

Derivations Tree:

It is useful to display derivations as trees. These pictures, called derivation (or generation or production or parse, syntax) trees.

Leaf: a vertex which has no sons, usually represent a terminal.

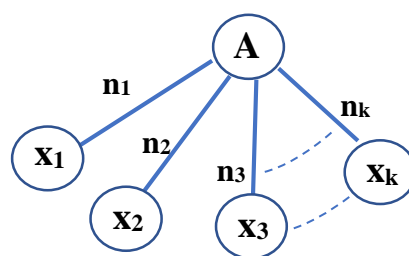
Interior vertex: a vertex with one or more sons usually belongs to a nonterminal.

Yield of the derivation tree: if we read the label of the leaves from left to right, we have a sentential form. We call this string the yield.

Definition:

Let $G=(N, T, P, S)$ be a grammar, a tree is a derivation (parse) tree for G if :

1. Every vertex (node) has a label, which is a symbol of $N \cup T \cup \{\epsilon\}$
2. The label of the root is S .
3. If a vertex n is interior and has label A , then A must be a symbol in N .
4. If a vertex n has label A and vertexes n_1, n_2, \dots, n_k are the sons of the vertex n , in order from the left, with labels $x_1, x_2, x_3, \dots, x_k$ respectively, then $A \rightarrow x_1, x_2, x_3, \dots, x_k$ must be a production in P .
5. If a vertex n has the label ϵ , then the vertex n is a leaf and is the only son of its father.



Example /

Consider the grammar $G=(\{S,A\},\{a,b\},P,S)$, where P consist of

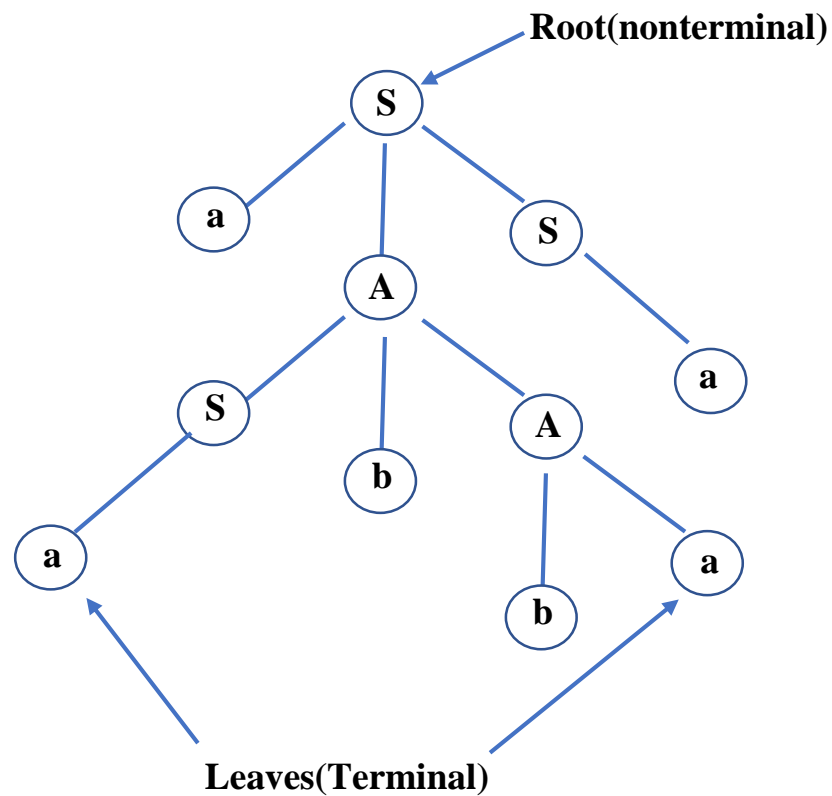
$S \rightarrow aAS / a$

$A \rightarrow SbA/SS/ba$

Draw the derivation tree for the string (**aabbaa**)

Answer /

$S \rightarrow aAS \rightarrow aSbAS \rightarrow aabAS \rightarrow aabbaS \rightarrow aabbaa$



H.W/

Let $G = (\{ S, A, B \}, \{ a, b \}, P, S)$, where

$P = \{ S \rightarrow AB, A \rightarrow aA / a, B \rightarrow bB / b \}$

Draw the derivation tree for the string :

1. $W / W = (a^2 b^3)$
2. $W / W = (a^3 b^2)$
3. **(aaabbb)**

Ambiguity

A grammar can lead to the generation of an identical string or sentence by two or more different derivations. Such a grammar is known as ambiguous grammar, otherwise, it is said to be unambiguous.

