Basrah University

Engineering College

Civil Engineering Department

Fluid Mechanics

2nd Stage - Second Semester

Edited by

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An Introduction to Fluid Mechanics Course

The course Fluid Mechanics is designed to introduce students to the fundamental engineering science concepts related to the mechanics of fluids. This includes basic fluid properties, fluid statics, fluid dynamics, fluid viscosity and turbulence, introduction to flow in closed conduits, pumps and pumping.
The aim of this course is to provide students with an understanding of the basic principles of fluid mechanics and of their application to Petroleum engineering problems. There is a strong focus on water and oil in the course as they of the most important fluids for engineering practice.

Objectives :

- The course will introduce fluid mechanics and establish its relevance in Petroleum engineering.
- Recognition of and develop the knowledge about the fundamental hydraulic definitions and the principle fluid properties underlying the subject.
- Establish how these definitions and properties are utilized to solve hydrostatical and hydro dynamical problems that may face the Petroleum engineer.

CONTENTS FOR THE 2-ND. SEMESTER

Or $$Q_{theo} = A_1 \sqrt{\frac{2g(\frac{P_1-P_2}{\gamma}+z_1-z_2)}{(\frac{A_1}{A_2})^2-1}} = A_1 \sqrt{\frac{2g(\frac{P_1-P_2}{\gamma}+z_1-z_2)}{(\frac{D_1}{D_2})^4-1}}$$

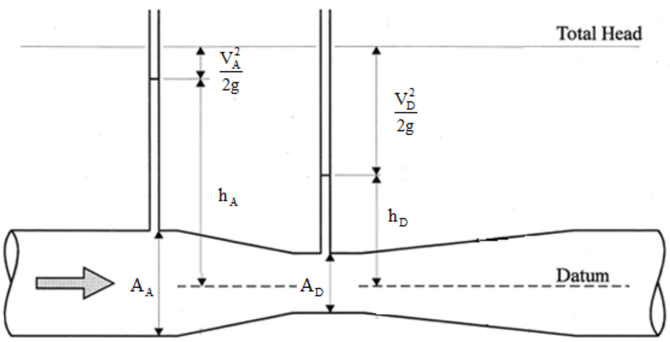
Actual discharge takes into account the losses due to friction, include a coefficient of discharge (Cd ≈ 0.9)

$$Q_{actual} = C_d A_1 \sqrt{\frac{2g(\frac{P_1 - P_2}{\gamma} + z_1 - z_2)}{(\frac{D_1}{D_2})^4 - 1}}$$

If the device was horezental and with two peizometers

$$Q_{real} = C_d \cdot \frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2 \cdot g \cdot \Delta H}$$

where Q = discharge,
Cd = discharge coefficient,
Δh = piezometer heads difference at upstream section & throat section.
A_1,A_2 = pipe cross-sectional area at upstream section, and A_2 = pipe cross-sectional area at throat section.
The discharge coefficient (Cd), otherwise known as the coefficient of the Venturi meter, typically has a value between 0.92 and 0.99. The actual value is dependent on a given Venturi meter, and even then it may change with flow rate.



Ideal conditions for a Venturi meter

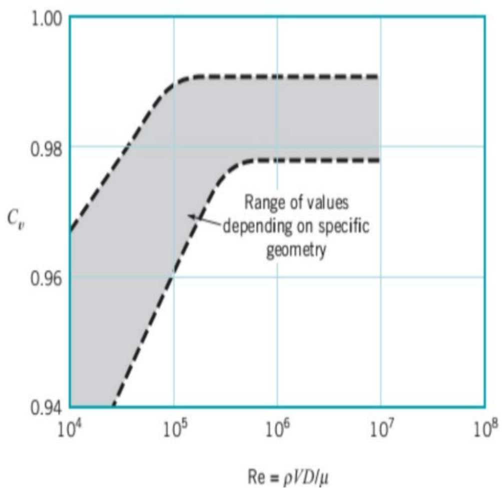


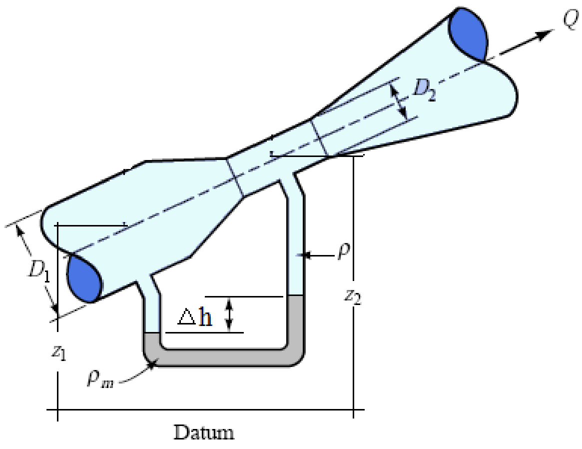
Figure . The co-efficient of discharge of a venturi meter

In terms of the manometer readings

$$P_1 + \gamma z_1 - \gamma_m \Delta h - \gamma(z_2 - \Delta h) = P_2$$

$$P_1 - P_2 + \gamma z_1 - \gamma z_2 = \gamma_m \Delta h - \gamma \Delta h = \Delta h(\gamma_m - \gamma)$$

$$\frac{P_1 - P_2}{\gamma} + z_1 - z_2 = \Delta h(\frac{\gamma_m}{\gamma} - 1)$$



Giving $$Q_{actual} = C_d A_1 \sqrt{\frac{2g\Delta h(\frac{\gamma_m}{\gamma_w} - 1)}{(\frac{D_1}{D_2})^4 - 1}}$$

- This expression does not include any elevation terms. (z_1 or z_2) When used with a manometer, the Venturimeter can be used without knowing its angle

Example -1

A horizontal Venturi meter with d1 = 20 cm, and d2 = 10 cm, is used to measure the flow rate of oil of sp.gr. = 0.8, the discharge through venture meter is 60 lit/s. find the reading of (oil-Hg) differential Take Cd = 0.98

Solution:

$Q = u_2 A_2 = 60$ lit/s (m^3/1000lit) = 0.06 m^3/s

$$Q = u_2 A_2 = 0.06 = Cd\sqrt{\frac{2R(\rho_m - \rho)g}{\rho}}\frac{A_1 A_2}{\sqrt{A_1^2 - A_2^2}}$$

$$= 0.98\sqrt{\frac{2R(1366 - 800)g}{800}}\frac{(\pi/4)^2(0.1)^2(0.2)^2}{\sqrt{(\pi/4)^2[(0.2)^4 - (0.1)^4]}}$$

⇒ R = 0.1815 m Hg = 18.15 cm Hg

Example -2

A horizontal Venturi meter is used to measure the flow rate of water through the piping system of 20 cm I.D, where the diameter of throat in the meter is d_2 = 10 cm. The pressure at inlet is 17.658 N/cm^2 gauge and the vacuum pressure of 35 cm Hg at throat. Find the discharge of water. Take Cd = 0.98.

Solution:

$P_1 = 17.658\ N/cm^2\ (100\ cm / m)^2 = 176580\ Pa$

$P_2 = -35\ cm\ Hg\ (m / 100\ cm)\ 9.81\ (13600) = -46695.6\ Pa$

$P_1 - P_2 = 176580-(-46695.6) = 223275.6\ Pa$

$$Q = u_2A_2 = Cd\sqrt{\left(\frac{2(-\Delta P)}{\rho}\right)}\ \frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}}$$

$$= 0.98\sqrt{\frac{2(223275.6)}{1000}}\ \frac{0.2^2[\pi/4](0.1)^2}{\sqrt{[(0.2)^4 - (0.1)^4]}}$$

$$= 0.168\ m^3/s$$

Example -3

A Venturi meter is to be fitted to a 25 cm diameter pipe, in which the maximum flow is 7200 lit/min and the pressure head is 6 m of water. What is the maximum diameter of throat, so that there is non-negative head on it?

Solution:

Since the pressure head at the throat is not to be negative, or maximum it can be zero (i.e. h_2 = zero). Therefore;

$\Delta h = h_1 - h_2 = 6 - 0 = 6\ m\ H_2O$

$Q = u_2\ A_2 = 7200\ lit/min\ (m^3/1000lit)\ (min / 60\ s) = 0.12\ m^3/s$

$$0.12 = Cd\sqrt{2g\Delta h}\ \frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}} = 1.0\sqrt{2(9.81)(6)}\ \frac{0.25^2[\pi/4](d_2)^2}{\sqrt{[(0.25)^4 - (d_2)^4]}}$$

$$0.225 = \frac{d_2^2}{\sqrt{(0.25)^4 - (d_2)^4}} \Rightarrow 0.0507 = \frac{d_2^4}{(0.25)^4 - (d_2)^4}$$

$$d_2^4 + 0.507 d_2^4 - 1.983 * 10^{-4} = 0 \Longrightarrow d_2 = 0.1172\ m = 11.72\ cm$$

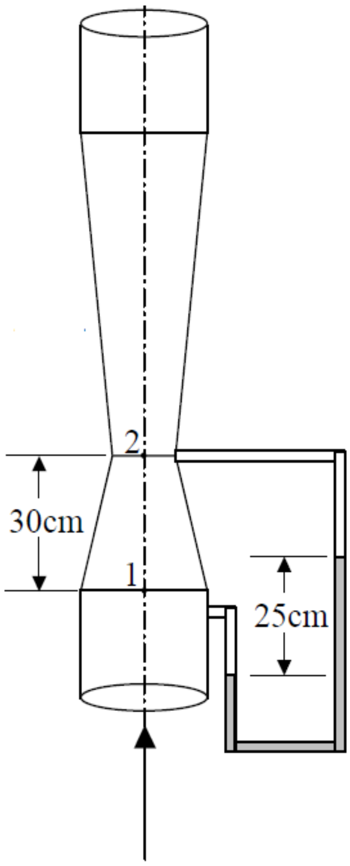
Example -4

A (30cm x 15cm) Venturi meter is provided in a vertical pipe-line carrying oil of sp.gr. = 0.9. The flow being upwards and the difference in elevations of throat section and entrance section of the venture meter is 30 cm. The differential U-tube mercury manometer shows a gauge deflection of 25 cm. Take Cd = 0.98 and calculate: -

i-The discharge of oil

ii-The pressure difference between the entrance and throat sections.

Solution:



$$i-\ Q = u_2A_2 = Cd\sqrt{\frac{2R(\rho_m-\rho)g}{\rho}}\ \frac{A_1A_2}{\sqrt{A_1^2-A_2^2}}$$

$$= 0.98\sqrt{\frac{2(0.25)(12700)9.81}{900}}\left[\frac{0.3^2[(\pi/4)(0.15)^2]}{\sqrt{0.3^4-0.15^4}}\right] \quad = 0.1488\ m^3/s$$

ii- Applying Bernoulli's equation at points 1 and 2

$$\frac{P_1}{\rho g}+\frac{u_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{u_2^2}{2g}+z_2$$

$$\frac{P_1-P_2}{\rho g}=z_2+\frac{u_2^2-u_1^2}{2g}$$

$u_1 = 0.1488/(\pi/4\ 0.3^2) = 2.1$ m/s,

$u_2 = 0.1488/(\pi/4\ 0.15^2) = 8.42$ m/s

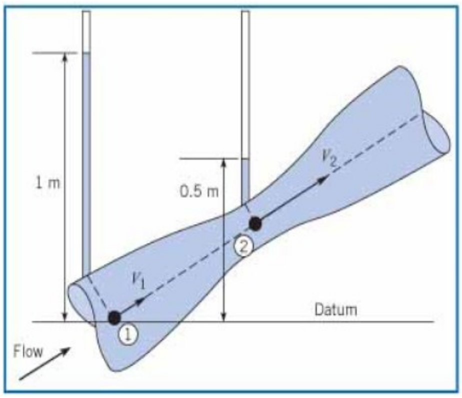
$P_1 - P_2 = 900\ (9.81)\ [0.3 + (8.422 - 2.12)/2(9.81)]$

= 32.5675 kPa

% but $P_1 - P_2 = 0.25\ (13600 - 900)(9.81)$ = 31.1467 kPa % error = 4.36

H.W 1

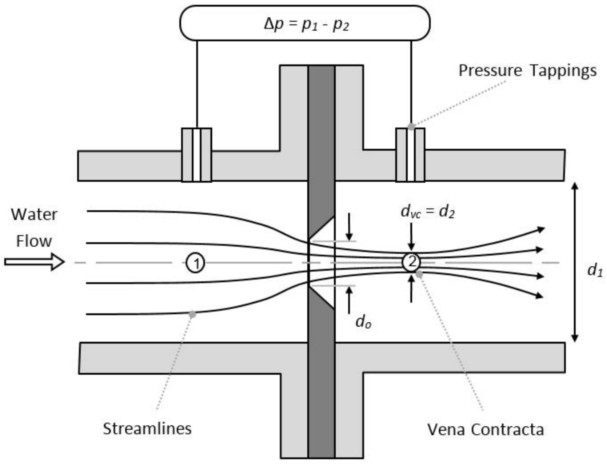
Piezometric tubes are tapped into a Venturi section as shown in the figure. The liquid is incompressible. The upstream piezometric head is 1 m, and the piezometric head at the throat is 0.5 m. The velocity in the throat section is twice large as in the approach section. Find the velocity in the throat section

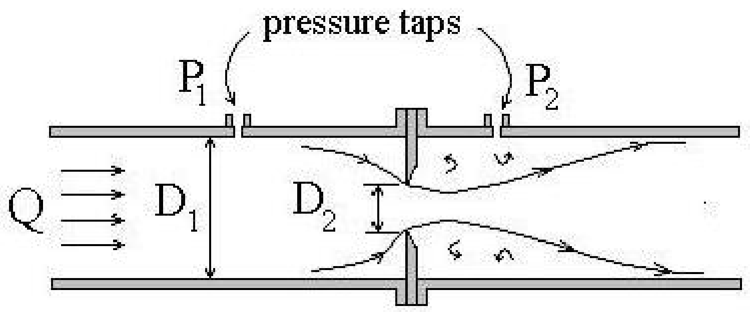


anc v =3.62 *m/s*

1.2#Orifice Meter

The primary element of an orifice meter is simply a flat plate containing a drilled located in a pipe perpendicular to the direction of fluid flow as shown in Figure;





At point 2 in the pipe the fluid attains its maximum mean linear velocity u2 and its smallest cross-sectional flow area A2. This point is known as ***"the vena contracta".*** *It is occurs at about one-half to two pipe diameters downstream from the orifice plate.*

Because of relatively the large friction losses from the eddies generated by the expanding jet below vena contracta, the pressure recovery in orifice meter is poor.

- From continuity equation $A_1u_1 = A_2u_2 \Rightarrow u_1 = (A_2/A_1)\, u_2$

- From Bernoulli's equation between points 1 and 2

$$\frac{P_1}{\rho g} + \frac{u_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{u_2^2}{2g} + z_2$$

$$\frac{P_1 - P_2}{\rho g} = \frac{u_2^2 - u_1^2}{2g} = \frac{u_2^2}{2g}\left[1 - \left(\frac{A_2}{A_1}\right)^2\right] = \frac{u_2^2}{2g}\left[\frac{A_1^2 A_2^2}{A_1^2}\right]$$

But **$C_c = A_2/A_o \Rightarrow A_2 = C_cA_o$**

C_c: coefficient of contraction [0.6 – 1.0] common value is 0.67

A_2: cross-sectional area at vena contracta

A_o: cross-sectional area of orifice

$$\Rightarrow \quad \frac{P_1 - P_2}{\rho} = \frac{u_2^2}{2}\left[1 - \left(\frac{C_C A_0}{A_1}\right)^2\right] = \frac{u_2^2}{2}\left[\frac{A_1^2 - (C_C A_0)^2}{A_1^2}\right]$$

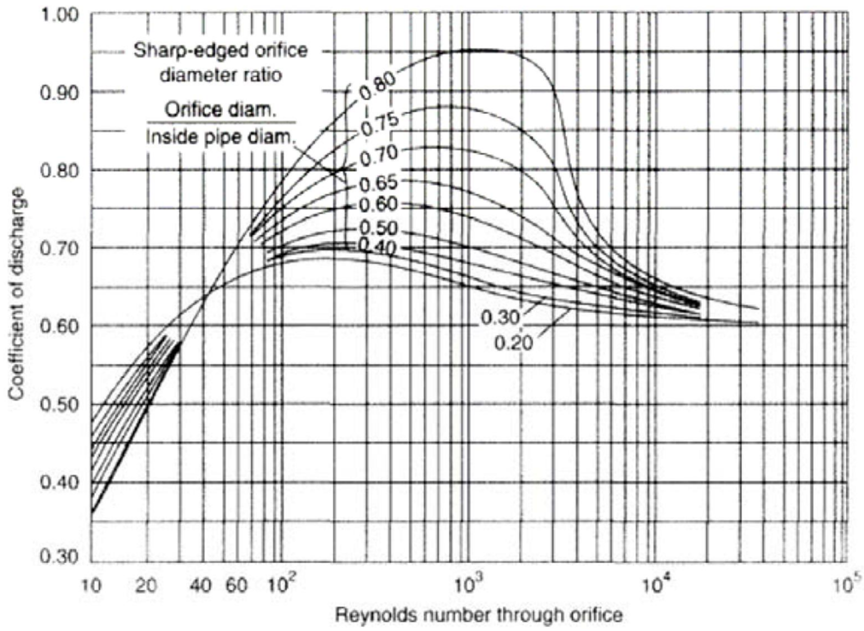
Using a coefficient of discharge Cd to take into account the frictional losses in the meter and of parameters C_c, α_1, and α_2. Thus the velocity at orifice or the discharge through the meter is;

$$Q = Cd\sqrt{\left(\frac{2(-\Delta P)}{\rho}\right)\left[\frac{A_0^2}{1 - (A_0/A_1)^2}\right]} = Cd\sqrt{\left(\frac{2(-\Delta P)}{\rho}\right)}\frac{A_1 A_0}{\sqrt{A_1^2 - A_0^2}}$$

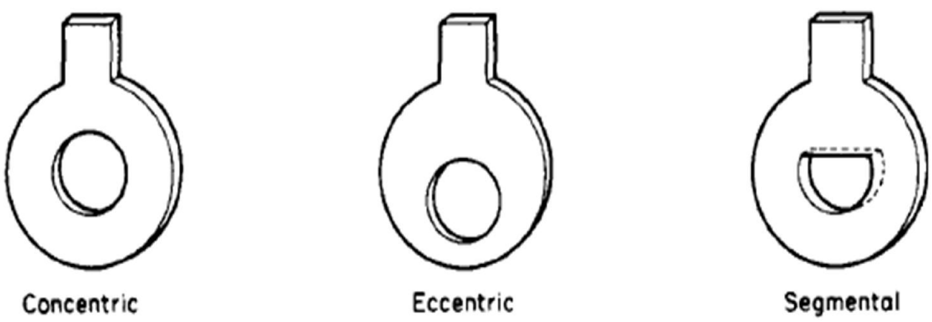
$$Or\ Q = Cd\sqrt{2g\Delta h}\left[\frac{A_0^2}{1 - (A_0/A_1)^2}\right] = Cd\sqrt{2g\Delta h}\frac{A_1 A_0}{\sqrt{A_1^2 - A_0^2}}$$

$$Or\ Q = Cd\sqrt{\frac{2R(\rho_m - \rho)g}{\rho}\left[\frac{A_0^2}{1 - (A_0/A_1)^2}\right]} = Cd\sqrt{\frac{2R(\rho_m - \rho)g}{\rho}}\frac{A_1 A_0}{\sqrt{A_1^2 - A_0^2}}$$

For Reo > 10^4 Cd = 0.61 And for Reo > 10^4 Cd From Figure below



The holes in orifice plates may be ***concentric***, ***eccentric*** or ***segmental*** as shown in Figure. Orifice plates are prone to damage by erosion.



Example -1

An orifice meter consisting of 10 cm diameter orifice in a 25 cm diameter pipe has Cd = 0.65. The pipe delivers oil of sp.gr. = 0.8. The pressure difference on the two sides of the orifice plate is measured by mercury oil differential manometer. If the differential gauge is 80 cm Hg, find the rate of flow.

Solution:

$$Q = Cd\sqrt{\frac{2R(\rho_m-\rho)g}{\rho}}\ \frac{A_1A_0}{\sqrt{A_1^2-A_0^2}}$$

$$= 0.65\sqrt{\frac{2(0.8)(13600-800)(9.81)}{800}}\left[\frac{(\pi/4)(0.1)^2(0.25)^2}{\sqrt{[(0.25)^4-(0.1)^4]}}\right]$$

$$= 0.08196\ m^3/s$$

Example -2

Water flow through an orifice meter of 25 mm diameter situated in a 75 mm diameter pipe at a rate of 300 cm^3/s, what will be the difference in pressure head across the meter μ = 1.0 mPa.s. CD=0.61

Solution:

Q = 300 x 10-6 m^3/s ⇒ u = (300 x 10-6 m3/s) / (π/4 0.25^2) = 0.611 m/s

$$Q = Cd\sqrt{2g\Delta h}\ \frac{A_1A_0}{\sqrt{A_1^2-A_0^2}}$$

$$Re_0 - \frac{\rho u_0 d_0}{\mu} - \frac{1000(0.611)(0.025)}{1 * 10^{-3}} - 1.528 * 10^4 \rightarrow C_d - 0.61$$

$$300 * 10^{-6}\ m^3/s - 0.61\sqrt{2(9.81)(\Delta h)}\left|\frac{(\pi/4)(0.025)^2(0.075)^2}{\sqrt{[(0.075)^4-(0.025)^4]}}\right|$$

$$\sqrt{\Delta h} = 0.2248 \rightarrow \Delta h = 0.05\,m\,H_2O = 50\,mm\,H_2O$$

Example -3

Water flow at between 3000-4000 cm^3/s through a 75 mm diameter pipe and is metered by means of an orifice. Suggest a suitable size of orifice if the pressure difference is to be measured with a simple water manometer. What approximately is the pressure difference recorded at the maximum flow rate? Cd = 0.6. The largest practicable height of a water manometer is 1.0 m

Solution:

The largest practicable height of a water manometer is 1.0 m

The maximum flow rate = 4 x 10^{-3} m^3/s

$$Q = Cd\sqrt{2g\Delta h}\ \frac{A_1 A_o}{\sqrt{A_1^2 - A_o^2}} \Rightarrow 4 * 10^{-3}\ m^3/s = 0.6\sqrt{2(9.81)1.0}\left[\frac{(\pi/4)(0.05)^2(d_0)^2}{\sqrt{[(0.05)^4 - d_0^{\,4}]}}\right]$$

$$\frac{d_0^{\,2}}{\sqrt{[(0.05)^4 - d_0^{\,4}]}} = 0.7665 \rightarrow d_0^4 = 3.67 * 10^{-6} - 0.5875\, d_0^4$$

$\Rightarrow$ do = 0.039 m = 39 mm

$(P_1 - P_2) = \Delta h\, \rho g = 1.0\,(1000)(9.81) = 9810$ Pa.

Example -4

17.79 **Oil flows through a pipe as shown in Fig. 17-17. The coefficient of discharge for the orifice in the pipe is 0.63. What is the discharge of oil in the pipe?**

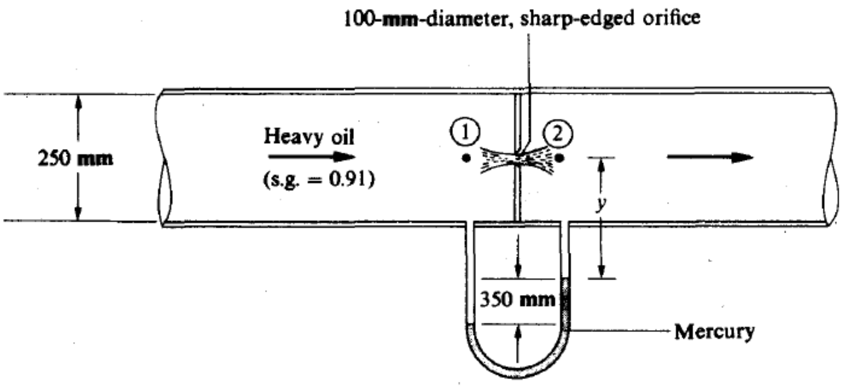


Fig. 17-17

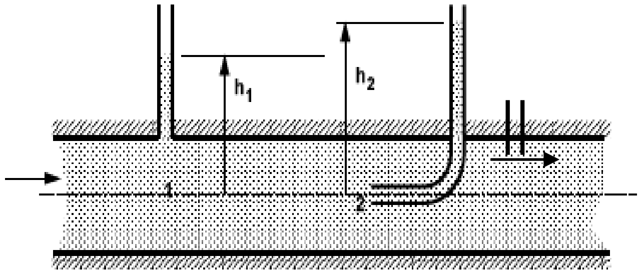
$$Q = CA\sqrt{2g[(p_1 - p_2)/\gamma]}[1 + (C^2/2)(D/D_p)^4] \qquad A = (\pi)(0.100)^2/4 = 0.007854\ m^2$$

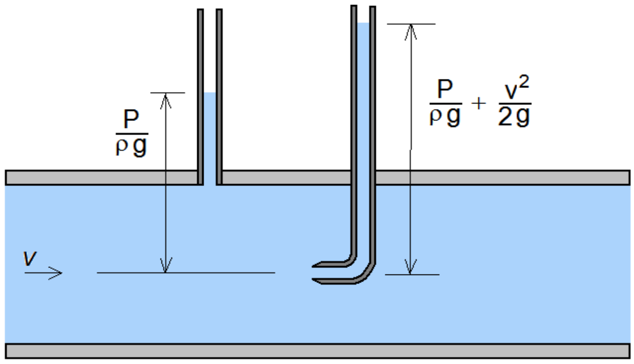
$$p_1/\gamma + y + 0.350 - (13.6/0.91)(0.350) - y = p_2/\gamma \qquad (p_1 - p_2)/\gamma = 4.881 \text{ m of oil}$$

$$Q = (0.63)(0.007854)\sqrt{(2)(9.807)(4.881)}[1 + (0.63^2/2)(\tfrac{100}{250})^4] = 0.0487\ m^3/s$$

1.3# Pitot Tube

Two piezometers, one as normal and one as a Pitot tube within the pipe can be used as shown below to measure velocity of flow





By applying Bernoulli's eq.,

$$\frac{p_1}{\rho g}+\frac{v_1^2}{2g}+\not{z}_1=\frac{p_2}{\rho g}+\frac{\not{v}_2^2}{\not{2}g}+\not{z}_2$$

we have the equation for p_2 ,

$$p_2 = p_1 + \frac{1}{2}\rho u_1^2$$

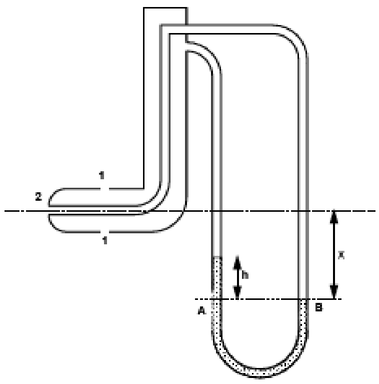
$$\rho g h_2 = \rho g h_1 + \frac{1}{2}\rho u_1^2$$

$$\boxed{u = \sqrt{2g(h_2 - h_1)}}$$

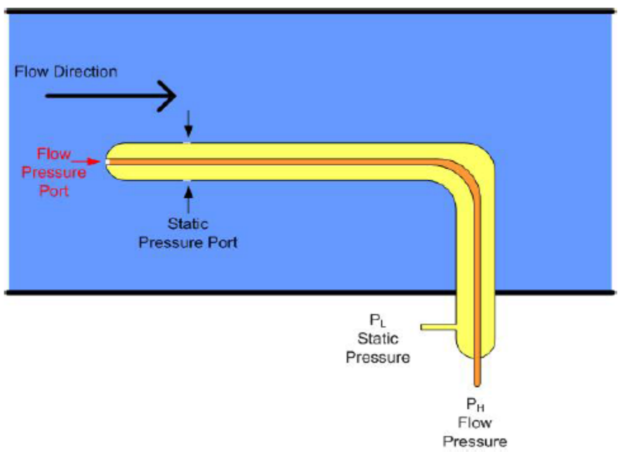
$$\boxed{u = \sqrt{2g\left(\frac{\Delta P}{\gamma}\right)}}$$

We now have an expression for velocity from two pressure measurements and the application of the Bernoulli equation.

The new shape for Pitot tube is compact
The holes on the side connect to one side of a manometer, while the central hole connects to the other side of the manometer
,Using the theory of the manometer



[Note: the diagram of the Pitot tube is not to scale. In reality its diameter is very small and can be ignored i.e. points 1 and 2 are considered to be at the same level]

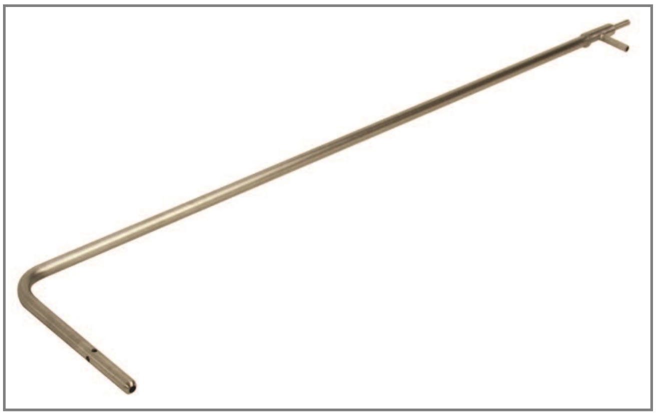


$$p_A = p_1 + \rho g(X - h) + \rho_{man} gh$$
$$p_B = p_2 + \rho g X$$
$$p_A = p_B$$
$$p_2 + \rho g X = p_1 + \rho g(X - h) + \rho_{man} gh$$

We know that $p_2 = p_1 + \frac{1}{2}\rho u_1^2$, giving

$$p_1 + hg(\rho_{man} - \rho) = p_1 + \frac{\rho u_1^2}{2}$$

$$u_1 = \sqrt{\frac{2gh(\rho_m - \rho)}{\rho}}$$



The Pitot/Pitot-static is:

- Simple to use (and analyse)
- Gives velocities (not discharge)

Since the Pitot tube measures velocity at one point only in the flow, several methods can be used to obtain the average velocity in the pipe;

The first method, the velocity is measured at the exact center of the tube to obtain u_{max}. then by using the Figure, the average velocity can be obtained.

The second method, readings are taken at several known positions in the pipe cross section and then a graphical or numerical integration is performed to obtain the average velocity, from the following equation;

$$u = \frac{\iint u_x dA}{A}$$

<u>Example -1</u>

Find the local velocity of the flow of an oil of sp.gr. =0.8 through a pipe, when the difference of mercury level in differential U-tube manometer connected to the two tapping of the Pitot tube is 10 cm Hg. Take Cp = 0.98.

<u>Solution:</u>

$$u_x = CP\sqrt{\frac{2R(\rho_m - \rho)g}{\rho}} \qquad u_x = 0.98\sqrt{\frac{2(0.1)(13600 - 1000)9.81}{800}} = 5.49\ ^{m}/_{s}$$

Example -2

A Pitot tube is placed at a center of a 30 cm I.D. pipe line has one orifice pointing upstream and other perpendicular to it. The mean velocity in the pipe is 0.84 of the center velocity (i.e. u/u_x =0.84). Find the discharge through the pipe if: -

i-The fluid flow through the pipe is water and the pressure difference between orifice is 6 cm H_2O.

ii-The fluid flow through the pipe is oil of sp.gr. = 0.78 and the reading manometer is 6 cm H_2O. Take Cp = 0.98.

Solution:

i- $u_x = CP\sqrt{2g\Delta h} \quad = 0.98\sqrt{2(9.81)(0.06)} = 1.063\, m/s$

u = 0.84 (1.063) = 0.893 m/s, Q = A.u = $\pi/4(0.3)^2$ (0.893) = 0.063 m^3/s

ii- $u_x = CP\sqrt{\dfrac{2R(\rho_m - \rho)g}{\rho}} \quad = 0.98\sqrt{\dfrac{2(0.06)(13600 - 780)9.81}{780}} = 0.565\, m/s$

u = 0.84 (0.565) = 0.475 m/s, Q = A.u = $\pi/4(0.3)^2$ (0.475) = 0.0335 m^3/s

Example -3

A Pitot tube is inserted in the pipe of 30 cm I.D. The static pressure head is 10 cm Hg vacuum, and the stagnation pressure at center of the pipe is 0.981 N/cm^2 gauge. Calculate the discharge of water through the pipe if u/umax = 0.85. Take Cp = 0.98.

