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Electric Circuits Analysis

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Chapter Four
Part 1

Three Phase Circuits

- ***Three Phase System***
- ***Balanced Three-Phase Voltages***
- ***Balanced Wye-Wye Connection***
- ***Balanced Wye-Delta Connection***

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Chapter Four

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4.1 Three Phase System

The three-phase system is an economical way of bulk power transmission over long distances and for distribution. The three-phase system consists of a three-phase voltage source (with phase equal to 120° between them) connected to a three-phase load by means of transformers and transmission lines. Three-phase systems are important for at least three reasons. **First, nearly all electric power is generated and distributed in three-phase, at the operating frequency of 60 Hz in the United States or 50 Hz in some other parts of the world**. When one-phase or two-phase inputs are required, they are taken from the three-phase system rather than generated independently. Even when more than three phases are needed—such as in the aluminum industry, where 48 phases are required for melting purposes—they can be provided by manipulating the three phases supplied. **Second, the instantaneous power in a three-phase system can be constant (not pulsating), as we will see in the next section** . This results in uniform power transmission and less vibration of three-phase machines. **Third, for the same amount of power, the three-phase system is more economical than the single phase**. The amount of wire required for a three-phase system is less than that required for an equivalent single-phase system.

4.2 Balanced Three-Phase Voltages

Three-phase voltages are often produced with a three-phase ac generator (or alternator) whose cross-sectional view is shown in Fig. 4.1. The generator basically consists of a rotating magnet (called the rotor) surrounded by a stationary winding (called the stator). Three separate windings or coils with terminals $a - a^-, b - b^-$ & $c - c^-$ and are physically placed 120° apart around the stator. As the rotor rotates, its magnetic field "cuts" the flux from the three coils and induces voltages in the coils. Because the coils are placed 120° apart, the induced voltages in the coils are equal in magnitude but out of phase by 120° as shown in Fig. 4.2. A three-phase system is equivalent to three single-phase circuits. The voltage sources can be either wye-connected as shown in Fig. 4.3(a) or delta-connected as in Fig. 4.3(b).
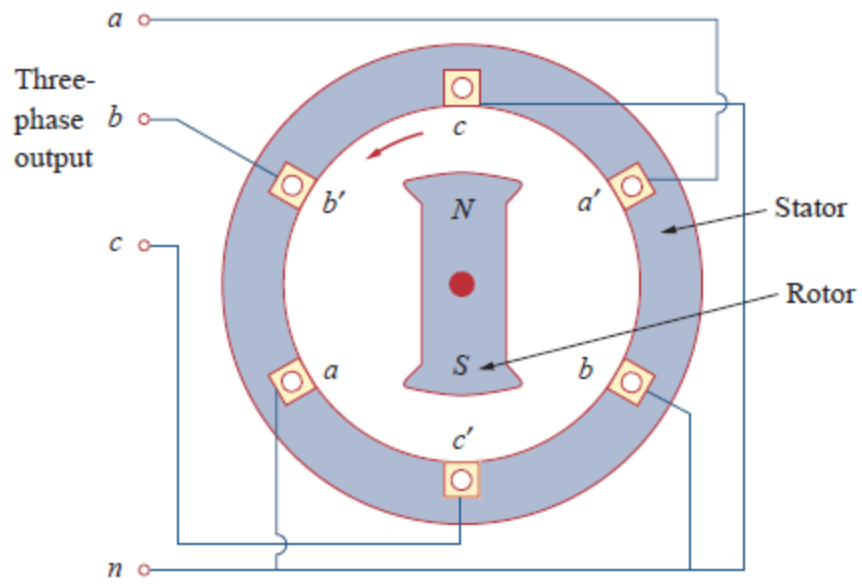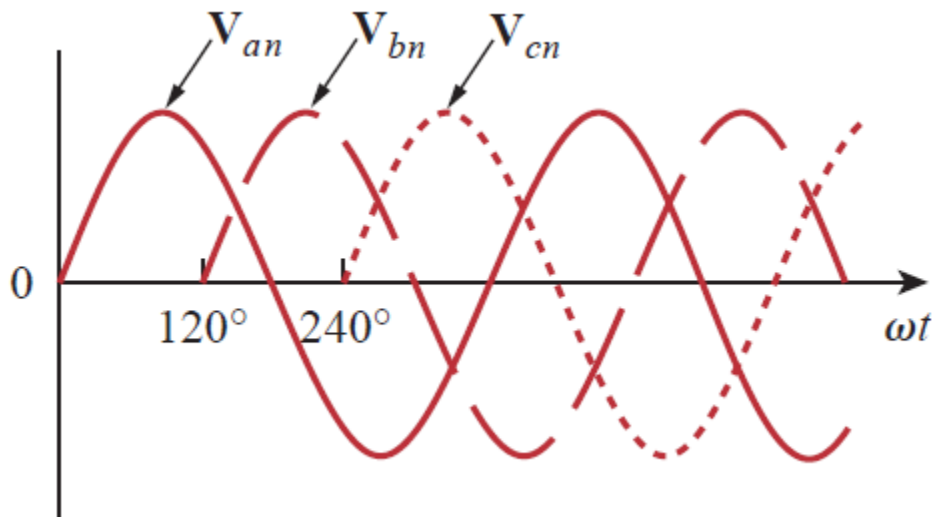
Fig 4.1 A three-phase generator.

