85\ x\ \frac{27}{1646}(300 - 100)\ x112 = 312\text{ mm}^2$$

And the steel acting with the web is:

$$A_{pw} = A_{ps} - A_{pf} = 494 - 312 = 182\text{ mm}^2$$

The actual stress block depth is now found from:

$$a = \frac{A_{pw}.f_{ps}}{0.85\ f_c{}'.b_w} = \frac{182\text{ x } 1646}{0.85\ x\ 27x\ 100} = 131\ mm$$

check ratio $\rho_p \frac{f_{ps}}{f_c{}'}$ ……. $\rho_p = \frac{A_{pw}}{b_w.d_p}$, then

$$\frac{A_{pw}}{b_w.d_p}\frac{f_{ps}}{f_c{}'} = \frac{182\ \text{ x } 1646}{100\ x\ 430x27} = 0.26 < 0.3 \text{ (under reinforced section)}$$

steel ratio balance the compression in the web

$M_u = \emptyset.\left[A_{pw} \cdot f_{ps}\left(d_p - \frac{a}{2}\right) + 0.85\ f_c{}'(b - b_w)\, h_f\left(d_p - \frac{h_f}{2}\right)\right]$

$M_u = 0.9\ x\left[182\ x 1646\left(430 - \frac{131}{2}\right) + 0.85\ x\ 27\ x\ (300 - 100)\ x\ 112\left(430 - \frac{112}{2}\right)\right]$

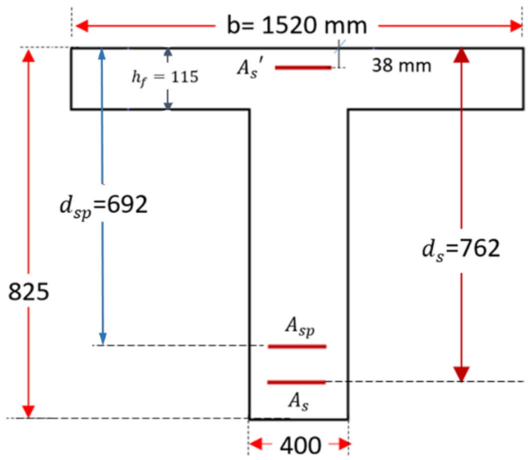
$M_u = 0.9x[109.2\ x\ 10^6 + 192.3x10^6] = 271.4$ kN.m

Example (10-6):

Find the ultimate felxrural capacity of the bonded posttentioned beam shown below.

$f_c{}' = 27$ N/mm², $f_{pu=}$ 1863 N/mm², $f_{y=}$ 410 N/mm², $A_s{}' = 645\ mm^2$ $A_{sp} = 2260\ mm^2$

$A_s = 2580\ mm^2$



Solution:

$\rho_p = \frac{2260}{1520\ x\ 692} = 2.4x\ 10^{-3}$

$f_{ps} = f_{pu}\left(1 - \frac{0.5\ .\rho_p.f_{pu}}{f_c{}'}\right)$

$f_{ps} = 1863(1 - \frac{0.5\ x2.4x\ 10^{-3}x1863}{27}) = 1725\ MPa$

When ordinary and prestressed steel exist, $d_{average}$ must be calculated using the following form:

$d_{average} \times (A_s \times f_y + A_{sp} \times f_{ps}) = A_s \times f_y \times d_s + A_{sp} \times f_{ps} \times d_{sp}$

∴Average depth of the tensile reinforcement:

$d = \frac{2260\ x\ 1725\ x\ 692 + 2580\ x\ 410\ x\ 762}{2260\ x\ 1725 + 2580\ x\ 410} = \frac{3503.9\ x\ 10^6}{4.69\ x\ 10^6} = 706\ mm$

Check ratio

- $\rho_p \frac{f_{ps}}{f_c{}'} = \frac{2260\ x\ 1725}{1520x\ 706\ x\ 27} = 0.13$
- $\rho\ \frac{f_y}{f_c{}'} = \frac{2580\ x\ 410}{1520x\ 706\ x\ 27} = 0.036$
- $\rho'\ \frac{f_y}{f_c{}'} = \frac{645\ x\ 410}{1520x\ 706\ x\ 27} = 0.009$

$\rho_p \frac{f_{ps}}{f_c{}'} + \rho\ \frac{f_y}{f_c{}'} - \rho'\ \frac{f_y}{f_c{}'} = 0.13 + 0.036 - 0.009 = 0.157 < 0.3$ under reinforcement

Compressive force in the flange = $0.85\ x\ f_c{}'.b.h_f + A_s{}'.f_y$

$= 0.85\ x\ 27\ x\ 1520\ x\ 115 + 645\ x\ 410 = 4276\ kN$

Total tensile force = $A_{ps} \cdot f_{ps} + A_s.f_y$

$= 2260\ x\ 1725 + 2580\ x\ 410 = 4960\ kN > 4276\ kN$

$\therefore flanged\ section$

Compression force in the overhanging portion of the flange and compression steel

$= 0.85\ x\ 27\ x\ (1520 - 400) x\ 115 + 645\ x\ 410 = 3220\ kN$

Force to be developed by the web = $A_{ps} \cdot f_{ps} + A_s . f_y - 3220$

$$= 4960 - 3220 = 1740\ kN$$

Check steel ratio of the web (in this case b = 400 mm)

$\rho_p \frac{f_{ps}}{f_c{}'} + \rho \frac{f_y}{f_c{}'} - \rho' \frac{f_y}{f_c{}'} = \frac{1740\ x\ 10^3}{400\ x\ 706\ x\ 27} = 0.23 < 0.3$ The web is under reinforcement

$$M_u = \emptyset . \begin{bmatrix} A_{pw} \cdot f_{ps} \cdot d \left(1 - \frac{0.59 . A_{pw} \cdot f_{ps}}{b_w\ x d\ x\ f_c{}'}\right) + \\ 0.85\ f_c{}'(b - b_w)\ h_f \left(d - \frac{h_f}{2}\right) + \\ A_s{}'\ f_y (d - d') \end{bmatrix}$$

$$M_u = 0.9 \begin{bmatrix} 1740\ x\ 10^3\ x\ 706 \left(1 - \frac{0.59\ x 1740\ x\ 10^3}{400\ x\ 706\ x\ 27}\right) + \\ 0.85\ x\ 27 x\ (1520 - 400) x\ 115 x \left(706 - \frac{115}{2}\right) + \\ 645 x\ 410 x (706 - 38) \end{bmatrix}$$

$M_u = 0.9[1063 + 1916.9 + 176.65] = 2841$ kN.m

11- Shear in Prestressed Concrete Beams

In prestressed concrete beams at service load, there are two factors that greatly reduce the intensity of diagonal tensile stresses, compared with stresses that would exist if no prestress force were present, see Figure (11). ***The first of these results from the* combination of longitudinal compressive stress and shearing stress.**

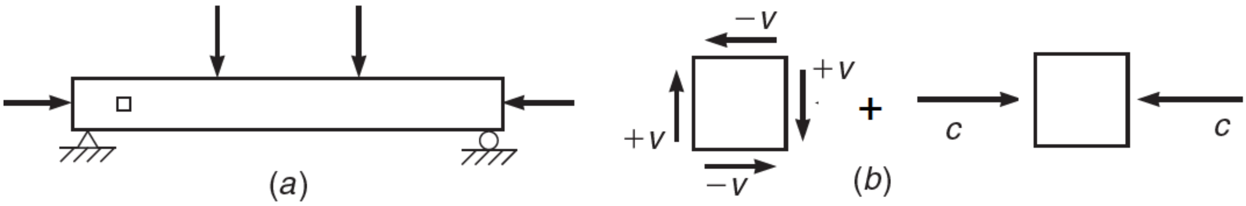
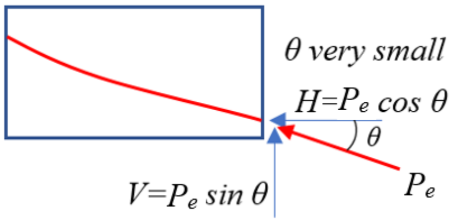


Figure 11: Shear in prestressed concrete

***The second factor working to reduce the intensity of the diagonal tension at service loads results from* the slope of the tendons.** Normally, this slope is such as to produce a shear due to the prestress force that is opposite in direction to the load-imposed shear. The magnitude of this *counter shear* is $V_p = P_e \sin\theta$, where θ is the slope of the tendon at the considered section.