Solution:

$P_1 = -10$ cm Hg (13600) 9.81 (m / 100 cm) = –13.3416 kPa

$P_2 = 0.981\ N/cm^2\ (m / 100\ cm)^2 = 9.81$ kPa

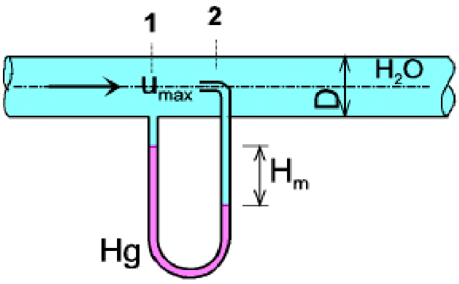
$\Delta P = P_2 - P_1 = 9.81 - (-13.3416) = 23.1516$ kPa

$$u_x - CP\sqrt{\frac{2(-\Delta P)}{\rho}} \quad = 0.98\sqrt{\frac{2 * 23.1516 * 10^3}{1000}} = 6.67\, m/s$$

u = 0.85 (6.67) = 5.67 m/s, Q = A.u = $\pi/4(0.3)^2$ (5.67) = 0.4 m^3/s

Example -4

Water flows in the pipeline (see fig.). Calculate maximum velocity $\mathbf{u_{max}}$ in the pipe axis and discharge **Q**. The mercury differential manometer (ρ_{Hg} = 13600 kgm^{-3}) shows the difference between levels in Pitot tube $\mathbf{H_m}$ **= 0,02 m**. Diameter of the pipe is **D = 0,15 .m**. Mean velocity is considered to be **v = 0,84** $\mathbf{u_{max}}$



Solution

Determination of point velocity (from Bernoulli equation for profiles 1 and 2, datum level at pipe axis):

,after equation arrangement comes to

$$u_{max} = .\sqrt{2.g\frac{p_2 - p_1}{\rho \cdot g}}$$

$$u_{max} = \varphi.\sqrt{2.g.H_m\left(\frac{\rho_{Hg}}{\rho_w} - 1\right)} = 1.\sqrt{2.9{,}81.0{,}02.(13{,}6 - 1)} = \underline{2{,}224\,m.s^{-1}}$$

Using the relation between point and mean velocity v = 0,84 u_{max}, mean velocity will be determined and, consequently, continuity equation will be used to calculate discharge Q.

$$v = 0{,}84.u_{max} = 1{,}868\,m.s^{-1}$$

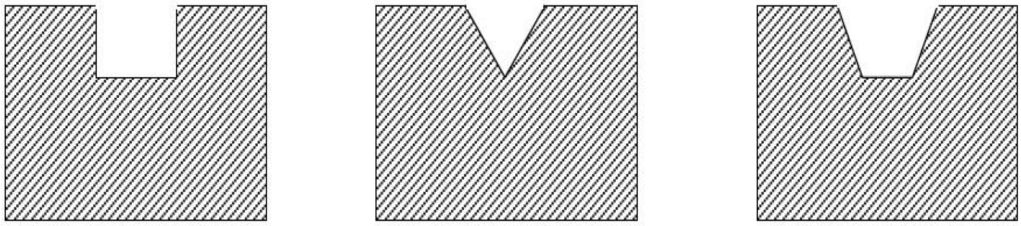
$$Q = v.S = 1{,}868.\frac{\pi.D^2}{4} = \underline{0{,}033\,m^3.s^{-1}}$$

Lec:3

1.4#The Notch or Weir

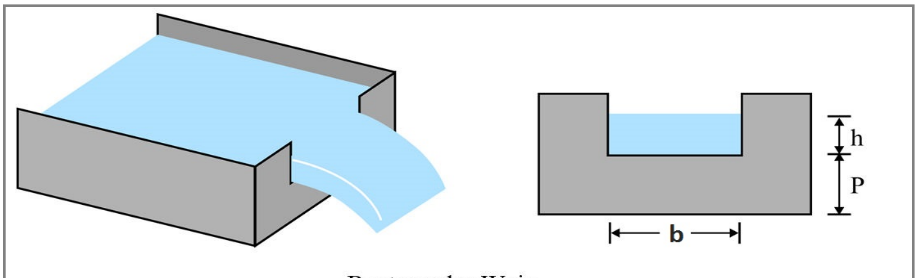
The flow of liquid presenting a free surface (open channels) can be measured by means of a weir. The pressure energy converted into kinetic energy as it flows over the weir, which may or may not cover the full width of the stream, and a calming screen may be fitted before the weir. Then the height of the weir crest gives a measure of the rate of flow. The velocity with which the liquid leaves depends on its initial depth below the surface

Many shapes of notch are available of which three shapes are given here as shown in ,Figures



Rectangular Notch

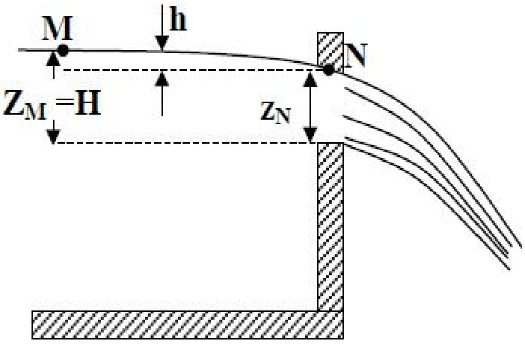
The rectangularweirs also classified according its crest width to sharp crested weir, broad crested weir and ogee weir. The sharp crested rectangular weir may be contracted when the length of weir opening is less than the channel width or suppress when the length of weir is equal to channel width



Rectangular Weir

H: height of liquid above base of the notch
h: depth of liquid from its level
b: width or length of notch
Consider a horizontal strip of liquid of thickness (dh) at depth (h).
The theoretical velocity of liquid flow through the strip = $\sqrt{2gh}$

To prove this equation applies Bernoulli's equation between points M and N as shown in Figure



$$\frac{P_M}{\rho g}+\frac{u_M^2}{2g}+z_M-\frac{P_N}{\rho g}+\frac{u_N^2}{2g}+z_N$$

The cross sectional area of flow at point M is larger than that at notch (point N), then $u_M \approx 0$

$P_M = P_N = P_O$ atmospheric pressure

$$z_M - z_N - \frac{u_N^2}{2g} \therefore u_N = \sqrt{2gh}$$

The area of the strip **dA = b.dh** The discharge through the strip

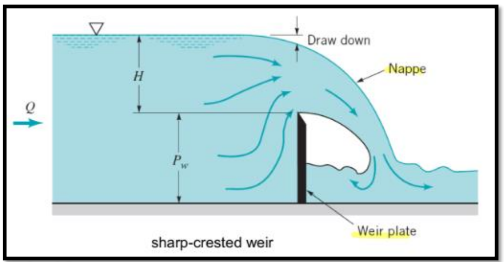
$$\rightarrow \int_0^Q dQ = C_d\sqrt{2g}\int_0^H h^{1/2}\,dh \Rightarrow Q = C_d b\sqrt{2g}\frac{H^{3/2}}{3/2} = \frac{2}{3}C_d\, b\sqrt{2g}\, H^{3/2}$$

To get the actual discharge we introduce a coefficient of discharge, C_d, to account for losses at the edges of the weir and contractions in the area of flow,

The practical estimation of Q for the sharp crested rectangular weir is

$$Q = \frac{2}{3}C_D\sqrt{2g}BH^{3/2}$$

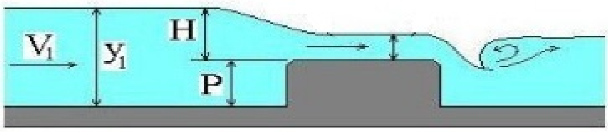
- $C_D = 0.602 + 0.083\frac{H}{P}$ *or estimated from labretory*



- Where:
  - Q (m³/s) is the volumetric flow rate over the weir
  - C_D is the discharge coefficient usually ranging from 0.60 to 0.62
  - H (m) is the head over the weir(from the weir crest to the upstream water surface)
  - P (m) is the height of the weir plate
  - B (m) is the width of the contracted notch (rectangular), or the width of the channel (suppressed)
  - g is the acceleration of gravity (9.81 m/s²)

The practical estimation of Q for the broad crested rectangular weir is

$$Q=C.B.H^{3/2}$$



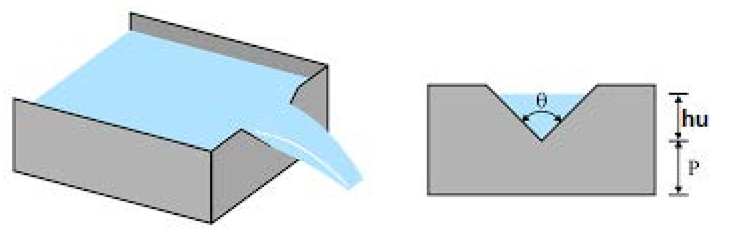
Broad Crested Weir

- Where:
  - Q (m³/s) is the volumetric flow rate over the weir
  - C is the discharge coefficient usually ranging from 1.6 -1.8 *or estimated from labretory*
  - H (m) is the head over the weir(from the weir crest to the upstream water surface)
  - B (m) is the width of the contracted notch (rectangular), or the width of the channel

Triangular Notch

- Calculating discharge across a V-Notch weir is more complicated:

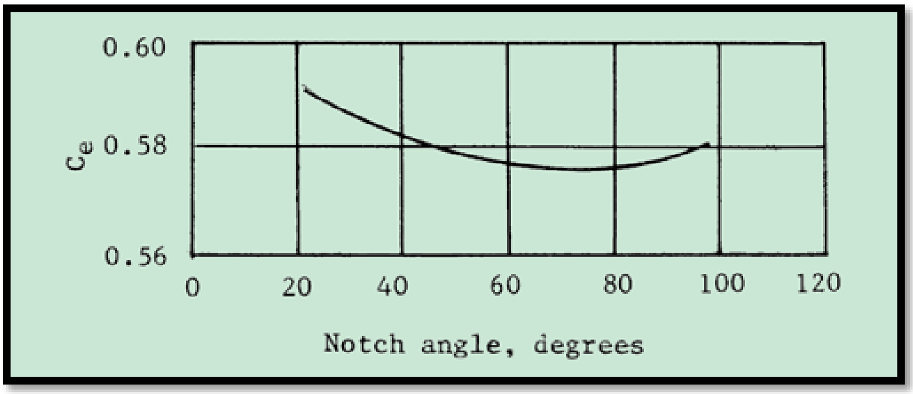
- $Q = \frac{8}{15}\sqrt{2g}C_e \tan\left(\frac{\theta}{2}\right) h_e^{5/2}$
- $h_e = h_u + K_h$

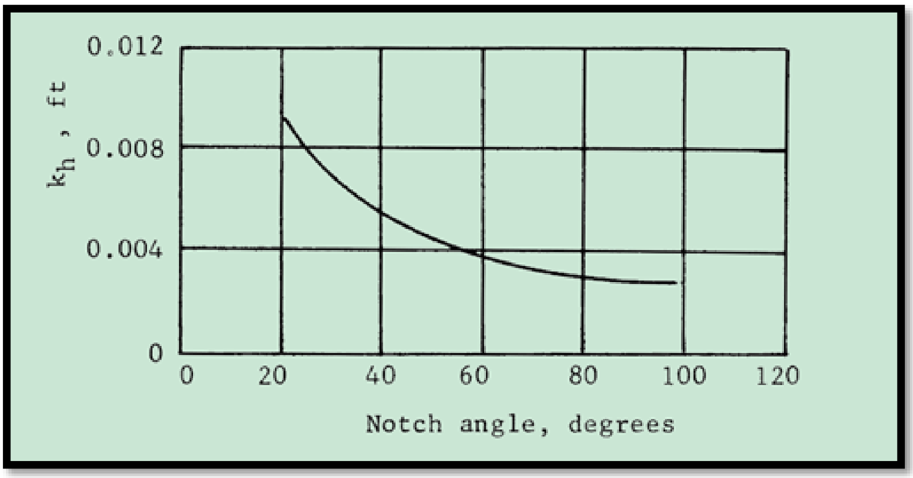


V-Notch Weir

- Where:
  - Q (m³/s) is flow over V-Notch weir
  - C_e , K_h can be found using the graphs to the right
  - h_u (m) is the head flowing through the notch
  - θ (degrees) is the notch angle
  - g is the acceleration of gravity(9.81 m/s²)

- When θ=90° this equation can be simplified to:
  - $Q = 2.49 h_e^{2.48}$
  - for 0.2 ft < h_e < 1.25 ft





Example 1

Water flows over a sharp-crested weir 600 mm wide. The measured head (relative to the crest) is 155 mm at a point where the cross-sectional area of the stream is 0.26 m^2 (see Fig. 3.28). .Calculate the discharge, assuming that $C_d = 0.61$

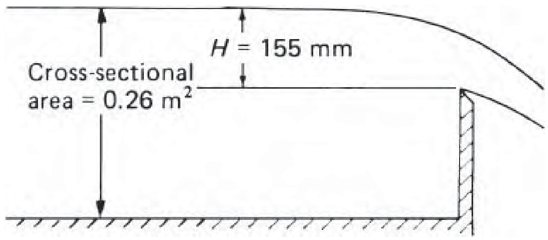


Fig. 3.28

Solution

As a first approximation,

$$Q = \frac{2}{3} C_d \sqrt{(2g)} b H^{3/2}$$

$$= \frac{2}{3} 0.61 \sqrt{(19.62 \text{ m} \cdot \text{s}^{-2})} 0.6 \text{ m} (0.155 \text{ m})^{3/2}$$

$$= 0.0660 \text{ m}^3 \cdot \text{s}^{-1}$$

Example 2

A rectangular notch 2.5 m wide has a constant head of 40 cm, find the discharge over the notch where Cd = 0.62

Solution:

$$Q = \frac{2}{3} C_d \, b \sqrt{2g} \, H^{3/2}$$ **= 2/3 (0.62) (2.5) (2 x 9.81) $(0.4)^{3/2}$ =1.16 m^3/s**

Example 3

A rectangular notch has a discharge of 21.5 m3/min, when the head of water is half the length of the notch. Find the length of the notch where Cd = 0.6.

Solution:

$$Q = \frac{2}{3} C_d \, b \sqrt{2g} \, H^{3/2}$$ ⇒ **21.5/60 = 2/3 (0.6) (b) $(2 \times 9.81)^{0.5}$ $(0.5\, b)^{3/2}$**

⇒ $b^{5/2}$ = 0.572 ⇒ b = $(0.572)^{2/5}$ = 0.8 m

Example 4

During an experiment in a laboratory, 50 liters of water flowing over a right-angled notch was collected in one minute. If the head of still is 50mm. Calculate the coefficient of discharge of the notch. cd

Solution:

$$Q = \frac{8}{15} C_d \tan(\theta/2)\sqrt{2g}\, H^{5/2}$$

Q= 50 lit/min (m^3/1000lit)(min/60s) = 8.334 x 10^{-4} m^3/s

⇒ Cd = (8.334 x 10^{-4})/[(8/15)$(2x9.81)^{0.5}$ tan(θ/2)$(0.05)^{5/2}$]

⇒ Cd = 0.63

Example 5

A rectangular channel 1.5 m wide is used to carry 0.2 m^3/s water. The rate of flow is measured by placing a 90^o V-notch weir. If the maximum depth of water is not to exceed 1.2 m, find the position of the apex of the notch from the bed of channel. Cd = 0.6.

Solution:

Q = 1.417 $H^{5/2}$ ⇒ $H^{5/2}$ = (0.2 m^3/s)/1.417 ⇒ H = 0.46 m

The maximum depth of water in channel = 1.2 m

H is the height of water above the apex of notch.

Apex of triangular notch is to be kept at distance = 1.2 –0.46

= 0.74 m from the bed of channel.

Example 6

Water enters the Millwood flood storage area via a rectangular weir when the river height exceeds the weir crest. For design purposes a flow rate of 0.163 m^3/s over the weir can be assumed

1. Assuming a height over the crest of 200mm and C_d=0.2, what is the necessary width, B, of the weir?
2. What will be the velocity over the weir at this design?

Solution:

Given: Q=0.163 m^3/s., H=200mm = 0.2 m., Cd=0.2.

Required B and V.

$$Q = Cd.\frac{2}{3}.B\sqrt{2g}.H^{\frac{3}{2}} \rightarrow 0.163 = 0.2x\frac{2}{3}xBx\sqrt{19.62}x0.2^{\frac{3}{2}}$$

$$\therefore B = 3.08\,m$$

$$V = \sqrt{2gh} \rightarrow V = \sqrt{19.62x0.2} = 1.98\,\frac{m}{s}$$

Example 7

Water is flowing over a 90o 'V' Notch weir into a tank with a is cross-sectional area of 0.6m^2. After 30s the depth of the water in the tank 1.5m. If the discharge coefficient for the weir is 0.8, what is the height of the water above the weir

Solution:

Given: θ=90^o , At=0.6m2, T=30s, Ht=1.5m, Cd=0.8.

Required H.

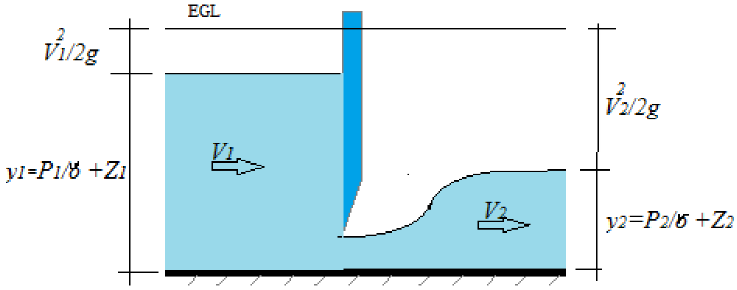
$$Q = \frac{Vt}{T} = \frac{0.6x1.5}{30} = 0.03\frac{m^3}{s}$$

$$Q = Cd.\frac{8}{15}.\sqrt{2g}.\tan(\frac{\theta}{2}).H^{\frac{5}{2}} \rightarrow 0.03 = 0.8x\frac{8}{15}x\sqrt{19.62}x\tan(\frac{90}{2})x\,H^{\frac{3}{2}}$$

$$\therefore H = 0.06\,m$$

1.5#Flow under the gates:

The sluice gate is a hydraulic structure for regulate discharge in open channels. It is cross the flow section which increases the velocity of flow at down stream of the gate and hence reduces the pressure. Apply Bernoulli along the streamline from sction 1 to section 2 shown . .in Fig



$$\frac{p_1}{\rho g}+z_1+\frac{v_1^2}{2g}=\frac{p_2}{\rho g}+z_2+\frac{v_2^2}{2g}$$

$$y_1+\frac{V_1^2}{2g}=y_2+\frac{V_2^2}{2g}$$

$$\therefore \qquad y_1-y_2=\frac{V_2^2}{2g}-\frac{V_1^2}{2g}$$

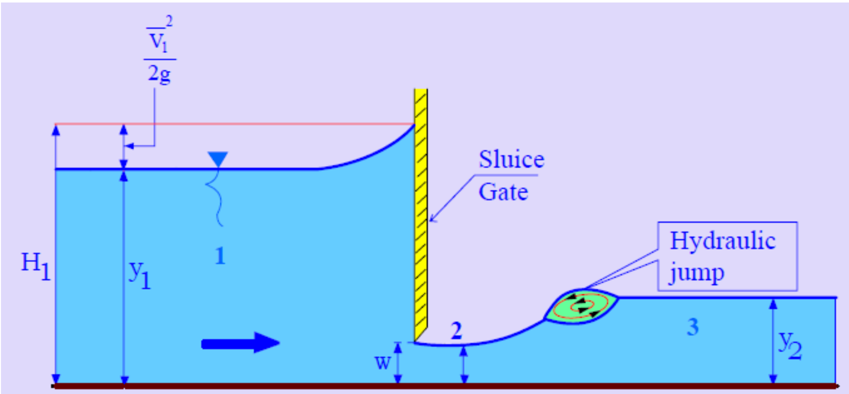
For unit length of gate, the continuity equation can be written as:

$$q=y_1\times 1\times V_1=y_2\times 1\times V_2 \qquad V_1=\frac{y_2}{y_1}V_2$$

$$2g(y_1-y_2)=V_2^2-V_2^2\left(\frac{y_2}{y_1}\right)^2=V_2^2\left[1-\left(\frac{y_2}{y_1}\right)^2\right] \qquad \therefore \quad V_2=\sqrt{\frac{2g(y_1-y_2)}{1-\left(\frac{y_2}{y_1}\right)^2}}$$

The practical estimation of Q below a Sluice gate is

$$Q=C.B.W.\sqrt{2g(y_1-y_2)}$$



Where:

- *Q (m3/s) is the flow under the gate*
- *C the discharge coefficient ranges between 0.6 to 0.5*
- *B (m) the gate width*
- *W (m) the gate opening*
- *y_1 & y_2 water depth at upstream and downstream of the gate*

Example 1

17.235 A rectangular channel 2.0 m wide contains a sluice gate which extends across the width of the channel. If the gate produces free flow when it is open 0.15 m with an upstream depth of 1.15 m, find the rate of discharge, assuming $C_d = 0.60$ and $C_c = 0.62$.

Solution $Q = y_2 = 0.0930\text{ m}\ \ y_2 + V_1^2/2g)]^{1/2}$ $y_2 = C_c x W = 0.62x0.15 = 0.093m$

Find a trial Q by neglecting the $V_1^2/2g$ term in the equation above.

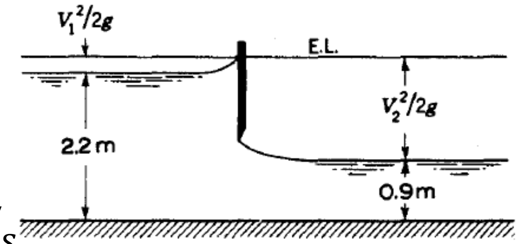
$$Q = (0.60)(2.0)(0.15)[(2)(9.807)(1.15 - 0.0930 + 0)]^{1/2} = 0.820\text{ m}^3/\text{s}$$

Example 2

For the sluice gate shown if C=0.62 and gate opening of 1.45m what will be the flow rate
If the Q must be increased by 20% what will be the opening

Solution

$$Q = C.B.W.\sqrt{2g(y_1 - y_2)}$$



We will take one meter width of gate (W=1)

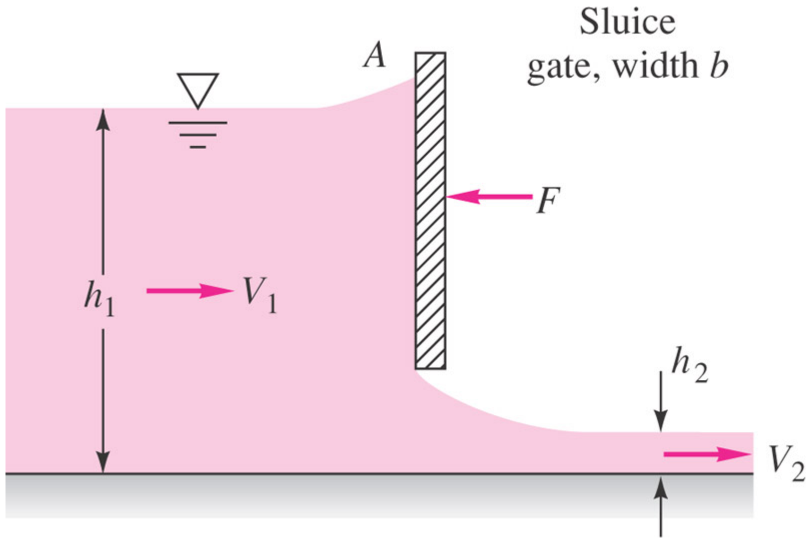
Then: $Q = 0.62x1x1.45x\sqrt{2g(2.2 - 0.9)} = 4.54\,{m^3}/{s}$

If Q increased by 20% then Q=1.2x4.54=5.45m^3/s

$\therefore 5.45 = 0.62x1xwx\sqrt{2g(2.2 - 0.9)}\ \ \rightarrow W = 1.74m$

EXAMPLE 3 H.W

A sluice gate controls flow in open channels. At sections 1 and 2, the flow is uniform and the pressure is hydrostatic. Neglecting bottom fiction and atmospheric pressure, find Q under the gate if $h_1 = 6$ m , $h_2 = 1$ m , Cd=0.65 and $b = 5$ m and opening =1.5m



Ans. 48.3m^3/s

Lec:4

2) Flow through orifice

a) With constant head

When an open tank fill with liquid and drains through a port at the bottom of the tank. The elevation of the liquid in the tank is constant above the drain. The drain port is at atmospheric pressure. The flow is steady, viscous effects are unimportant and velocity at liquid surface is zero. The Bernoulli equation between points 1 and 2 on streamline:

$$\frac{p_1}{\rho g}+\frac{v_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{v_2^2}{2g}+z_2$$

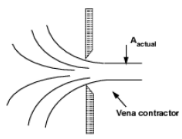
$p_1 = p_2$ because the pressure at the outlet and the tank surface are the same (atmospheric).

The velocity at the tank surface zero, then:

$$0+0+(z_1-z_2)=0+\frac{V_2^2}{2g}$$

$$V_2=\sqrt{2g(z_1-z_2)}=\sqrt{2gH}$$

$$Q_{theo}=V_2A_2$$



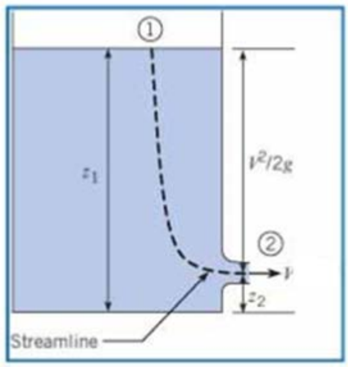
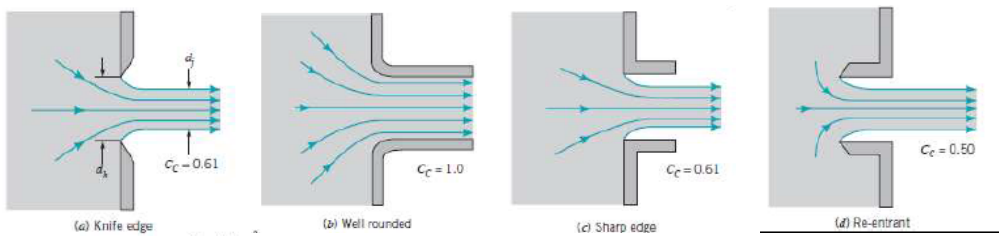


Fig.4.8

$$Q_{actual}=C_d\times Q_{theo}$$

$where\ C_d\ is\ coefficient\ of\ discharge$ (0.64)



- If the tank is closed with interior pressure of $\boldsymbol{P_o}$ then Bernoulli's equation can be expressed as:

$$\frac{P_o}{\gamma}+0+(z_1-z_2)=0+\frac{V_2^2}{2g} \quad \text{then} \quad V_2=\sqrt{2g\left(\frac{P_o}{\gamma}+H\right)}$$

b) With variable head (time for the tank to empty)

For cylindrical tank, the tank cross sectional area is **A**. In a time ***dt*** the level falls by ***dH***

$$Q = -A\frac{dH}{dt}$$

We have an expression for the discharge from the tank

$$Q_{act} = C_d A_2 \sqrt{2gH}$$

This discharge out of the orifice is the same as the flow in the tank so,

$$-A\frac{dH}{dt} = C_d A_2 \sqrt{2gH}$$

Integrating between the initial level, h_1, and final level, h_2, gives the time it takes to fall this height:

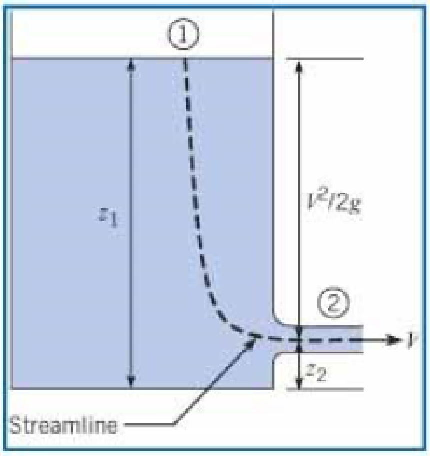
$$\int_{H_1}^{H_2} H^{-0.5} dH = \frac{-C_d A_2 \sqrt{2g}}{A} \int_{t_1}^{t_2} dt$$

$$\left[2H^{0.5}\right]_{H_1}^{H_2} = \frac{-C_d A_2 \sqrt{2g}}{A} \Delta t$$

$$\sqrt{H_1} - \sqrt{H_2} = \frac{C_d A_2 \sqrt{2g}}{2A} \Delta t$$

Example: (Example 4.7 outlet velocity from draining tank)

An open tank filled with water and drains through a port at the bottom of the tank. **The elevation of the water in the tank is 10 m above the drain**. The drain port is at atmospheric pressure. Find the velocity of the liquid in the drain port.



Assumptions:

1. Flow is steady.
2. Viscous effects are unimportant.
3. Velocity at liquid surface is much less than velocity in drain port.

Solution

1. The Bernoulli equation between points 1 and 2 on streamline:

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

2. The pressure at the outlet and the tank surface are the same (atmospheric), so $p_1 = p_2$. The velocity at the tank surface is much less than in the drain port so $V_1 >> V_2$. Solution for V_2:

$$0+0+(z_1-z_2)=0+\frac{V_2^2}{2g}$$

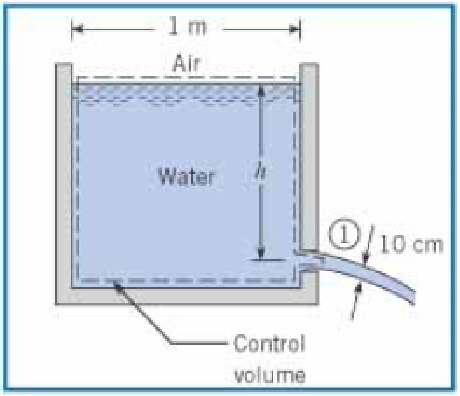
4. Velocity calculation

$$V_2=\sqrt{2g(z_1-z_2)}=\sqrt{2gH}=\sqrt{2\times 9.81\times 10}=14m/s$$

which states that the speed of efflux is equal to the speed of free fall from the surface of the reservoir. This is known as *Torricelli's theorem.*

Example2: (Example 5.6 water level drop rate in draining tank)

A 10 cm jet of water issues from a 1 m diameter tank. Assume that the velocity in the jet is m/s where h is the elevation of the water surface above the outlet jet. How long will it take for the water surface in the tank to drop from h_1 = 2 m to h_2 = 0.50 m? Take C_d = 0.98



Solution

We have an expression for the discharge from the tank

$$Q_{actual}=C_d A_{nominal}\cdot\sqrt{2gh}$$

We can use this to calculate how long of time it will take for the level to fall As the tank empties the level of water surface falls and the discharge will also drop.