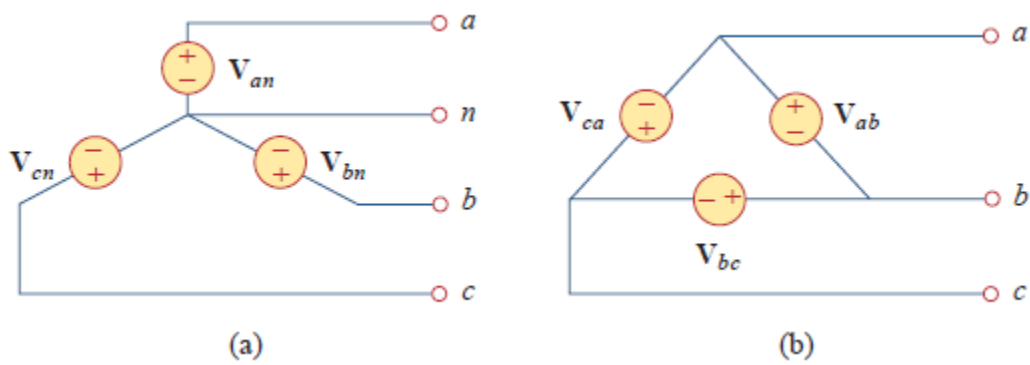Fig 4.2 The generated voltages are apart $120°$ from each

Fig 4.3 Three-phase voltage sources: (a) Y-connected source, (b) Δ -connected

Let us consider the wye-connected voltages in Fig. 4.3(a) for now. The voltages V_{an}, V_{bn} and V_{cn} are respectively between lines a, b, and c, and the neutral line n. These voltages are called phase voltages. If the voltage sources have the same amplitude and frequency ω and are out of phase with each other by $120°$ the voltages are said to be balanced. This implies that

$$V_{an} + V_{bn} + V_{cn} = 0$$
$$|V_{an}| = |V_{bn}| = |V_{cn}| \qquad \qquad ...(4.1)$$