Hint: The two factors are not considered as safety measure or additional factors of safety.



Types of shear:

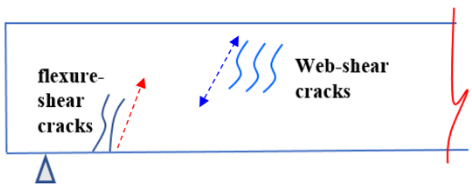
Two types of diagonal cracks have been observed in tests of prestressed concrete beams:

1. Flexure-shear cracks:

Occurring at nominal shear V_{ci}, start as nearly vertical flexural cracks at the tension face of the beam, then spread diagonally upward (under the influence of diagonal tension) toward the compression face. *These are common in beams with a low value of prestress force.*

2. Web-shear cracks:

Occurring at nominal shear V_{cw}, start in the web due to high diagonal tension, then spread diagonally both upward and downward. *These are often found in beams with thin webs and high prestress force.*



ACI Code:

Shear causing flexural share cracking:

$$V_{ci} = 0.05\sqrt{f_c{}'}\; b_w . d_p + V_{cr,o+d+l}$$

$V_{cr,o+D+L}$: Shear force due to total load at which the flexural crack forms at the considered section.

$0.05\sqrt{f_c{}'}\; b_w . d_p$: Represents an additional shear force required to transform the flexural crack into an inclined crack.

It is convenient to separate the total shear $V_{cr,o+D+L}$ into:

V_o: shear force caused by the beam self-weight (without load factor)

V_{cr}: additional shear force due to superimposed dead and live loads, corresponding to flexural cracking, thus:

$$V_{ci} = 0.05\sqrt{f_c{}'}\; b_w . d_p + V_o + V_{cr} \qquad \text{and} \qquad V_{cr} = \frac{V_{d+l}}{M_{d+l}} M_{cre}$$

$\frac{V_{d+l}}{M_{d+l}}$: the ratio of superimposed dead and live load shear to moment (remains constant as the load increases to the cracking load), and the equation for M_{cre}, the moment causing flexural cracking due to wxternal loads, is given by

$$M_{cre} = \frac{I}{y_t}(0.5\,\lambda\sqrt{f_c{}'} + f_{pe} - f_o)$$

I: Second moment of area

y_t: Distance from concrete centroid to tension face.

f_{pe}: Compressive stress in concrete due to effective prestress (after losses) at exterem fiber where external loads causes tension.

f_o: stress due to beam self-weight (unfactored) at the extrem fiber as (f_{pe})

(D.L $_{beam}$+ superimposed D.L)

λ: modification factor reflecting the lower tensile strength of lightweight concrete compared with normal weight concrete of the same compressive strength.

Aggregate type	λ
All lightweight concrete	0.75
Sand lightweight concrete	0.85
Normal weight concrete	1.0

Therefore,

$$V_{ci} = 0.05\,\lambda\sqrt{f_c{}'}\; b_w . d_p + V_o + \frac{V_{d+l}}{M_{d+l}} M_{cre} \quad \textbf{\textit{not less than}}\ 0.14\,\lambda\sqrt{f_c{}'}\; b_w . d_p$$

d_p: should be not less than $0.8\,h$

According to the ACI Code, the shear force causing web-shear cracking can be found using the following approximation expression:

$$V_{cw} = (0.29\,\lambda\,\sqrt{f_c{}'} + 0.3\,f_{pc})b_w.d_p + V_p$$

V_p: the vertical component of the effective prestress force.

f_{pc}: the compressive stress in the concrete, after losses, at the centroid of the concrete section (or at the junction of the web and the flange when the centroid lies in the flange)

In a pretensioned beam, the $0.3\,f_{pc}$ contribution to V_{cw} should be adjusted from zero at the beam end to its full value one transfer length in from the end of the beam.

After V_{ci} and V_{cw} have been calculated, then V_c , the shear resistance provided by the concrete, is taken equal to the smaller of the two values.

To shorten the calculation required, the ACI Code includes, as a conservative alternative to the above procedure, an equation for finding the concrete shear resistance V_c directly:

$$V_c = (0.05\,\lambda\sqrt{f_c{}'} + 4.8\,\frac{V_u\,d_p}{M_u})\,b_w.d$$

M_u: the bending moment occurring simultaneously with shear force V_u and

$\frac{V_u\,d_p}{M_u}$ ***should be* not greater than** 1

d: the effective depth including prestressed and non-prestressed reinforcement.

$$0.17\,\lambda\,\sqrt{f_c{}'}\,b_w.d_p < V_c < 0.42\,\lambda\,\sqrt{f_c{}'}\,b_w.d_p$$

When shear reinforcement perpendicular to the axis of the beam is used, its contribution to shear strength of a prestressed beam is (same as for a non-prestressed member):

$V_s = \frac{A_v\,f_{yt}.d}{s}$ ***but not greater than*** $0.66\,\sqrt{f_c{}'}\,b_w.d$

The total nominal shear strength V_n is found by summing the contributions of the concrete and steel:

$V_n = V_c + \frac{A_v\,f_{yt}.d}{s}$ ……. Then,

$V_u = \emptyset\,V_n = \emptyset\,(V_c + V_s)$ ……. Thus,

$$V_u = \emptyset\,(V_c + \frac{A_v\,f_{yt}.d}{s})$$

The required cross-sectional area of one stirrup A_v can be calculated by suitable transposition of the above equation:

$$A_v = \frac{(V_u - \emptyset\,V_c)}{\emptyset\,f_{yt}.d}\,s$$

A_v = area of two legs.

Normally, in practical design, the engineer will select a trial stirrup size, for which the required spacing is found. Thus, a more convenient form of the last equation is:

$$s = \frac{\phi\, A_v\, f_{yt}.d}{V_u - \phi\, V_c}$$

Minimum amount of steel:

ACI Code: A minimum area of shear reinforcement is required in all prestressed concrete members where the total factored shear force is greater than $\frac{V_c}{2}$

The minimum area of shear reinforcement to be provided in all other cases is equal to the smaller of

$$A_{v,min} = 0.062\sqrt{f_c'}\ \frac{b_w.s}{f_{yt}} \le 0.35\ \frac{b_w.s}{f_{yt}}$$

or $A_{v,min} = \frac{A_{ps}}{80}\ \frac{f_{pu}}{f_{yt}}\ \frac{s}{d}\sqrt{\frac{d}{b_w}}$

A_{ps}: the cross-sectional area of the prestressing steel,

f_{pu}: the tensile strength of the prestressing steel

Maximum spacing:

For prestressed members, this maximum spacing is not to exceed the smaller $\frac{3}{4}\,h$ or 600 mm

If the value V_s exceeds $0.33\ \sqrt{f_c'}\ b_w.d_p$ these limits are reduced by one-half.

How to solve problems for shear in Prestressed Concrete Beams?

1. Find the shear force that causing flexural -shear cracking.

$$V_{ci} = 0.05\,\lambda\,\sqrt{f_c{}'}\,b_w.d_p + V_o + \frac{V_{d+l}}{M_{d+l}}M_{cre} \;\;\not< 0.14\,\lambda\,\sqrt{f_c{}'}\,b_w.d_p$$

2. Find the shear force that causing web-shear cracking.

$$V_{cw} = (0.29\,\lambda\,\sqrt{f_c{}'} + 0.3\,f_{pc})b_w.d_p + V_p$$

3. The shear resistance provided by the concrete V_c, is taken equal to the smaller of the V_{ci} *and* V_{cw}.

4. Find V_u and M_u for the considered section.

5. From the equation ($A_v = \frac{(V_u - \emptyset\,V_c)}{\emptyset\,f_{yt}.d}\,s$), assume rebar diameter and find

   A_v = area of two legs.