The tank has a cross sectional area of A.
In a time dt the level falls by dh
The continuity equation is written for tank as: $$\boldsymbol{Q_{in}-Q_{out}-\frac{d\forall}{dt}=0}$$

$$0-C_d\cdot A_o\cdot\sqrt{2gh}-A\frac{dh}{dt}=0$$

$$C_d\cdot A_o\cdot\sqrt{2gh}=-A\frac{dh}{dt}$$

$$dt=\frac{-A\cdot dh}{C_d\cdot A_o\cdot\sqrt{2gh}}$$

Integrating between the initial level, h_1, and final level, h_2, gives the time it takes to fall this height

$$t = \frac{-A}{C_d \cdot A_o \cdot \sqrt{2g}} \int_{h_1}^{h_2} \frac{dh}{\sqrt{h}}$$

$$\left(\int \frac{1}{\sqrt{h}} = \int h^{-1/2} = 2h^{1/2} = 2\sqrt{h} \right)$$

$$t = \frac{-A}{C_d A_o \sqrt{2g}} \left[2\sqrt{h} \right]_{h_1}^{h_2}$$

$$= \frac{-2A}{C_d A_o \sqrt{2g}} \left[\sqrt{h_2} - \sqrt{h_1} \right]$$

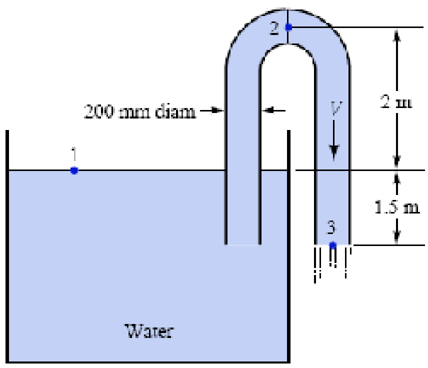
$$t = \frac{-2A}{C_d A_o \sqrt{2g}} \left[\sqrt{h_2} - \sqrt{h_1} \right]$$

$$t = \frac{-2 \times 1}{0.98 \times 0.01_o \sqrt{2 \times 9.81}} \left[\sqrt{0.5} - \sqrt{2} \right] = 32.55\, s$$

3) Syphon

Example 1 The siphon of Fig. is filled with water

Find the pressure at point 2



SOLUTION The energy equation is first applied to all the water in the system upstream from point 3, with elevation datum at point 3

$$\frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 = \frac{V_3^2}{2g} + \frac{p_3}{\gamma} + z_3$$

$$0 + 0 + 1.5 = \frac{V_3^2}{2g} + 0 + 0$$

$V_3 = 5.42$ m/s

The energy equation applied between points 1 and 2, with elevation datum at point 1

$$0 + 0 + 0 = \frac{p_2}{\gamma} + 2 + \frac{V_2^2}{2g}$$

$V_2 = V_3 = 5.42$ m/s

$$0 + 0 + 0 = \frac{p_2}{\gamma} + 2 + \frac{5.42^2}{2g}$$

$P_2 = -34.3$ kPa

4 Nozzle flow

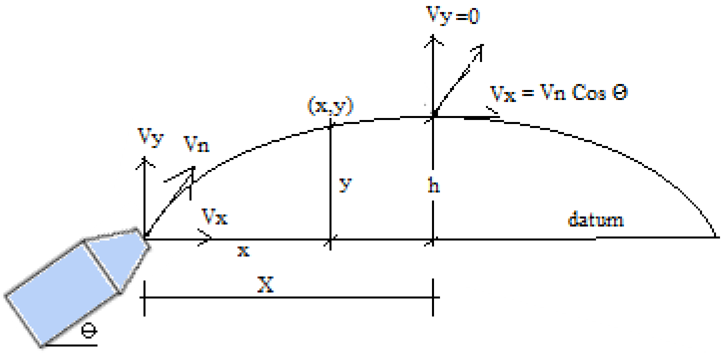


Fig. 4.9

The pressure of all points of liquid jet outside the nozzle is equal to atmospheric pressure. So, it will be equal to zero when the atmospheric pressure is the reference pressure. Then, the Bernoulli's equation between points 1 and 2 in Fig. 4.9 can be written as:

$$\frac{V_1^2}{2g} = \frac{V_2^2}{2g} + z_2$$

The velocity components for ***Vn*** are:

$$V_x = V_n \cos\theta \quad \text{and} \quad V_y = V_n \sin\theta$$

Therefore, for point 1: $V_{1x} = V_n \cos\theta$ and $V_{1y} = V_n \sin\theta$

For point 2: $V_{2x} = V_n \cos\theta$ and $V_{2y} = 0$, so, $V_2 = V_n \cos\theta$, which mean that the horizontal velocity component is constant along the nozzle jet path.

$$\therefore \frac{V_{1x}^2}{2g} + \frac{V_{1y}^2}{2g} = h + \frac{V_2^2}{2g}$$

$$\frac{(V_n \cos\theta)^2}{2g} + \frac{(V_n \sin\theta)^2}{2g} = h + \frac{(V_n \cos\theta)^2}{2g}$$

Therefore the maximum height of nozzle jet will be: $h = \frac{(V_n \sin\theta)^2}{2g}$

The vertical component of velocity ***Vy*** is varied along the nozzle jet path as below:

$$V_y = V_n \sin\theta - g.t$$

The horizontal and vertical axes(x,y) of any point along the nozzle jet path as below:

$$y = V_n \sin\theta \,.\, t - \frac{1}{2} g t^2$$

$$x = V_x . t = V_n \cos\theta . t$$

at the highest point of jet ***Vy*** reaches to zero, so:

$$h = \frac{1}{2} g t^2 \quad \rightarrow \quad t_2 = \sqrt{\frac{2h}{g}}$$

and the horizontal distance of highest point determined as:

$$X = V_x . t_2 = V_n \cos\theta . t_2$$

Example 1: Determine the vertical and horizontal distance of highest point of water jet from nozzle with velocity of 20m/s. Also, find the diameter of the jet at the highest point if the diameter of nozzle is 2cm. The jet is inclined at 60^{o} with horizontal. Neglect air resistance.

Solution: $h = \frac{(V_n \cos\theta)^2}{2g} = \frac{(20 \sin 60)^2}{2 \times 9.81}$=15.29m

$$t_2 = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 15.29}{9.81}} = 1.766 sec$$

$$V_x = V_n \cos\theta = 20 \cos 60 = 10 m/s$$

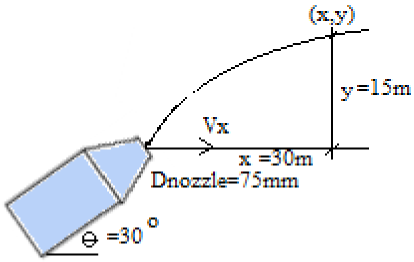
$$X = V_x . t_2 = 10 \times 1.766 = 17.66 m$$

$Q = V_n \times A_{nozzle} = 20.\pi/4(0.02)^2 = 6.28 \times 10^{-3} m^3/s$

$A2 = Q/V_x = 6.28/10 = 6.28 \times 10^{-4} m^2$

$D = 0.028m = 2.8cm$

Example 2: Point (b) is located on the stream line. Determine the flowrate



Solution

$$x = V_x.t = V_n \cos\theta.t$$

$$V_n = \frac{x}{cos\,\theta.t} = \frac{42.43}{t} \quad \ldots\ldots\ldots..1$$

$$y = V_n\, sin\,\theta\,.t - \frac{1}{2}gt^2$$

$$V_n = \frac{y + \frac{1}{2}gt^2}{sin\,\theta.t} = \frac{21.21}{t} + 6.94t \quad \ldots\ldots\ldots\ldots\ldots 2$$

$$\frac{42.43}{t} = \frac{21.21}{t} + 6.94t \quad \rightarrow \quad 6.94t = \frac{21.21}{t} \quad \rightarrow \quad t = 1.749sec$$

Vn=42.43/1.749=24.26m/s

$Q = Vn.A_{nozzle} = 0.107m^3/s$

5 Pumps and Turbines

The energy line of liquid flow through turbine drops down directly due to consumption of energy by turbine which call turbine head (h_t) . While the energy line of liquid flow through pump rises up directly due to adding of energy to the flow by pump which call pump head (h_p). So, the Bernoulli's equation will be:

$$\frac{p_1}{\rho g}+\frac{v_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{v_2^2}{2g}+z_2+h_t-h_p$$

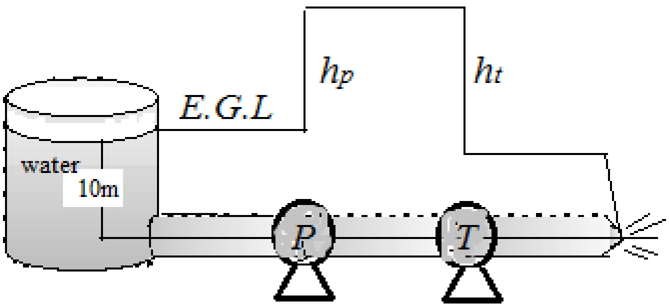


Fig. 4.10

Pumps and turbines power

The head is the energy of unit weight: $h=\frac{E}{W}$ *then,* $E=W\times h$

Power is the energy per unit time: $Power=\frac{Wh}{t}$ $while\ Q_W=\frac{W}{t}=Q\gamma$

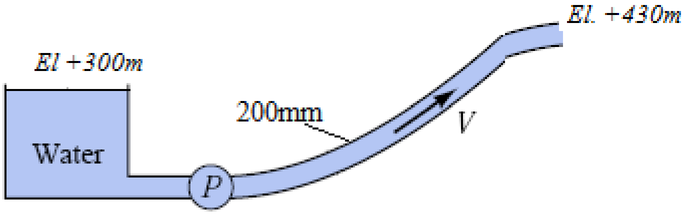
$$\therefore Power=Q\gamma h$$

and $\boxed{Pump\ power=Q\gamma h_p}$ at Watt

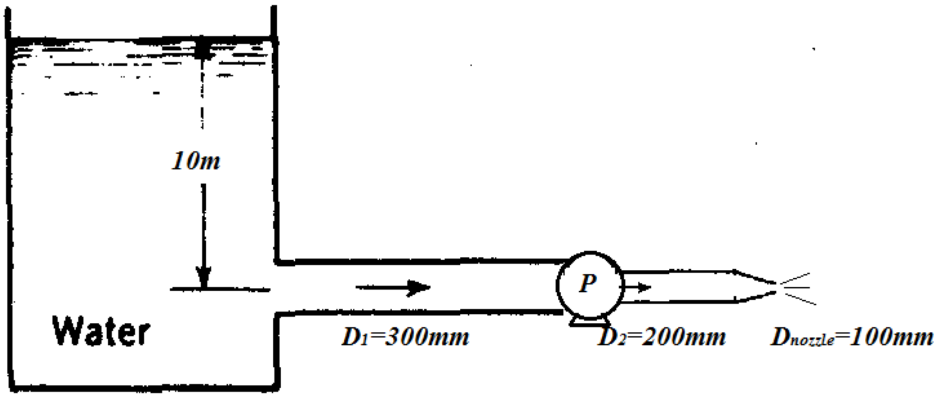
$\boxed{Turbine\ power=Q\gamma h_t}$ at Watt

Pw(hp) = Pw(Watt) /746

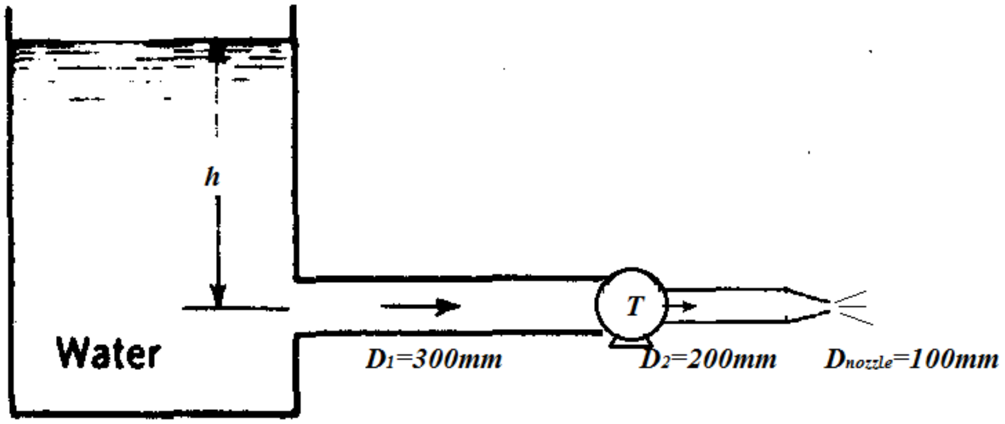
Example 1: Draw the E.G.L. and H.G.L. of the pipe system in Fig. and determine the power of pump. The discharge is $0.15 m^3/s$. neglect the friction of pipe.



Example 2: The depth of water in tank shown in Fig. is 10m and discharge required through the system is $0.15 m^3/s$. Determine the velocity and the pressure in each pipe, the power of the pump. Plot E.G.L. and H.G.L.



Example 3: Calculate the depth of water in tank shown in Fig. which will produce a discharge of 85 l/s. The input power of the turbine is 15kW. What flowrate may be expected if the turbine is removed?



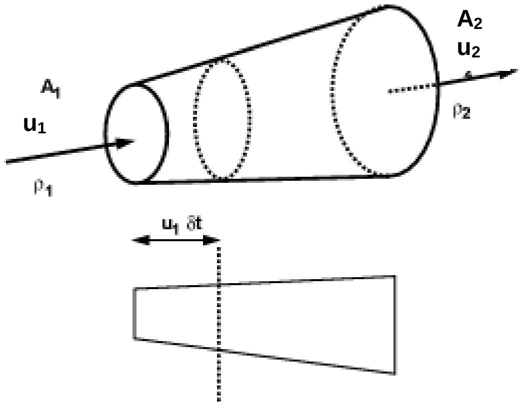
Momentum Equation

Lec:5

The ***Momentum equation*** is a statement of Newton's Second Law. It relates the sum of the forces to the acceleration or rate of change of momentum. From solid mechanics you will recognize

$$F = ma$$

What mass of moving fluid we should use?
We use a different form of the equation.
Consider a stream tube and assume steady non-uniform flow:



In time δt a volume of the fluid moves from the inlet a distance $v_1\delta t$, so

volume entering the stream tube = area × distance

$$=A_1 v_1 \delta t$$

The mass entering,
mass entering stream tube = volume density

$$=\rho\, A_1\, v_1\, \delta t$$

And momentum
momentum entering stream tube = mass velocity

$$= \rho\, A_1\, v_1\, \delta t\, v_1$$

Similarly, at the exit, we get the expression:

momentum leaving stream tube = $\rho\, A_2\, v_2\, \delta t\, v_2$

By another reading of Newton's 2nd Law.
where Momentum = m x v,

Force = mass x acceleration $= m\dfrac{dv}{dt} = \dfrac{dmv}{dt} =$ rate of change of momentum

$$F = \frac{(\rho_2 A_2 u_2 \delta t\, u_2 - \rho_1 A_1 u_1 \delta t\, u_1)}{\delta t}$$

We know from continuity that

$$Q = A_1 u_1 = A_2 u_2$$

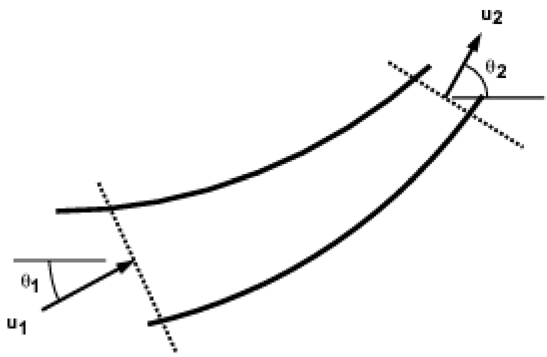
And if we have a fluid of constant density,

$$F = Q\rho(u_2 - u_1)$$

The Momentum equation

This force acts on the fluid in the direction of the flow of the fluid

The previous analysis assumed the inlet and outlet velocities in the same direction (i.e. a one dimensional system). What happens when this is not the case?



We consider the forces by resolving in the directions of the co-ordinate axes.

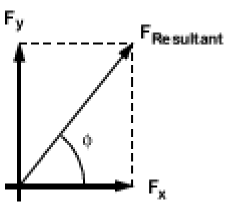
The force in the x-direction

$$F_x = \rho Q(u_2 \cos\theta_2 - u_1 \cos\theta_1)$$
$$= \rho Q(u_{2x} - u_{1x})$$

And the force in the y-direction

$$F_y = \rho Q(u_2 \sin\theta_2 - u_1 \sin\theta_1)$$
$$= \rho Q(u_{2y} - u_{1y})$$

The resultant force can be found by combining these components



$$F_{resultant} = \sqrt{F_x^2 + F_y^2}$$

And the angle of this force

$$\phi = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

This hydrodynamic force exerted on fluid mass due to time rate of change of the linear momentum of the system is countered with other external forces exist within control volume (i.e. pressure forces and body forces and thrust to result an excess action force that exerted on any solid body touching the control volume, ***R***:

F_B = Force exerted due to fluid body (e.g. gravity)

F_P = Force exerted on the fluid control volume due to fluid pressure at the open fluid edges of the control volume

F_R = Force exerted on the fluid by any solid body touching the control volume

So we say that the total force, F_T, is given by the sum of these forces:

$$F_T = F_R + F_B + F_P$$

The force exerted by the fluid on the solid body touching the control volume is opposite to F_R ***(action force)***.

So the reaction force, ***R***, is given by

$$R = -F_R$$




Applications of Momentum Equation:

1) Pipe reducer and nozzle

In pipe reducer and nozzle, the inlet and outlet velocities are in the same direction as shown in Fig. 4.8 which represent reducer fitting in pipe line, equation 4.12 written as:

$P_1A_1 - P_2A_2 - R_x = \rho.Q.(V_2 - V_1)$, then $\boldsymbol{R_x}$ can be found.

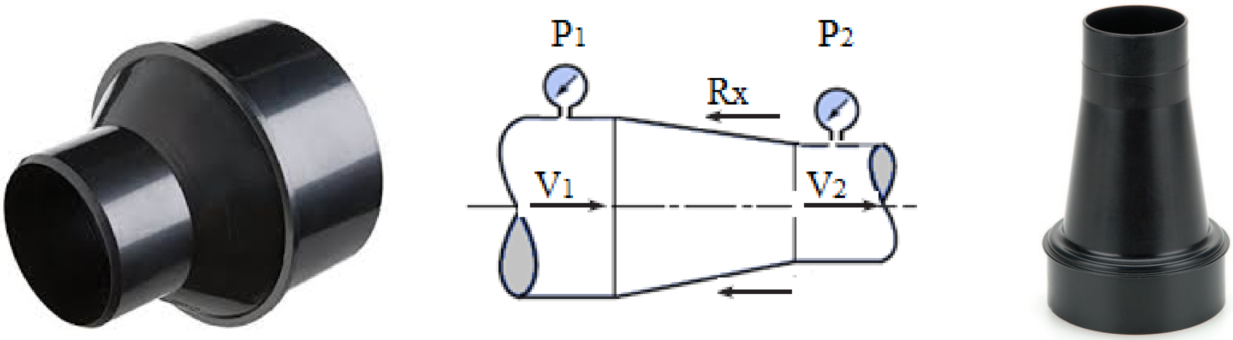


Fig. 4.8

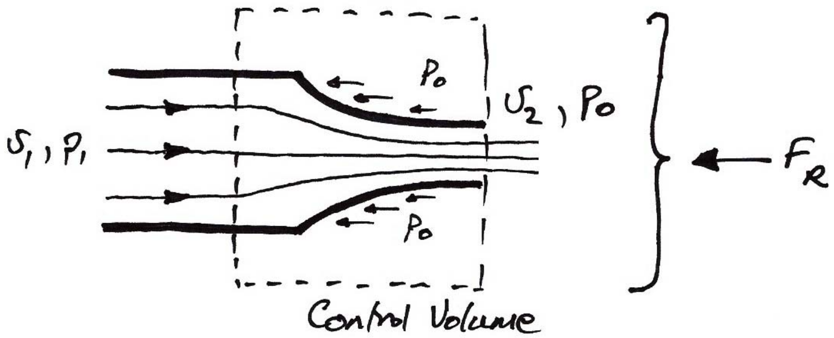
Application – Force exerted by a firehose

Example 1

A firehose discharges 5 l/s. The nozzle inlet and outlet diameters are 75 and 25 mm respectively. Calculate the force required to hold the hose in place.

Solution

The control volume is taken as shown:



There are three forces in the x-direction:

- The reaction force F_R provided by the fireman;
- Pressure forces F_P: $p_1 A_1$ at the left side and $p_0 A_0$ at the right hand side;
- The momentum force F_M caused by the change in velocity.

So we have: $F_M = F_P + F_R$

The momentum force is: $F_M = \rho Q(v_2 - v_1)$

Therefore, we need to establish the velocities from continuity:

$$v_1 = \frac{Q}{A_1} = \frac{5\times10^{-3}}{\pi(0.075)^2/4} = 1.13 \text{ m/s}$$

And

$$v_2 = \frac{5\times10^{-3}}{\pi(0.025)^2/4} = 10.19 \text{ m/s}$$

Hence:

$$F_M = \rho Q(v_2 - v_1) = 10^3\left(5\times10^{-3}\right)(10.19 - 1.13) = 45 \text{ N}$$

The pressure force is: $F_P = p_1 A_1 - p_0 A_0$

If we consider gauge pressure only, the $p_0 = 0$ and we must only find p_1. Using Bernoulli's Equation between the left and right side of the control volume:

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} = \left(\frac{p_0}{\rho g}\right)_{=0} + \frac{v_0^2}{2g}$$

Thus:

$$p_1 = \left(\frac{\rho}{2}\right)\left(v_1^2 - v_0^2\right) = \left(\frac{10^3}{2}\right)\left(10.19^2 - 1.13^2\right) = 51.28 \text{ kN/m}^2$$

Hence

$$F_P = p_1 A_1 - p_0 A_0$$
$$= \left(51.28\times10^3\right)\left(\frac{\pi(0.075)^2}{4}\right) - 0 = 226 \text{ N}$$

Hence the reaction force is:

$$F_R = F_M - F_P = 45 - 226 = -181 \text{ N}$$

This is about a fifth of an average body weight – not inconsequential.

2) Pipe Bends

Calculating the force on pipe bends is important to design the support system. In pipe bend the inlet and outlet velocities are in different directions. There are two cases of pipe bend can be illustrated as below:

Case 1: pipe bend in horizontal plan

According Fig. 4.9, equation 4.12 written as:

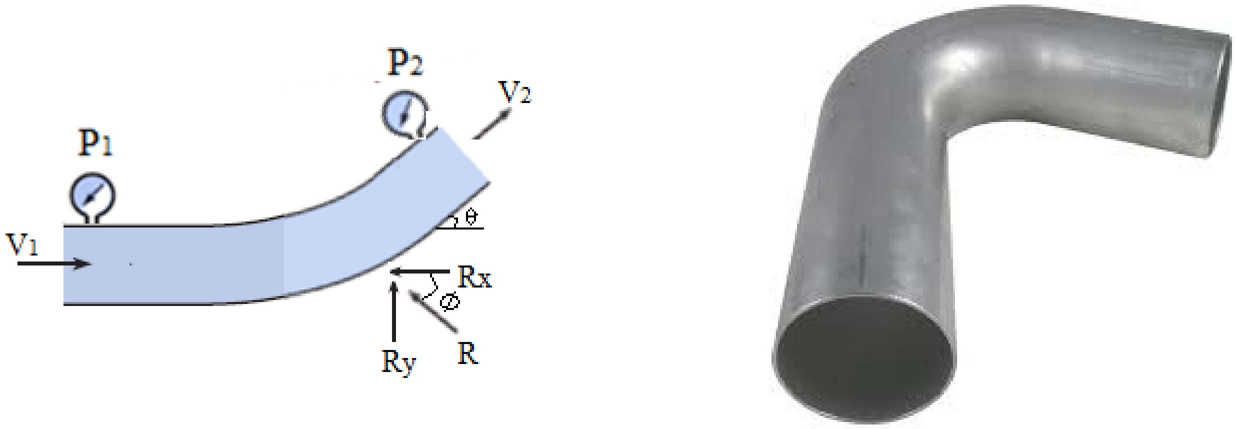


Fig. 4.9

$\sum F_x = P_1A_1 - P_2A_2\cos\theta - R_x = \rho.Q.(V_2\cos\theta - V_1)$, to find $\boldsymbol{R_x}$

$\sum F_y = 0 - P_2A_2\sin\theta - R_y = \rho.Q.(V_2\sin\theta - 0)$, to find $\boldsymbol{R_y}$

The resultant can be get by: $R = \sqrt{R_x^2 + R_y^2}$

The resultant inclined with horizontal with angle of: $\emptyset = \tan^{-1}\frac{R_x}{R_y}$

Case 2: pipe bend in perpendicular plan

According Fig. 4.10, the summation of forces in x-direction is same as in case of horizontal plan. The summations of forces in y-direction include the effect of fluid weight in pipe bend W_f.

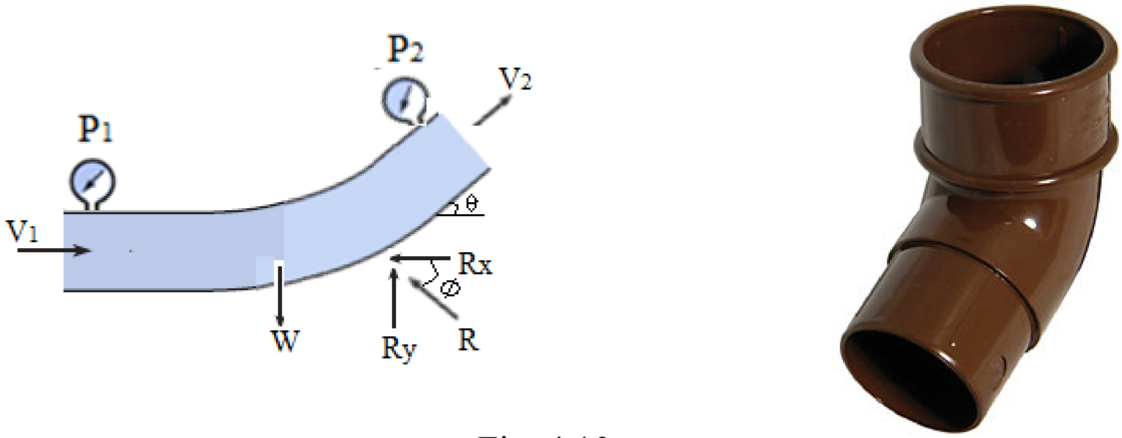


Fig. 4.10

$$\sum F_y = 0 - P_2 A_2 \cos\theta - W_f - R_y = \rho.Q.(V_2 \sin\theta - 0) \text{ , to find } \boldsymbol{R_y}$$

The weight of fluid can be founded as below:

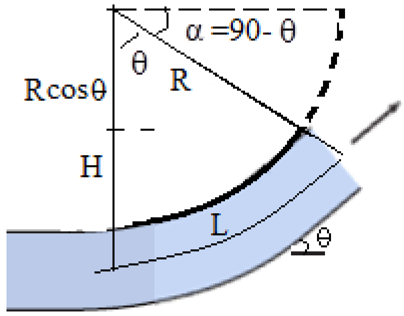


Fig. 4.11

$$W_f = \gamma \forall$$

the volume of fluid is a cross section area times the length of center for the pipe bend. The length of center line can be calculated as below:

For pipe bend of $\theta \leq 90^o$ as shown in Fig. 4.11, the radius for center line of bend given by:

$$H = R - R\cos\theta = R(1 - \cos\theta)$$

$$\sum F_x = P_1 A_1 - P_2 A_2 \cos\theta - R_x = \rho.Q.(V_2 \cos\theta - V_1) \text{ , to find } \boldsymbol{R_x}$$

$$\sum F_y = 0 - P_2 A_2 \sin\theta - W_f - R_y = \rho.Q.(V_2 \sin\theta - 0) \text{ , to find } \boldsymbol{R_y}$$

The resultant can be get by: $R = \sqrt{R_x^2 + R_y^2}$

The resultant inclined with horizontal with angle of: $\emptyset = \tan^{-1}\frac{R_x}{R_y}$

$$\therefore R = \frac{H}{1 - \cos\theta}$$

For pipe bend of $\theta > 90^{o}$ as shown in Fig. 4.12, the radius for center line of bend given by:

$$y = R \sin\theta$$

$$R = H - y = H - R\sin\alpha$$

$$H = R(1 + \sin\alpha)$$

$$R = \frac{H}{1 + R\sin\alpha}$$

Then, length of pipe bend centerline

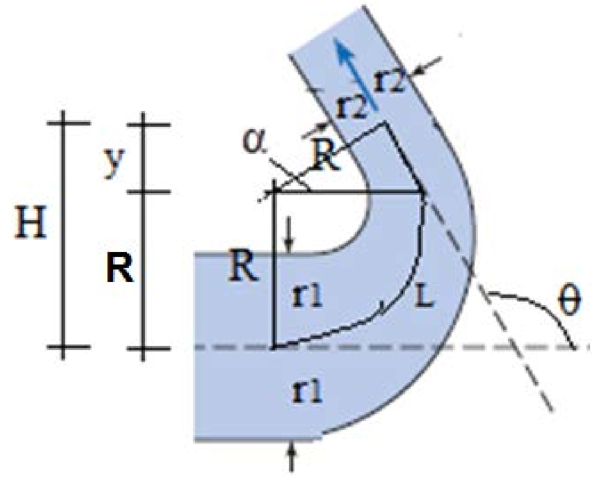


Fig. 4.12

is given by;

$$L = \pi R \frac{\theta}{180}$$

Volume of fluid in pipe bend given by;

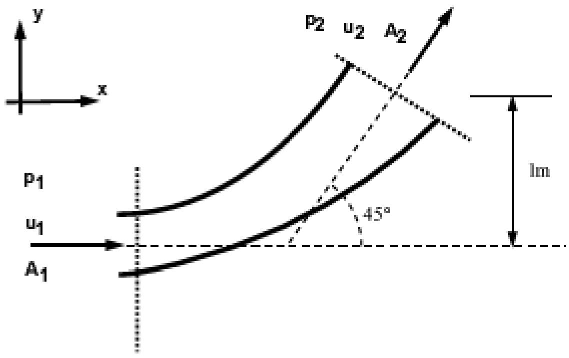
$$\forall = \frac{\pi}{3} L(r_1^2 + r_1 r_2 + r_2^2)$$

Example-1: Forces on a vertical Bend

The outlet pipe from a pump is a bend of 45° rising in the vertical plane (i.e. and internal angle of 135°). The bend is 150mm diameter at its inlet and 300mm diameter at its outlet. The pipe axis at the inlet is horizontal and at the outlet it is 1m higher. By neglecting friction, calculate the force and its direction if the inlet pressure is 100kN/m^2 and the flow of water through the pipe is 0.3m^3/s. The volume of the pipe is 0.075m^3.

Solution:

1&2 Draw the control volume and the axis System



p_1 = 100 kN/m^2,
Q = 0.3 m^3/s
θ = 45°

d_1 = 0.15 m $\quad$ d_2 = 0.3 m

A_1 = 0.177 m^2 $\quad$ A_2 = 0.0707 m^2

3. Calculate the total force
in the x direction

$$\sum F_x = \rho\left(\sum_{cs} Q_{out}v_{x\,out} - \sum_{cs} Q_{in}v_{x\,in}\right) = F_{Tx}$$

$$F_{Tx} = \rho\left(\sum_{cs} Q_{out}v_{x\,out} - \sum_{cs} Q_{in}v_{x\,in}\right)$$
$$F_{Tx} = \rho Q(v_{2x} - v_{1x})$$
$$F_{Tx} = \rho Q(v_2\cos\theta - v_1)$$

by continuity: $A_1v_1 = A_2v_2 = Q$

$$v_1 = \frac{0.3}{\pi\left(\frac{0.15^2}{4}\right)} = 16.98\frac{m}{s}$$
$$v_2 = \frac{0.3}{0.0707} = 4.24\frac{m}{s}$$

$$F_{Tx} = 1000\times 0.3(4.24\cos 45 - 16.98)$$
$$= -4193.68\ N$$

and in the y-direction
$$F_{Ty} = \rho Q(v_{2y} - v_{1y})$$
$$F_{Ty} = \rho Q(v_2\sin\theta - 0)$$

$$= 1000\times 0.3(4.24\sin 45)$$
$$= 899.44\ N$$

4. Calculate the pressure force.

$$F_p = \text{pressure force at 1 - pressure force at 2}$$

$$\theta_1 = 0, \qquad \theta_2 = \theta$$

$$F_{P_x} = p_1 A_1 \cos 0 - p_2 A_2 \cos\theta = p_1 A_1 - p_2 A_2 \cos\theta$$

$$F_{P_y} = p_1 A_1 \sin 0 - p_2 A_2 \sin\theta = -p_2 A_2 \sin\theta$$

We know pressure at the inlet, but not at the outlet

we can use the Bernoulli equation to calculate this unknown pressure.