Since the three-phase voltages are $120°$ out of phase with each other, there are two possible combinations. One possibility is shown in Fig. 4.4(a) and expressed mathematically as

$$V_{an} = V_p\underline{/0°}$$
$$V_{bn} = V_p\underline{/-120°}$$
$$V_{cn} = V_p\underline{/-240°} = V_p\underline{/+120°}$$
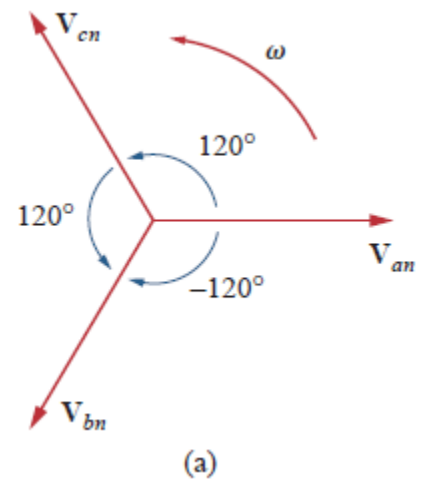
Where V_p is the effective or **rms** value of the phase voltages. This is known as the **abc** sequence or positive sequence.

The other possibility is shown in Fig. 4.4(b), it called **acb** and is given by

$$V_{an} = V_p\underline{/0°}$$
$$V_{cn} = V_p\underline{/-120°}$$
$$V_{bn} = V_p\underline{/-240°} = V_p\underline{/+120°}$$
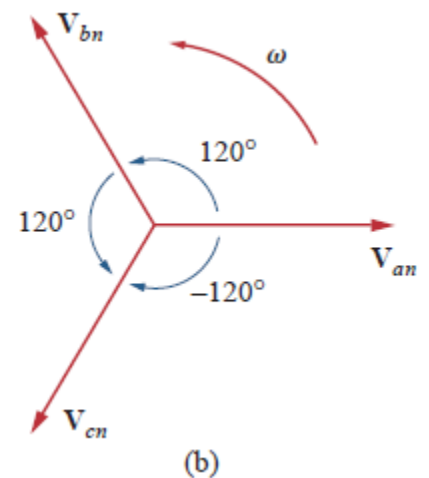

(a)

(b)

Fig 4.4 Phase sequences: (a) abc or positive sequence, (b) acb or negative sequence.

Like the generator connections, a three-phase load can be either wye-connected or delta-connected, depending on the end application. Figure 4.5(a) shows a wye-connected load, and Fig. 4.5(b) shows a delta-connected load. The neutral line in Fig. 4.5(a) may or may not be there, depending on whether the system is four- or three-wire. (And, of course, a neutral connection is topologically impossible for a delta connection.). **A wye- or delta-connected load is said to be unbalanced if the phase impedances are not equal in magnitude or phase.**

(a)

(b)

Fig 4.5. Two possible three-phase load configurations:(a) Y-connected load, (b)a Δ-connected load.

For a balanced wye-connected load,

$$\mathbf{Z}_1 = \mathbf{Z}_2 = \mathbf{Z}_3 = \mathbf{Z}_Y \qquad \dots (4.2)$$

For a balanced delta connected load,

$$\mathbf{Z}_a = \mathbf{Z}_b = \mathbf{Z}_c = \mathbf{Z}_\Delta \qquad \dots (4.3)$$

Where $\mathbf{Z}_Y$ & $\mathbf{Z}_\Delta$ are the load impedance per phase.

As it has mentioned before, the relationship between Y & Δ is

$$\mathbf{Z}_\Delta = 3\mathbf{Z}_Y \qquad \text{or} \qquad \mathbf{Z}_Y = \frac{1}{3}\mathbf{Z}_\Delta$$

So we know that a wye-connected load can be transformed into a delta connected load, or vice versa.