6. Find the maximum spacing from the following expressions:

$$A_{v,min} = 0.062\,\sqrt{f_c{}'}\,\frac{b_w.s}{f_{yt}} \leq 0.35\,\frac{b_w.s}{f_{yt}}$$

$$A_{v,min} = \frac{A_{ps}}{80}\,\frac{f_{pu}}{f_{yt}}\,\frac{s}{d}\,\sqrt{\frac{d}{b_w}}$$

use which is smaller

$\frac{3}{4}\,h$

600 mm

7. Or use the direct method to find the concrete shear resistance V_c

$$V_c = (0.05\,\lambda\sqrt{f_c{}'} + 4.8\,\frac{V_u\,d_p}{M_u})\,b_w.d \;\;\ldots\ldots.. \;\; \frac{V_u\,d_p}{M_u} \not> 1$$

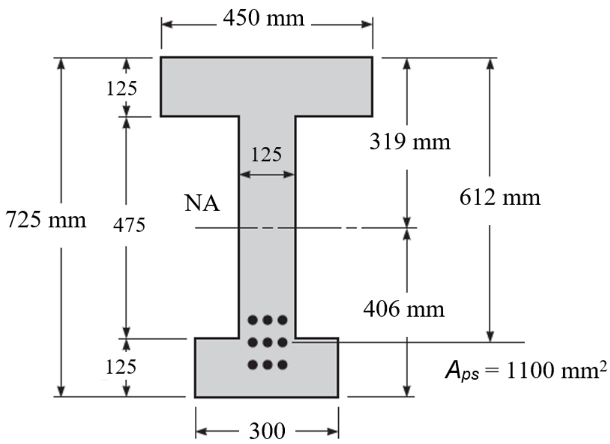
8. Check with the upper and lower limits

$$0.17\,\lambda\,\sqrt{f_c{}'}\,b_w.d_p < V_c < 0.42\,\lambda\,\sqrt{f_c{}'}\,b_w.d_p$$

9. Find the maximum spacing, use step (6).

Example (11-1):

The unsymmetrical I beam shown in the figure below carries an effective prestress force of 1280 kN and supports a superimposed dead load of 5 kN/m and service live load of 13 kN/m, in addition to its own weight of 3.6 kN/m, on a 15 m simple span. At the maximum moment section, the effective depth to the main steel is 612 mm, (eccentricity 293 mm). The strands are deflected upward starting 4.5 m from the support, and eccentricity is reduced linearly to zero at the support. If concrete with $f_c{}' = 35\ MPa$ and stirrups with $f_{yt} = 420\ MPa$ are used, and if the prestressed strands have strength $f_{pu} = 1860\ MPa$, **what is the required stirrup spacing at a point 3m from the support?** take $\lambda = 1$

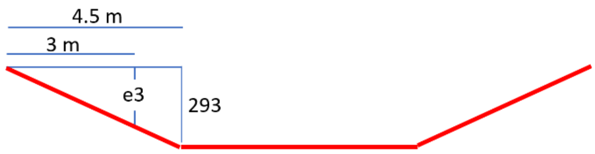


Solution:

I_c = 9.45 x 10⁹ mm⁴, A_c = 153 x 10³ mm²,

$r^2 = I_c / A_c$ = 61.8x10³ mm².

At a distance 3 m from the support centre-line, the tendon eccentricity is:



$e = 293 \text{ x } \frac{3}{4.5} = 195$ mm….. (293-195 = 98 mm)

corresponding to an effective depth d from the compression face of 612-98= 514 mm.

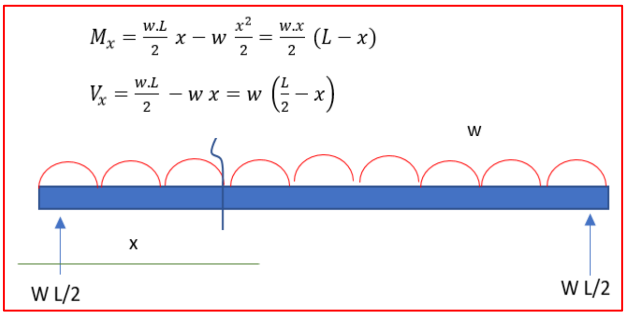
According to the ACI Code, the larger value of d = 0.80 × 725 = 580 mm will be used. The bottom-fiber stress due to effective prestress acting alone is:

$$f_{pe} = -\frac{P_e}{A_c}\left(1 + \frac{e.c_2}{r^2}\right) = -\frac{1280\ x\ 10^3}{153\ x\ 10^3}\left(1 + \frac{195\ x\ 406}{61.8\ x\ 10^3}\right) = -19.08\ MPa$$

The moment and shear at the section due to beam load alone are, respectively,

$$M_o = \frac{w_o.x}{2}\ (l - x) = \frac{3.6 \times 3}{2}\ (15 - 3) = 64.8\ kN.m$$

$$V_o = w_o\left(\frac{l}{2} - x\right) = 3.6\ \left(\frac{15}{2} - 3\right) = 16.2\ kN$$



and the bottom-fiber stress due to this load is

$$f_o = \frac{M_{o.10}\ x\ c_2}{I} = \frac{64.8x\ 10^6\ x406}{9.45\ x\ 10^9} = 2.78\ MPa$$

Then, from the equation of M_{cre},

$$M_{cre} = \frac{I}{y_t}\left(0.5\ \lambda\ \sqrt{f_c{}'} + f_{pe} - f_o\right) = \frac{9.45\ x\ 10^9}{406}(2.96 + 19.08 - 2.78)\ x\ 10^{-6}$$

$$= 448\ kN.m$$

The ratio of superimposed load shear to moment at the section is:

$$\frac{V_{d+l}}{M_{d+l}} = \frac{(l/2-x)}{x(l-x)/2} = \frac{l-2x}{x(l-x)} = \frac{15-2\times3}{3(15-3)} = 0.25\ m^{-1} \ldots\ldots\ldots\ldots\ldots\ldots.\frac{m}{m^2}$$

Equation of V_{ci} is then used to determine the shear force at which flexure-shear cracks can be expected to form:

$$V_{ci} = 0.05\,\lambda\,\sqrt{f_c{}'}\,b_w.d_p + V_o + \frac{V_{d+l}}{M_{d+l}}M_{cre}$$

$$= 0.05\ x\ 1x\sqrt{35}\ x\ (125x580)\ x\ 10^{-3} + 16.2 + 0.25x448 = 150\ kN$$

The lower limit of $0.14\,\lambda\,\sqrt{f_c{}'}\,b_w.d_p$

$= 0.14\ x1\ \sqrt{35}\ x\ (125x580)\ x\ 10^{-3} = 60\ kN$ does not control.

The slope θ of the tendons at the section under consideration is such that:

$\sin\theta \approx \tan\theta$ = 293/4500=0.065.

Consequently, the vertical component of the effective prestress force is:

V_P = 0.065 × 1280 = 83.2 kN

The concrete compressive stress at the section centroid is:

$$f_{pc} = \frac{P_e}{A_c} = \frac{1280\ x\ 10^3}{153\ x\ 10^3} = 8.37\ MPa$$

Now find the shear at which web-shear cracks should occur.

$$V_{cw} = \left(0.29\,\lambda\,\sqrt{f_c{}'} + 0.3\,f_{pc}\right)b_w.d_p + V_p$$

$$= \left(0.29\ \sqrt{35} + 0.3\ x\ 8.37\right)125\ x\ 580\ x\ \ 10^{-3} + 83.2 = 390\ kN$$

Thus, in the present case,

Take the lower value of $V_c = V_{ci} = 150\ kN$

At the section considered, the total shear force at factored loads is

$V_u = 1.2\ x\ 8.6\ x\ 4.5 + 1.6x13\ x\ 4.5 = 140\ kN$

When ϕ10 U stirrups are used, for which Av = 2 × 78.5 = 157 mm^2, the required spacing is found from:

$$s = \frac{\emptyset\,A_v\,f_{yt}.d}{(V_u-\emptyset\,V_c)} = \frac{0.75x\ 157\ x\ 420\ x\ 580}{(140-0.75\ x\ 150)x\ 10^3} = 1043\ mm$$

Applying following equation applied to establish a maximum spacing criterion.

$$A_{v,min} = \frac{A_{ps}}{80}\ \frac{f_{pu}}{f_y}\ \frac{s}{d}\ \sqrt{\frac{d}{b_w}} \Longrightarrow 157 = \frac{1100}{80}\ x\ \frac{1860}{420}x\ \frac{s}{580}\ \sqrt{\frac{580}{125}} = 0.22\ s$$

$$s = \frac{157}{0.22} = 713\ mm$$

The other criteria for maximum spacing, $\frac{3}{4}\ x\ 725 = 544\ mm\ and\ 600\ mm$, however, control here. Open U stirrups will be used, at a spacing of 540 mm.

For comparison, the concrete shear will be calculated on the basis of following equation. Note that ratio *Vu/Mu* is 0.25, and

$$V_c = \left(0.05\,\lambda\sqrt{f_c{}'} + 4.8\,\frac{V_u\,d_p}{M_u}\right) b_w . d_p$$

$$= \left(0.05\sqrt{35} + 4.8\,x0.25x0.58\right)(125x580)x\,10^{-3} = 71.9\,kN$$

The lower and upper limits,

$0.17\ \sqrt{35}\,(125\ x\ 580)x10^{-3} = 72.9\ kN$ and

$0.42\ \sqrt{35}\,(125\ x\ 580)x10^{-3} = 180\ kN$, do not control.

The required spacing of ϕ10 U stirrups is:

$$s = \frac{\emptyset\,A_v\,f_{yt}.d}{(V_u - \emptyset\,V_c)} = \frac{0.75x\ 157\ x\ 420\ x\ 580}{(140 - 0.75\ x\ 71.9)x\ 10^3} = 333\ mm$$

For the present case, an I-section beam of intermediate span, nearly 2 times the web steel is required at the location investigated if the alternative expression giving V_c directly is used.

Homework:

For a simply supported prestressed beam of 12 m span, design the required shear reinforcement at 4 m from the support, consider the figure shown below.

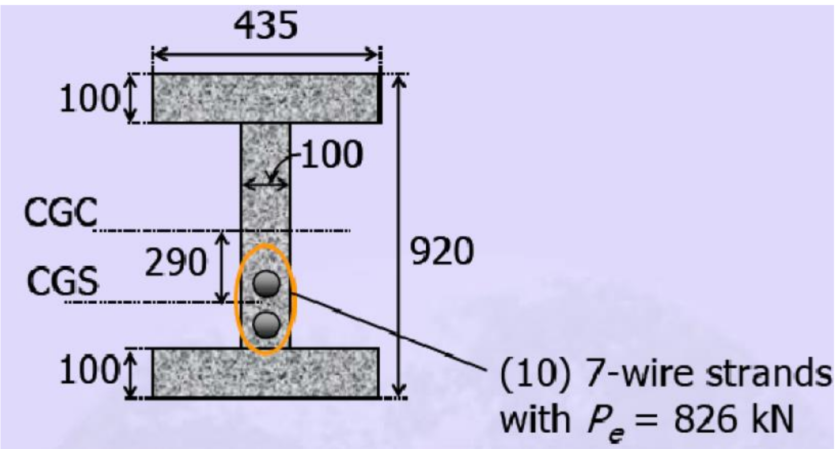
The properties of the section:

A_c = 159 x 10^3 mm^2, I = 17808 x 10^{10} mm^4, A_{ps} = 960 mm^2, $\lambda = 1$, $f_c{}'$= 35 MPa,

f_{pu} =1470 MPa, f_{pe} = 860 MPa, and $f_{yt} = 420\ MPa$.

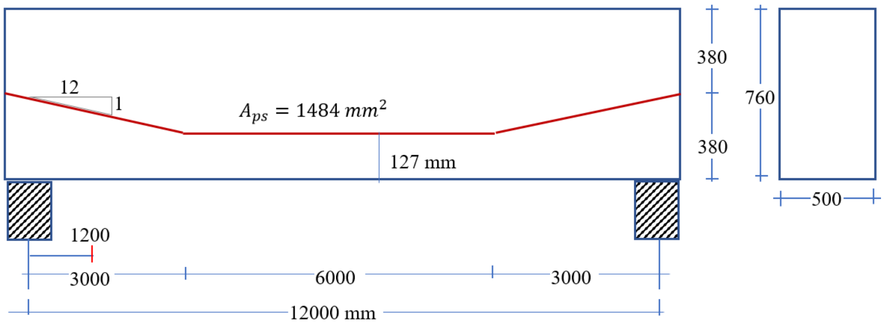
The service dead load is 15 kN/m and the live load is 18 kN/m, use 30 mm as a cover to the longitudinal reinforcement (ϕ 12).

Hint: assume any missing data.



Example (11-2):

For the simply supported prestressed concrete beam shown in the figure below, find $v_u = v_c$ at section located 1200 mm from the support center. Also, find the maximum stirrup spacing at the same section. P_e= 1449 kN, L.L. = 20 kN/mm^2, Beam self-weight= 9 kN/mm^2, $f_c{}'$ = 34 N/mm^2, $f_{pu=}$ 1725 N/mm^2, $f_{yt=}$ 275 N/mm^2, U stirrups of 10 mm diameter are to be used, $\lambda = 1$.



Solution:

$$w_u = 1.2\ x\ 9\ + 1.6\ x\ 20 = 42.8\ kN/m$$

$$V_u = w_u\left(\frac{l}{2} - x\right) = \ 42.8\ x\ (12/2 - 1.2) = 205.4\ kN$$

$$M_u = w_u \frac{x}{2}\ (l - x) = \ 42.8\ x \frac{1.2}{2}\ x\ (12 - 1.2) = 277.3\ kN.m$$

$d = 380 + 1200\ x\ 1/12 = 480\ mm$ Code ⇒ $d = 0.8\ x\ h = 0.8\ x\ 760 = 608\ mm$	The larger value of d will be used

$\therefore\ use\ d = 608\ mm$

1- Find v_c using the Direct method:

$$\frac{V_u.d_p}{M_u} = \frac{205.4\ x10^3 x\ 608}{277.3\ x10^6} = 0.45\ < 1.0 \quad \text{o.k.}$$

ACI Code formula for nominal stress:

$$v_c = 0.05\sqrt{f_c{}'} + 4.8\ \frac{V_u}{M_u} d_p = 0.05\sqrt{34} + 4.8\ x\ 0.45 = 2.45\ \text{N/mm}^2$$

Lower limit = $0.17\sqrt{f_c{}'} = 0.17\sqrt{34}$ = 0.99 N/mm^2 < 2.45 N/mm^2

Upper limit = $0.42\sqrt{f_c{}'} = 0.42\sqrt{34}$ = 2.45 N/mm^2 = 2.45 N/mm^2

$\therefore\ take\ v_c = 2.45$ N/mm^2

2- Find v_c using the lower value of $v_{ci}\ and\ v_{cw}$:

a- Flextural shear stresses:

$$I = \frac{1}{12}\ x\ 500\ x\ 760^3 = 18.3\ x\ 10^9\ mm^4\ ,\ c = c1 = c2 = 380\ mm,$$

$$e = \frac{1200}{12} = 100\ mm\ \text{ from the slope}$$

$$M_{cre} = \frac{I}{c}\left(0.5\sqrt{f_c{}'} + f_p - f_d\right)$$

f_p: Compressive stress in concrete due to effective prestress (after losses).

$f_p = -\frac{P_e}{A} - \frac{P_e.e}{I/c} = -\frac{1449x10^3}{500\ x\ 760} - \frac{1449\ x\ 100x10^3}{18.3\ x\ 10^9/\ 380} = -3.8 - 3 = 6.8\ N/mm^2$

f_o: Unfactored dead load stress at the same extrem fiber as (f_p) (D.L $_{beam}$+ superimposed D.L)

$M_{o\ (unfactored)} = 9\ x\ 1.2\ (6 - 0.6) = 58.3\ kN.m$

$f_o = \frac{M_o}{I/c} = \frac{58.3\ x10^6}{18.3\ x\ 10^9/\ 380} = 1.2\ N/mm^2$

$M_{cre} = \frac{I}{c}\left(0.5\sqrt{f_c{'}} + f_p - f_o\right) = \frac{18.3\ x\ 10^9}{380}\left(0.5\ \sqrt{34} + 6.8 - 1.2\right)x\ 10^{-6} = 410\ kN.m$

Factored ***Live Load*** moment and shear:

$M_L = 1.6\ x\ 20\ x\ 1.2\ (6 - 0.6) = 207.36\ kN.m$

$V_L = 1.6\ x\ 20\ (6 - 1.2) = 153.6\ kN$

$\frac{V_L}{M_L}\ M_{cre} = \frac{153.6}{207.36}\ x\ 410 = 303.7\ kN$

$\frac{L-2x}{x\ (L-x)} = \frac{12-2\times1.2}{1.2\ (12-1.2)} = 0.74, \quad \frac{V_L}{M_L} = \frac{153.6}{207.36} = 0.74$

Dead load shear $V_o = 9\ (6 - 1.2) = 43.2\ kN$, $d = 608\ mm$ Code (d = 0.8 h)

$v_{ci} = 0.05\sqrt{f_c{'}} + \frac{\frac{V_l}{M_l} M_{cre} + V_o}{b_w.d_p} = 0.05\sqrt{34} + \frac{(303.7+43.2)\ x\ 10^3}{500\ x\ 608} = 1.43\ N/mm^2$

Minimum $v_{ci} = 0.14\ \sqrt{f_c{'}} = 0.14\ \sqrt{34} = 0.82 < 1.43\ N/mm^2$ o.k

b- Web shear cracking stress:

$v_{cw} = 0.29\sqrt{f_c{'}} + 0.3 f_{pc} + \frac{V_P}{b_w.d_p}$

f_{pc}: The compressive stress in the concrete, after losses, at the centroid of the concrete section (or at the junction of the web and the flange when the centroid lies in the flange).

$f_{pc} = \frac{1449\ x\ 10^3}{500\ x\ 760} = 3.8\ N/mm^2$

For small value of $\theta \Rightarrow \sin\theta = \tan\theta$

$V_p = \frac{1}{12} x\ 1449 = 120.75\ kN$

$v_{cw} = 0.29x\ \sqrt{34} + 0.3x3.8 + \frac{120.75\ x\ 10^3}{500\ x\ 608} = 3.23\ N/mm^2$

$v_{ci} < v_{cw}$ take the smaller value

$\therefore v_{ci}$ controls = 1.43 N/mm^2 (or take v_c directly = 2.45 N/mm^2)

Shear stress at the considered section (at factored load) is v_u.

$v_u = \frac{V_u}{b_w.d_p} = \frac{205.4\ x\ 10^3}{500\ x\ 608} = 0.676\ N/mm^2$

$\emptyset\ \frac{v_c}{2} = \frac{0.75\ x\ 1.43}{2} = 0.54$

$\therefore v_u > \emptyset\ \frac{v_c}{2}$ (nominal web reinforcement is required)

3- Find the maximum spacing:

$\therefore A_v = 0.35\ \frac{b_w.s}{f_{yt}}$

For two legs $2\ x\ 78.5 = 0.35\ \frac{500\ x\ s}{275} \Longrightarrow s = \frac{157}{0.636} \Longrightarrow \therefore s \approx 247\ mm$

Maximum spacing = $\frac{3h}{4} = 0.75\ x\ 760 = 570\ mm$ or $600\ mm$

$\therefore take\ s = 250\ mm$

Example (11-3):

For a simply supported prestressed concrete beam of 16 m span, design the required shear reinforcement at a distance 1.8 m from the support, consider the figure shown below.

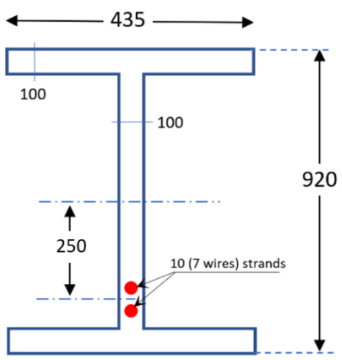
Section properties:

A_c = 159 x 10^3 mm^2, I = 17.808 x 10^9 mm^4, A_{ps} = 1200 mm^2, $\lambda = 1$, $f_c{}'$= 35 MPa,

$P_e = 950\ kN$, f_{pu} =1470 MPa, and $f_{yt} = 420\ MPa$, r^2 =112x10^3 mm^2

The service dead load is 9 kN/m and the live load is 12 kN/m.

At the considered section tan θ = 0.059.



Solution:

$w_o = 159\ \times 24\ x\ 10^{-3} = 3.82$ kN/m

$w_u = 1.2\ \times\ (3.82 + 9) + 1.6\ x\ 12 = 34.6\ kN/m$

$V_u = w_u\left(\frac{l}{2} - x\right) = 34.6\ \times\ (16/2 - 1.8) = 214.5\ kN$

$M_u = w_u\frac{x}{2}\ (l - x) = 34.6\ \times \frac{1.8}{2}\ \times\ (16 - 1.8) = 442.2\ kN.m$

At considered section: d = 250 + 920/2 = 710 mm

ACI Code: d = 0.80 × 920 = 736 mm

According to the ACI Code, use the larger d:

$\therefore d$ = 736 mm

1- Flexural-shear stress:

$$v_{ci} = 0.05\,\lambda\sqrt{f_c{}'} + \frac{V_o + \frac{V_{d+l}}{M_{d+l}}M_{cre}}{b_w.d_p}$$

$$M_{cre} = \frac{I}{c}\left(0.5\,\lambda\sqrt{f_c{}'} + f_p - f_o\right)$$

The bottom-fiber stress due to effective prestress acting alone is:

$f_p = -\ \frac{P_e}{A_c}\left(1 + \frac{e.c}{r^2}\right)$ ……. c = c1 = c2 = 920/2 = 460 mm

$$f_p = -\frac{950 \times 10^3}{159 \times 10^3}\left(1 + \frac{250 \times 460}{112 \times 10^3}\right) = -12.11\ MPa$$

The moment and shear at the section due to beam load self-weight:

$M_o = \frac{w_o . x}{2}\ (l - x) = \frac{3.82 \times 1.8}{2}\ (16 - 1.8) = 48.82\ kN.m$ (without load factor) used to find f_o.

$V_o = w_o\left(\frac{l}{2} - x\right) = 3.82\ \left(\frac{16}{2} - 1.8\right) = 23.68\ kN$ (without load factor) used in v_{ci} (V_{ci}) equation.

and the bottom-fiber stress due to this load is:

$$f_o = \frac{M_o}{I/c} = \frac{48.82 \times 10^6 \times 460}{17.808 \times 10^9} = 1.26\ MPa$$

Then, from the equation of M_{cre},

$$M_{cre} = \frac{I}{c}\left(0.5\,\lambda\sqrt{f_c{}'} + f_p - f_o\right)$$

$$M_{cre} = \frac{17.808\ x\ 10^9}{460}\left(0.5\ \times \sqrt{35} + 12.11 - 1.26\right) \times 10^{-6} = 534.55\ kN.m$$

The ratio of superimposed load shear to moment at the section is:

$$\frac{V_{d+l}}{M_{d+l}} = \frac{\left(\frac{l}{2} - x\right)}{\frac{x}{2}(l - x)} = \frac{8 - 1.8}{0.9 \times (16 - 1.8)} = 0.485\ m^{-1}$$

Equation of v_{ci} is then used to determine the shear stress at which flexure-shear cracks can be expected to form:

$$v_{ci} = 0.05\,\lambda\sqrt{f_c{}'} + \frac{V_o + \frac{V_{d+l}}{M_{d+l}}M_{cre}}{b_w . d_p}$$

$$= 0.05 \times \sqrt{35} + \frac{23.68 + 0.485 \times 534.55}{100 \times 736} \times 10^3 = 4.14\ MPa$$

The lower limit of $0.14\,\lambda\sqrt{f_c{}'} = 0.14\sqrt{35} = 0.83\ MPa$

$v_{ci} = 4.14\ MPa$ will be used

2- Web-shear stress:

The slope θ of the tendons at the section under consideration is such that:

$\sin\theta \approx \tan\theta = 0.059$

Consequently, the vertical component of the effective prestress force is:

$V_p = P_e \times \tan\theta = 950 \times 0.059 = 56.1$ kN

The concrete compressive stress at the section centroid is:

$$f_{pc} = \frac{P_e}{A_c} = \frac{950 \times 10^3}{159 \times 10^3} = 5.97\ MPa$$

Now find the shear at which web-shear cracks should occur.

$$v_{cw} = \left(0.29\,\lambda\sqrt{f_c{}'} + 0.3\ f_{pc}\right) + \frac{V_p}{b_{w.}d_p}$$

$$= \left(0.29\ \sqrt{35} + 0.3\ \times\ 5.97\right) + \frac{56.1 \times 10^3}{100 \times 736} = 4.27\ MPa$$

Thus, in the present case,

Take the lower value of $v_c = v_{ci} = 4.14\ MPa$

3- Direct method:

$v_c = 0.05\ \lambda\sqrt{f_c{'}} + 4.8\ \frac{V_u\ d_p}{M_u}$ ……… $\frac{V_u}{M_u} = 0.485\ m^{-1}$

$= 0.05\sqrt{35} + 4.8\ \times 0.485 \times 0.736\ = 2.01\ MPa$

The lower and upper limits,

$0.17\ \sqrt{35}\ = 1.01\ MPa$ $0.42\ \sqrt{35}\ = 2.48\ MPa$

Therefore, $v_c = 2.03$ MPa

Finally

$v_c = 4.14$ MPa or

$v_c = 2.03$ MPa from the direct method

4- Maximum stirrups spacing:

Using stirrups 10 mm rebar $A_v = 157\ mm^2$ for two legs

Applying following equation applied to establish a maximum spacing criterion.

$$A_{v,min} = \frac{A_{ps}}{80}\ \frac{f_{pu}}{f_y}\ \frac{s}{d}\ \sqrt{\frac{d}{b_w}}$$

$$157 = \frac{1200}{80} \times \frac{1470}{420} \times \frac{s}{736}\ \sqrt{\frac{736}{100}} \Longrightarrow 157 = 0.194\ s$$

$s = \frac{157}{0.194} = 809\ mm$

$s = \frac{3}{4}\ x\ 920 = 690\ mm$

$s = 600\ mm$

Use the lowest value of s

∴ Use U stirrups Ø 10 mm at a spacing of 600 mm.