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + z_2$$

The height of the pipe at the outlet is 1m above the inlet.
Taking the inlet level as the datum:

$z_1 = 0$, $z_2 = 1\text{m}$

So the Bernoulli equation becomes:

$$\frac{100000}{1000\times 9.81} + \frac{16.98^2}{2\times 9.81} + 0 = \frac{p_2}{1000\times 9.81} + \frac{4.24^2}{2\times 9.81} + 1.0$$

$$p_2 = 225361.4 \ N/m^2$$

$$F_{P_x} = 100000\times 0.0177 - 225361.4\cos 45\times 0.0707$$
$$= 1770 - 11266.34 = -9496.37\ kN$$

$$F_{P_y} = -225361.4\sin 45\times 0.0707$$
$$= -11266.37$$

5. Calculate the body force

The body force is the force due to gravity. That is the weight acting in the -ve y direction.

$$F_{B_y} = -\rho g \times volume$$
$$= -1000\times 9.81\times 0.075$$
$$F_{By} = -735.75N$$

There are no body forces in the x direction,

$$F_{B_x} = 0$$

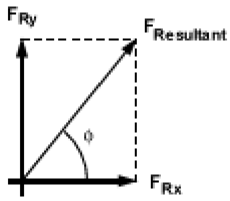
6. Calculate the resultant force

$$F_{Tx} = F_{Rx} + F_{Px} + F_{Bx}$$
$$F_{Ty} = F_{Ry} + F_{Py} + F_{By}$$

$$\begin{aligned} F_{Rx} &= F_{Tx} - F_{Px} - F_{Bx} \\ &= -4193.6 + 9496.37 \\ &= 5302.7\,N \end{aligned}$$

$$\begin{aligned} F_{Ry} &= F_{Ty} - F_{Py} - F_{By} \\ &= 899.44 + 11266.37 + 735.75 \\ &= 12901.56\,N \end{aligned}$$

And the resultant force on the fluid is given by



$$\begin{aligned} F_R &= \sqrt{F_{Rx}^2 - F_{Ry}^2} \\ &= \sqrt{5302.7^2 + 12901.56^2} \\ &= 13.95\,kN \end{aligned}$$

And the direction of application is

$$\begin{aligned} \phi &= \tan^{-1}\left(\frac{F_{R_y}}{F_{R_x}}\right) \\ &= \tan^{-1}\left(\frac{12901.56}{5302.7}\right) \\ &= 67.66^\circ = 67^\circ\,39' \end{aligned}$$

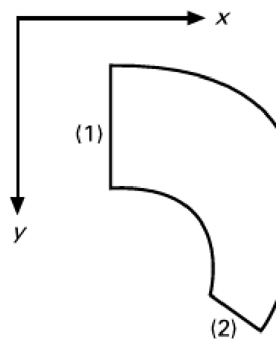
The reaction force on bend is the same magnitude but in the opposite direction

$$R = -F_R = -13.95\,kN$$

Example-2

The diameter of a pipe-bend is 300 mm at inlet and 150 mm at outlet and the flow is turned through 120° in a vertical plane. The axis at inlet is horizontal and the centre of the outlet section is 1.4 m below the centre of the inlet section. The total volume of fluid contained in the bend is 0.085 m^3. Neglecting friction, calculate the magnitude and direction of the net force exerted on the bend by water flowing through it at 0.23 $m^3 \cdot s^{-1}$ when the inlet gauge pressure is 140 kPa.

solution



$$u_1 = \frac{0.23}{(\pi/4)(0.3)^2}\ \mathrm{m \cdot s^{-1}} = 3.254\ \mathrm{m \cdot s^{-1}}$$

$$u_2 = 4u_1$$

$$\text{Energy eqn: } \frac{1.4 \times 10^5}{1000 \times 9.81}\ \mathrm{m} + \frac{3.254^2}{19.62}\ \mathrm{m} + 1.4\ \mathrm{m} = \frac{p_2}{1000 \times 9.81\ \mathrm{N \cdot m^{-3}}} + \frac{16 \times 3.254^2}{19.62}\ \mathrm{m}$$

whence $p_2 = 74\,300$ Pa

$$F_x + p_1 A_1 + p_2 A_2 \cos 60^\circ = \varrho Q(u_2 \cos 120^\circ - u_1)$$

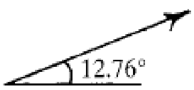
$$\begin{aligned}\therefore F_x &= 1000\ \mathrm{kg \cdot m^{-3}} \times 0.23\ \mathrm{m^3 \cdot s^{-1}} \\ &\quad \times (-4 \times 3.254 \times 0.5 - 3.254)\ \mathrm{m \cdot s^{-1}} \\ &\quad - 1.4 \times 10^5\ \mathrm{N \cdot m^{-2}} \times \frac{\pi}{4}(0.3\ \mathrm{m})^2 \\ &\quad - 74\,300\ \mathrm{N \cdot m^{-2}} \frac{\pi}{4} \times (0.15\ \mathrm{m})^2 0.5 \\ &= -12\,800\ \mathrm{N}\end{aligned}$$

$$F_y - p_2 A_2 \sin 60^\circ + W = \varrho Q(u_2 \sin 120^\circ - 0)$$

$$\begin{aligned}\therefore F_y &= 1000\ \mathrm{kg \cdot m^{-3}} \times 0.23\ \mathrm{m^3 \cdot s^{-1}} \times 4 \times 3.254 \times \frac{\sqrt{3}}{2}\ \mathrm{m \cdot s^{-1}} \\ &\quad + 74\,300\ \mathrm{N \cdot m^{-2}} \times \frac{\pi}{4}(0.15\ \mathrm{m})^2 \frac{\sqrt{3}}{2} \\ &\quad - 0.085\ \mathrm{m^3} \times 1000\ \mathrm{kg \cdot m^{-3}} \times 9.81\ \mathrm{N \cdot kg^{-1}} \\ &= 2896\ \mathrm{N}\end{aligned}$$

$$F = \sqrt{12\,800^2 + 2896^2}\ \mathrm{N} = \mathbf{13\,120\ N}$$

$$\tan\theta = \frac{2896}{-12\,800} \qquad \therefore \theta = 180^\circ - 12.76^\circ$$

Force on *bend* is equal and opposite to this, that is, 

3) Momentum force of flow through diversion

Lec:6

For flow through diversion shown in Fig.13, the momentum equation become as follow:

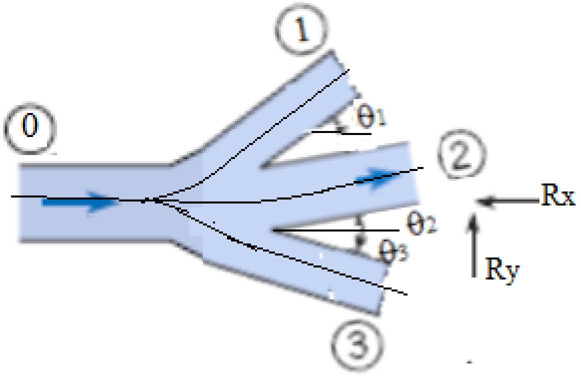


Fig. 4.13

$$\sum F_x = (momentum\ out)_x - (momentum\ in)_x$$

$$\sum F_y = (momentum\ out)_y - (momentum\ in)_y$$

The main pipe supply each branching pipe according to its diameter as follow:

$Q_1 = \frac{D_1^2}{D_1^2+D_2^2+D_3^2} \times Q_0$, $Q_2 = \frac{D_2^2}{D_1^2+D_2^2+D_3^2} \times Q_0$, $Q_3 = \frac{D_3^2}{D_1^2+D_2^2+D_3^2} \times Q_0$, ….etc.

$$\therefore \sum F_x = \sum \rho Q V_{x_{out}} - \rho Q V_{x_{in}}$$

$F_o - F_1 \cos\theta_1 - F_2 \cos\theta_2 - F_3 \cos\theta_3 - R_x = \rho[Q_1 V_1 \cos\theta_1 + Q_2 V_2 \cos\theta_2 + Q_3 V_3 \cos\theta_3] - \rho[Q_o V_o]$, then $\boldsymbol{R_x}$ can be founded.

In the same way:

$$\therefore \sum F_y = \sum \rho Q V_{y_{out}} - \rho Q V_{y_{in}} \qquad -W$$

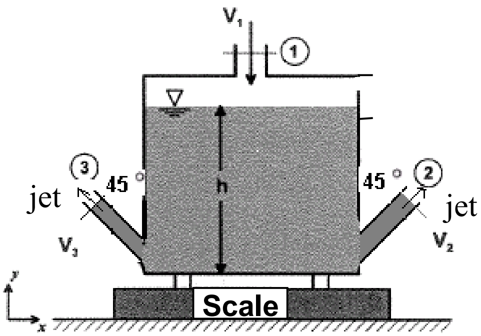
$0 - F_1 \sin\theta_1 - F_2 \sin\theta_2 + F_3 \sin\theta_3 + R_y - W = \rho[Q_1 V_1 \sin\theta_1 + Q_2 V_2 \sin\theta_2 - Q_3 V_3 \sin\theta_3] - \rho[0]$, then $\boldsymbol{R_y}$ can be founded.

The resultant can be get by: $R = \sqrt{R_x^2 + R_y^2}$

The resultant inclined with horizontal with angle of: $\emptyset = \tan^{-1}\frac{R_x}{R_y}$

Example-1

A cylindrical metal container 60 cm high with an inside diameter of 27 cm, weights 22N when empty. The container is placed on a scale and water flows in through an opening in the top and out through the two equal area 45°-deflection openings in the sides as shown in the diagram. Under steady flow conditions the height of the water in the tank is h = 58 cm. Your friend claims that the scale will read the weight of the volume of water in the tank plus the tank weight, i.e., that we can treat this as a simple statics problem. You disagree, claiming that a flow analysis is required. Who is right, and what is the scale reading in Newtons



$A_1 = 95\ cm^2$

$\mathbf{V_1} = -305\hat{j}\ cm/s$

$A_2 = A_3 = 43\ cm^2$

Solution $Q_{in} = Q_{out}$

$$V_1A_1 = V_2A_2 + V_3A_3 \qquad (\because A_2 = A_3\ \&\ symmetry \Rightarrow V_2 = V_3)$$

$$V_1A_1 = 2V_2A_2 = 2V_3A_3$$

$$\therefore V_2 = V_1\frac{A_1}{2A_2} = 3.05 \times \frac{0.0095}{2 \times .0043} = 3.37\,m/s$$

For CV shown

$$\sum F_y = (\sum \rho Q V_{y-out} - \sum \rho Q V_{y-in})$$

$$V_{in}\big|_{water-surface} = 0$$

$$\therefore \sum F_y = 1000 \times 3.37^2\ x43/10000\ cos\,45 \times 2 - 0 = 69.06N$$

$$\sum F_y = \sum F_{py} + F_{Ry} + \sum F_{By}$$

$$F_{Ry} = \sum F_y - \sum F_{py} - \sum F_{By}$$

$$\sum F_{By} = -22 - 0.27^2/4 \times \pi \times 0.58 \times 9810 = -347.77N$$

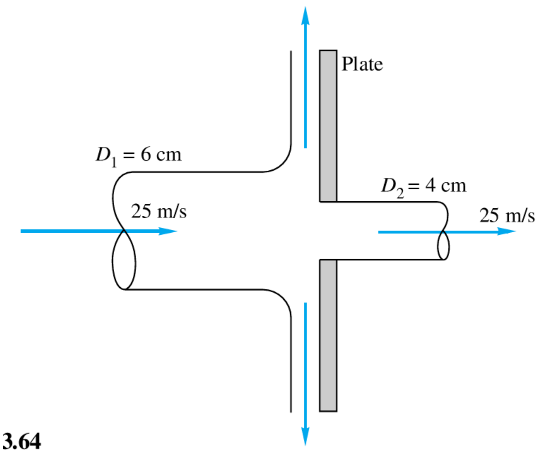
$$p_1 = 0, p_2 = p_3 = 0 \Rightarrow \sum F_{py} = 0$$

$$F_{Ry} = 69.06N + 347.77N = 416.83N = scale - reading$$

Then scale reading not equal the static loads only, but with addition value of dynamic effects

:Example-2

The 6-cm-diameter 20°C water jet in Fig. strikes a plate containing a hole of 4-cm diameter. Part of the jet passes through the hole, and part is deflected. Determine the horizontal force required to hold the plat



3.64

Solution

$$Q_{in} = \frac{\pi}{4}(0.06)^2 25 = 0.0707\ m^3/s$$

$$Q_{hole} = \frac{\pi}{4}(0.04)^2 25 = 0.0314\ m^3/s$$

$$F = \rho Q(V_{out} - V_{in})$$

for devided or branched flow

$$F_x = \rho(\sum Q_{out}V_{outx} - \sum Q_{in}V_{inx})$$

$$F_x = 998(0 + 0 + 0.0314 \times 25 - 0.0707 \times 25)$$

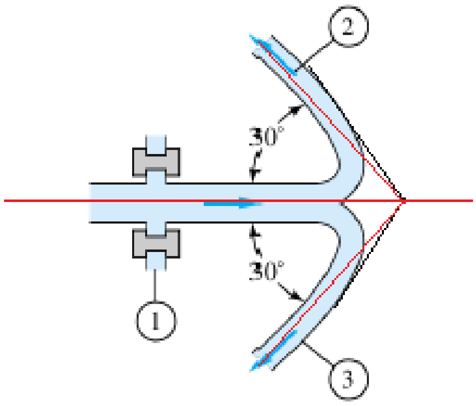
$$F_x = -980\ N$$

$$R_x = 980N$$

$$F_y = 0$$

: **Example-3**

Water at $20°C$ exits to the standard sea-level atmosphere through the split nozzle in Fig. Duct areas are A_1 = 0.02 m^2 and A_2 = A_3 = 0.008 m^2. If p_1= 135 kPa (absolute) and the flow rate is Q_2 = Q3 = 275 m^3/h, compute the force on the flange bolts at section 1.



Solution: With the known flow rates, we can compute the various velocities:

$$V_2 = V_3 = \frac{275/3600\ m^3/s}{0.008\ m^2} = 9.55\ \frac{m}{s}; \quad V_1 = \frac{550/3600}{0.02} = 7.64\ \frac{m}{s}$$

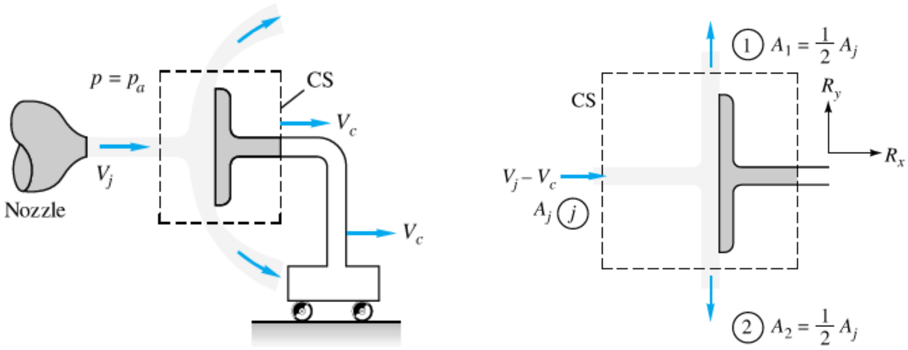
The CV encloses the split nozzle and cuts through the flange. The balance of forces is

$$\sum F_x = -F_{bolts} + p_{1,gage}A_1 = \rho Q_2(-V_2\cos 30°) + \rho Q_3(-V_3\cos 30°) - \rho Q_1(+V_1),$$

or: $$F_{bolts} = 2(998)\left(\frac{275}{3600}\right)(9.55\cos 30°) + 998\left(\frac{550}{3600}\right)(7.64) + (135000 - 101350)(0.02)$$

$$= 1261 + 1165 + 673 \approx \mathbf{3100\ N} \quad \textit{Ans.}$$

Example 4: A water jet of velocity Vj impinges normal to a flat plate which moves to the right at velocity Vc, as shown in Fig. Find the force required to keep the plate moving at constant velocity if the jet density is 1000 kg/m^3, the jet area is 3 cm^2, and Vj and Vc are 20 and 15 m/s, respectively. Neglect the weight of the jet and plate, and assume steady flow with respect to the moving plate with the jet splitting into an equal upward and downward half-jet.



Solution

For moving control volume with *V*=*Vc* we have

$V_{in} = V_j - V_c = 20 - 15 = 5\,m/s$

By continuity equation we have:

$Q_{in} = Q_{out}$

$$A_j V_{in} = A_1 V_1 + A_2 V_2,\ A_1 = A_2 = \frac{1}{2} A_j$$

$$V_{in} = \frac{1}{2} V_1 + \frac{1}{2} V_2,\quad but\ from\ symmetry\ and\ neglacting\ the\ wight: V_1 = V_2$$

$V_{in} = V_1 = V_2$

$F = \rho Q(V_{out} - V_{in})$

for devided or branched flow

$F_x = \rho(\sum Q_{out} V_{outx} - \sum Q_{in} V_{inx})$

$F_x = \rho(\sum A_{out} V_{out} V_{outx} - \sum A_{in} V_{in} V_{inx})$

$F_x = \ (0 - 1000 \times 0.0003 \times 5 \times 5)$

$F_x = -7.5\ N$

$F_x = F_{px} + F_{Rx}$

$F_{Rx} = F_x - F_{px} = -7.5 - 0 = -7.5$

$R_x = -7.5N$

$F_y = 0$

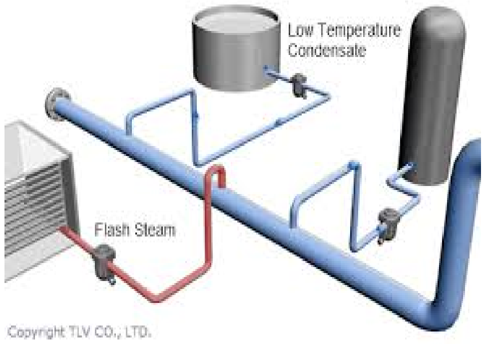
Lec:7

REAL FLOW IN PIPES

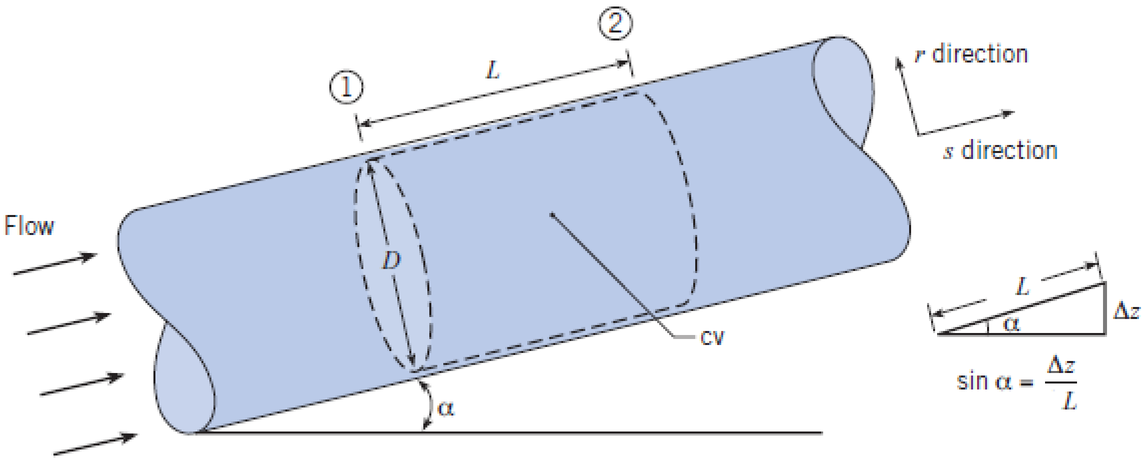
Viscous Flow in Ducts (Flow in Pipes)

The pipe defined as:

1. **Same container sectional area along whole length (L)**
2. **Same container material along whole length (L)**
3. **Circular sectional area along whole length (L)**
4. **Straightforward along whole length (L)**
5. **Full flow (closed conduit) along whole length (L)**



CV Analysis: For pipe segment control volume below



Continuity:

$$\rho Q_1 = \rho Q_2 = const.$$

$$i.e.\, V_1 = V_2 \quad \text{since} \quad A_1 = A_2,\, \rho = const.,\, and\ V = V_{ave}$$

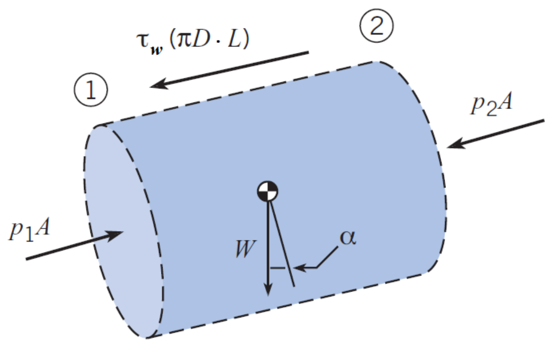
Energy (Bernoulli's) equation in real fluid flow conditions:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + \boldsymbol{h_L}$$

where h_L represents the head loss between two sections 1 and 2 which divided into two parts h_f (head losses due to friction, called major losses), and h_m (head losses due to fitting, valves, and any other source of losses along individual pipe, called miner losses)

$$h_L = h_f + h_m$$

$$h_L = \frac{\Delta p}{\gamma} + \Delta z = \Delta(\frac{p}{\gamma} + z) = \Delta H$$



For a single straightforward part of pipe **without miner losses**, Momentum become:

$$h_f = \frac{\Delta p}{\gamma} + \Delta z = \Delta(\frac{p}{\gamma} + z) = \Delta H$$

$$F_{sT} = \rho Q \overbrace{(V_{2s} - V_{1s})}^{=0} = 0$$

$$F_{sT} = \overbrace{(p_1 - p_2)}^{\Delta p}\pi\frac{D^2}{4} + \overbrace{\gamma\pi\frac{D^2}{4}L}^{W} \cdot \overbrace{\sin\vartheta}^{\Delta z/L} - \overbrace{\tau_w \pi DL}^{\text{friction on walls}}$$

$$\Delta p \pi \frac{D^2}{4} + \gamma\pi\frac{D^2}{4}\Delta z - \tau_w \pi DL = 0$$

$$\Delta p + \gamma\Delta z = \frac{4\tau_w L}{D}$$

$$\frac{\Delta p}{\gamma} + \Delta z = \frac{4\tau_w L}{\gamma D}$$

$$\frac{\Delta p}{\gamma} + \Delta z = \Delta H = h_f = \frac{4\tau_w L}{\gamma D}$$

$$\therefore h_f = \Delta H = \frac{4\tau_w L}{\gamma D} \quad (1)$$

or

$$\tau_w = \frac{D\gamma}{4}\frac{\Delta H}{L} = -\frac{D\gamma}{4}\frac{dh}{ds}$$

i.e. shear stress varies linearly in r across pipe for either laminar or turbulent flow

∴ once τ_w is known, we can determine head drop (pressure drop).

In general,

$$\tau_w = \tau_w(\rho, V, \mu, D, \varepsilon)$$

roughness

By rearrange the right side of Eq. (1).

$$h_f = \left\{\frac{L}{D}\right\}\left\{\frac{4\tau_w}{\rho V^2/2}\right\}\left\{\frac{\rho V^2/2}{\gamma}\right\}$$

$$h_f = \left\{\frac{4\tau_w}{\rho V^2/2}\right\}\frac{L}{D}\frac{V^2}{2g}$$

and by dimensional Analysis we obtain:

$$\tau_w = Cf \cdot \rho\frac{V^2}{2}$$

Where C_f is dimensionless coefficient (Coefficient of Friction). By substitute this value of τ_w in eq. 1 above, we find:

$$\therefore h_f = \Delta H = Cf \cdot \frac{V^2}{2}\frac{4L}{gD} = 4Cf \cdot \frac{L}{D}\frac{V^2}{2g}$$

$$4 \times Cf = f = friction\ factor$$

$f = f(\mu, \varepsilon / D)$ and ***V***

Then

$$\Delta H = h_f = f\frac{L}{D}\frac{V^2}{2g}$$

(Darcy-Weisbach Equation, 1857)

Which define a new factor called the **friction factor *f*** that gives the ratio of wall shear stress $\boldsymbol{\tau_w}$ to kinetic pressure $\rho V^2/2$:

$$\boldsymbol{f} \equiv \frac{(4 \cdot \tau_w)}{\rho V^2/2} \cong \frac{\boldsymbol{shear\ force\ acting\ on\ the\ wall}}{\boldsymbol{kinetic\ pressure}}$$

Darcy-Weisbach Equation, is applied for any flow statues (Laminar or turbulent).

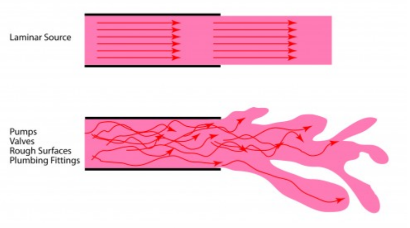
- It is clear that if we want to calculate the value of ***head loss (h_f)*** we must first find the value of ***f.***

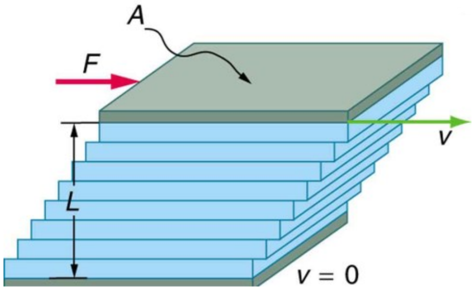
<u>What is the value of *f*?</u>

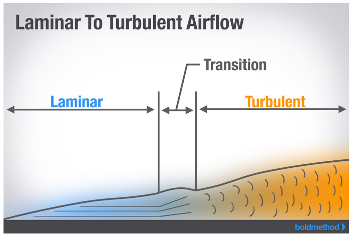
- Before we can select the value of ***f*** , we first of all must defines the regime of flow whether is ***<u>Laminar or turbulent flow</u>***

Laminar flow:

- *Laminar flow* is a flow regime in which fluid motion is smooth, the flow occurs in layers (laminae), and the
- Mixing between layers occurs by molecular diffusion, a process that is much slower than turbulent mixing.





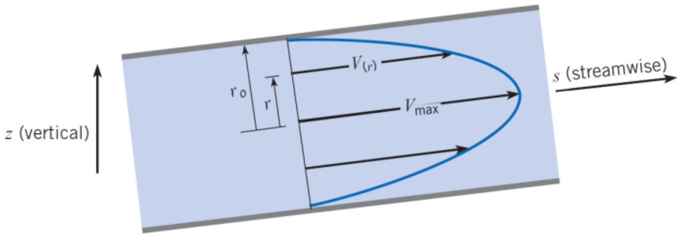


- Reynolds number, **Re** (dimensionless group), is the index that used to distinguish between whether the flow is laminar or turbulent:

$$Re = \frac{\rho VD}{\mu} = \frac{inertia}{viscosity}$$ **(Reynolds Number, 1883)**

Reynolds number for various flow regimes	
Flow Regime	**Reynolds Number**
Laminar	< 2000
Transitional	2000-4000
Turbulent	> 4000

- Laminar flow occurs when Re ≤ 2000. Laminar flow in a round tube is called *Poiseuille* flow or *Hagen-Poiseuille* flow in honor of pioneering researchers who studied low-speed flows in the 1840s.



* For **laminar flow only** in pipes, shear stresses through flow is related by Newton law of viscosity

$$\tau = \mu \frac{dV}{dy}$$

where y is the distance from the pipe wall. Change variables by letting $y = r_0 - r$, where r_0 is pipe radius and r is the radial coordinate. Next, use the chain rule of calculus:

$$\tau = \mu \frac{dV}{dy} = \mu \frac{dV}{dr}\frac{dr}{dy} = -\mu \frac{dV}{dr}$$

From eq(1) with r=D/2

$$h_f = \Delta H = \frac{4\tau_w L}{\gamma D} = \frac{\tau_w 2L}{\gamma r} = -\mu \frac{dv}{dr} 2 \frac{L}{r\gamma}$$

$$dv = -\frac{h_f \gamma}{2\mu L} r dr$$

By integral with boundary condition of $\boldsymbol{r = 0, V = V_{max}}$

$$V = V_{\max} - \frac{h_f \gamma}{4\mu L} r^2$$ (**Parabolic velocity profile**)

From this equation we find that (at $\boldsymbol{r = r_o = \frac{D}{2}, V = 0}$):

$$V_{\max} = \frac{h_f \gamma}{16\mu L} D^2$$

$$the\ mean\ velocity = V_{mean} = \frac{Q}{A}$$

We find discharge by integration of VdA overall the section area and we arrived to following result:

$$V_{mean} = \frac{1}{2} V_{max}$$

$$V_{mean} = \frac{h_f \gamma}{32\mu L} D^2$$

$$\Rightarrow h_f = 32 \frac{\mu L}{\gamma D^2} V$$ (**Hagen-Poisseuille equation, 1839,40**)

By comparing this equation with Darcy equation, we find that:

$$\Rightarrow h_f = f \frac{L}{D} \frac{V^2}{2g} = 32 \frac{\mu L}{\gamma D^2} V$$

and if $Re = \frac{\rho V D}{\mu}$

Then

$$\boldsymbol{f = \frac{64}{Re}}$$

Re is the Reynolds number based on D, not L!, thus for **<u>horizontal pipes (Δz = 0):</u>**

$$\Delta P = \Delta H \times \rho g = \frac{64}{\mathrm{Re}} \frac{\rho V^2}{2} \frac{L}{D} = \frac{64\mu}{\rho V D} \frac{\rho V^2}{2} \frac{L}{D} = \frac{32\mu V L}{D^2}$$

- Sometimes it's more convenient to deal with volume flow rate (Q) rather than velocity (V). Thus we can write one last relation:
- $$hf = \frac{128. L. Q. \mu}{\pi. \gamma. D^4}$$ (**laminar flow only in horizontal pipes!**)

Note the significance of this result: if you double the flow rate **Q** or the length of the pipe **L**, the pressure drop doubles (makes sense.) Also, for a given flow rate Q, if you double the diameter of the tube, the pressure drop decreases by a factor of 16! So use a little bigger pipe in your plumbing design!

(**EXAMPLE** : (EX. 10.2 HEAD LOSS FOR LAMINAR FLOW

Oil (S = 0.85) with a kinematic viscosity of 6×10^{-4} m^2/s flows in a 15 cm pipe at a rate of 0.020 m^3/s. What is the head loss per 100 m length of pipe?

Problem Definition

1. Oil is flowing in a pipe at a flow rate of Q = 0.02 m^3/s.
2. Pipe diameter is D = 0.15 m.

Find: Head loss (in meters) for a pipe length of 100 m.

. **Properties:** Oil: S = 0.85, ν = 6×10^{-4} m^2/s

Solution

1. Mean velocity

$$V = \frac{Q}{A} = \frac{0.020\ \text{m}^3/\text{s}}{(\pi D^2)/4} = \frac{0.020\ \text{m}^3/\text{s}}{\pi((0.15\ \text{m})^2/4)} = 1.13\ \text{m/s}$$

2. Reynolds number

$$\text{Re} = \frac{VD}{\nu} = \frac{(1.13\ \text{m/s})(0.15\ \text{m})}{6 \times 10^{-4}\ \text{m}^2/\text{s}} = 283$$

3. Since Re < 2000, the flow is laminar.
4. Head loss (laminar flow).

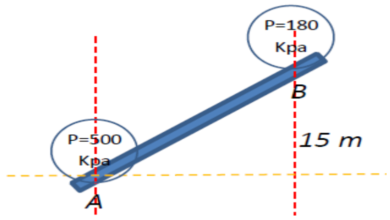
$$h_f = \frac{32\mu LV}{\gamma D^2} = \frac{32\rho\nu LV}{\rho g D^2} = \frac{32\nu LV}{gD^2}$$
$$= \frac{32(6 \times 10^{-4}\ \text{m}^2/\text{s})(100\ \text{m})(1.13\ \text{m/s})}{(9.81\ \text{m/s}^2)(0.15\ \text{m})^2} = \boxed{9.83\ \text{m}}$$

Review: Tip! An alternative way to calculate head loss for laminar flow is to use the Darcy-Weisbach equation as follows:

$$f = \frac{64}{\text{Re}} = \frac{64}{283} = 0.226$$
$$h_f = f\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right) = 0.226\left(\frac{100\ \text{m}}{0.15\ \text{m}}\right)\left(\frac{(1.13\ \text{m/s})^2}{2 \times 9.81\ \text{m/s}^2}\right)$$
$$= 9.83\ \text{m}$$

Example: SAE 30 oil at 20°C flows in the 3-cm-diameter pipe in Fig. which slopes at 37°. For the pressure measurements shown, determine (a) whether the flow is up or down and (b) the flow rate in m^3/h

ρ=891 Kg/m3 μ=0.29 Pa.s



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$$HGL_B = \frac{p_B}{\rho g} + z_B = \frac{180000}{891(9.81)} + 15 = 35.6 \text{ m}; \quad HGL_A = \frac{500000}{891(9.81)} + 0 = 57.2 \text{ m}$$

$$\text{Since} \quad HGL_A > HGL_B \text{ the } \textbf{flow is } \textbf{\textit{up}} \quad \textit{Ans.}(a)$$

The head loss is the difference between hydraulic grade levels:

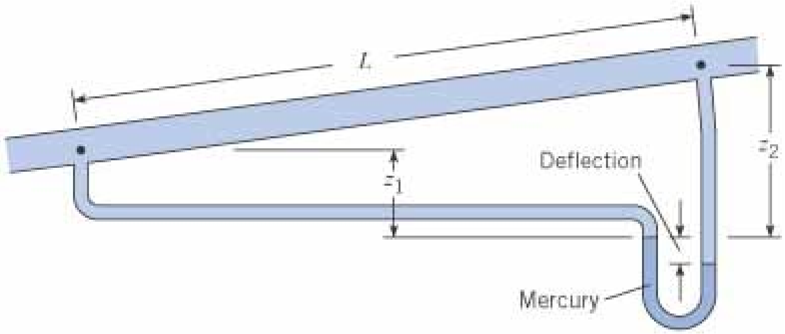
$$L = \frac{15}{Sin(37)} = 24.95 \approx 25m$$

$$h_f = 57.2 - 35.6 = 21.6 \text{ m} = \frac{128\mu LQ}{\pi\rho g d^4} = \frac{128(0.29)(25)Q}{\pi(891)(9.81)(0.03)^4}$$

$$\text{Solve for} \quad Q = 0.000518 \text{ m}^3/\text{s} \approx \mathbf{1.86 \ m^3/h} \quad \textit{Ans.} \text{ (b)}$$

$$\text{Finally, check} \quad Re = 4\rho Q/(\pi\mu d) \approx 68 \text{ (OK, laminar flow).}$$

Example The velocity of oil (S = 0.8) through the 5 cm smooth pipe is 1.2 m/s. were L = 12 m, z_1 = 1 m, z_2 = 2 m, and the manometer deflection is 10 cm. Determine the flow direction, the resistance coefficient f, whether the flow is laminar or turbulent, and the viscosity of the oil.



Solution:

Based on the deflection on the manometer, the piezometric head on the right side of the pipe is larger than that on the left side. Since the velocity at 1 and 2 is the same, the energy at location 2 is higher than the energy at location 1. Since the a fluid will move from a location of high energy to a location of low energy, the flow is downward (from right to left).

From energy princeles:

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_f$$

$$h_f = \frac{\Delta p}{\gamma} + \Delta z = f \frac{L}{D} \frac{V^2}{2g} \tag{1}$$

Manometer equation

$$p_2 + (2\,\mathrm{m})\,\gamma_{\mathrm{oil}} + (0.1\,\mathrm{m})\,\gamma_{\mathrm{oil}} - (0.1\,\mathrm{m})\,\gamma_{\mathrm{Hg}} - (1\,\mathrm{m})\,\gamma_{\mathrm{oil}} = p_1$$

Algebra gives

$$\begin{aligned}
\frac{p_2 - p_1}{\gamma_{\mathrm{oil}}} &= -(2\,\mathrm{m}) - (0.1\,\mathrm{m}) + (0.1\,\mathrm{m})\frac{\gamma_{\mathrm{Hg}}}{\gamma_{\mathrm{oil}}} + (1\,\mathrm{m}) \\
&= -(1\,\mathrm{m}) + (0.1\,\mathrm{m})\left(\frac{S_{\mathrm{Hg}}}{S_{\mathrm{oil}}} - 1\right) \\
&= -(1\,\mathrm{m}) + (0.1\,\mathrm{m})\left(\frac{13.6}{0.8} - 1\right) \\
\frac{p_2 - p_1}{\gamma_{\mathrm{oil}}} &= 0.6\,\mathrm{m} \qquad (2)
\end{aligned}$$

Substituting Eq. (2) into (1) gives

$$\begin{aligned}
(0.6\,\mathrm{m}) &= (-1\,\mathrm{m}) + f\frac{L}{D}\frac{V^2}{2g} \\
&\text{or} \\
f &= 1.6\left(\frac{D}{L}\right)\left(\frac{2g}{V^2}\right) \\
&= 1.6\left(\frac{0.05}{12}\right)\left(\frac{2 \times 9.81}{1.2^2}\right) \\
&\boxed{f = 0.0908}
\end{aligned}$$

$$\begin{aligned}
f &= \frac{64}{\mathrm{Re}} \\
0.0908 &= \frac{64\mu}{\rho V D} \\
&\text{or} \\
\mu &= \frac{0.0908\rho V D}{64} \\
&= \frac{0.0908 \times (0.8 \times 1000) \times 1.2 \times 0.05}{64} \\
&= \boxed{0.068 \quad \mathrm{N \cdot s/m^2}}
\end{aligned}$$

$$\begin{aligned}
\mathrm{Re} &= \frac{V D \rho}{\mu} \\
&= \frac{1.2 \times 0.05 \times (0.8 \times 1000)}{0.068} \\
&= 706
\end{aligned}$$

Thus, flow is $\boxed{\text{laminar.}}$

Example

An oil with $\rho = 900\ kg/m^3$ and $\nu = 0.0002\ m^2/s$ flows upward through an inclined pipe as shown in Fig. 9-1. Assuming steady laminar flow, (*a*) verify that the flow is up and find the (*b*) head loss between section 1 and section 2, (*c*) flow rate, (*d*) velocity, and (*e*) Reynolds number. Is the flow really laminar?

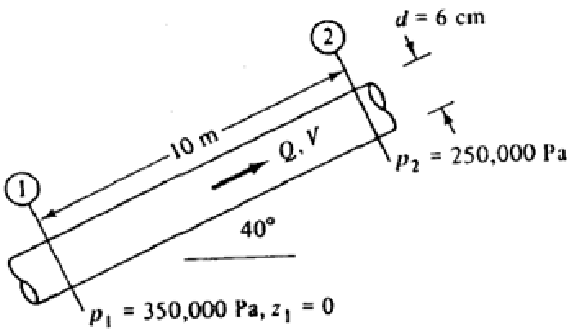


Fig. 9-1

solusion

(*a*) $$HGL = z + p/\rho g \qquad HGL_1 = 0 + 350\,000/[(900)(9.807)] = 39.65\ m$$

$$HGL_2 = (10)(\sin 40°) + 250\,000/[(900)(9.807)] = 34.75\ m$$

Since $HGL_1 > HGL_2$, the flow is upward.

(*b*) $$h_f = HGL_1 - HGL_2 = 39.65 - 34.75 = 4.90\ m$$

(*c*) $$\mu = \rho\nu = (900)(0.0002) = 0.180\ kg/(m \cdot s)$$

$$Q = \frac{\pi \rho g d^4 h_f}{128 \mu L} = \frac{(\pi)(900)(9.807)(\frac{6}{100})^4(4.90)}{(128)(0.180)(10)} = 0.00764\ m^3/s$$

(*d*) $$v = Q/A = 0.00764/[(\pi)(\tfrac{6}{100})^2/4] = 2.70\ m/s$$

(*e*) $$N_R = dv/\nu = (\tfrac{6}{100})(2.70)/0.0002 = 810$$

This value of N_R is well within the laminar range; hence, the flow is most likely laminar.

Example

How much power is lost per kilometer of length when a viscous fluid ($\mu = 0.20\ Pa \cdot s$) flows in a 200-mm-diameter pipeline at 1.00 L/s? The fluid has a density of $840\ kg/m^3$.

solusion

$$v = Q/A = (1.00 \times 10^{-3})/[(\pi)(0.200)^2/4] = 0.03183\ m/s$$

$$N_R = \rho dv/\mu = (840)(0.200)(0.03183)/0.20 = 26.74$$

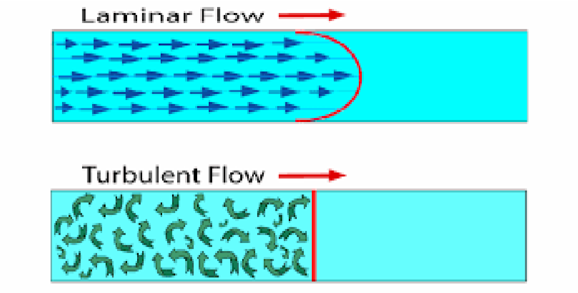
$$h_f/L = (f)(1/d)(v^2/2g)$$

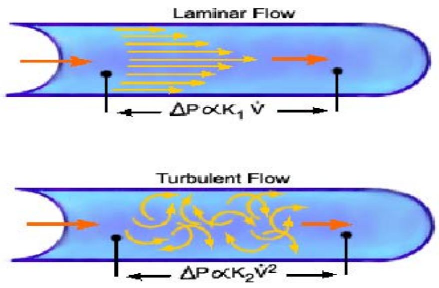
Since $N_R < 2000$, the flow is laminar and $f = 64/N_R = 64/26.74 = 2.393$, $h_f/L = 2.393[1/(0.200)]\{0.03183^2/[(2)(9.807)]\} = 0.0006180\ m$, $P/L = Q\gamma h_f/L = Q\rho g h_f/L = (1.00 \times 10^{-3})(840)(9.807)(0.0006180) = 0.00509\ W/m = 5.09\ W/km$.

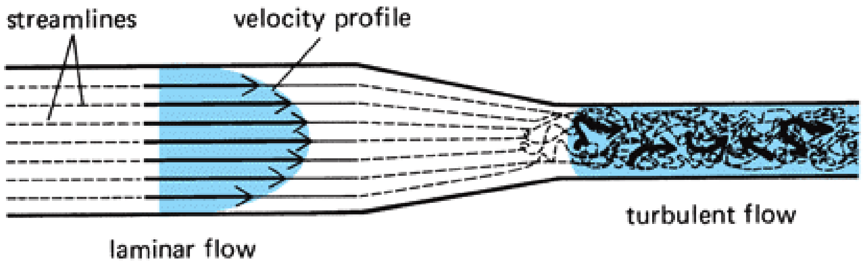
Lec:8

Turbulent flow

*For **turbulent flow**, the friction factor depends not only on **Re** but also the roughness of the pipe wall, which is characterized by a ***roughness factor* = ε/d**, where ε is a measure of the roughness (i.e. height of the bumps on the wall) and **d** is (as always) the pipe diameter. The combined effects of roughness and Re are presented in terms of the ***Moody chart* (1944)** (Re, ε/D) still needs to be determined. For laminar flow, there is an exact solution for *f* since laminar pipe flow has an exact solution. For turbulent flow, approximate solution for *f* using log-law as per Moody diagram and discussed late or other approximations.







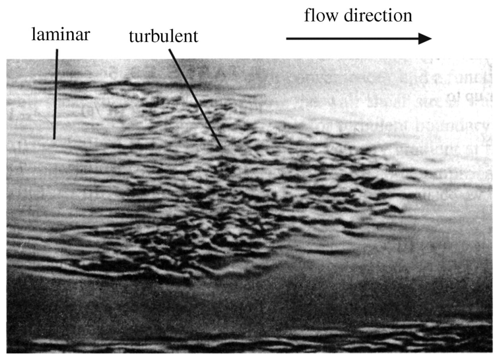
1) **Blasius Equation** $f = \dfrac{0.316}{Re^{0.25}}$ $Re \le 10^5$

2) **Burka Equation** $f = \dfrac{0.21}{Re^{0.21}}$

3) **Von Karman-Prandtl Equations**

$$\frac{1}{\sqrt{f}} = 2 \cdot log\left(\frac{Re \cdot \sqrt{f}}{2.51}\right)$$

$$\frac{1}{\sqrt{f}} = 2 \cdot log\left(\frac{3.7 \cdot D}{\varepsilon}\right)$$



4) **Barr Equation** $\dfrac{1}{\sqrt{f}} = -2 \cdot log\left(\dfrac{\varepsilon}{3.7 \cdot D} + \dfrac{5.1286}{Re^{0.89}}\right)$

5) **Colebrook-White Equation, 1939** $\dfrac{1}{\sqrt{f}} = -2 \cdot log\left(\dfrac{\varepsilon}{3.7 \cdot D} + \dfrac{2.51}{Re \cdot \sqrt{f}}\right)$

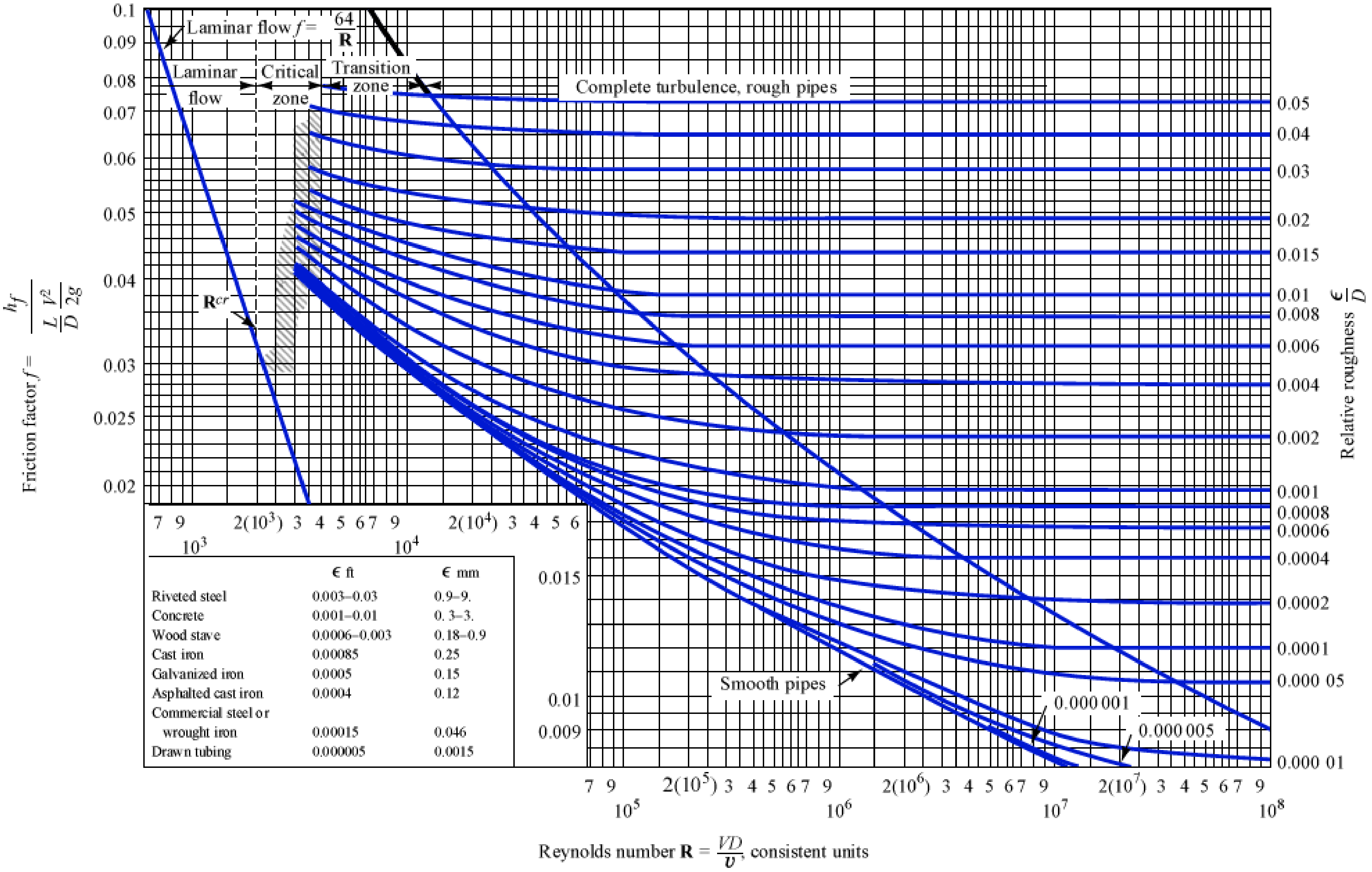
6) Swamee Equation $f = \dfrac{0.25}{\left[log\left(\dfrac{\varepsilon}{3.7 \cdot D} + \dfrac{5.74}{Re^{0.9}} \right) \right]^2}$ $3 \times 10^3 \le Re \le 3 \times 10^8$ $10^{-6} \le \dfrac{\varepsilon}{D} \le 2 \times 10^{-2}$

Moody Diagram

Colebrook work by acquiring data for commercial pipes and then developing an empirical equation, called the Colebrook-White formula, for the friction factor. Moody used the Colebrook-White formula to generate a design chart similar to that shown in Fig. 6.4. This chart is now known as the *Moody diagram* for commercial pipes.

In the Moody diagram, the variable *e* denote the *relative roughness.* In the Moody diagram, the abscissa is the Reynolds number *Re*, and the ordinate is the resistance coefficient *f*. To find *f*, given *Re* and *e/D* one goes to the right to find the correct elative roughness curve. Then one looks at the bottom of the chart to find the given value of *Re* and, with this value of Re, moves vertically upward until the given curve is reached. Finally, from this point one move horizontally to the left scale to read the value of *f*. If the curve for the given value of is not plotted, then one simply finds the proper position on the graph by interpolation between the curves that bracket the given *e/D*.

By using the Colebrook-White formula, Swamee and Jain developed an explicit equation for friction factor:



Laminar flow $f = \frac{64}{\mathbf{R}}$
Laminar flow
Critical zone
Transition zone
Complete turbulence, rough pipes
$\mathbf{R}_{cr}$
Smooth pipes
Friction factor $f = \frac{h_f}{\frac{L}{D}\frac{V^2}{2g}}$
Relative roughness $\frac{\epsilon}{D}$
Reynolds number $\mathbf{R} = \frac{VD}{\nu}$, consistent units
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.025
0.02
0.015
0.01
0.009
0.05
0.04
0.03
0.02
0.015
0.01
0.008
0.006
0.004
0.002
0.001
0.0008
0.0006
0.0004
0.0002
0.0001
0.000 05
0.000 01
0.000 001
0.000 005
7 9 10^3 $2(10^3)$ 3 4 5 6 7 9 10^4 $2(10^4)$ 3 4 5 6
7 9 10^5 $2(10^5)$ 3 4 5 6 7 9 10^6 $2(10^6)$ 3 4 5 6 7 9 10^7 $2(10^7)$ 3 4 5 6 7 9 10^8
€ ft — € mm
Riveted steel 0.003–0.03 0.9–9.
Concrete 0.001–0.01 0. 3–3.
Wood stave 0.0006–0.003 0.18–0.9
Cast iron 0.00085 0.25
Galvanized iron 0.0005 0.15
Asphalted cast iron 0.0004 0.12
Commercial steel or wrought iron 0.00015 0.046
Drawn tubing 0.000005 0.0015

Simple Pipe Problems

Pipe Flow Problem Types

Variable	Type I	Type II	Type III
a. Fluid Density Viscosity	 Given Given	 Given Given	 Given Given
b. Pipe Diameter Length Roughness	 Given Given Given	 Given Given Given	 **Determine** Given Given
c. Flow Flowrate or Average Velocity	Given	**Determine**	Given
d. Pressure Pressure Drop or Head Loss	**Determine**	Given	Given

Type I

Example1 Determine the head lost to friction when water at 15 ∘C, ν = 1.14 $mm^2 \cdot s^{-1}$, flows through 300 m of 150 mm diameter galvanized steel pipe (ε = 0.15 mm) at 50 $L \cdot s^{-1}$.

Solution

$V = 50 \times 10^{-3} m^3 \cdot s^{-1}/((\pi/4)(0.15)^2 m^2) = 2.83\ m\cdot s^{-1}$

$Re = VD/\nu = 2.83\ m\cdot s^{-1} \times 0.15\ m/(1.14 \times 10^{-6}\ m^2 \cdot s^{-1}) = 3.72 \times 10^5$

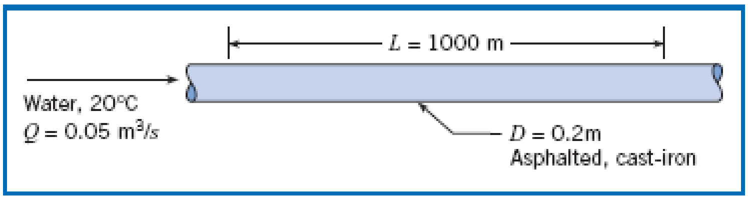
For galvanized steel, ε /D = 0.001

From Moody Diagram f = 0.0208, so

$$hf = 0.0208x\frac{300}{0.15}x\frac{2.83^2}{2x9.81} = 16.98m$$

Example 2 :Case 1

Water (T = 20°C) flows at a rate of 0.05 m^3/s in a 20 cm asphalted cast-iron pipe. What is the head loss per kilometer of pipe?



ϵ = 0.12mm , $\upsilon = 1x10^{-6}\ m^2/s$

Solution

1. Mean velocity

$$V = \frac{Q}{A} = \frac{0.05\ \text{m}^3/\text{s}}{(\pi/4)(0.02\ \text{m})^2} = 1.59\ \text{m/s}$$

2. Reynolds number

$$\text{Re} = \frac{VD}{\nu} = \frac{(1.59\ \text{m/s})(0.20\ \text{m})}{10^{-6}\ \text{m}^2/\text{s}} = 3.18 \times 10^5$$

3. Resistance coefficient

- Equivalent sand roughness $e = 0.12$ mm
- Relative roughness:
  $e/D = (0.00012\ \text{m})/(0.2\ \text{m}) = 0.0006$
- Look up f on the Moody diagram for $\text{Re} = 3.18 \times 10^5$ and $e/D = 0.0006$:

$$f = 0.019$$

4. Darcy-Weisbach equation

$$h_f = f\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right) = 0.019\left(\frac{1000\ \text{m}}{0.20\ \text{m}}\right)\left(\frac{1.59^2\ \text{m}^2/\text{s}^2}{2(9.81\ \text{m/s}^2)}\right)$$

$$= \boxed{12.2\ \text{m}}$$

Example 3 :Case 1

Gasoline at 20 °C is pumped at 0.2 m^3/s through 16 km of 180-mm-diameter cast iron pipe. Compute the power required if the pumps are 75 percent efficient. *ρ=719 kg/m^3 , ϵ = 0.26mm , μ=2.92x10^{-4} Pa.s*

solusion

$$h_f = (f)(L/d)(v^2/2g)$$

$$v = Q/A = 0.2/[(\pi)(0.180)^2/4] = 7.860\ \text{m/s}$$

$$N_R = \rho d v/\mu = (719)(0.180)(7.860)/(2.92 \times 10^{-4}) = 3.48 \times 10^6$$

$$\epsilon/d = 0.00026/(0.180) = 0.00144$$

From Fig. A-5, $f = 0.0216$.

$$h_f = 0.0216[(16)(1000)/(0.180)]\{7.860^2/[(2)(9.807)]\} = 6048\ \text{m}$$

$$P = \rho g Q h_f/\eta = (719)(9.807)(0.2)(6048)/0.75$$

$$= 11.37 \times 10^6\ \text{W} \quad \text{or} \quad 11.37\ \text{MW}$$

Type II

Example 1

A plastic pipe, 10 km long and 300 mm diameter, conveys water from a reservoir (water level 850 m above datum) to a water treatment plant (inlet level 700 m above datum). Assuming the reservoir remains full, estimate the discharge using the following methods:

1. the Moody diagram;

Take the kinematic viscosity to be 1.13×10^{-6} m²/s. $\epsilon = 0.03mm$

Solution:

Given: hL= 850-700 = 150 m.

L=10000m , D=0.3m , $\upsilon = 1.13x10^{-6}\ m^2/s$, $\epsilon = 0.03mm$

$$hf = f x \frac{L}{D} x \frac{v^2}{2g} \quad \rightarrow v = \sqrt{\frac{2g.hl.\frac{D}{L}}{f}}$$

$$\rightarrow v = \sqrt{\frac{0.08829}{f}}$$

€/D = (0.00003/0.3) = 0.0001

Assume f =0.02 → V= 2.1 m/s → Re = 2.1x0.3/($1.13x10^{-6}$) = 5.5 x10^5

→ from mody Digram f = 0.015

Arrange the solution by table as follow:

F	V	Re	€/D	New (f)
0.02	2.1	5.5 x10^5	0.0001	0.015
0.015	2.42	6.4 x10^5	0.0001	0.014
0.014	2.51	6.6 x10^5	0.0001	0.014

Use f = 0.014 → $150 = 0.014x\frac{10000}{0.3}x\frac{v^2}{19.62}$ → V = 2.51 m/s

Then Q=V.A = 2.51 (π/4x0.3^2) =0.1775 m^3/s

Type II

Example: 2

Oil, with ρ =950 kg/m^3 and ν = 2 E-5 m^2/s, flows through a 30-cm-diameter pipe 100 m long with a head loss of 8 m. The roughness ratio is ε/d = 0.0002. Find the average velocity and flow rate.

Solution:

By definition, the friction factor is known except for V:

$$f = h_f \frac{d}{L}\frac{2g}{V^2} = (8\text{ m})\left(\frac{0.3\text{ m}}{100\text{ m}}\right)\left[\frac{2(9.81\text{ m/s}^2)}{V^2}\right] \quad \text{or} \quad fV^2 \approx 0.471 \quad \text{(SI units)}$$

To get started, we only need to guess f, compute $V = \sqrt{0.471/f}$, then get Re_d, compute a better f from the Moody chart, and repeat. The process converges fairly rapidly. A good first guess is the "fully rough" value for $\epsilon/d = 0.0002$, or $f \approx 0.014$ from Fig. 6.13. The iteration would be as follows:

Guess $f \approx 0.014$, then $V = \sqrt{0.471/0.014} = 5.80$ m/s and $Re_d = Vd/\nu \approx 87{,}000$. At $Re_d = 87{,}000$ and $\epsilon/d = 0.0002$, compute $f_{new} \approx 0.0195$ [Eq. (6.64)].

New $f \approx 0.0195$, $V = \sqrt{0.481/0.0195} = 4.91$ m/s and $Re_d = Vd/\nu = 73{,}700$. At $Re_d = 73{,}700$ and $\epsilon/d = 0.0002$, compute $f_{new} \approx 0.0201$ [Eq. (6.64)].

Better $f \approx 0.0201$, $V = \sqrt{0.471/0.0201} = 4.84$ m/s and $Re_d \approx 72{,}600$. At $Re_d = 72{,}600$ and $\epsilon/d = 0.0002$, compute $f_{new} \approx 0.0201$ [Eq. (6.64)].

We have converged to three significant figures. Thus our iterative solution is

$$V = 4.84 \text{ m/s}$$

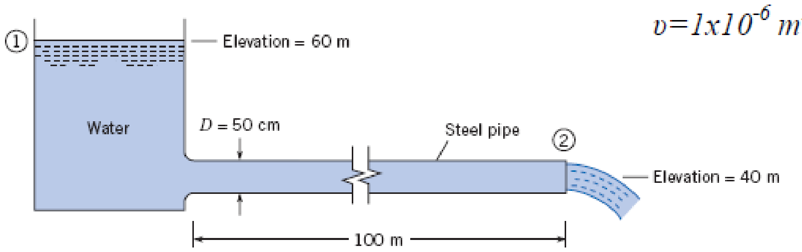
$$Q = V\left(\frac{\pi}{4}\right)d^2 = (4.84)\left(\frac{\pi}{4}\right)(0.3)^2 \approx 0.342 \text{ m}^3/\text{s} \qquad \textit{Ans.}$$

f	v	Re	New f
0.014	5.8	87000	0.0195
0.0195	4.91	73700	0.0201
0.0201	4.84	72600	0.0201

Example 3 :Case 2

Water (T = 20°C) flows from a tank through a 50 cm diameter steel pipe. Determine the discharge of water.

$\upsilon = 1x10^{-6}\ m^2/s$, $\epsilon = 0.046mm$



Solution:

Given: hL= 60-40 = 20 m.

L=100m , D=0.5m , $\upsilon = 1x10^{-6}\ m^2/s$, $\epsilon = 0.046mm$

$$hf = f x \frac{L}{D} x \frac{v^2}{2g} \quad \rightarrow v = \sqrt{\frac{2g.hl.\frac{D}{L}}{f}} \quad \rightarrow v = \sqrt{\frac{1.962}{f}}$$

€/D = (0.000046/0.5) = 0.000092

Assume f =0.02 → V= 9.9 m/s → Re = 9.9x0.5/(*1x10^-6^*) = 4.9 x10^6

→ from mody Digram f = 0.013

Arrange the solution by table as follow:

F	V	Re	€/D	New (f)
0.02	9.9	4.9 x10^6	0.00009	0.013
0.013	12.28	6.1 x10^6	0.00009	0.014
0.014	11.83	5.9 x10^6	0.00009	0.014

Use f = 0.014 → $20 = 0.014x\frac{100}{0.5}x\frac{v^2}{19.62}$ → V = 11.4 m/s

Then Q=V.A = 11.4 (π/4x0.5^2) = 2.23 m^3/s

Example 4 :Case 2

The head loss per kilometer of 20 cm asphalted cast-iron pipe is 12.2 m. What is the flow rate of water through the pipe?

$\epsilon = 0.12mm$, $\upsilon=1x10^{-6}\ m^2/s$

Solution:

Given: hL= 12.2 m.

L=1000m , D=0.2m , $\upsilon=1x10^{-6}\ m^2/s$, $\epsilon = 0.12mm$

$$\rightarrow v = \sqrt{\frac{2g.hl.\frac{D}{L}}{f}} \qquad \rightarrow v = \sqrt{\frac{0.047}{f}}$$

€/D = (0.00012/0.2) = 0.0006

Assume f =0.02 → V= 1.532 m/s → Re = 1.532x0.2/($1x10^{-6}$) = 3.0 x10^5

→ from mody Digram f = 0.018

Arrange the solution by table as follow:

F	V	Re	€/D	New (f)
0.02	1.532	3.0 x10^5	0.0006	0.018
0.018	1.615	3.2 x10^5	0.0006	0.017
0.017	1.66	3.3 x10^5	0.0006	0.017

Use f = 0.017 → $12.2 = 0.017x\frac{1000}{0.2}x\frac{v^2}{19.62}$ → V = 1.66 m/s

Then Q=V.A = 1.66 (π/4x0.2^2) = 0.0521 m^3/s

Type III

Example: 1

Work previous Example backward, assuming that Q = 0.342 m^3/s and ε = 0.06 mm are known but that d (30 cm) is unknown. Recall L = 100 m, ρ = 950 kg/m^3, ν = 2 E-5 m^2/s, and h_f = 8m

Solution:

First write the diameter in terms of the friction factor: $f = h_f \frac{d}{L}\frac{2g}{V^2} = h_f . \frac{d}{l} . \frac{2g}{(Q/\frac{\pi}{4}.d^2)^2} = h_f . \frac{\pi^2}{16}\frac{d^5}{l} . \frac{2g}{Q^2}$

$$f = \frac{\pi^2}{8}\frac{(9.81\ \text{m/s}^2)(8\ \text{m})d^5}{(100\ \text{m})(0.342\ \text{m}^3/\text{s})^2} = 8.28d^5 \qquad \text{or} \qquad d \approx 0.655f^{1/5} \qquad (1)$$

in SI units. Also write the Reynolds number and roughness ratio in terms of the diameter:

$$\text{Re}_d = \frac{4(0.342\ \text{m}^3/\text{s})}{\pi(2\ \text{E-5}\ \text{m}^2/\text{s})d} = \frac{21{,}800}{d} \qquad (2)$$

$$\frac{\epsilon}{d} = \frac{6\ \text{E-5 m}}{d} \qquad (3)$$

Guess f, compute d from (1), then compute Re_d from (2) and ϵ/d from (3), and compute a better f from the Moody chart or Eq. (6.64). Repeat until (fairly rapid) convergence. Having no initial estimate for f, the writer guesses $f \approx 0.03$ (about in the middle of the turbulent portion of the Moody chart). The following calculations result:

$$f \approx 0.03 \qquad d \approx 0.655(0.03)^{1/5} \approx 0.325\ \text{m}$$

$$\text{Re}_d \approx \frac{21{,}800}{0.325} \approx 67{,}000 \qquad \frac{\epsilon}{d} \approx 1.85\ \text{E-4}$$

$$f_{\text{new}} \approx 0.0203 \qquad \text{then} \qquad d_{\text{new}} \approx 0.301\ \text{m}$$

$$\text{Re}_{d,\text{new}} \approx 72{,}500 \qquad \frac{\epsilon}{d} \approx 2.0\ \text{E-4}$$

$$f_{\text{better}} \approx 0.0201 \qquad \text{and} \qquad d = 0.300\ \text{m} \qquad \textit{Ans.}$$

F	D	Re	€/D	New (f)
0.03	0.325	6.7 x10^4	0.000185	0.020
0.020	1.30	7.2 x10^4	0.0002	0.020

Use f = 0.020 →D=0.3m /s

Example 2 :Case 3

What size of asphalted cast-iron pipe is required to carry water (60°F) at a discharge of 3 cfs and with a head loss of 4 ft per 1000 ft of pipe?