Since both the three-phase source and the three-phase load can be either wye- or delta-connected, we have four possible connections:

    A- Y-Y connection

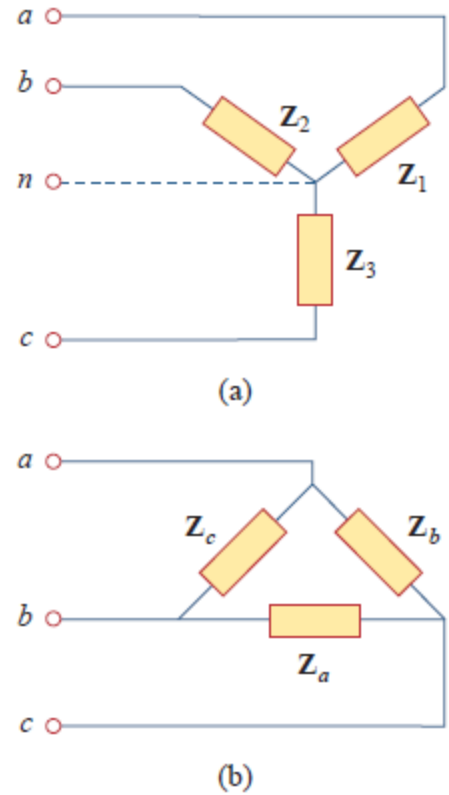   B- Y-Δ connection

    C- Δ-Δ connection

    D- Δ-Y connection

A- Balanced Wye-Wye Connection

We begin with the Y-Y system, because any balanced three-phase system can be reduced to an equivalent Y-Y system. Therefore, analysis of this system should be regarded as the key to solving all balanced three-phase systems. Consider the balanced four-wire Y-Y system of Fig. 4.6, where a Y-connected load is connected to a Y-connected source. Where Z_S, Z_l & Z_L are source impedance, line impedance and load impedance respectively, let Z_Y is the total impedance, and as Z_N is the neutral (which is small value, the we can neglected), the new system is shown in Fig. 4.7 :-

Fig 4.6. A balanced Y-Y system, showing the source, line, and load impedances.

$$Z_Y = Z_s + Z_l + Z_L \qquad \dots (4.4)$$

Assuming the positive sequence, the phase voltages (or line-to neutral voltages) are

$$\mathbf{V}_{an} = V_P\underline{/0^\circ}$$
$$\mathbf{V}_{bn} = V_P\underline{/-120^\circ}, \qquad \mathbf{V}_{cn} = V_P\underline{/+120^\circ} \qquad \dots (4.5)$$

The line-to-line voltages or simply line voltages V_{ab}, V_{bc} & V_{ca} and are related to the phase voltages. For example,

Fig 4.7. Balanced Y-Y connection.

$$\mathbf{V_{ab}} = \mathbf{V_{an}} - \mathbf{V_{bn}} = V_P\angle 0^\circ - V_P\angle -120^\circ = V_P\left(1 + \frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = \sqrt{3}\,V_P\angle 30^\circ \qquad \dots (4.6)$$

$$\mathbf{V_{bc}} = \mathbf{V_{bn}} - \mathbf{V_{cn}} = V_P\angle -120^\circ - V_P\angle 120^\circ = \sqrt{3}\,V_P\angle -90^\circ \qquad \dots (4.7)$$

$$\mathbf{V_{ca}} = \mathbf{V_{cn}} - \mathbf{V_{an}} = V_P\angle 120^\circ - V_P\angle 0^\circ = \sqrt{3}\,V_P\angle -210^\circ \qquad \dots (4.8)$$

Thus, the magnitude of the line voltages V_L is $\sqrt{3}$ times the magnitude of the phase voltages V_P

