## Regular expression (RE) -:

Regular expressions consist of constants, which denote sets of strings, and operator symbols, which denote operations over these sets.

The rules defining regular expression can be summarized as follows: -

1) $\boldsymbol{\emptyset}$ is a regular expression and denotes the empty set.
2) $\boldsymbol{\epsilon}$ is a regular expression and denotes the set $\{\boldsymbol{\epsilon}\}$.
3) For all $\mathbf{a} \boldsymbol{\epsilon T ,}$, is a regular expression and denotes the set $\{\mathbf{a}\}$.
4) If $\mathbf{r}, \mathbf{s}$ are regular expressions denoting the languages $\mathbf{L}_{\mathbf{r}}$ and $\mathbf{L}_{\mathbf{s}}$ then: -
a. $(\mathbf{r})+(\mathbf{s})$ is $R E$ denotes $\mathbf{L}_{\mathbf{r}} \mathbf{U} \mathbf{L}_{\mathbf{s}}$.
b. (r). (s) is RE denotes $\mathbf{L}_{r}$. $\mathbf{L}_{\mathbf{s}}$.
c. (r)* is RE denotes $\mathbf{L}_{\mathbf{r}}$

## Example/

$$
(a+b)^{*} b a(b a)^{*}
$$

## Graph: -

A graph, denoted $\mathrm{G}=(\mathrm{V}, \mathrm{E})$, consists of a finite set of vertices (or node)
V and a set of pairs of vertices $\boldsymbol{E}$ called edges, this graph is known undirected graph.

## Example /


$\mathrm{V}=\{1,2,3\}$
$\mathrm{E}=\{(1,2),(2,3)\}$

A path in a graph is a sequence of vertices $v_{1}, v_{2}, \ldots . v_{k}, k \geq 1$, such there edge $\left(v_{i}, v_{i+1}\right)$ for each $i, 1 \leq i \leq k$, the length of the path is $k-1$.

For example, $1,2,3$ is a path of length 2.
A directed graph, also denoted $\mathbf{G}=(\mathbf{V}, \mathbf{E})$, consists of a finite set of vertices $\mathbf{V}$ and a set of pairs of vertices $\mathbf{E}$ called arcs.

## Example /

he graph $\mathrm{G}=(\mathrm{V}, \mathrm{E})$ where ,
$V_{1}=\{v, w, x\}$ and $E_{1}=\{(v, w),(v, x),(w, x),(x, v),(x, x)\}$.


A path in a directed graph is a sequences of vertices $\mathrm{v}_{1}, \mathrm{v}_{2}, \ldots . \mathrm{v}_{\mathrm{k}}, \mathrm{k} \geq 1$,
Such that $v_{i} \rightarrow v_{i+1}$, is an arc for each $i, 1 \leq i \leq k$.
If $\mathbf{v} \rightarrow \mathbf{w}$ is an arc, we say $\mathbf{v}$ is predecessor of $\mathbf{w}$ and $\mathbf{w}$ is successor of $\mathbf{v}$.

## Transition graph to RE:-

The transition graph consists of:-

1. Nodes.
2. Directly labeled edge
3. Initial state denoted as:
4. Final state denoted as:


## Example1

Draw the transition graph for the following $\mathrm{RF}=0+1$


The production is $\mathrm{P}=\{\mathrm{S} \rightarrow 0 / 1\}$

## Example2/

Draw the transition graph for the following $\mathrm{RE}=01+10$
Answer/


The production is $\mathrm{P}=\{\mathrm{S} \rightarrow 0 \mathrm{~A} / 1 \mathrm{C}, \mathrm{A} \rightarrow 1, \mathrm{C} \rightarrow 0$

## Example3 /

Draw the transition graph for the following $\mathrm{RE}=\mathrm{a}^{*}$

## Answer/



## Example4 /

Draw the transition graph for the following $\mathrm{RE}=(\mathrm{ab}+\mathrm{ba})^{*}$

## Answer/



## Example5 /

Draw the transition graph for the following $\mathrm{RE}=(\mathrm{a}+\mathrm{b})^{*} \mathrm{ba}(\mathrm{ba})^{*}$
Answer/


## Examples5 /

1. $\mathrm{RE}=(\mathrm{a}+\mathrm{b})$

$$
\text { Words= }\{\mathrm{a}, \mathrm{~b}\}
$$

2. $R E=(a+b)(a+b)$

Words $=\{a a, a b, b a, b b\}$
3. $R E=(a+b)(a+b)(a+b)$

$$
\text { Words }=\{a a a, a b b, a b a, \ldots\}
$$

4. $\mathrm{RE}=\mathrm{a}^{*}=\{\mathrm{a}, \mathrm{aa}, \mathrm{aaa}, \mathrm{aaaa}, \ldots .$.

## Examples6/

5. language: alphabet (a,b) All the words must start with $b$

Answer/

$$
b(a+b)^{*}
$$

## Notel

$$
b a^{*} \neq(b a)^{*}
$$

## Examples6/

At least one a
Answer

$$
(a+b)^{*} a(a+b)^{*}
$$

At least two a's

## Answer/

$$
(a+b)^{*} a(a+b)^{*} a(a+b)^{*}
$$

## H.W/

Draw the transition graph for the following: -

1. $\mathrm{RE}=00^{*} 1$
2. $\mathrm{RE}=(\mathrm{a}+\mathrm{b})^{*} \mathrm{abb}$
3. $\mathrm{RE}=\left((\mathrm{abc})^{*}(\mathrm{cba})^{*}\right)^{*}$
4. $\mathrm{RE}=\mathrm{a} \cdot(\mathrm{ba}+\mathrm{b})^{*}+\mathrm{b}$
5. $\mathrm{RE}=0 * 10 * 1(0+1)^{*}$
6. Exactly two a's?
7. At least one $a$ or at least one $b$ ?