1. Iteration 1
   - Guess $f = 0.015$.
   - Solve for diameter using eq. $D^5 = \frac{fLQ^2}{0.785^2(2gh_f)}$

$$D^5 = \frac{0.015(1000\ \text{ft})(3\ \text{ft}^3/\text{s})^2}{0.785^2(64.4\ \text{ft}/\text{s}^2)(4\ \text{ft})} = 0.852\ \text{ft}^5$$

$$D = 0.968\ \text{ft}$$

   - Find parameters needed for calculating f:

$$V = \frac{Q}{A} = \frac{3\ \text{ft}^3/\text{s}}{(\pi/4)(0.968^2\ \text{ft}^2)} = 4.08\ \text{ft}/\text{s}$$

$$\text{Re} = \frac{VD}{\nu} = \frac{(4.08\ \text{ft}/\text{s})(0.968\ \text{ft})}{1.22 \times 10^{-5}\ \text{ft}^2/\text{s}} = 3.26 \times 10^5$$

$$e/D = 0.005/(0.97 \times 12) = 0.00043$$

   - Calculate f using Eq. 6.25 or Moody diagram : $f = 0.0178$.

2. In the table below, the first row contains the values from iteration 1. The value of $f = 0.0178$ from iteration 1 is used for the initial value for iteration 2. Notice how the solution has converged by iteration 3.

Iteration #	Initial f	D	V	Re	k_s/D	New f
		(ft)	(ft/s)			
1	0.0150	0.968	4.08	3.26E+05	4.3E−04	0.0178
2	0.0178	1.002	3.81	3.15E+05	4.2E−04	0.0178
3	0.0178	1.001	3.81	3.15E+05	4.2E−04	0.0178
4	0.0178	1.001	3.81	3.15E+05	4.2E−04	0.0178

Specify a pipe with a 12-inch inside diameter.

Example 3 :Case 3

Gasoline is being discharged from a pipe, as shown in Fig. 9-19. The pipe roughness (ϵ) is 0.500 mm, and the pressure at point 1 is 2500 kPa. Find the pipe diameter needed to discharge gasoline at a rate of 0.10 m^3/s. Neglect any minor losses.

$\rho = 719\ kg/m^3$, $\mu = 2.92x10^{-4}$ Pa.s ,

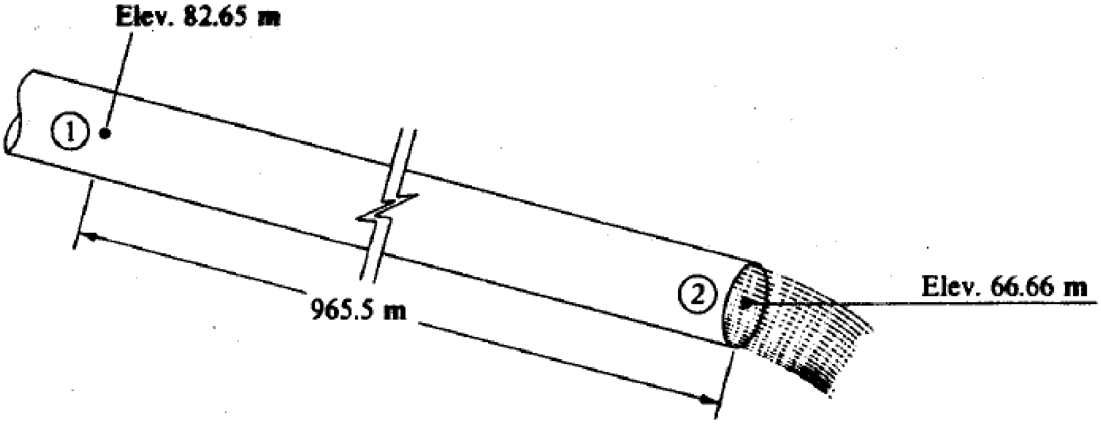


Fig. 9-19

Solution

$$p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L$$

$$h_L = h_f = (f)(L/d)(v^2/2g) = (f)(965.5/d)\{v_2^2/[(2)(9.807)]\} = 49.23fv_2^2/d$$

$$2.500/7.05 + v_1^2/2g + 82.65 = 0 + v_2^2/2g + 66.66 + 49.23fv_2^2/d \qquad v_1^2/2g = v_2^2/2g$$

$$fv_2^2/d = 0.3320 \qquad v_2 = Q/A_2 = 0.10/(\pi d^2/4) = 0.1273/d^2 \qquad (f)(0.1273/d^2)^2/d = 0.3320 \qquad d = (0.04881f)^{1/5}$$

Assume $f = 0.0200$. $d = [(0.04881)(0.0200)]^{1/5} = 0.2500$ m, $v_2 = 0.1273/0.2500^2 = 2.037$ m/s; $N_R = \rho dv/\mu = (719)(0.2500)(2.037)/(2.92 \times 10^{-4}) = 1.25 \times 10^6$. From Table A-9, $\epsilon = 0.00050$ m. $\epsilon/d = 0.00050/0.2500 = 0.0020$. From Fig. A-5, $f = 0.0235$. Evidently, the assumed value of f of 0.0200 was not the correct one. Try a value of f of 0.0235.

$$d = [(0.04881)(0.0235)]^{1/5} = 0.2582\ \text{m} \qquad v = 0.1273/0.2582^2 = 1.909\ \text{m/s}$$

$$N_R = (719)(0.2582)(1.909)/(2.92 \times 10^{-4}) = 1.21 \times 10^6$$

$$\epsilon/d = 0.00050/0.2582 = 0.00194 \qquad f = 0.0235$$

Hence, 0.0235 must be the correct value of f, and $d = 0.2582$ m.

F	D	Re	€/D	New (f)
0.02	0. 250	1.25 x10^6	0.0020	0.023
0.023	0.258	1.21 x10^6	0.0019	0.023

Lec:9

Head Minor Losses

For any pipe system, in addition to the Moody-type friction loss (referred as *Major losses*) computed for the length of pipe, there are additional so-called *minor losses* due to:

1. Pipe entrance or exit
2. Sudden expansion or contraction
3. Bends, elbows, tees, and other fittings
4. Valves, open or partially closed
5. Gradual expansions or contractions

These losses may not be so minor in its effects; e.g., a partially closed valve can cause a greater pressure drop than a long pipe.

Since the flow pattern in fittings and valves is quite complex, the theory is very weak. The losses are commonly measured experimentally and correlated with the pipe flow parameters.
The data, especially for valves, are somewhat dependent upon the particular manufacturer's design, so that the values listed here must be taken as average design estimates
The measured minor loss is usually given as a ratio of the head loss *hm* through the device to the velocity head ($\frac{V^2}{2g}$) of the associated piping system

Head Loss in Conduits and Pipes

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 + h_f + h_{L,\text{fitting}}$$

Minor loss coefficient, $K = \dfrac{h_m}{V^2/2g}$

A single pipe system may have many minor losses. Since all are correlated with $V^2/(2g)$, they can be summed into a single total system loss if the pipe has constant diameter

$$\{\text{Total head loss}\} = \{\text{Pipe head loss}\} + \{\text{Component head loss}\}$$

$$hL = hf + \sum hm = f\frac{L}{D}.\frac{v^2}{2g} + \sum k.\frac{v^2}{2g}$$

$$h_L = h_f + \sum h_m = \frac{V^2}{2g}\left(\frac{fL}{D} + \sum K\right)$$

Equivalent Length or Diameter for Pipes

This method depending on replace the total system of piping and fittings by one : equivalent single straight pipe

With **equivalent length** for **specified diameter** or

With **equivalent diameter** for **specified length**

For any pipe system with variable head losses and discharges :

The equivalent pipe must have unique value for **hf and Q**

1-Replace one straight pipe without minor losses by another equivalent standards pipe:

To replace any pipe with length $\mathbf{L_1}$ and dia. $\mathbf{D_1}$ & **Q1=Q** & **hf**
With equivalent pipe with **dia. =De** at same **Q** and **hf**
Then equivalent pipe will have length=Le and diameter= **De** and with same **hf** and **Q**

Depending on Darcy–Weisbach equation:

$$hf = f\frac{L}{D}.\frac{v^2}{2g} \quad \text{re arrange in term of Q}$$

$$\Rightarrow \quad hf = f.\frac{L}{D^5}.\frac{8Q^2}{g\pi^2}$$

$$\therefore f_e.\frac{L_e}{D_e^5}.\frac{8Q^2}{g\pi^2} = f_1.\frac{L_1}{D_1^5}.\frac{8Q^2}{g\pi^2} \rightarrow \boxed{L_e = .\frac{f_1}{f_e}.\left(\frac{D_e}{D_1}\right)^5 . L_1}$$

$$\& \quad \boxed{D_e = \sqrt[5]{\left(\frac{f_1}{f_e}.\frac{l_e}{l_1}D_1^5\right)}}$$

2-Replace pipe with minor losses for fittings by another equivalent one standards pipe:

To replace any pipe with length $\mathbf{L_1}$ and dia. $\mathbf{D_1}$ and have many minor losses for fittings with **Q1=Q** & **hm**
By one equivalent pipe with **dia. =De** at same **Q** and one **hf**
Then equivalent pipe will have length=**Le** and diameter= **De** and with head-loss = **hf** and **Q**

The minor losses in pipe system due to fittings is **hm**

$$hm = \sum K \cdot \left(\frac{v^2}{2g}\right)$$

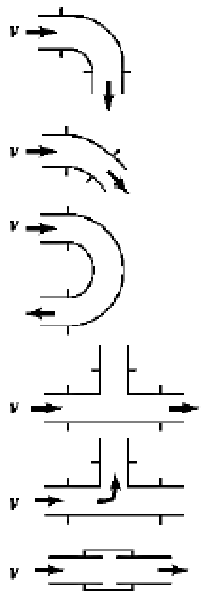
The equivalent pipe will have **hf = hm** for same **Q** or **V**
Then:

$$f_e \frac{L_e}{D_e} \cdot \frac{v^2}{2g} = \sum K \cdot \left(\frac{v^2}{2g}\right)$$

$$\therefore \boxed{L_e = \frac{\sum K}{f_e} \cdot D_e} \quad \& \quad \boxed{D_e = \frac{L_e \cdot f_e}{\sum K}}$$

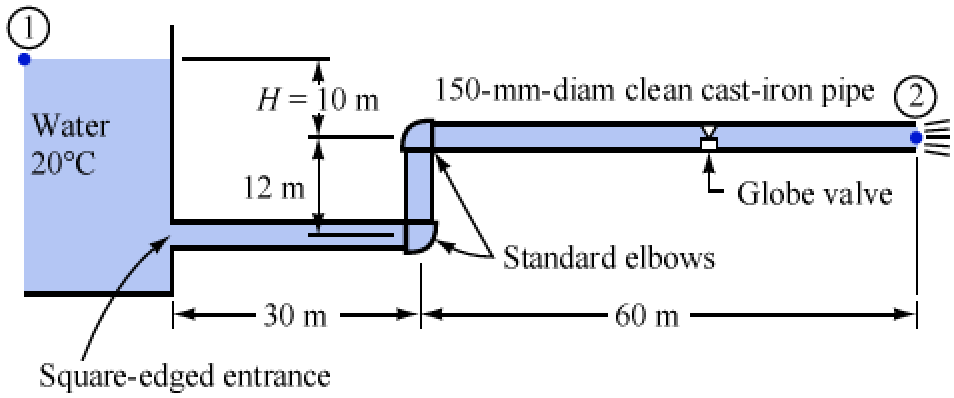
Loss Coefficients for Pipe Components $\left(h_L = K_L \frac{V^2}{2g}\right)$

Component	K_L
a. Elbows	
Regular 90°, flanged	0.3
Regular 90°, threaded	1.5
Long radius 90°, flanged	0.2
Long radius 90°, threaded	0.7
Long radius 45°, flanged	0.2
Regular 45°, threaded	0.4
b. 180° return bends	
180° return bend, flanged	0.2
180° return bend, threaded	1.5
c. Tees	
Line flow, flanged	0.2
Line flow, threaded	0.9
Branch flow, flanged	1.0
Branch flow, threaded	2.0
d. Union, threaded	0.08
*e. Valves	
Globe, fully open	10
Angle, fully open	2
Gate, fully open	0.15
Gate, $\frac{1}{4}$ closed	0.26
Gate, $\frac{1}{2}$ closed	2.1
Gate, $\frac{3}{4}$ closed	17
Swing check, forward flow	2
Swing check, backward flow	∞
Ball valve, fully open	0.05
Ball valve, $\frac{1}{3}$ closed	5.5
Ball valve, $\frac{2}{3}$ closed	210



Example 1 Find the discharge through the pipeline in Fig for $H = 10$ m and determine the head loss H for $Q = 60$ L/s. $\nu = 1.01\ \mu\,\text{m}^2/\text{s}$ $\frac{\epsilon}{D} = 0.0017$

in which the entrance loss coefficient is $\frac{1}{2}$, each elbow is 0.9, and the globe valve is 10.



SOLUTION The energy equation applied between points 1 and 2, including all the losses, can be written as $\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_f + h_{L,\text{fitting}}$

$$H_1 + 0 + 0 = \frac{V_2^2}{2g} + 0 + 0 + \frac{1}{2}\frac{V_2^2}{2g} + f\frac{102\text{ m}}{0.15\text{ m}}\frac{V_2^2}{2g} + 2(0.9)\frac{V_2^2}{2g} + 10\frac{V_2^2}{2g}$$

in which the entrance loss coefficient is $\frac{1}{2}$, each elbow is 0.9, and the globe valve is 10. Then

$$H_1 = \frac{V_2^2}{2g}(13.3 + 680f) \qquad \text{For H1=10} \rightarrow v = \sqrt{\frac{20x9.81}{(13.3+680f)}}$$

When the head is given, this problem is solved as the second type of simple pipe problem. If $f = 0.022$,

$$10 = \frac{V_2^2}{2g}[13.3 + 680(0.022)]$$

and $V_2 = 2.63$ m/s. From Appendix C,

$$\nu = 1.01\ \mu\,\text{m}^2/\text{s} \qquad \frac{\epsilon}{D} = 0.0017 \qquad \mathbf{R} = \frac{(2.63\text{ m/s})(0.15\text{ m})}{1.01\times10^{-6}\text{ m}^2/\text{s}} = 391{,}000$$

From Fi; Moody chart $f = 0.023$. Repeating the procedure gives $V_2 = 2.60$ m/s, $\mathbf{R} = 380{,}000$, and $f = 0.023$. The discharge is

$$Q = V_2A_2 = (2.60\text{ m/s})\frac{\pi}{4}(0.15\text{ m})^2 = 45.9\text{ L/s}$$

F	V	Re	e/D	New F
0.022	2.63	3.91E5	0.0017	0.023
0.023	2.60	3.8E5	0.0017	0.023

For the second part, with Q known, the solution is straightforward:

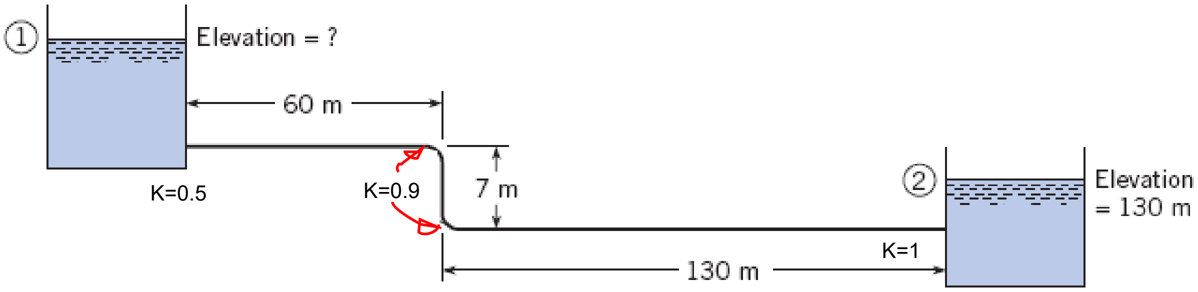
$$V_2 = \frac{Q}{A} = \frac{0.06\text{ m}^3/\text{s}}{(\pi/4)(0.15\text{ m})^2} = 3.40\text{ m/s} \qquad \mathbf{R} = 505{,}000 \qquad f = 0.023$$

and

$$H_1 = \frac{(3.4\text{ m/s})^2}{2(9.806\text{ m/s}^2)}[13.3 + 680(0.023)] = 17.06\text{ m}$$

Example 2 PIPE SYSTEM WITH COMBINED HEAD LOSS

If oil ($\nu = 4 \times 10^{-5}$ m^2/s; $S = 0.9$) flows from the upper to the lower reservoir at a rate of 0.028 m^3/s in the 15 cm smooth pipe, what is the elevation of the oil surface in the upper reservoir?



Solution: Energy equation and term-by-term analysis

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$$

$$0 + 0 + z_1 = 0 + 0 + z_2 + h_L$$

$$z_1 = z_2 + h_L \qquad \text{........(1)}$$

Combined head loss equation

$$h_L = \sum_{\text{pipes}} f\frac{L}{D}\frac{V^2}{2g} + \sum_{\text{components}} K\frac{V^2}{2g}$$

$$h_L = f\frac{L}{D}\frac{V^2}{2g} + \left(2K_b\frac{V^2}{2g} + K_e\frac{V^2}{2g} + K_E\frac{V^2}{2g}\right)$$

$$= \frac{V^2}{2g}\left(f\frac{L}{D} + 2K_b + K_e + K_E\right) \qquad \text{........(2)}$$

Combine eqs. (1) and (2).

$$z_1 = z_2 + \frac{V^2}{2g}\left(f\frac{L}{D} + 2K_b + K_e + K_E\right)$$

- Flow rate $V = \frac{Q}{A} = \frac{(0.028\ \text{m}^3/\text{s})}{(\pi/4)(0.15\ \text{m})^2} = 1.58\ \text{m/s}$
- Reynolds number $\text{Re} = \frac{VD}{\nu} = \frac{1.58\ \text{m/s}(0.15\ \text{m})}{4 \times 10^{-5}\ \text{m}^2/\text{s}} = 5.93 \times 10^3$ Thus, flow is turbulent.
- Swamee-Jain equation 6.25

$$f = \frac{0.25}{\left[\log_{10}\left(\frac{k_s}{3.7D} + \frac{5.74}{\text{Re}^{0.9}}\right)\right]^2} = \frac{0.25}{\left[\log_{10}\left(0 + \frac{5.74}{5930^{0.9}}\right)\right]^2} = 0.036$$

$$z_1 = (130\ \text{m}) + \frac{(1.58\ \text{m/s})^2}{2(9.81)\text{m/s}^2}\left(0.036\frac{(197\ \text{m})}{(0.15\ \text{m})} + 2(0.9) + 0.5 + 1.0\right)$$

$z_1 = 136.4$m

Example 3 can be solve by equivalent length method as below:

$$L_e = \frac{\sum KD}{f} = \frac{3.3 \text{ x } 0.15}{0.036} = 13.75\text{m}$$

$$z_2 = 130 + f\frac{L + L_e}{D}\frac{V_2^2}{2g} = 0.036\ \frac{197+13.75}{0.15}\ \frac{1.58^2}{2\text{x}9.81} = 136.4\ \text{m}$$

Example 4

Two reservoirs containing water at 20 °C are connected by 800 m of 180-mm cast iron pipe, including a sharp entrance, a submerged exit, a gate valve 75 percent open, two 1-m-radius bends, and six regular 90° elbows. If the flow rate is 9 m^3/min, find the difference in reservoir elevations. $\epsilon = 0.00026$

Solution: $p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad h_L = h_f + h_m \qquad h_f = (f)(L/D)(v^2/2g)$

$v = Q/A = (9/60)/[(\pi)(0.180)^2/4] = 5.895\ \text{m/s} \qquad N_R = Dv/\nu = (0.180)(5.895)/(1.02\times10^{-6}) = 1.04\times10^6$

$\epsilon/D = 0.00026/0.180 = 0.00144$ — For sharp entrance, $K_1 = 0.5$. For elbows, $K_5 = (0.27)$. For exit, $K_2 = 1.0$. For gate valve 75 percent open, $K_3 = 0.3$. For bends, $K_4 = (0.15)$

From Fig. A-5, $f = 0.0217$. $h_f = 0.0217[800/0.180]\{5.895^2/[(2)(9.807)]\} = 170.9$ m. For sharp entrance, $K_1 = 0.5$. For exit, $K_2 = 1.0$. For gate valve 75 percent open, $K_3 = 0.3$. For bends, $K_4 = (2)(0.15) = 0.30$. For elbows, $K_5 = (6)(0.27) = 1.62$. $h_m = (0.5 + 1.0 + 0.3 + 0.30 + 1.62)\{5.895^2/[(2)(9.807)]\} = 6.6$ m, $h_L = 170.9 + 6.6 = 177.5$ m, $0 + 0 + z_1 = 0 + 0 + z_2 + 177.5$, $z_1 - z_2 = 177.5$ m.

Example 5

The system in Fig. 9-38 consists of 1000 m of 50-mm cast iron pipe, two 45° and four 90° flanged long-radius elbows, a fully open flanged globe valve, and a sharp exit into a reservoir. What gage pressure is required at point 1 to deliver 5 L/s of water at 20 °C into the reservoir, whose free surface lies 100 m above point 1?

For 45° elbows, $K_1 = (0.20)$ For 90° elbows, $K_2 = (0.30)$

For the open valve, $K_3 = 8.5$. For exit, $K_4 = 1.0$.

$\epsilon = 0.00026$, $\nu = (1.02\times10^{-6})$

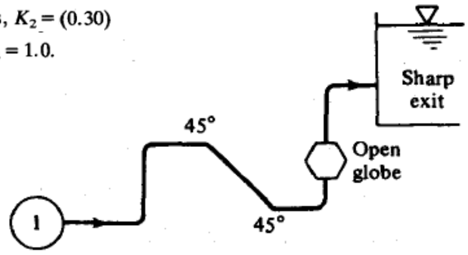


Fig. 9-38

Solution: $p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad h_L = h_f + h_m \qquad h_f = (f)(L/D)(v^2/2g)$

$v = Q/A = 0.005/[(\pi)(0.050)^2/4] = 2.546\ \text{m/s} \qquad N_R = Dv/\nu = (0.050)(2.546)/(1.02\times10^{-6}) = 1.25\times10^5$

$$\epsilon/D = 0.00026/0.050 = 0.00520$$

From Fig. A-5, $f = 0.0315$. $h_f = 0.0315[1000/0.050]\{2.546^2/[(2)(9.807)]\} = 208.2$ m. For 45° elbows, $K_1 = (2)(0.20) = 0.40$. For 90° elbows, $K_2 = (4)(0.30) = 1.20$. For the open valve, $K_3 = 8.5$. For exit, $K_4 = 1.0$. $h_m = (0.40 + 1.20 + 8.5 + 1.0)\{2.546^2/[(2)(9.807)]\} = 3.7$ m, $h_L = 208.2 + 3.7 = 211.9$ m, $p_1/9.79 + 2.546^2/[(2)(9.807)] + 0 = 0 + 0 + 100 + 211.9$, $p_1 = 3056$ kPa gage.

Example 6

A pipe system carries water from a reservoir and discharges it as a free jet, as shown in Fig. 9-40. How much flow is to be expected through a 200-mm steel commercial pipe with the fittings shown?

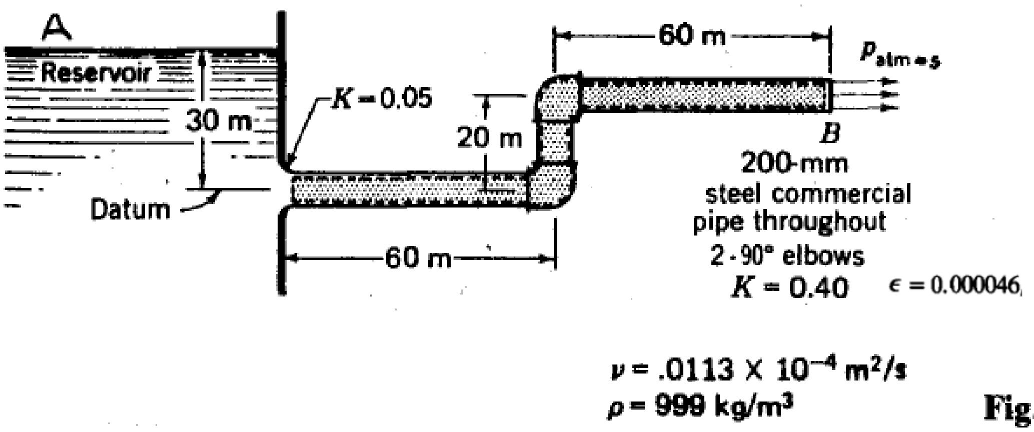


Fig. 9-40

Solution: $p_1/\gamma + v_1^2/2g + z_1 = p_2/\gamma + v_2^2/2g + z_2 + h_L \qquad h_L = h_f + h_m$

$$h_f = (f)(L/D)(v^2/2g) = f[(60 + 20 + 60)/(\tfrac{200}{1000})]\{v_2^2/[(2)(9.807)]\} = 35.69 f v_2^2$$

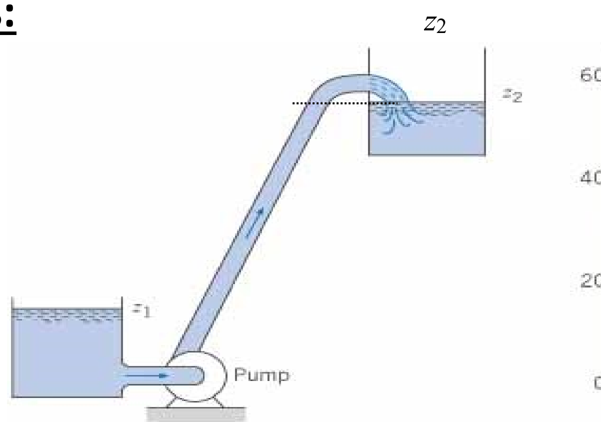
$$h_m = (K)(v^2/2g) = [0.05 + (2)(0.40)]\{v_2^2/[(2)(9.807)]\} = 0.04334 v_2^2$$

$$0 + 0 + 30 = 0 + v_2^2/[(2)(9.807)] + 20 + (35.69 f v_2^2 + 0.04334 v_2^2) \qquad v_2 = \sqrt{10/(35.69f + 0.09432)}$$

Try $f = 0.014$: $v_2 = \sqrt{10/[(35.69)(0.014) + 0.09432]} = 4.103$ m/s, $N_R = Dv/\nu = (\tfrac{200}{1000})(4.103)/(0.0113\times10^{-4}) = 7.26\times10^5$, $\epsilon/D = 0.000046/(\tfrac{200}{1000}) = 0.000230$. From Fig. A-5, $f = 0.0152$. Try $f = 0.0152$: $v_2 = \sqrt{10/[(35.69)(0.0152) + 0.09432]} = 3.963$ m/s, $N_R = (\tfrac{200}{1000})(3.963)/(0.0113\times10^{-4}) = 7.01\times10^5$, $f = 0.0152$ (O.K.); $Q = Av = [(\pi)(\tfrac{200}{1000})^2/4](3.963) = 0.125\ m^3/s$.

F	V	Re	e/D	New F
0.014	4.103	7.26E5	0.00023	0.0152
0.0152	3.963	7.01E5	0.00023	0.0152

Pumps and Turbines:

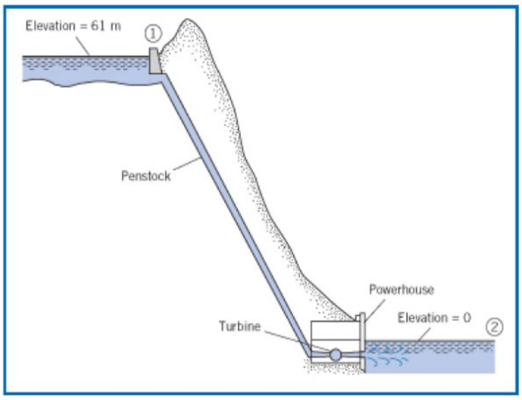


- Pumps and turbines represent the external source-sink term of energy head

- Apply energy equation from the reservoir water surface to the outlet stream is:

In Ideal Flow:

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1+h_p=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2$$



In real Flow with: For pump

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1+h_p=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+\sum K\frac{V_2^2}{2g}+\sum\frac{fL}{D}\frac{V_2^2}{2g}$$

While for turbine it become:

$$\frac{p_1}{\gamma}+\frac{V_1^2}{2g}+z_1-h_T=\frac{p_2}{\gamma}+\frac{V_2^2}{2g}+z_2+\sum K\frac{V_2^2}{2g}+\sum\frac{fL}{D}\frac{V_2^2}{2g}$$

Bern. Betw: 1 & 2

$$\rightarrow \frac{P1}{\gamma}+\frac{v1^2}{2g}+Z1+hP=\frac{P2}{\gamma}+\frac{v2^2}{2g}+Z2+hf+hm$$ For pump

$$\rightarrow \frac{P1}{\gamma}+\frac{v1^2}{2g}+Z1-hT=\frac{P2}{\gamma}+\frac{v2^2}{2g}+Z2+hf+hm$$ For turbine

$\text{Pump Power}=\rho gQh$ (Hydraulic power of pump, Power out)

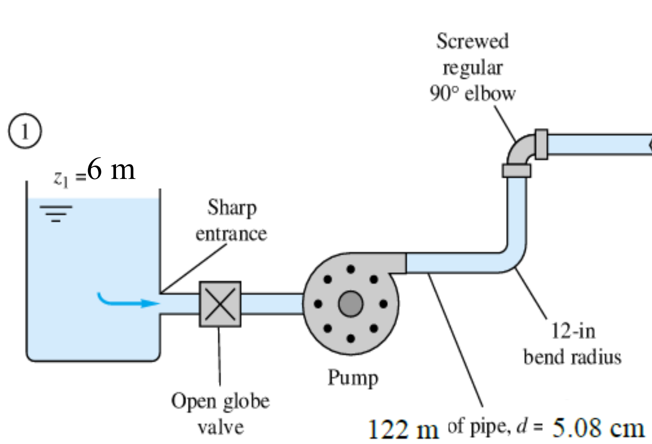
$\text{Turbine Power}=-\rho gQh$ (Hydraulic power of Turbine, Power in)

$$Efficiency, Pump=\frac{Power_{out}}{Power_{in}}=\frac{\rho gQh}{Power_{in}},\qquad Efficiency, Turbine=\frac{Power_{out}}{Power_{in}}=\frac{Power_{out}}{\rho gQh}$$

$$Power_{out}=\frac{\rho gQh}{Efficiency, Pump}$$

Example 1

Water, ρ=1000 kg/m^3 and ν = 1.02 ×10^{-6} , is pumped between two reservoirs at 0.0508 m^3/sthrough 122 m of 5.08 cm diameter pipe and several minor losses,as shown . The roughness ratio is ε/*d* = 0.001. Compute the pump power required. Take the following minor losses .



Loss	K
Sharp entrance (Fig. 6.21)	0.5
Open globe valve (2 in, Table 6.5)	6.9
12-in bend (Fig. 6.20)	0.15
Regular 90° elbow (Table 6.5)	0.95
Half-closed gate valve (from Fig. 6.18*b*)	2.7
Sharp exit (Fig. 6.21)	1.0
	Σ K = 12.2

Solution:

Write the steady-flow energy equation between sections 1 and 2, the two reservoir surfaces:

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \left(\frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2\right) + h_f + \sum h_s - h_p$$

where h_p is the head increase across the pump.

$A = \pi \times 0.0508^2 / 4$

$$V = \frac{Q}{A} \qquad V = 2.81 \text{ m/s}$$

With ε/D = 0.001 and $Re = \dfrac{VD}{\nu}$ **= 139000.95** → ***f= 0.0214***

But since $p_1 = p_2 = 0$ and $V_1 = V_2 = 0$, solve the above energy equation for the pump head :

$$h_p = Z_2 - Z_1 + h_f = Z_2 - Z_1 + \left\{ f\frac{L}{D} + \sum K_i \right\} \frac{V^2}{2g}$$

Z_2 = 36 m , Z_1 = 6 m , L = 122 m V=2.81 m/s → ***h_P = 55.78 m***

The power required to be delivered to the fluid is give by :

$$P_f = \rho Q g h_p \quad = 3119 \text{ W}$$

If the pump has an efficiency of 80 %, the power requirements would be specified

P_{in}= P_f/ η = 3119 /0.8
P_{in}= 3898.75 W

Example 2 Water flows from an upper reservoir to a lower one while passing through a turbine, as shown in Fig. 21-12. Find the power generated by the turbine. Neglect minor losses.

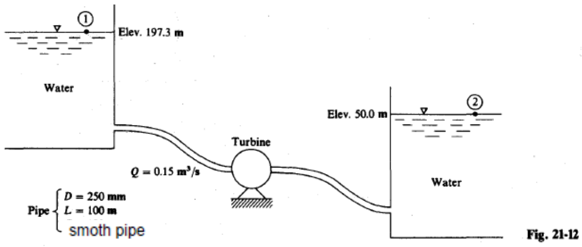


Fig. 21-12

Solution:

with $Q = 0.15\ m^3/s$ and $D = 250$ mm → v=3.05m/s

$p_1/\gamma + V_1^2/2g + z_1 - E_t = p_2/\gamma + V_2^2/2g + z_2 + h_L$. From Fig. A-14, with $Q = 0.15\ m^3/s$ and $D = 250$ mm,

$$Re = \frac{3.05x0.25}{1.14x10^{-6}} = 6.7x10^5$$

From mody dig. → $f = 0.019$

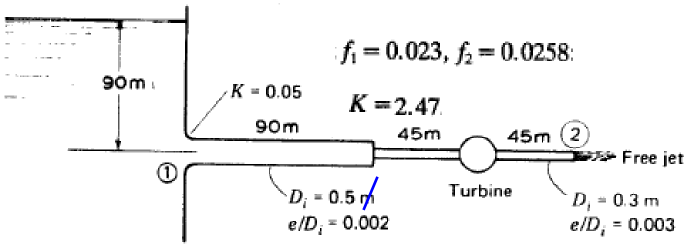
$$hf = f\frac{L}{D}.\frac{v^2}{2g} = 0.019\frac{100}{0.25}.\frac{3.05^2}{2g} = 3.7m$$

$$0 + 0 + 197.3 - E_t = 0 + 0 + 50.0 + 3.70$$

$E_t = 143.6$ m $\qquad P = Q\gamma E_t = (0.15)(9.79)(143.6) = 211$ kW

Example 3

Plot the hydraulic grade line and the energy grade line for the pipe shown in Fig. 22-1*a*. The turbine develops 45 kW; the water is at 5 °C.



(*a*)

Solution:

First compute *a*.

$$(V_1^2/2) + (p_1/\rho) + 0 - \text{Ht} = (V_2^2/2) + (p_2/\rho) + 0 + h_L \quad (1)$$

$$\text{Ht} = (45\,000)/[V_2(\pi/4)(0.3^2)(1000)] = 637/V_2 \quad (2)$$

$$h_L = f_1(90/0.5)(V_1^2/2) + f_2(90/0.3)(V_2^2/2) + 0.05(V_1^2/2) + K(V_1^2/2) \quad (3)$$

$f_1 = 0.023$, $f_2 = 0.0258$: $K = 2.47$. Substitute the above results into Eq. (*1*) noting that $V_1 = (0.3/0.5)^2 V_2 = 0.36V_2$,

$$[(0.36V_2)^2/2] + (p_1/\rho) - (637/V_2) = (V_2^2/2) + (0.023)(90/0.5)[(0.36V_2)^2/2] + (0.0258)(90/0.3)(V_2^2/2) + [(0.05)(0.36V_2)^2/2] + [2.47(0.36V_2)^2/2] \quad (4)$$

But, by Bernoulli's equation, $90g = (p_1/\rho) + [(0.36V_2)^2/2] + 0$. Substitute this expression for (p_1/ρ) in Eq. (*4*) to find: $4.80V_2^3 - 90gV_2 + 637 = 0$. Solve by trial and error: $V_2 = 13.18$ m/s, $V_1 = 4.74$ m/s.

At *A*:

$$(H_{Hyd})_a = 90 - (0.05)(4.74^2/2g) - (0.023)(90/0.5)(4.74^2/2g) \qquad (H_{Hyd})_a = 85.2 \text{ m}$$

After contraction:

$$(H_{Hyd})_{a'} = -(2.47)(4.74^2/2g) + 85.2 = 82.4 \text{ m}$$

At *B*:

$$(H_{Hyd})_b = 82.4 - (0.0258)(45/0.3)(13.18^2/2g) \qquad (H_{Hyd})_b = 48.1 \text{ m}$$

After turbine (b'):

$$(H_{Hyd})_{b'} = 48.1 - [637/(13.18)(g)] = 43.2 \text{ m}$$

At *C*:

$$(H_{Hyd})_c = 43.2 - (0.0258)(45/0.3)(13.18^2/2g) = 8.9 \text{ m}$$

This just equals $V_2^2/2g$, the energy head at exit, as it must. The graphs are shown in Fig. 22-1*b*.

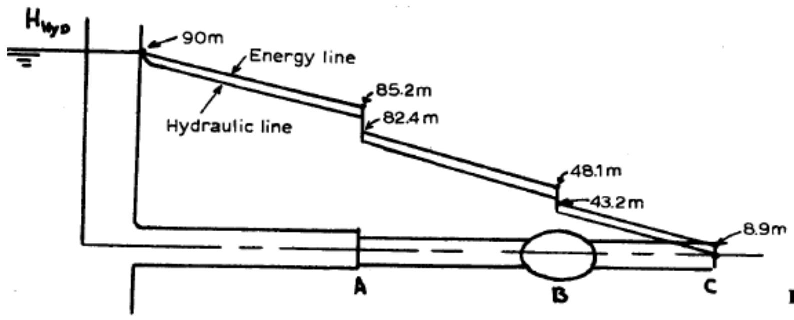


Fig. 22-1

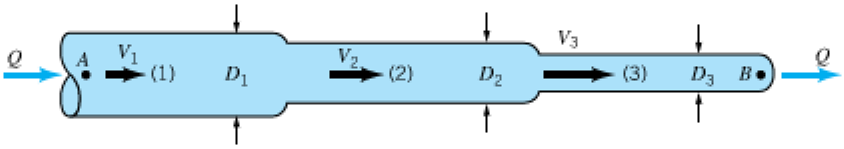
(*b*)

Pipe Systems

Pipes in series

When pipes of different diameters or material are connected end to end to form a pipe line, they are said to be in series. The total loss of energy (or head) will be the sum of the losses in each pipe plus local losses at connections (minor losses).

$H_L = \sum h_f$ $\qquad Q_1 = Q_2 = \cdots$ $\quad Q_1 = Q_2 = Q_3 = Q$



(a)

$$Q_1 = Q_2 = Q_3$$
$$h_{L_{A-B}} = h_{L_1} + h_{L_2} + h_{L_3}$$

$$h_{f,a-b} = h_{f,1} + h_{f,2} + h_{f,3}$$

$$h_{f,a-b} = \left(f\frac{L}{D} + \sum K_i\right)_{D_1}\frac{V_1^2}{2g} + \left(f\frac{L}{D} + \sum K_i\right)_{D_2}\frac{V_2^2}{2g} + \left(f\frac{L}{D} + \sum K_i\right)_{D_3}\frac{V_3^2}{2g}$$

There are two type of problems:

1- If the head lost (HL) is known and (Q) is required: the step of solution is:

- Record all the hf and hm in term of Q as follow:

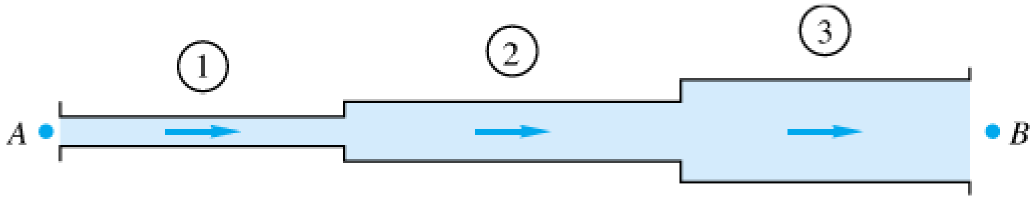
$$hf = f.\frac{L}{D}.\frac{v^2}{2g} \rightarrow \boxed{hf = \left(\frac{L.Q^2}{12.1.D^5}\right)f} \qquad \& \qquad hm = K.\frac{v^2}{2g} \rightarrow \boxed{hm = \left(\frac{Q^2}{12.1.D^4}\right)K}$$

- Solve the total bernolly equation to find Q.

2- If the (Q) is known and (head lost HL) is required: the step of solution is direct solution with (HL=∑hf+∑hm)

Example1

Given is a three-pipe series system, as in Fig. The total pressure drop is $p_A - p_B$ = 150,000 Pa, and the elevation drop is $z_A - z_B$ = 5 m. The pipe data are in table below. The fluid is water, ρ = 1000 kg/m3 and ν = 1.02 x 10 $^{-6}$ m^2/s. Calculate the flow rate Q in m^3/h through the system.



Pipe	L, m	d, cm	ϵ, mm	ϵ/d
1	100	8	0.24	0.003
2	150	6	0.12	0.002
3	80	4	0.20	0.005

$$f_1 = 0.0288 \qquad f_2 = 0.0260 \qquad f_3 = 0.0314$$

Solution:

The total head loss across the system is

$$\Delta h_{A\to B} = \frac{p_A - p_B}{\rho g} + z_A - z_B = \frac{150{,}000}{1000(9.81)} + 5\text{ m} = 20.3\text{ m}$$

From the continuity relation (6.105) the velocities are

$$V_2 = \frac{d_1^2}{d_2^2}V_1 = \frac{16}{9}V_1 \qquad V_3 = \frac{d_1^2}{d_3^2}V_1 = 4V_1$$

and

$$\text{Re}_2 = \frac{V_2 d_2}{V_1 d_1}\text{Re}_1 = \frac{4}{3}\text{Re}_1 \qquad \text{Re}_3 = 2\,\text{Re}_1$$

Neglecting minor losses and substituting into Eq. (6.107), we obtain

$$\Delta h_{A\to B} = \frac{V_1^2}{2g}\left[1250 f_1 + 2500\left(\frac{16}{9}\right)^2 f_2 + 2000(4)^2 f_3\right]$$

or

$$20.3\text{ m} = \frac{V_1^2}{2g}(1250 f_1 + 7900 f_2 + 32{,}000 f_3) \qquad (1)$$

$$f_1 = 0.0288 \qquad f_2 = 0.0260 \qquad f_3 = 0.0314$$

Substitution into Eq. (1) gives the better estimate

$$V_1 = 0.565\text{ m/s} \qquad Q = \tfrac{1}{4}\pi d_1^2 V_1 = 2.84 \times 10^{-3}\text{ m}^3/\text{s}$$

or

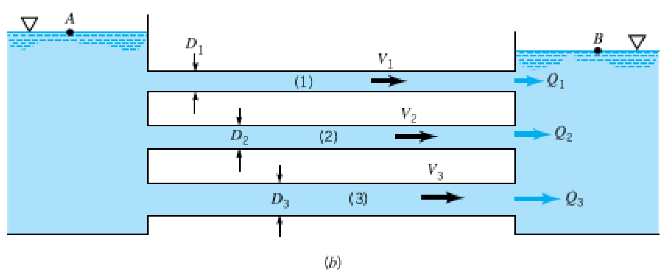
$$Q_1 = 10.2\text{ m}^3/\text{h} \qquad \textit{Ans.}$$

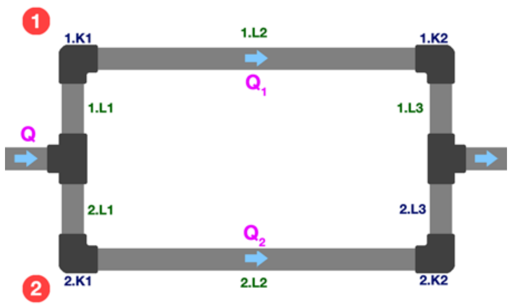
A second iteration gives $Q = 10.22\text{ m}^3/\text{h}$, a negligible change.

Pipes in parallel

When two or more pipes in parallel connect two reservoirs, as shown in Figure, for example, then the fluid may flow down any of the available pipes at possible different rates. But the head difference over each pipe will always be the same. The total volume flow rate will be the sum of the flow in each pipe. The analysis can be carried out by simply treating each pipe .individually and summing flow rates at the end

$Q=\sum Q_i \quad where\ i = 1,2,3,4, \dots \dots ..$ $\qquad h_{f_1} = h_{f_2} = \cdots$





$$Q_{A-B} = Q_1 + Q_2 + Q_3$$
$$h_{L_1} = h_{L_2} = h_{L_3}$$

There are two types of problems:

1. When the elevation of HGL at A and B known. The discharge is required. In this case, the discharges added to find the Q because h_f (drop of HGL) is known.
2. Q is known as well as pipe and fluid characteristics. The distribution of flow and head loss need to be calculated.

   For this type of problem the solution will be as follow:

   a. Assume discharge Q_1' through pipe 1.

   b. Solve for h_{f1}' using assumed discharge Q_1'.

   c. Using h_{f1}' find $Q_2', Q_3', \dots\dots$

   d. With these discharges (Q_1', Q_2', Q_3' ....) for common head loss, now , assume that given Q is split up among the pipes in the same proportion as Q_1', Q_2', Q_3' ...... thus;

   $$Q_1 = \frac{Q_1'}{\sum Q_i'}, Q_2 = \frac{Q_2'}{\sum Q_i'}, Q_3 = \frac{Q_3'}{\sum Q_i'} \dots\dots.$$

   e. Check the correctness of these discharges by computing $h_{f1}', h_{f2}', h_{f3}'$ for the computed $Q_1, Q_2, Q_3, \dots..$

Example2

Assume that the same three pipes in Example above are now in parallel with the same total head loss of 20.3 m. Compute the total flow rate Q, neglecting minor losses.

Pipe	L, m	d, cm	ϵ, mm	ϵ/d
1	100	8	0.24	0.003
2	150	6	0.12	0.002
3	80	4	0.20	0.005

$f_1 = 0.0262$
$f_2 \approx 0.0234$,
$f_3 \approx 0.0304$.

Solution:

$$20.3\text{ m} = \frac{V_1^2}{2g}1250f_1 = \frac{V_2^2}{2g}2500f_2 = \frac{V_3^2}{2g}2000f_3 \qquad (1)$$

Guess fully rough flow in pipe 1: $f_1 = 0.0262$, $V_1 = 3.49$ m/s; hence $\text{Re}_1 = V_1 d_1/\nu = 273{,}000$. From the Moody chart read $f_1 = 0.0267$; recompute $V_1 = 3.46$ m/s, $Q_1 = 62.5$ m^3/h. [This problem can also be solved from Eq. (6.66).]

Next guess for pipe 2: $f_2 \approx 0.0234$, $V_2 \approx 2.61$ m/s; then $\text{Re}_2 = 153{,}000$, and hence $f_2 = 0.0246$, $V_2 = 2.55$ m/s, $Q_2 = 25.9$ m^3/h.

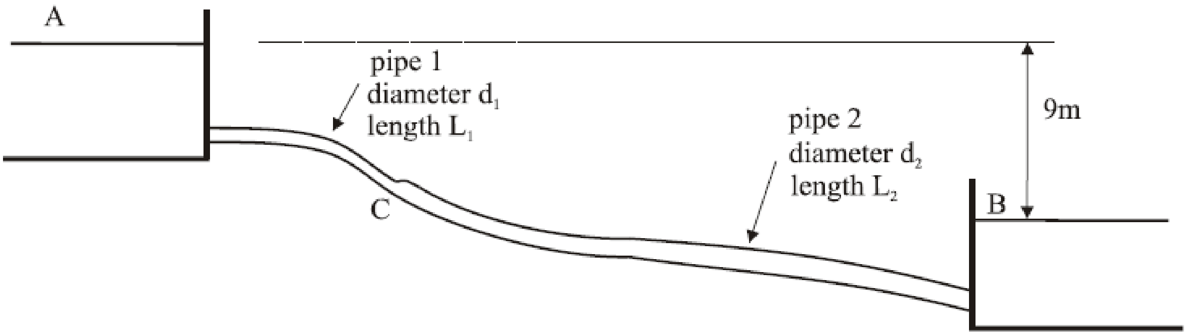
Finally guess for pipe 3: $f_3 \approx 0.0304$, $V_3 \approx 2.56$ m/s; then $\text{Re}_3 = 100{,}000$, and hence $f_3 = 0.0313$, $V_3 = 2.52$ m/s, $Q_3 = 11.4$ m^3/h.

This is satisfactory convergence. The total flow rate is

$$Q = Q_1 + Q_2 + Q_3 = 62.5 + 25.9 + 11.4 = 99.8\text{ m}^3/\text{h} \qquad \textit{Ans.}$$

Example 3: Pipes in Series Example

Consider the two reservoirs shown in figure, connected by a single pipe that changes diameter over its length. The surfaces of the two reservoirs have a difference in level of 9m. The pipe has a diameter of 200mm for the first 15m (from A to C) then a diameter of 250mm for the remaining 45m (from C to B). find Q.



For the entrance use $K_e = 0.5$ and the exit $K_E = 1.0$. The join at C is sudden. For .both pipes use $f = 0.3$

Solution:

Total head loss for the system H_L = height difference of reservoirs

h_{f1} = head loss for 200mm diameter section of pipe

h_{f2} = head loss for 250mm diameter section of pipe

hm_e = head loss at entry point

hm_j = head loss at join of the two pipes

hm_E = head loss at exit point

So $H_L = h_{f1} + h_{f2} + hm_e + hm_j + hm_E = 9\text{m}$ …….. (1)

All losses are, in terms of Q:

$$h_{f1} = \frac{fL_1Q^2}{12.1 D_1^5} = 0.371 \frac{Q^2}{D_1^5}$$

$$h_{f2} = \frac{fL_2Q^2}{12.1 D_2^5} = 1.115 \frac{Q^2}{D_2^5}$$

$$hm_e = 0.5\frac{V_1^2}{2g} = 0.5\frac{1}{2g}\left(\frac{4Q}{\pi D_1^2}\right)^2 = 0.5 \times 0.0826\frac{Q^2}{D_1^4} = 0.0413\frac{Q^2}{D_1^4}$$

$$hm_E = 1.0\frac{V_2^2}{2g} = 1.0 \times 0.0826\frac{Q^2}{D_2^4} = 0.0826\frac{Q^2}{D_2^4}$$

$$K_j = \left[1 - \left(\frac{D_1}{D_2}\right)^2\right]^2 = \left[1 - \left(\frac{200}{250}\right)^2\right]^2 = 0.1296$$

$$hm_j = Kj\frac{V^2}{2g} = Kj\left(\frac{4Q}{\pi}\right)^2\frac{\left(\frac{1}{D_1^2}\right)^2}{2g} = 0.0107\frac{Q^2}{D_1^4}$$

Substitute these into eq.1

and solve for Q, to give Q = 0.062 m^3/s

Example 4

Two concrete pipes are connected in series. The flow rate of water through the pipes is 0.14 m^3/s with a total friction loss of 14.10 m for both pipes. Each pipe has a length of 300 m. If one pipe has a diameter of 300 mm, what is the diameter of the other one? Neglect minor losses. *use f=0.021*

For first pipe: With $Q = 0.14\ m^3/s$ and $D = 300$ mm,

$V = Q/A = 1.98 m/s$

$$h_f = 0.021 \frac{300}{0.3} \frac{1.98^2}{19.62} = 4.20 \text{ m}.$$

For second pipe: $h_f = 14.10 - 4.20 = 9.90$ m,

$$9.9 = 0.021 \frac{300}{12.1} \frac{0.14^2}{D^5}$$

$D = 0.253m = 253mm$

Example 5

A 225-m-long, 300-mm-diameter concrete pipe and a 400-m-long, 500-mm-diameter concrete pipe are connected in series. Find the diameter of a 625-m-long equivalent pipe. Assume a flow rate of 0.1 m^3/s.

Solution:

For the 300-mm-diameter pipe from Q and D find V and Re by known (ϵ/d) find f1 and hf1=1.66m

For the 500-mm-diameter pipe from Q and D find V and Re by known (ϵ/d)ind f2 and hf2=0.256m

For a 625-m-long equivalent pipe with this head loss

.hf = 1.66+0.26 = 1.917 m and Q=0.1 m3/s equivalent De by Darcy eq $hf = f.\frac{L}{D^5}.\frac{8Q^2}{g\pi^2}$

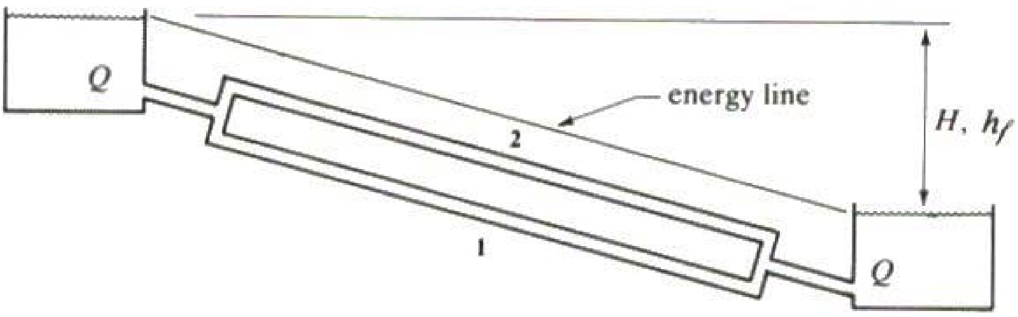
assume f then find D and V then Re then check f and repeat (D=360mm

Example 6: Pipes in Parallel Example

Two pipes connect two reservoirs (A and B) which have a height difference of 10m. Pipe 1 has diameter 50mm and length 100m. Pipe 2 has diameter 100mm and length 100m. Both have entry loss $K_L = 0.5$ and exit loss K_L=1.0 and Darcy *f* of 0.032. Calculate:

a) rate of flow for each pipe

b) the diameter D of a pipe 100m long that could replace the two pipes and provide the sameflow.



Solution

a)Apply Bernoulli to each pipe separately. For pipe 1:

$$z_A - z_B = \left(0.5 + \frac{fl}{D_1} + 1.0\right)\frac{V_1^2}{2g}$$

$$10 = \left(1.5 + \frac{0.032 \times 100}{0.05}\right)\frac{V_1^2}{2 \times 9.81}$$

$$V_1 = 1.731\, m/s$$

And flow rate is given by

$$Q_1 = V_1 \frac{\pi D_1^2}{4} = 0.0034\, m^3/s$$

For pipe 2:

$$z_A - z_B = \left(0.5 + \frac{fl}{D_2} + 1.0\right)\frac{V_2^2}{2g}$$

$$10 = \left(1.5 + \frac{0.032 \times 100}{0.1}\right)\frac{V_2^2}{2 \times 9.81}$$

$$V_2 = 2.42\, m/s$$

And flow rate is given by

$$Q_2 = V_2 \frac{\pi D_2^2}{4} = 0.0190\, m^3/s$$

b) Replacing the pipe, we need $Q = Q_1 + Q_2 = 0.0034 + 0.0190 = 0.0224\ m^3/s$

For this pipe, diameter D, velocity u , and making the same assumptions about entry/exit losses, we have:

$$z_A - z_B = \left(0.5 + \frac{fl}{D} + 1.0\right)\frac{u^2}{2g} \longrightarrow 10 = \left(1.5 + \frac{0.008 \times 100}{D}\right)\frac{u^2}{2 \times 9.81} \rightarrow 196.2 = \left(1.0 + \frac{3.2}{D}\right)u^2$$

The velocity can be obtained from Q i.e.

$$Q = Au = \frac{\pi D^2}{4}u$$

$$u = \frac{4Q}{\pi D^2} = \frac{0.02852}{D^2}$$

So

$$196.2 = \left(1.5 + \frac{3.2}{D}\right)\left(\frac{0.02852}{D^2}\right)^2$$

$$0 = 241212 D^5 - 1.5D - 3.2$$

which must be solved iteratively

An approximate answer can be obtained by dropping the second term:

$$0 = 241212 D^5 - 3.2$$

$$D = \sqrt[5]{3.2/241212}$$

$$D = 0.1058m$$

Example 7

Two reservoirs are connected by three clean cast iron pipes in series: $L_1 = 300$ m, $D_1 = 200$ mm; $L_2 = 400$ m, $D_2 = 300$ mm; $L_3 = 1200$ m, $D_3 = 450$ mm. If the flow is 360 m³/h of water at 20 °C, determine the difference in elevation of the reservoirs. $\upsilon = 1.02 \times 10^{-6}$ m²/s, $\epsilon = 0.26$mm

Solution $v = Q/A \quad v_1 = (360/3600)/[(\pi)(0.200)^2/4] = 3.183$ m/s

$v_2 = (360/3600)/[(\pi)(0.300)^2/4] = 1.415$ m/s $\quad v_3 = (360/3600)/[(\pi)(0.450)^2/4] = 0.6288$ m/s

$$N_R = Dv/\nu \quad (N_R)_1 = (0.200)(3.183)/(1.02 \times 10^{-6}) = 6.24 \times 10^5$$

$$(N_R)_2 = (0.300)(1.415)/(1.02 \times 10^{-6}) = 4.16 \times 10^5 \quad (N_R)_3 = (0.450)(0.6288)/(1.02 \times 10^{-6}) = 2.77 \times 10^5$$

$$(\epsilon/D)_1 = 0.00026/0.200 = 0.00130 \quad (\epsilon/D)_2 = 0.0026/0.300 = 0.000867$$

$$(\epsilon/D)_3 = 0.00026/0.450 = 0.000578$$

From Fig. A-5, $f_1 = 0.0215$, $f_2 = 0.020$, and $f_3 = 0.0185$. $H = h_f = (f)(L/D)(v^2/2g) = 0.0215[300/0.200]\{3.183^2/[(2)(9.807)]\} + 0.020[400/0.300]\{1.415^2/[(2)(9.807)]\} + 0.0185[1200/0.450]\{0.6288^2/[(2)(9.807)]\} = 20.37$ m.

Example 8

Solve Prob. 10.24 by the method of equivalent lengths.

Express pipes 2 and 3 in terms of pipe 1:

$$L_e = (f_2/f_1)(L_2)(D_1/D_2)^5$$

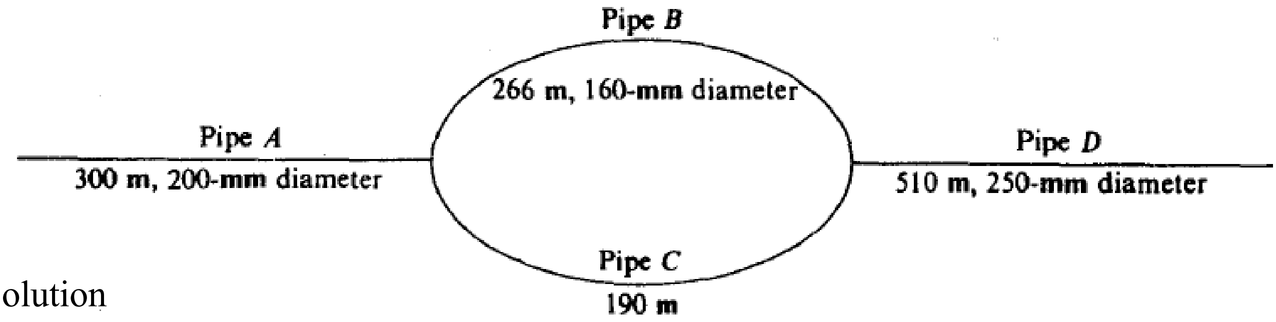
$$(L_e)_2 = (0.020/0.0215)(400)(\tfrac{200}{300})^5 = 49.00 \text{ m} \quad (L_e)_3 = (0.0185/0.0215)(1200)(\tfrac{200}{450})^5 = 17.91 \text{ m}$$

$$(L_e)_{total} = 300 + 49.00 + 17.91 = 366.9 \text{ m}$$

$$H = h_f = (f)(L/D)(v^2/2g) = 0.0215[366.9/0.200]\{3.183^2/[(2)(9.807)]\} = 20.37 \text{ m}$$

Example 9

If the flow rate of water through the pipe system shown in Fig. 11-4 is 0.050 m³/s under total head loss of 9.0 m, determine the diameter of pipe C. Assume a C coefficient f=0.023 for all pipes



Solution

With $Q_A = 0.050$ m³/s and $D_A = 200$ mm, L=300m f=0.023 $(h_f)_A = 4.50$ m.

With $Q_D = 0.050$ m³/s and $D_D = 250$ mm, L=510m f=0.023 $(h_f)_D = 2.50$ m.

$(h_f)_B = (h_f)_C = 9.0 - 4.50 - 2.50 = 2.00$ m. From Parallel pipes

from hfB=2m , L=266m f=0.023 , D=160mm → $Q_B = 0.019$ m³/s.

$$Q_C = 0.050 - 0.019 = 0.031 \text{ m}^3/\text{s}.$$

With $Q_C = 0.031$ m³/s hfc=2m , L=190m f=0.023 , → $D_C = 180$ mm.

Example 10

Refer to Fig. 11-26. Assume the water surface in the reservoir is at elevation 94 m. Pipes *A*, *B*, and *C* are all 840 m long, and they all have diameter of 0.7 m, with $f = 0.022$. Neglecting minor losses, find the flow rate in all pipes, supposing that the pump develops 9 m of head when the velocity in pipe *C* is 3.6 m/s.

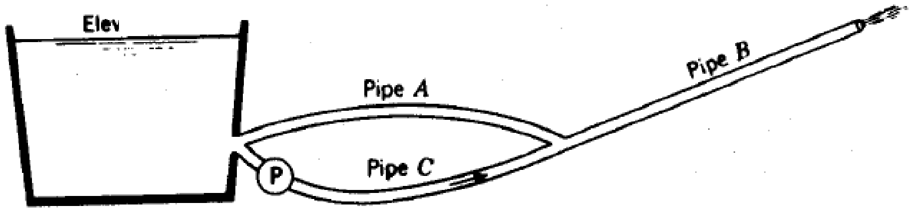


Fig. 11-26

Solution

$h_f = (f)(L/d)(v^2/2g)$ $(h_f)_A = 0.022[840/(0.7)]\{v_A^2/[(2)(9.807)]\} = 1.346v_A^2$ $(h_L)_C = (h_f)_C - h_{pump}$

$(h_f)_C = 0.022[840/(0.7)]\{3.6^2/[(2)(9.807)]\} = 17.44\text{ m}$ $(h_L)_C = 17.44 - 9 = 8.44\text{ m}$

$1.346v_A^2 = 8.44$ $v_A = 2.50\text{ m/s}$

$Q_A = A_A v_A = [(\pi)(0.7)^2/4](2.50) = 0.962\text{ m}^3/\text{s}$ (to the right)

$Q_C = [(\pi)(0.7)^2/4](3.6) = 1.385\text{ m}^3/\text{s}$ (to the right)

$Q_B = Q_A + Q_C = 0.962 + 1.385 = 2.347\text{ m}^3/\text{s}$ (to the right)

Example 11

In Prob. 11.50, find the elevation of pipe *B* at discharge.

Solution

$\text{El}_B = \text{El}_{\text{reservoir surface}} + h_{pump} - \sum h_f - v_B^2/2g$ $v_B = Q_B/A_B = 2.347/[(\pi)(0.7)^2/4] = 6.099\text{ m/s}$

$(h_f)_B = 0.022[840/(0.7)]\{6.099^2/[(2)(9.807)]\} = 50.07\text{ m}$ $\sum h_f = 17.44 + 50.07 = 67.51\text{ m}$

$\text{El}_B = 94 + 9 - 67.51 - 6.099^2/[(2)(9.807)] = 33.6\text{ m}$

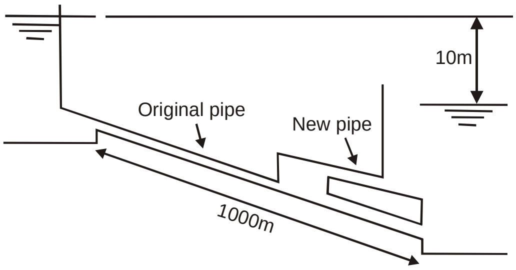
Example 12

Adding a parallel pipe example

A pipe joins two reservoirs whose head difference is 10m. The pipe is 0.2 m diameter, 1000m in length and has a f value of 0.008.

a) ?what is the flow in the pipeline

b) It is required to increase the flow to the downstre reservoir by 30%. This is to be done adding a second pipe of the same diameter that connects at some point along the old pipe and runs down to the lower reservoir. Assuming the diameter and the friction factor are the same as the old pipe, how long should the new pipe be?



Solution

a) $h_f = \dfrac{fLQ^2}{3d^5}$ $10 = \dfrac{0.008 \times 1000 Q^2}{3 \times 0.2^5}$ $Q = 0.0346\, m^3/s$

$Q = 34.6\, litres/s$

b) $H = 10 = h_{f1} + h_{f2} = h_{f1} + h_{f3}$

$\therefore\ h_{f2} = h_{f3}$ $\dfrac{f_2 L_2 Q_2^2}{3d_2^5} = \dfrac{f_3 L_3 Q_3^2}{3d_3^5}$

as the pipes 2 and 3 are the same f, same length and the same diameter then $Q_2 = Q_3$.

By continuity $Q_1 = Q_2 + Q_3 = 2Q_2 = 2Q_3$ So $Q_2 = \dfrac{Q_1}{2}$ and $L_2 = 1000 - L_1$

Then $10 = h_{f1} + h_{f2}$

$$10 = \frac{f_1 L_1 Q_1^2}{2d_1^2} + \frac{f_2 L_2 Q_2^2}{2d_2^2}$$

$$10 = \frac{f_1 L_1 Q_1^2}{2d_1^2} + \frac{f_2 (1000 - L_1)(Q_1/2)^2}{2d_2^2}$$

As $f_1 = f_2$, $d_1 = d_2$ $$10 = \frac{f_1 Q_1^2}{3d_1^2}\left(L_1 + \frac{(1000 - L_1)}{4}\right)$$

The new Q_1 is to be 30% greater than before so $Q_1 = 1.3 \times 0.034 = 0.442\text{ m}^3/\text{s}$ Solve for L to give

$L_1 = 455.6\text{m}$ $L_2 = 1000 - 455.6 = 544.4\text{ m}$

Open channel flow and its classifications:

Introduction :

A passage through which water flows with its free surface in contact with the atmosphere is known as "Open Channel". The water therefore runs under the atmospheric pressure throughout the open channel open at the top.

Classification of Channels:

Open channel may be classified as described below:

a- Channel of regular section or irregular section.

The rectangular, trapezoidal, circular or semicircular section are examples of channels of regular section. The example of channels of irregular section are stream or rivers.

b- Natural or artificial channel

Open channel may be either natural or artificial. Streams or rivers are example of natural channels. Artificial channel are man made.

c- Prismatic or non- prismatic channels

In prismatic channels, the cross-section and slope remain uniform throughout its length. Artificial channels are prismatic channels. The bed slope and cross section of non-prismatic channels do not remain uniform throughout the length. Natural channels are non-prismatic channels.

Classification of Flow:

The flow through the channel may be classified in different types as in the case of the pipe flow:

a- Uniform Flow

The flow is to be uniform when the velocity of the flow dose note change both in magnitude and direction from one section to another in part of channel of uniform cross-section. The Depth of flow must also remain constant throughout its length for uniform flow.

b- Non-Uniform Flow

A flow is said to be non-uniform when the velocity and depth of flow varies from one section to another.

c- Steady flow

A flow with constant rate of discharge is known as steady flow. Also, steady flow defined as the flow where the characteristics at a point are not change with time.

d- Unsteady Flow

A flow in which the rate of discharge dose not remain constant.

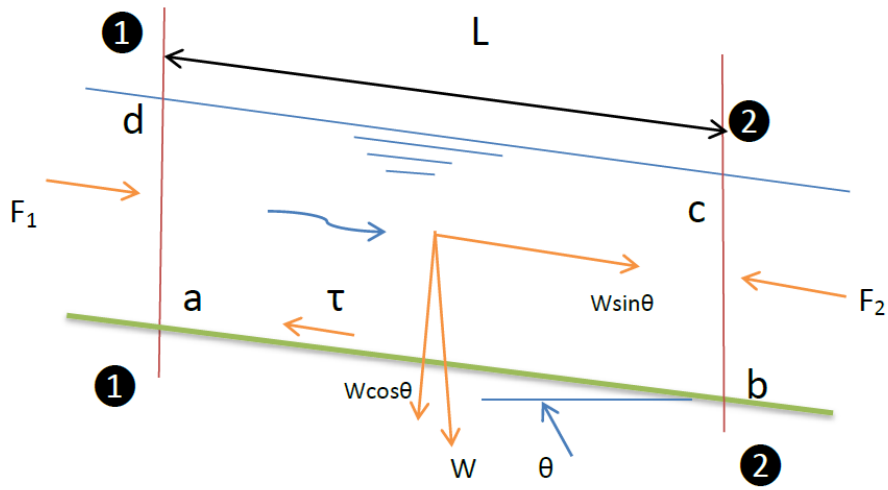
e- Gradually Varied Flow

When the change in depth of flow is gradual the flow is said gradually varied flow.

f- Rapidly Varied Flow

if the change in the depth of flow is abrupt and the transition is conferred to short length only. It is said to be rabidly varied flow.

Chezy's Formula for Uniform Flow



Assuming a section of length L between section (1-1) and section (2-2) moving along the channel as shown in the figure. The water body (abcd) is subjected to the following forces:

- Hydrostatic force (F1) upstream end of the body.

- Hydrostatic force (F2) downstream end of the body.

- Weight of water body (W)

- Frictional resistance force (τ*PL)

Since the water through the channel flows with uniform velocity, therefore the net accelerating or retarding force is equal to zero.

Hence:-

$$F_1 - F_2 + W\sin\theta - \tau \times PL = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

$F_1 = F_2$ (Because the depth of the flow is constant)

$$W = A \times L \times \gamma = AL \times \rho g$$

$$\tau_0 = C_f \times \rho \times \frac{v^2}{2}$$

$$Sin\ \theta = \frac{h_L}{L} = S_0$$

Substituting values of W and τ in the equation (1)

$$AL\rho g \sin\theta - \frac{1}{2} C_f \rho v^2 PL = 0$$

$$v^2 = \frac{2Ag \sin\theta}{C_f P} = \frac{2g}{C_f} \times \frac{A}{P} \sin\theta$$

$$\frac{A}{P} = R_h$$

$$\frac{2g}{C_f} = C$$

$v = C\sqrt{R \times S_0}$ .... Chezy Equation

$$Q = A \times v$$

$$Q = CA\sqrt{R_h S_0}$$

Where:

Q=Discharge

A=Cross section area

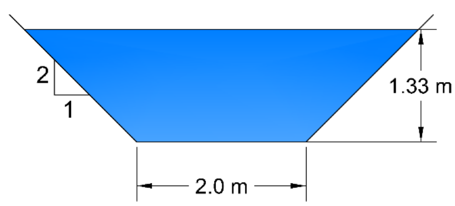
C= Chezy Coefficient

R_h= Hydraulic radius

S_0= Longitudinal Slope

Example :A trapezoidal channel has base width of 2m and side slope of 1 horizontal to 2 vertical. The depth of flow in the channel is 1.33m. Find Chezy constant if the discharge through this channel is 7200 liter/sec and the longitudinal slope of this channel is 1 to 530.

Sol. :



$$A = By + zy^2$$

$$A = 2(1.33) + 0.5(1.33)^2 = 3.544\ m2$$

$$P = B + 2y\sqrt{1+z^2}$$

$$P = 2 + 2(1.33)\sqrt{1+(0.5)^2} = 4.98\ m$$

$$R_h = \frac{A}{P} = \frac{3.544}{4.98} = 0.713m$$

$$Q = \frac{7200}{1000} = 7.2\ \frac{m3}{sec}$$

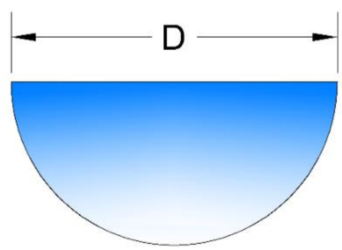
$$Q = CA\sqrt{R_h\ S_0}$$

$$7.2 = C(3.544)\sqrt{(0.713)\ (\frac{1}{530})}$$

$$C = 55.3$$

Example: The discharge through a semi-circular open channel is 10m3/sec. The bed slope is (1 to 1650) and the channel is running full. Find the diameter of this channel if the Chezy coefficient (C) is 70.

Sol.:



$$Q = CA\sqrt{R_h\ S_0}$$

$$A = \frac{\pi R^2}{2} \quad , \quad P = \pi R$$

$$R_h = \frac{0.5\pi R^2}{\pi R} = \frac{R}{2}$$

$$10 = 70(\frac{\pi R^2}{2})\sqrt{(\frac{R}{2})(\frac{1}{1650})}$$

$$R^{\frac{5}{2}} = \frac{10}{35\pi(0.017)}$$

$$R = 1.95m$$

$$D = 2 \times 1.95 = 3.9m$$

The Manning's Formula:

One of the best as well as one of the most widely used formula for uniform flow in open channels is that published by the Irish engineer (Robert Manning). Manning found from many tests, that the value of (C) in the Chezy formula varied approximately as ($R^{1/6}$), so he proposed the following relation:

$$C = \frac{1}{n} R_h^{\frac{1}{6}}$$

Where n is constant and depends on the channel material. Now, Substituting the value of C in the following equation:-

$$V = C\sqrt{R_h S_0}$$

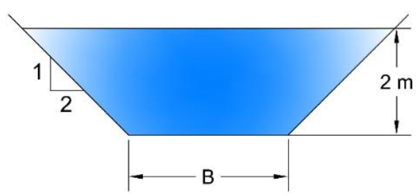
$$V = \frac{1}{n} R_h^{\frac{1}{6}} \sqrt{R_h S_0}$$

$Q = \frac{1}{n} A R_h^{\frac{2}{3}} S_0^{\frac{1}{2}}$ ------------------ In SI Units System (m , kg,…).

$Q = \frac{1.49}{n} A R_h^{\frac{2}{3}} S_0^{\frac{1}{2}}$ ------------------- In UK Units System (ft, Ib,….).

Example: An open channel of trapezoidal section has side slope of 2 horizontal to 1 vertical and carries water of rate equal to 15 m^3/sec. The bed slope of the channel is 0.5 per one kilometer and depth of flow is 2m .

a- Find the bed width of channel assuming n =0.018.
b- If the channel at the above cross section is used to discharge 6 m^3/sec of water at the velocity of 0.5 m/sec and the depth of flow being 2m. determine the bed width and bed slope of the channel.



Sol.:

a- $Q=15\ m^3/sec$, $S = \frac{0.5}{1000} = 0.0005$

$$Q = \frac{1}{n} A R^{\frac{2}{3}} S^{\frac{1}{2}}$$

$$A = By + Zy^2$$

$A = 2B + (2)(2)^2 = 2B + 8$

P= $B + 2y\sqrt{1 + Z^2}$

P= $B + 2(2)\sqrt{1 + (2)^2} = B + 4\sqrt{5}$

$R = \frac{A}{P} = \frac{2B + 8}{B + 4\sqrt{5}}$

$15 = \frac{1}{0.018}\ (2B + 8)(\frac{2B + 8}{B + 4\sqrt{5}})^{\frac{2}{3}}\ (0.0005)^{\frac{1}{2}}$

$\frac{15 \times 0.018}{(0.0005)^{\frac{1}{2}}} = \frac{(2B + 8\ )^{\frac{5}{3}}}{(B + 4\sqrt{5}\ )^{\frac{2}{3}}}$

$B = 1.75\ m$

b- Q= 6 m^3/sec, V=0.5 m/sec , y=2m , n=0.018 , z=2m , S=?

$A = \frac{Q}{v} = \frac{6}{0.5} = 12\ m^2$
$A = By + Zy^2$
$12 = 2B + 2(2)^2 = 2B + 8$
$B = 2\ m$

$Q = \frac{1}{n}\ A\ R^{\frac{2}{3}}\ S^{\frac{1}{2}}$

P= $B + 2y\sqrt{1 + Z^2}$

P= $2 + 2(2)\sqrt{1 + (2)^2} = 10.94\ m$

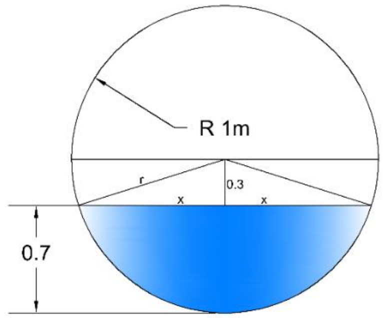
$R = \frac{A}{P} = \frac{12}{10.94} = 1.096\ m$

$6 = \frac{1}{0.018}\ 12 \times\ (1.096)^{\frac{2}{3}}\ S^{\frac{1}{2}}$

$S = 0.0000715$

Example: A circular Sewage of 1 m radius has longitudinal slope of (1 to 250) .find the discharge though the sewage if depth of flow is 700 mm assuming (n) equal to 0.015

Sol.:



$$Q = \frac{1}{n} A R^{\frac{2}{3}} S^{\frac{1}{2}}$$

$$A = Area\ of\ sector - Area\ of\ triangle$$

$$Area\ of\ sector = Area\ of\ circle\ \times \frac{2\theta}{360}$$

$$\cos\theta = \frac{h}{r}$$

$$r = d + h$$

$$h = r - d = 1 - 0.7 = 0.3\ m$$

$$\cos\theta = \frac{0.3}{1}\ \ ,\ \theta = 72.54°$$

$$\sin\theta = \frac{x}{r}$$

$$\text{Sin}(72.54) = \frac{x}{1}\ ,\qquad x = 0.95\ m$$

$$Area\ of\ sector = \pi r^2 \times \frac{2\theta}{360}$$

$$Area\ of\ sector = \pi(1)^2 \times \frac{2(72.54)}{360} = 1.266\ m^2$$

$$Area\ of\ triangle\ = \frac{1}{2} \times 2x \times h$$

$$Area\ of\ triangle\ = \frac{1}{2} \times 2(0.95) \times (0.3) = 0.285\ m^2$$

$$A = 1.266 - 0.285 = 0.981\ m^2$$

$$P\ of\ sector = 2\pi r \times \frac{2\theta}{360}$$

$$P = 2\pi(1) \times \frac{(72.54)}{180} = 2.53\ m$$

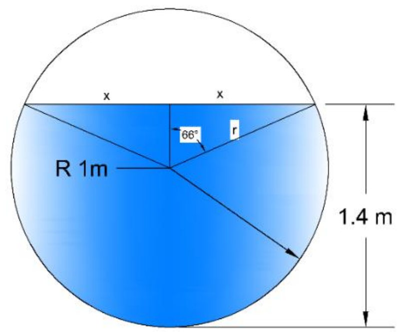
$$R = \frac{A}{P} = \frac{0.981}{2.53} = 0.387m$$

$$Q = \frac{1}{0.015}\ (0.981)\ (0.387)^{\frac{2}{3}}\ (\frac{1}{250})^{\frac{1}{2}}$$

$$Q = 2.196\ m^3/sec$$

Example: Use the Same data of the previous example with depth of flow (y) equal to 1.4 m. find the discharge ?

Sol.:



$$Q = \frac{1}{n}\ A\ R^{\frac{2}{3}}\ S^{\frac{1}{2}}$$

$$\cos\theta = \frac{h}{r}$$

$$h = d - r = 1.4\ - 1 = 0.4\ m$$

$$\cos\theta = \frac{0.4}{1}\quad ,\quad \theta = 66°$$

$$\sin\theta = \frac{x}{r}$$

$$\text{Sin}(66) = \frac{x}{1}\ ,\qquad x = 0.91\ m$$

$$2\theta = 2 \times 66 = 132$$

$$2\alpha = 360 - 2\theta = 360 - 132 = 228°$$

Method 1

$$Area\ of\ flow = Area\ of\ circle - Area\ sector(2\theta) + Area\ of\ triangle$$

$$Area\ of\ flow = \pi r^2 - \pi r^2 \times \frac{2\theta}{360} + \frac{1}{2} \times 2x \times h$$

$$Area\ of\ flow = \pi(1)^2 - \pi(1)^2 \times \frac{132}{360} + \frac{1}{2} \times 2(0.91) \times (0.4) = 2.35\ m^2$$

Method 2

$$Area\ of\ flow = Area\ sector(2\alpha) + Area\ of\ triangle$$

$$Area\ of\ flow = \pi r^2 \times \frac{2\alpha}{360} + \frac{1}{2} \times 2x \times h$$

$$Area\ of\ flow = \pi(1)^2 \times \frac{228}{360} + \frac{1}{2} \times 2(0.91) \times 0.4 = 2.35\ m^2$$

Method 1

$$P = watted\ perimeter\ of\ circle - watted\ perimeter\ of\ sector(2\theta)$$

$$P = 2\pi r - 2\pi r\frac{2\theta}{360} = 2\pi(1) - 2\pi(1)\frac{132}{360} = 3.97\ m$$

Method 2

$$P = watted\ perimeter\ of\ sector(2\alpha)$$

$$P = 2\pi r\frac{2\alpha}{360} = 2\pi(1)\frac{228}{360} = 3.97\ m$$

$$R = \frac{A}{P} = \frac{2.35}{3.97} = 0.59\ m$$

$$Q = \frac{1}{0.015}\ (2.35)\ (0.59)^{\frac{2}{3}}\ (\frac{1}{250})^{\frac{1}{2}}$$

$$Q = 6.97\ \ m^3/sec$$

Condition of the Most Economical Cross Section :

The main purpose of channel is to transport water therefore the cross section of any geometrical shape channel which gives maximum discharge is known as most economical cross section.

In other words the channel of most efficient cross section needs minimum of excavation work for the given discharge through the channel is given by:

$Q = A \times v$ and $v = \frac{1}{n} R_h^{\frac{2}{3}} S^{\frac{1}{2}}$

For a given value of roughness factor (n); area of flow (A) and the hydraulic slope (S) the discharge is maximum if the hydraulic radius (R_h) is maximum but since:-

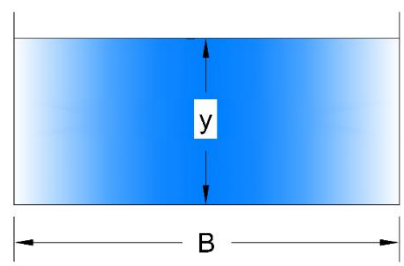
$$R_h = \frac{A}{P}$$

Hence , R_h is maximum if wetted parameter (P) is minimum.

Condition of the Most Economical Rectangular Section :

Consider a channel of rectangular section as shown the figure:

B= bed width
y= Depth of flow



$$A = B \times y$$
$$B = \frac{A}{y}$$
$$P = B + 2y$$
$$P = \frac{A}{y} + 2y$$
$$\frac{dP}{dy} = \frac{-A}{y^2} + 2 = 0$$
$$A = 2y^2$$
$$B \times y = 2y^2$$
$$B = 2y$$

This is the width of channel should be twice water depth for maximum discharge.

That is mean the most economical rectangular section is one half of square.

$$R_h = \frac{A}{P} = \frac{B.y}{B+2y}$$

$$R_h = \frac{2y.y}{2y+2y} = \frac{2y^2}{4y}$$

$$R_h = \frac{y}{2}$$

Example: find the discharge and best properties for a rectangular channel having cross section area 4.5 m2 the bed slope is 0.001 and n is 0.013.

Sol.:

$$Q = \frac{1}{n} A\, R_h^{\frac{2}{3}} S^{\frac{1}{2}}$$

$$A = B.y$$

$$R_h = \frac{y}{2}$$

$$4.5 = 2y.y$$

$$y^2 = \frac{4.5}{2} = 2.25\, m$$

$$y = 1.5\, m$$

$$B = 2y = 2(1.5) = 3\, m$$

$$Q = \frac{1}{0.013}(4.5)\left(\frac{1.5}{2}\right)^{\frac{2}{3}}(0.01)^{\frac{1}{2}} = 9.03\, \frac{m^2}{sec}$$

Example: find the best properties for rectangular channel to carry 1.5 m3/sec of water when the bed slope is (1to 3000) taken n equal to 0.015?

Sol.:

$$Q = \frac{1}{n} A\, R_h^{\frac{2}{3}} S^{\frac{1}{2}}$$

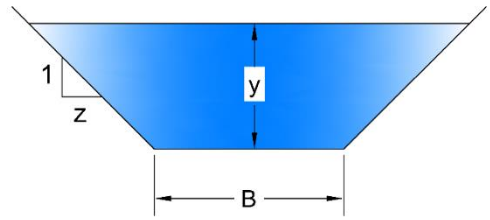
$$A = B.y = 2y^2$$

$$1.5 = \frac{1}{0.015}(2y^2)\left(\frac{y}{2}\right)^{\frac{2}{3}}\left(\frac{1}{3000}\right)^{\frac{1}{2}}$$

$$y = 0.92\, m \qquad , B = 2(0.92) = 1.84\, m$$

Condition of the Most Economical Trapezoidal Section :

Consider a channel of trapezoidal section as shown the figure:



$$A = By + Zy^2$$

$$B = \frac{A}{y} - Zy$$

$$P = B + 2y\sqrt{1+Z^2}$$

$$P = \frac{A}{y} - Zy + 2y\sqrt{1+Z^2}$$

$$\frac{dP}{dy} = \frac{-A}{y^2} - Z + 2\sqrt{1+Z^2}$$

$$\therefore \frac{A}{y^2} + Z = 2\sqrt{1+Z^2}$$

$$\frac{By + Zy^2}{y^2} + Z = 2\sqrt{1+Z^2}$$

$$\frac{y(B+Zy)}{y^2} + Z = 2\sqrt{1+Z^2}$$

$$\frac{(B+Zy+Zy)}{y} = 2\sqrt{1+Z^2}$$

$$\frac{(B+2Zy)}{y} = 2\sqrt{1+Z^2}$$

$$R_h = \frac{A}{P} = \frac{By+Zy^2}{B+2y\sqrt{1+Z^2}} = \frac{y(B+Zy)}{B+(B+2Zy)}$$

$$= \frac{y(B+Zy)}{2B+2Zy}$$

$$R_h = \frac{y(B+Zy)}{2(B+Zy)}$$

$$\therefore R_h = \frac{y}{2}$$

The most economical trapezoidal section is one half of hexagon $,\theta = 60°$

Example: find the width of the best efficient channel if it has side slope and bed slope (1:1) and (1 to 1000) respectively the discharge is 15 m3/sec and Chezy coefficient C=60?

Sol.:

$$Q = CA\sqrt{R_h\,S_0}$$

$$\frac{B+2Zy}{y} = 2\sqrt{1+Z^2}$$

$$B = 2\sqrt{2y} - 2y \quad ,B = 0.828y$$

$$15 = 60(By + Zy^2)\sqrt{\frac{y}{2} \times \frac{1}{1000}}$$

$$\frac{15}{60} = (0.828y^2 + y^2)\sqrt{\frac{y}{2} \times \frac{1}{1000}}$$

$$\frac{0.25}{\sqrt{0.0005}} = 1.828y^{\frac{5}{2}}$$

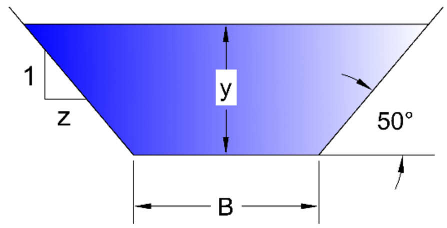
$$11.18 = 1.828y^{\frac{5}{2}}$$

$$y = 2.064\,m$$

$$B = 0.828(2.064) = 1.71\,m$$

Example: A trapezoidal channel having side slope equal to 50° with the horizontal as shown in the figure and laid on a slope of (1to 1000) the cross section area of the channel is 2 m^2 .find the discharge of this channel for the most economical cross section? Use C=66

Sol:



$$\tan 50° = \frac{1}{Z}$$

$$Z = 0.839$$

$$\frac{B + 2Zy}{y} = 2\sqrt{1 + Z^2}$$

$$B = 2y\sqrt{1 + Z^2} - 2Zy$$

$$B = 2y\sqrt{1 + (0.839)^2} - 2(0.839)y$$

$B = 0.932y$ ------------------------------------(1)

$$A = By + Zy^2$$

$$2 = (0.932y)y + (0.839)y^2$$

$$y = 1.0627\ m$$

$$B = 0.932(1.0627) = 0.99\ m$$

$$R_h = \frac{y}{2} = \frac{1.0627}{2} = 0.531m$$

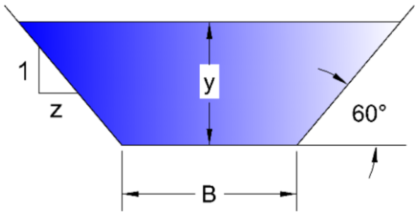
$$Q = CA\sqrt{R_h\ S_o}$$

$$Q = 66 \times 2\sqrt{(0531)(\frac{1}{1000})}$$

$$Q = 3.043\ m^3/sec$$

Example: A trapezoidal channel having side slope equal to 60° with the horizontal as shown in the figure and laid on a slope of (1to 750) carries discharge of 10 m^3/sec. find the width of the base and depth of flow for the most economical section, Take C=66.

Sol.:



$$\tan 60° = \frac{1}{Z}$$

$$Z = 0.5774$$

$$\frac{B + 2Zy}{y} = 2\sqrt{1 + Z^2}$$

$$B = 2y\sqrt{1 + Z^2} - 2Zy$$

$$B = 2y\sqrt{1 + (0.5774)^2} - 2(0.5774)y$$

$$B = 1.155y \quad \text{-----------------------------------}(1)$$

$$A = By + Zy^2$$

$$A = (1.155y)y + (0.577)y^2$$

$$A = 1.7325y^2 \quad \text{-----------------------------------}(2)$$

$$R_h = \frac{y}{2}$$

$$Q = CA\sqrt{R_h \ S_o}$$

$$10 = 66 \times (1.7325y^2)\sqrt{(\frac{y}{2})(\frac{1}{750})}$$

$$y = 1.625\ m$$

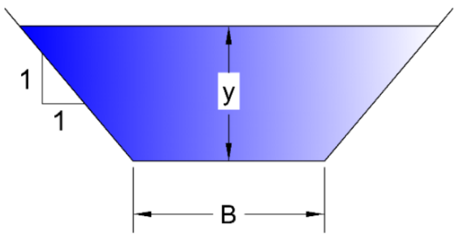
$$B = 1.155y$$

$$B = 1.155(1.625) = 1.876\ m$$

Example: A trapezoidal channel having side slope (1:1) it is required to discharge 13.75 m3/sec of water with a bed slope (1 to 1000). If this channel is unlined and the value of C=44 and when this channel is lined with concrete the value C=60. The cost per cubic meter of excavation is four times the cost per square meter of lining. The channel is to be the most efficient one find whether the lined channel or the unlined channel will be cheaper what will be the dimensions of the economical channel?

Sol.:

1- When the channel is unlined for most economical section.



$Q = CA\sqrt{R_h \ S_o}$

$B + 2Zy = 2y\sqrt{1 + Z^2}$

$B + 2y = 2y\sqrt{2}$

$B = 0.828y$

$A = By + Zy^2$

$A = (0.828y)y + y^2 = 1.828y2$

$R_h = \dfrac{y}{2}$

$$13.75 = 44 \times (1.828y^2)\sqrt{(\frac{y}{2})(\frac{1}{1000})}$$

$y = 2.256\, m$

$B = 0.828(2.256) = 1.876 \ m$

$A = 1.828(2.256)^2 = 9.303 \ m^2$

Let the cost of lining of square meters of concrete =x

Cost of excavation per m^3= 4 x

The cost of excavation per (1m) length of channel = volume of excavation =4x.

$Area \ \times 1 \times 4x = 9.303 \times 1 \times 4x = 37.212x$

2-When the channel is lined

$Q = CA\sqrt{R_h \ S_o}$

$$13.75 = 60 \times (1.828y^2)\sqrt{(\frac{y}{2})(\frac{1}{1000})}$$

$y = 1.993\, m$

$B = 0.828(1.993\,) = 1.65 \ m$

$A = 1.828(1.993)^2 = 7.26\ m^2$

The cost of lined channel = cost of excavation + cost of lining

$(A \times 1 \times 4x) + (P * 1 * x)$

$P = B + 2y\sqrt{1 + Z^2}$

$P = 1.65 + 2(1.993)\sqrt{1 + 1}$

$P = 7.287\ m$

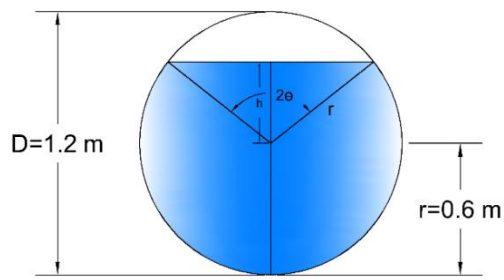
$(7.26 * 1 * 4x) + (7.287\ * 1 * x) = 36.327\ x$

The lined channel is cheaper

Example: A Circular channel of 1.2 m diameter is laid on a slope 1to 1500 . find the discharge through the channel when velocity of flow is maximum . Take n=0.015

Sol.:

For maximum velocity y=0.81 D



$$y = 0.81(1.2) = 0.972\ m$$

$$h = y - r$$

$$h = 0.972 - 0.6 = 0.372\ m$$

$$\cos\theta = \frac{h}{r}$$

$$\cos\theta = \frac{0.372}{0.6} = 0.62 \quad ,\theta = 51.68°$$

$$\sin\theta = \frac{x}{r}$$

$$\sin 51.68 = \frac{x}{0.6} \quad , \quad x = 0.47$$

$$2\theta = 2 \times 51.68 = 103.36°$$

$$2\alpha = 360 - 103.36 = 256.64°$$

Area= Area of sector + Area of triangle

$$= \pi r^2 \frac{2\alpha}{360} + \frac{1}{2} 2x \times h$$

$$= \pi(0.6)^2 \frac{256.64}{360} + 0.47 \times 0.372$$

$$= 0.806 + 0.175 = 0.98\ m^2$$

$$P = watted\ perimeter\ of\ sector$$

$$P = 2\pi r \frac{2\alpha}{360}$$

$$P = 2\pi(0.6)\frac{256.64}{360}$$

$$P = 2.69\ m$$

$$R = \frac{A}{P}$$

$$R = \frac{0.98}{2.69} = 0.36\ m$$

$$Q = \frac{1}{n} AR^{\frac{2}{3}} S^{\frac{1}{2}}$$

$$Q = \frac{1}{0.015}(0.98)(0.36)^{\frac{2}{3}}(\frac{1}{1500})^{\frac{1}{2}}$$

$$Q = 0.853\ m^3/sec$$