## Concatenation of languages

This is an operation on string that allows us to construct languages.

Let $L_1$ , $L_2$ are two formal languages over an alphabet $\sum$ , the concatenation of $L_1$ with $L_2$ as the following:

$L_3 = L_1.L_2 = \{ x.y \,/\, x \in L_1 , y \in L_2 \}$

**Example1:**

Let $\sum = \{0, 1\}$

$L1 = \{ 01, 0 \}$

$L2 = \{ \varepsilon, 0, 10 \}$

$L3 = L2.L1 = \{ 01, 0, 001, 00, 1001, 100 \}$

$L1.L2 = ?$

**Example2:**

Let $\sum = \{a, b, \ldots, z\}$

L1 = {in, out}

L2 = {come, low, door }

L3 = L1.L2 = { income, inlow, indoor, outcome, outlow, outdoor }

## Power of language: -

$L^k = L.L.L.L\ldots\ldots L$, where L an language and $k \geq 0$

(i.e the set of all string that can be obtained by concatenating k elements

of L)

$L.L = L^2$

$L = L^1$

$L^0 = \{\varepsilon\}$

## Kleene closure operation

The closure or (Kleene - star operation) "*" is defined on language L and will be denoted by L* and consists of all strings that can be obtained by concatenating any number of strings from L.

Let L be a language also $L^0 = \{ \varepsilon \}$ and $L^i = L.L^{i-1}$ for $i \geq 1$, where $L^2 = L.L$ means that the language string is concatenated in various possible combinations.

Then Kleene closure of L denoted by L* where,

$L^* = \bigcup_{i=0}^{\infty} L^i = L^0 \cup L^1 \cup L^2 \cup \ldots\ldots..$

and

$L^+ = \bigcup_{i=1}^{\infty} L^i = L^1 \cup L^2 \cup L^3 \cup \ldots\ldots..$

where $L^+$ is the set of all strings obtainable by concatenating one or more

elements of L and $L^+ = L^* - \{\varepsilon\}$

**Example3:**

Let L= {a, b} find $\bigcup_{i=0}^{3} L^i$

$\bigcup_{i=0}^{3} L^i = L^0 \cup L^1 \cup L^2 \cup L^3$

$=\{\varepsilon\} \cup \{a, b\} \cup \{a, b\}^2 \cup \{a, b\}^3$

$=\{\varepsilon\} \cup \{a, b\} \cup \{a, b\}.\{a, b\} \cup \{a, b\}.\{a, b\}.\{a, b\}$

$= \{\varepsilon\} \cup \{a, b\} \cup \{aa, ab, ba, bb\} \cup \{a, b\}.\{a, b\}.\{a, b\}$

$= \{\varepsilon\} \cup \{a, b\} \cup \{aa, ab, ba, bb\} \cup \{a, b\}.\{aa, ab, ba, bb\}$

$= \{\varepsilon\} \cup \{a, b\} \cup \{aa, ab, ba, bb\} \cup \{aaa, aab, aba, abb, baa, bab, bba,$

$bbb\}$

$= \{\varepsilon, a, b, aa, ab, ba, bb, aaa, aab, aba, abb, baa, bab, bba, bbb\}$

## Theorem-1

Prove that L+ = L.L*

Ans/

$L+ = \bigcup_{i=1}^{\infty} L^i = L^1 \cup L^2 \cup L^3 \cup \ldots\ldots$

$L.L* = L. \bigcup_{i=0}^{\infty} L^i = \bigcup_{i=0}^{\infty} L^{i+1} = \bigcup_{i=1}^{\infty} L^i = L^+$

## Theorem-2

Prove that L* = L*.L*

Ans/

$L* = \bigcup_{i=0}^{\infty} L^i = ( L^0 \cup L^1 \cup L^2 \cup \ldots\ldots)$

$L*.L* = \bigcup_{i=0}^{\infty} L^i . \bigcup_{i=0}^{\infty} L^i$

$= \bigcup_{i=1}^{\infty} L^{2i}$

$= L^0 . L^2 . L^4 . \ldots\ldots$

$= L*$

## Theorem-3

Prove that (L*)* = L*

Ans/

$(L*)* = ( \bigcup_{i=0}^{\infty} L^i)* = \bigcup_{i=0}^{\infty}(\bigcup_{i=0}^{\infty} L^i )^i$

$= (\bigcup_{i=0}^{\infty} L^i)^0 \cup (\bigcup_{i=0}^{\infty} L^i)^1 \cup (\bigcup_{i=0}^{\infty} L^i)^2 \cup \ldots\ldots$

$= \varepsilon \cup (L^0 \cup L^1 \cup L2 \cup \ldots\ldots) \cup (L^0 \cup L^1 \cup L^2 \cup \ldots\ldots)^2 \cup \ldots\ldots$

$= \varepsilon \cup L* \cup (L* )^2 \cup \ldots\ldots$

$= L*$