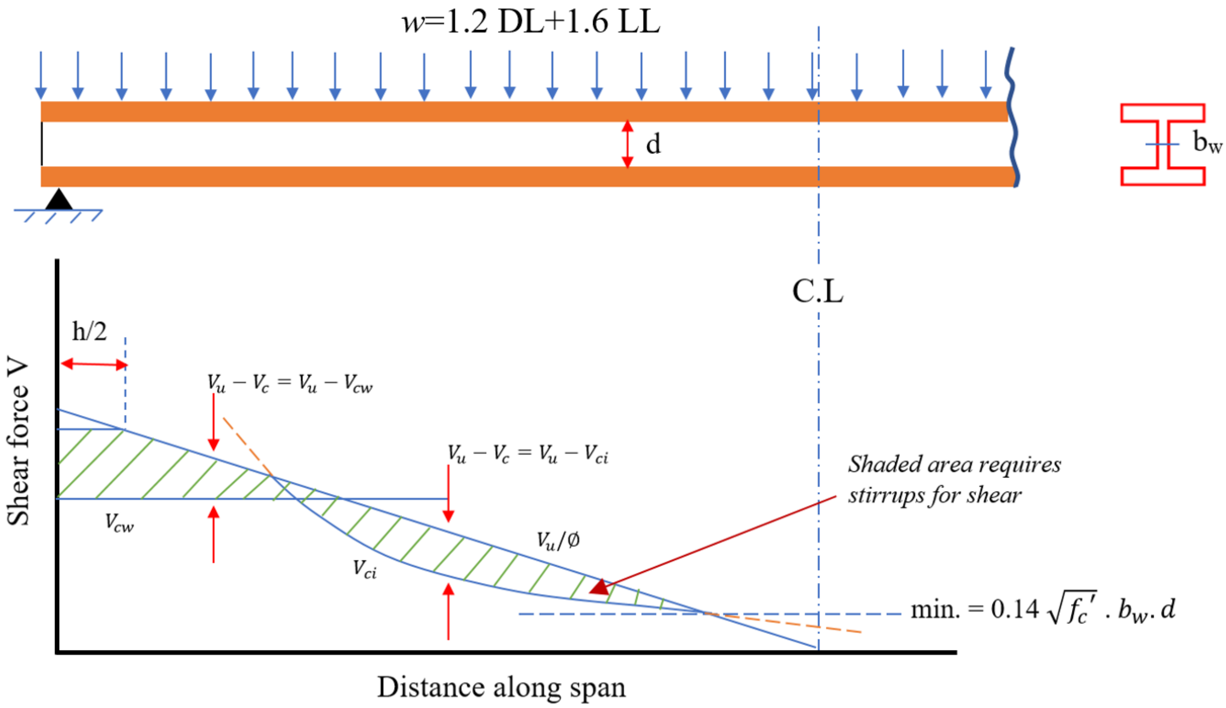


Figure 12: ACI analysis for shear strength distribtuion of shear along the span

12- Camber and Deflections

Deflection of the slender relatively flexible beams that are made possible by prestressing must be predicted with care. Many members have proved to be unserviceable because of excessive deformation. In some cases, the absolute amount of deflection is excessive. Often, it is the differential deformation between adjacent members (for instance: precast roof-deck units) that causes problems. More often than not, any difficulties that occur are associated with upward deflection due to the sustained prestress load. Such difficulties are easily avoided by proper consideration in design.

When the prestress force is first applied, a beam will normally camber upward. With the passage of time, concrete shrinkage and creep will cause a gradual reduction of prestress force. In spite of this, the upward deflection usually will increase, due to the differential creep, affecting the highly stressed bottom fibres more than the top. With the application of superimposed dead and live loads, this upward deflection will be partially or completely overcome, and zero or downward deflection obtained. Clearly, in computing deformation, careful attention must be paid to both the age of the concrete at the time of load application and the duration of the loading.

- ❖ Prestressed concrete beams are slender more than reinforced concrete beams and having high span/depth ratios; thus, more deflection.
- ❖ Curvature induced by prestressing causes the member to deform resulting in camber, ***which is the upward displacement of a flexural member resulting from an eccentric prestress force***. Camber may important, and it may increase due to concrete creep with time.
  - ➢ The bridge camber may induce the pavement to be uneven and dangerous.
  - ➢ Excessive roof camber may create drainage problems.
  - ➢ Excessive floor camber → partition cracking and other non-structural cracking.
- ❖ The total deflection is a resultant of the upward deflection due to prestressing force and downward deflection due to the gravity loads.
- ❖ Only the flexural deformation is considered and any shear deformation is neglected in the calculation of deflection.

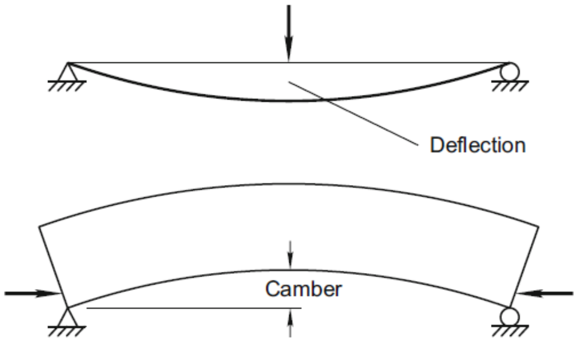


Figure 13: Deflection and camber in a beam

❖ The deflection of a member is calculated at least for two cases:

- ***Short term deflection at transfer***: The short term deflection at transfer is due to the initial prestressing force and self-weight neglecting the effect of creep and shrinkage of concrete.
- ***Long term deflection at service loading***: The long term deflection under service loads is due to the effective prestressing force and the total gravity loads

Table 5: Maximum Permissible Computed Deflections (ACI Code)

Type of member	**Deflection to be considered**	**Deflection limitation**
Flat roofs not supporting or attached to non-structural elements likely to be damaged by large deflections	Immediate deflection due to live load	$L/180$
Floors not supporting or attached to non-structural elements likely to be damaged by large deflections		$L/360$
Roof or floor construction supporting or attached to non-structural elements likely to be damaged by large deflections	That part of the total deflection occurring after attachment of non-structural elements (sum of the long-term deflection due to all sustained loads and the immediate deflection due to any additional live load)	$L/480$
Roof or floor construction supporting or attached to non-structural elements not likely to be damaged by large deflections		$L/240$

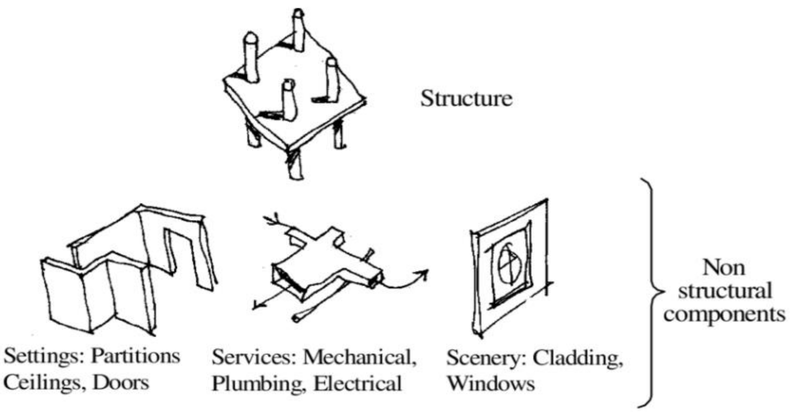


Table 6: Section properties to be used in computing deflections based on member classification

Condition	Class U	Class T	Class C
Assumed behavior	Uncracked	Transition between cracked and uncracked	Cracked
Deflection calculation basis	Gross section	Cracked section—bilinear behavior	Cracked section—bilinear behavior

Moment of Inertia

- Class U: $f_t \leq 0.62\sqrt{f_c'}$……….. Use gross section moment of inertia, I_g
- Class T: $0.62\sqrt{f_c'} \leq f_t \leq \sqrt{f_c'}$ ……….. Use effective moment of inertia, I_e
- Class C: $f_t > \sqrt{f_c'}$ ……….. Use effective moment of inertia, I_e

f_t : concrete tensile strength.

Alternatively, use long-term multipliers from the Precast/Prestressed Concrete Institute PCI (Table 7).

Table 7: PCI (Prestress Concrete Institute) multipliers for long-term deflection and camber

At Erection	**Without composite topping**	**With composite deflection**
Deflection (downward) component: apply to the elastic deflection due to the member weight at release of prestress	1.85	1.85
Camber (upward) component: apply to the elastic camber due to prestress at the time of release of prestress	1.8	1.8
Final		
Deflection (downward) component: apply to the elastic deflection due to the member weight at release of prestress	2.70	2.40
Camber (upward) component: apply to the elastic camber due to prestress at the time of release of prestress	2.45	2.20
Deflection (downward): apply to the elastic deflection due to the superimposed dead load only	3.00	3.00
Deflection (downward): apply to the elastic deflection caused by the composite topping	--	2.30

Multipliers have derived for precast prestressed members with or without composite topping slabs, to estimate the total long-term effects of prestressing, member self-weight, and the composite slab if present. To use these multipliers, which result in total (immediate plus time-dependent) deflection components; the upward and downward components of the initial elastic camber should be separated to take into account the effects of loss of prestressing, which affects only the upward component. In Table (7), the term “erection” refers to the time the precast member is placed in the structure, assumed to be from 30 to 60 days following casting; thus, both immediate and partial time-dependent effects are included at this stage. The term “final” refers to the stage at which all loss and creep effects have occurred.

Deflection Due to dead and live loads:

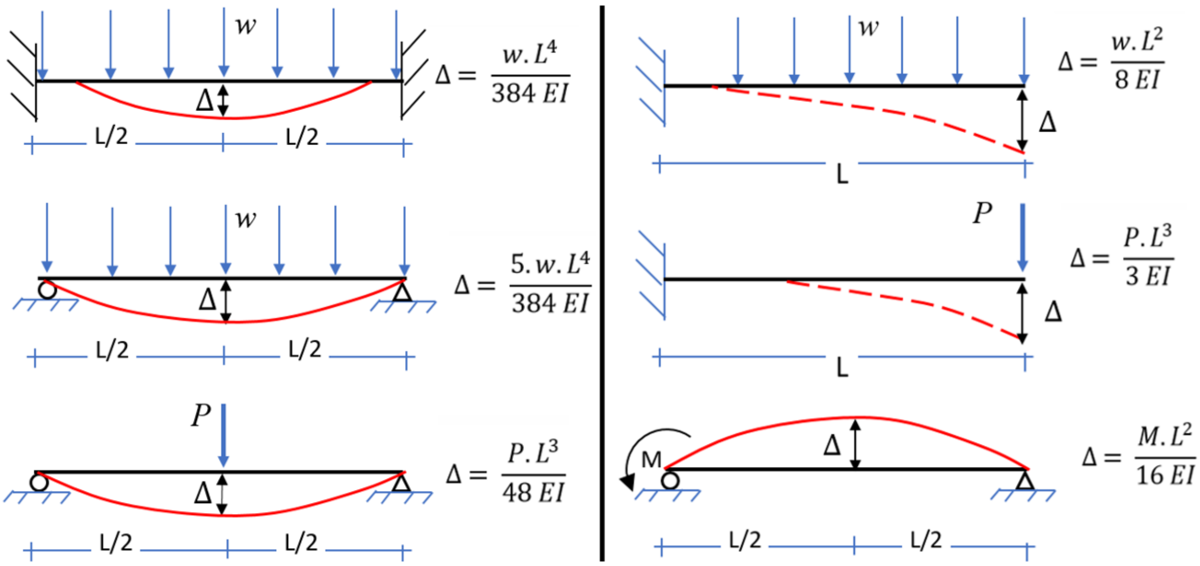


Figure 14: Deflection formula due to different cases

Deflection due to Prestressing Force

The prestressing force causes a deflection only if the CGS (*Center of gravity of prestressing tendon)* is eccentric ($e \neq 0$) to the CGC (*Center of gravity of concrete section)*.

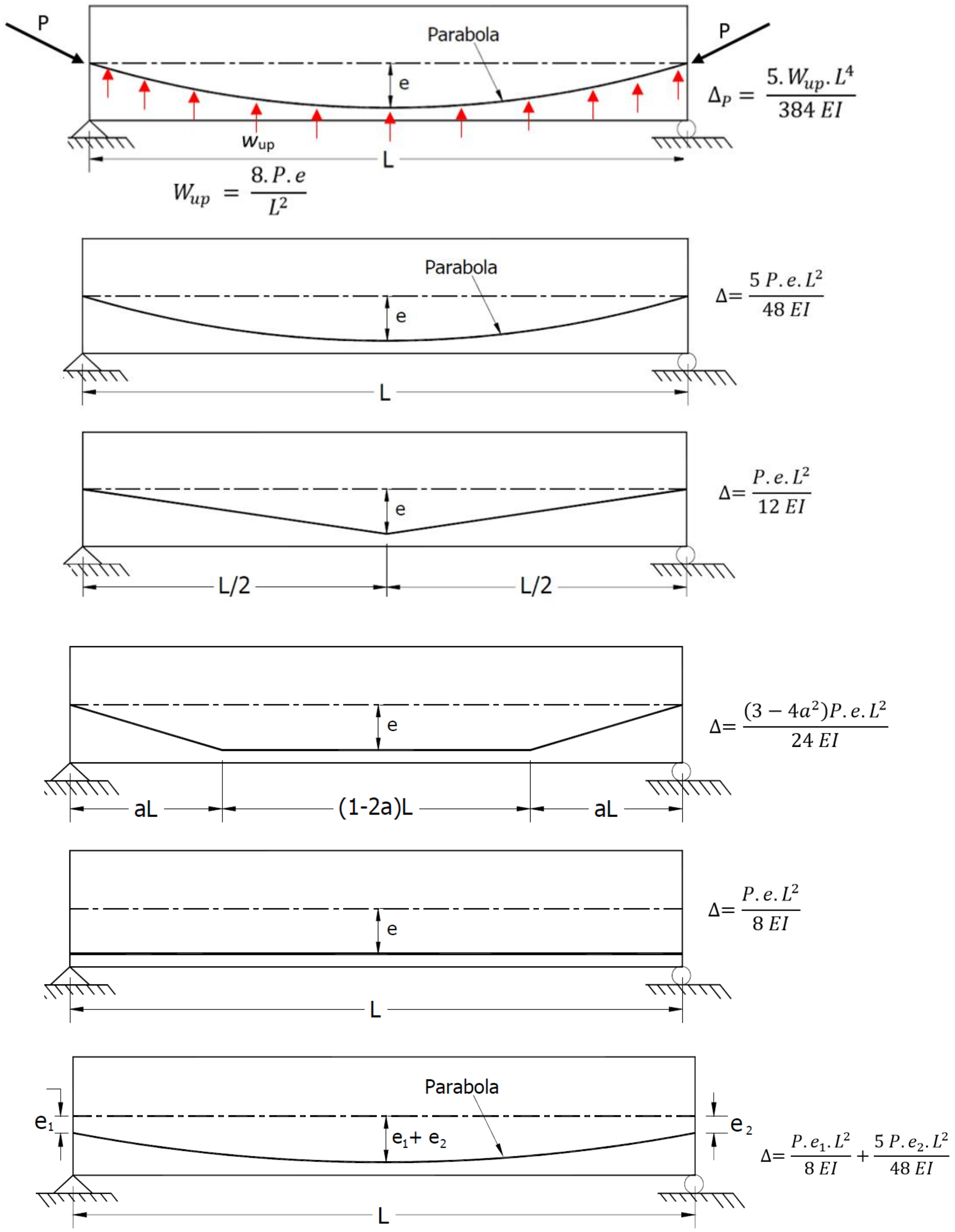


Figure 15: Deflection due to some prestressing force cases

Example (12-1):

A concrete beam with a cross sectional area of 32000 mm^2 and radius of gyration of 72 mm, is prestressed by a parabolic cable carrying an effective stress of 1000 N/mm^2. The span of the beam is 8 m, and the cable composed of 6 (7 mm diameter) wires, has an eccentricity of 50 mm at the centre and zero at the supports. Neglecting all losses, find the central deflection of the beam at:

a- Self-weight + prestress; and

b- Self-weight + prestress + live load of 2 kN/m.

Solution:

Assume Ec = 38 kN/mm^2, and concrete unit weight = 24 kN/m^3

A= 32 x 10^3 mm^2, radius of gyration (r) = 72 mm, L = 8000 mm, e = 50 mm

I = A. r^2 = 32 x 10^3 x (72)^2 = 165.9 x 10^6 mm^4

$$P = 6\,x\,\left(7^2 x \frac{\pi}{4}\right)\,x\,1000 = 231000\text{ N} = 231\text{ kN}$$

$$w = \frac{32x10^3}{10^6}\,x\,24 = 0.77\text{ kN/m} = 0.77\,x\,10^{-3}\text{ kN/ mm}$$

downward deflection due to self-weight $= \frac{5.w.L^4}{384\,EI} = \frac{5\,x\,0.77\,x\,10^{-3}\,x\,8000^4}{384\,x\,38\,x\,165.9\,\,x\,10^6} = 6.5\,mm\ \downarrow$ deflection

upward deflection due prestressing force $= \frac{5.P.e.L^2}{48\,EI} = \frac{5\,x\,231\,x\,50\,x\,8000^2}{48\,x\,38\,x\,165.9\,\,x\,10^6} = 12.2\,mm\ \uparrow$ camber

downward deflection due to live load $= \frac{2}{0.77} x\,6.5 = 16.9\,mm\ \downarrow$ deflection

a- Self-weight + prestress= $\Delta_1 = 12.2 - 6.5 = 5.7\,mm \uparrow$

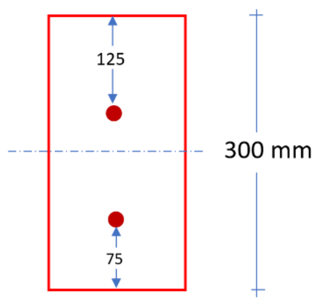
b- Self-weight + prestress + live load of 2 kN/m = $\Delta_2 = 6.5 - 12.2 + 16.9 = 11.2\,mm\ \downarrow$

Example (12-2):

A rectangular concrete beam of cross section 150 mm wide and 300 mm deep is simply supported over a span of 8 m and prestressed by means of a symmetric parabolic cable, at a distance of 75 mm from the bottom of the beam at mid-span and 125 mm from the top of the beam at support sections. If the force in the cable is 350 kN and the modulus of elasticity of concrete is 38 kN/mm^2, calculate:

a- The deflection at mid-span, consider beam self-weight only, and

b- The concentrated load which must be applied at mid-span to restore it to the level of supports.

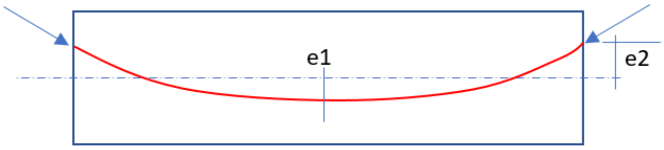


Solution:

P = 350 kN, Ec = 38 kN/mm^2, L = 8000 mm,

e1 = 150- 75 = 75 mm, e2=150 - 125 = 25 mm,

I = $\frac{150}{12}\ x\ 300^3$ = 337.5 x 10^6 mm^4



$\Delta = \frac{P.L^2}{48\ EI}(-5e1 + e2)$

Net deflection due to prestressing force

$$\Delta = \frac{P.L^2}{48\ EI}(-5e1 + e2) = \frac{350\ x\ 8000^2}{48\ x\ 38\ x\ 337.5\ \ x\ 10^6}(-5x75 + 25) = -12.7\ mm = 12.7\ mm\ \uparrow$$

Self-weight (w) = 0.15 x 0.3 x 24 = 1.08 kN/m= $1.08\ x\ 10^{-3}$ kN/ mm

downward deflection due to self-weight = $\frac{5.w.L^4}{384\ EI} = \frac{5\ x\ 1.08\ x\ 10^{-3}\ x\ 8000^4}{384\ x\ 38\ x\ 337.5\ \ x\ 10^6} = 4.5\ mm\ \downarrow$

a- Deflection due to (prestress + self-weight) = $-12.7 + 4.5 = \ -8.2\ mm = 8.2\ mm\ \uparrow$

b- If Q = concentrated load required at the centre of span, then:

$$\Delta = \frac{Q\ x\ L^3}{48\ EI} = 8.2\ \text{, Therefore. } Q = \frac{8.2\ x\ 48\ x\ 38\ x\ 337.5\ x10^6}{8000^3} = 9.86\ kN$$

Example (12-3):

A simply supported beam with a uniform cross section spanning over 6 m is posttensioned by two cables, both of which have an eccentricity of 100 mm below the centroid of the section at mid-span. The first cable is parabolic and is anchored at an eccentricity of 100 mm above the centroid at each end, the second cable is straight and parallel to the line joining the supports. The cross-sectional area of each cable is 100 mm^2 and they carry an initial stress of 1200 N/mm^2. The concrete cross-section of 2 x 10^4 mm^2 and a radius of gyration is 120 mm. The beam supported two concentrated loads of 20 kN each at the third points of the span, Ec = 38 kN/mm^2. Calculate:

a- The instantaneous deflection at the centre of span; and

b- The deflection at the centre of span after 2 years, assuming 20 % loss in prestress and the effective modulus of elasticity to be 1/3 of the short term modulus of elasticity.

Solution:

A= 2 x 10^4 mm^2, radius of gyration (r) = 120 mm, L = 6000 mm, e1 =e2= 100 mm

I = A. r^2 = 2 x 10^4 mm^2 x (120)^2 = 288 x 10^6 mm^4, P = 1200 x 100 x10^{-3}=120 kN,

Self-weight w = 2 x 10^4 x 24 x 10^{-6} =0.48 kN/m = $0.48\ x\ 10^{-3}$ kN/ mm,

concentrated loads at the third points of the span = 20 kN

a- instantaneous deflection at the centre of span:

downward deflection due to self-weight = $\frac{5.w.L^4}{384\ EI} = \frac{5\ x\ 0.48\ x\ 10^{-3} x\ 6000^4}{384\ x\ 38\ x\ 288\ x\ 10^6} = 0.74\ mm\ \downarrow$

downward deflection due to concentrated loads = $\frac{23\ Q\ L^3}{648\ EI} = \frac{23\ x\ 20\ x\ 6000^3}{648\ x\ 38\ x\ 288\ x\ 10^6} = 14\ mm\ \downarrow$

deflection due to prestressing force

deflection due to parabolic cable = $\frac{P.L^2}{48\ EI}\ (-5e1 + e2)$

$$= \frac{120\ x\ 6000^2}{48\ x\ 38\ x\ 288\ x\ 10^6}(-5x100 + 100) = \ 3.29\ mm\ \uparrow$$

Deflection due to straight cable = $-\ \frac{P.e.L^2}{8\ EI} = -\frac{120\ x100\ x\ 6000^2}{8\ x\ 38\ x288x\ 10^6} = \ 4.93\ mm\ \uparrow$

Instantaneous deflection $= (-3.29 - 4.93) + 0.74 + 14 = 6.52\ mm\ \downarrow$

b- The deflection at the centre of span after 2 years

$E_{ce} = \frac{E}{3}\ and\ losses\ of\ prestress = 20\%$

deflection due to prestressing force (losses and new modulus of elasticity will be accounted)

Upward deflection = $3\ [(1 - 0.2)\ x\ (3.29 + 4.93)] = 19.73\ mm\ \uparrow$

deflection due to self-weight and external loads (only new modulus of elasticity will be accounted)

Downward deflection = $3\ (0.74 + 14\ ) = 44.22\ mm\ \downarrow$

Δ_{net} = 44.22 – 19.73 = 24.49 $mm\ \downarrow$