or $$V_L = \sqrt{3}\,V_P \qquad \dots (4.9)$$

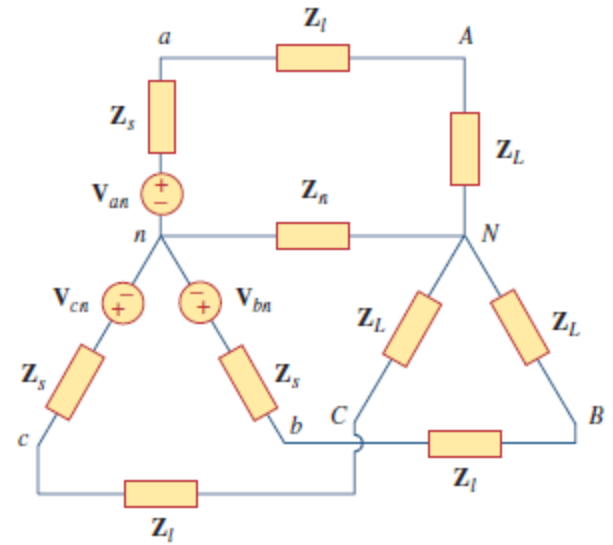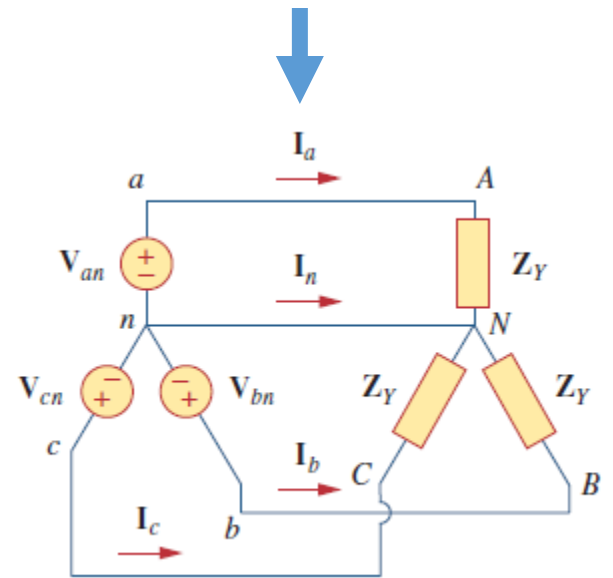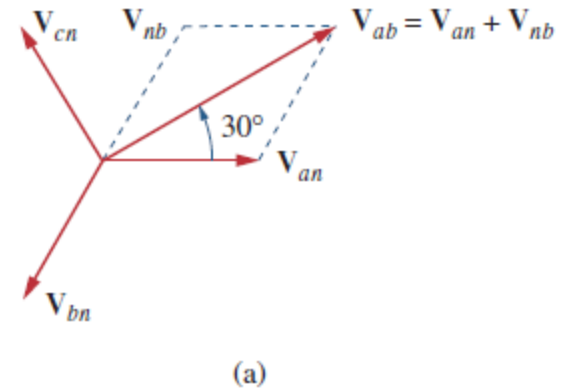Also the line voltages lead their corresponding phase voltages by $30°$. Figure 4.8(a) illustrates this. Figure 4.8(a) also shows how to determine $\mathbf{V_{ab}}$ from the phase voltages, while Fig. 4.8(b) shows the same for the three line voltages. Notice that $\mathbf{V_{ab}}$ leads $\mathbf{V_{bc}}$ by $120°$ and $\mathbf{V_{bc}}$ leads $\mathbf{V_{ca}}$ by $120°$ so that the line voltages sum up to zero as do the phase voltages.

(a)

Applying KVL to each phase in Fig. 4.7, we obtain the line currents as

$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y},$$

$$\mathbf{I}_b = \frac{\mathbf{V}_{bn}}{\mathbf{Z}_Y} = \frac{\mathbf{V}_{an}\underline{/-120°}}{\mathbf{Z}_Y} = \mathbf{I}_a\underline{/-120°}$$

$$\mathbf{I}_c = \frac{\mathbf{V}_{cn}}{\mathbf{Z}_Y} = \frac{\mathbf{V}_{an}\underline{/-240°}}{\mathbf{Z}_Y} = \mathbf{I}_a\underline{/-240°}$$
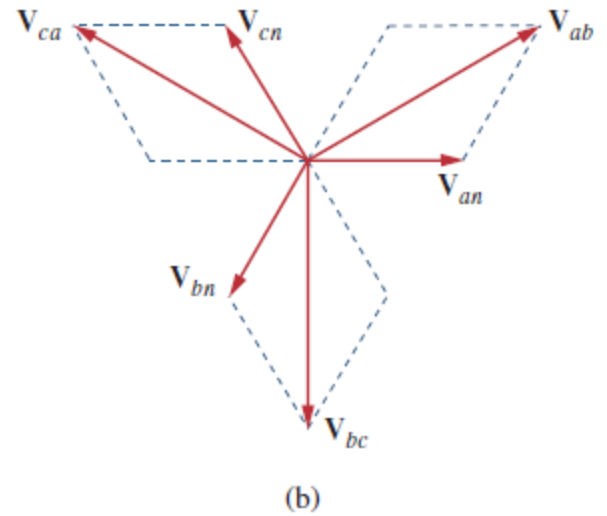
... (4.10)

(b)

Fig 4.8. Phasor diagrams illustrating the relationship between line voltages and phase voltages.

We can readily infer that the line currents add up to zero,

$$\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c = 0 \quad \Longrightarrow \quad \mathbf{I}_n = -(\mathbf{I}_a + \mathbf{I}_b + \mathbf{I}_c) = 0 \quad \Longrightarrow \quad \mathbf{V}_{nN} = \mathbf{Z}_n\mathbf{I}_n = 0$$

While the line current is the current in each line, the phase current is the current in each phase of the source or load. In the Y-Y system, the line current is the same as the phase current. We will use single subscripts as shown in Fig 4.9 for line currents because it is natural and conventional to assume that line currents flow from the source to the load. So the current will be

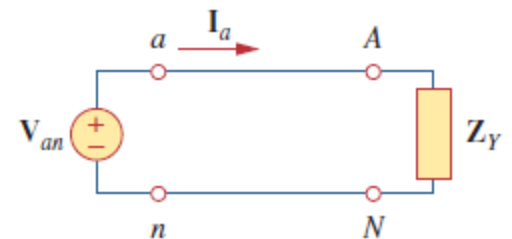$$\mathbf{I}_a = \frac{\mathbf{V}_{an}}{\mathbf{Z}_Y}$$

... (4.11)

Fig 4.9. A single-phase equivalent circuit.

Example 4.1/ Calculate the line currents in the three-wire Y-Y system of Fig. 4.10.
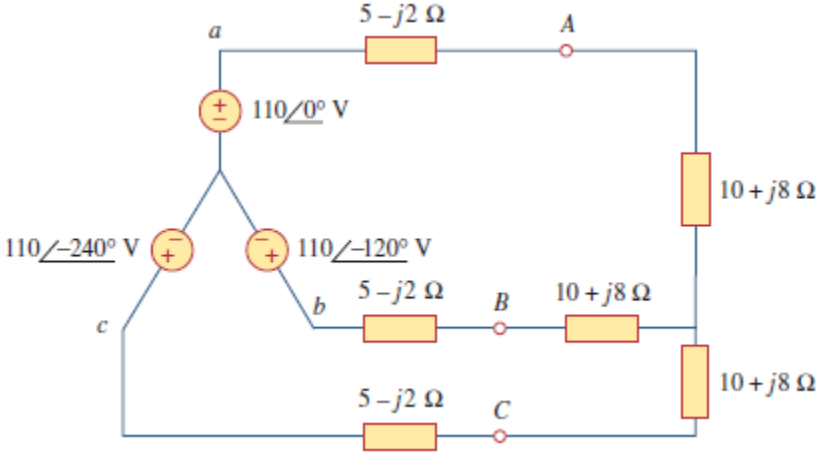

Fig 4.10. For Example 4.1

B- Balanced Wye-Delta Connection

The balanced Y-delta system is shown in Fig. 4.11, where the source is Y-connected and the load is Δ -connected. There is, of course, no neutral connection from source to load for this case. Assuming the positive sequence, the phase voltages are again

Fig 4.11. Balanced Y-Δ connection.

$$\mathbf{V}_{an} = V_P\angle 0°$$
$$\mathbf{V}_{bn} = V_P\angle -120°, \qquad \mathbf{V}_{cn} = V_P\angle +120° \qquad \dots (4.12)$$

As shown in Section 12.3, the line voltages are

$$\mathbf{V}_{ab} = \sqrt{3}V_P\angle 30° = \mathbf{V}_{AB}, \qquad \mathbf{V}_{bc} = \sqrt{3}V_P\angle -90° = \mathbf{V}_{BC} \qquad \dots (4.13)$$
$$\mathbf{V}_{ca} = \sqrt{3}V_P\angle -150° = \mathbf{V}_{CA}$$

The above equation is showing that the line voltages are equal to the voltages across the load impedances for this system configuration. From these voltages, we can obtain the phase currents as

$$\mathbf{I}_{AB} = \frac{\mathbf{V}_{AB}}{\mathbf{Z}_\Delta}, \qquad \mathbf{I}_{BC} = \frac{\mathbf{V}_{BC}}{\mathbf{Z}_\Delta}, \qquad \mathbf{I}_{CA} = \frac{\mathbf{V}_{CA}}{\mathbf{Z}_\Delta} \qquad \dots (4.14)$$

These currents have the same magnitude but are out of phase with each other by $120°$.

The line currents are obtained from the phase currents by applying KCL at nodes A, B, and C. Thus,

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA}, \qquad \mathbf{I}_b = \mathbf{I}_{BC} - \mathbf{I}_{AB}, \qquad \mathbf{I}_c = \mathbf{I}_{CA} - \mathbf{I}_{BC} \qquad \dots (4.15)$$

Since $\mathbf{I}_{CA} = \mathbf{I}_{AB}\angle -240°$

$$\mathbf{I}_a = \mathbf{I}_{AB} - \mathbf{I}_{CA} = \mathbf{I}_{AB}(1 - 1\angle -240°)$$
$$= \mathbf{I}_{AB}(1 + 0.5 - j0.866) = \mathbf{I}_{AB}\sqrt{3}\angle -30° \qquad \dots (4.16)$$

Thus, the magnitude of the line current I_L is $\sqrt{3}$ times the magnitude of the phase current I_P or
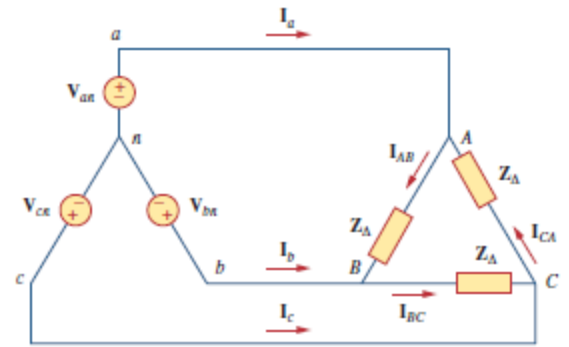
$$I_L = \sqrt{3}\, I_P \qquad \dots (4.17)$$

Where

$$I_L = |\mathbf{I}_a| = |\mathbf{I}_b| = |\mathbf{I}_c|$$

&

$$I_p = |\mathbf{I}_{AB}| = |\mathbf{I}_{BC}| = |\mathbf{I}_{CA}|$$

Also, the line currents lag the corresponding phase currents by **30°** assuming the positive sequence. Figure 4.12 is a phasor diagram illustrating the relationship between the phase and line currents.
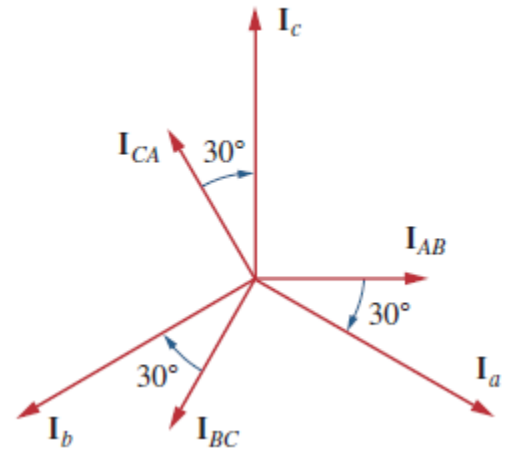
Fig 4.12. Phasor diagram illustrating the relationship between phase and line currents.
Example

An alternative way of analyzing the Y- Δ circuit is to transform the Δ-connected load to an equivalent Y-connected load. Using the below transformation formula ,

$$\mathbf{Z}_Y = \frac{\mathbf{Z}_\Delta}{3} \qquad \qquad \dots (4.17)$$

After this transformation, we now have a Y-Y system as in Fig. 4.7. The three-phase Y- Δ system in Fig. 4.11 can be replaced by the single phase equivalent circuit in Fig. 4.13. This allows us to calculate only the line currents. The phase currents are obtained using Eq. (4.17) and utilizing the fact that each of the phase currents leads the corresponding line current by **30°**.
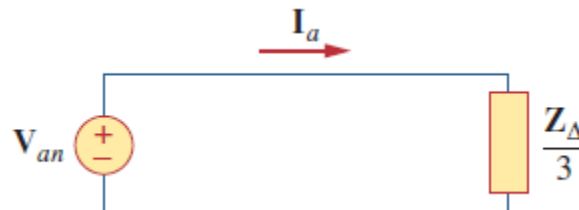

Fig 4.13. A single-phase equivalent circuit of a balanced Y-Δ circuit.

Example 4.2*/** A balanced ***abc-sequence Y-Δ connected source with $V_{an} = 100\angle 10° \, V$ is connected to a Δ -connected balanced load ($8 + j4)\Omega$ per phase. Calculate the phase and line currents.

H1:- If $V_{ab} = 400\ v$ in a balanced Y-connected three phase generator, find the phase voltages, assuming the phase sequence is:

   A- abc   B- acb

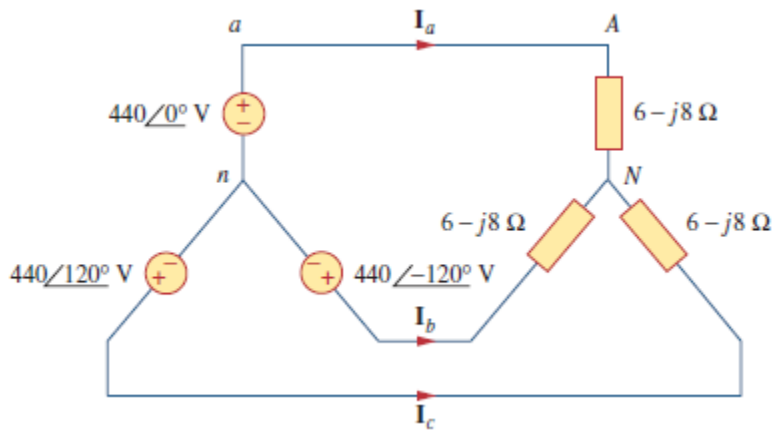*H2:-* Obtain the line currents in the three-phase circuit of Fig. 4.14.

Fig 4.14. For H1.

H3:- A balanced *positive*-sequence Y-Δ connected source with $V_{an} = 240\angle 0°\ V$ is connected to a Δ -connected balanced load $(2 - j3)\Omega$ per phase. Calculate the line voltage and the line currents.

H4:- A balanced Y-Y four-wire system has phase voltages

$$V_{an} = 120\underline{/0°}, \qquad V_{bn} = 120\underline{/-120°}$$
$$V_{cn} = 120\underline{/120°}\ V$$

The load impedance per phase is and the line impedance per phase is Solve for the line currents and neutral current.