Example1 /

Consider the grammar $G = (\{S, A, B\}, \{a, b\}, P, S)$, where

$P = \{ S \rightarrow AB, B \rightarrow ab / b, A \rightarrow aa / a \}$

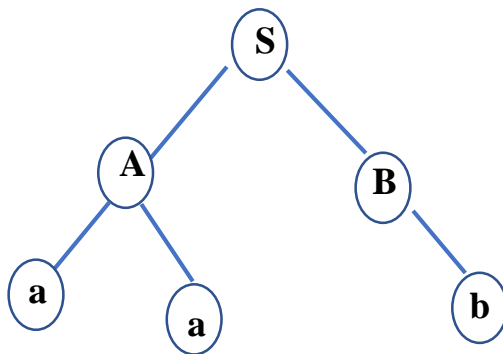
Is the Grammar G ambiguous for the string "aab" ?

Answer /

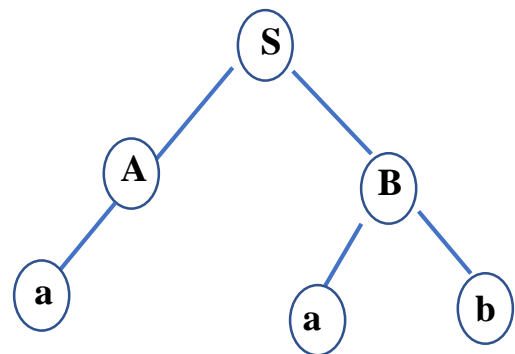
$S \rightarrow \underline{A}B \rightarrow aa\underline{B} \rightarrow aab \dots\dots\dots(1)$

$S \rightarrow \underline{A}B \rightarrow a\underline{B} \rightarrow aab \dots\dots\dots(2)$

(1)



(2)



The grammar G is ambiguous

Example2 /

Let $G = (\{S\}, \{a, b, c\}, P, S)$, where $P = \{ S \rightarrow SbS / ScS / a \}$

Is the Grammar G ambiguous for the string "abaca" ?

Binary operation on language

Reverse operation العمليات المعكوسة

Suppose L is given language generated by $G : (N, T, P, S)$ then the reversal grammar that generates L^R is grammar $G^R : (N, T, P^R, S)$

where $L^R = \{ \alpha^R / \alpha \in L \}$ the set of production p^R is obtained from P as

$$\{ \alpha^R \rightarrow B^R / \alpha \rightarrow B \in P \}$$

Example /

Let $G = (\{S, A, B\}, \{a, b\}, P, S)$, where

$$P = \{ S \rightarrow AB, B \rightarrow ab / b, A \rightarrow aa / a \}$$

Then, we will get the string ($W = \underline{a}a\underline{a}b$) as: $S \rightarrow \underline{A}B \rightarrow \underline{aa}B \rightarrow \underline{aa}ab$

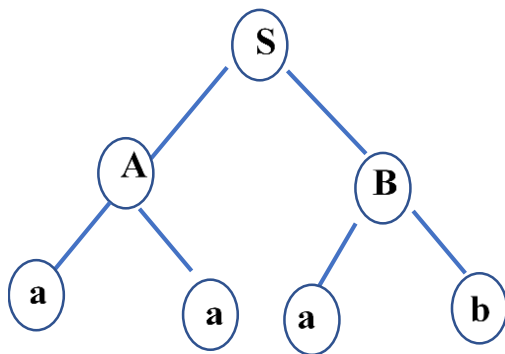
, thus will be in $L(G)$.

If we use the grammar $G^R = (N, T, P^R, S)$ where,

$$P^R = \{ S \rightarrow BA, B \rightarrow ba / b, A \rightarrow aa / a \}$$

Then we will get ($W^R = \underline{b}a\underline{a}a$) as : $S \rightarrow \underline{B}A \rightarrow \underline{ba}A \rightarrow \underline{ba}aa$

W:



W^R:

