Basrah University

Engineering College

Civil Engineering Department

Fluid Mechanics/1

2nd Stage - First Semester

Edited by

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An Introduction to Fluid Mechanics Course

The course Fluid Mechanics is designed to introduce students to the fundamental engineering science concepts related to the mechanics of fluids. This includes basic fluid properties, fluid statics, fluid dynamics, fluid viscosity and turbulence, introduction to flow in closed conduits, pumps and pumping.
The aim of this course is to provide students with an understanding of the basic principles of fluid mechanics and of their application to Petroleum engineering problems. There is a strong focus on water and oil in the course as they of the most important fluids for engineering practice.

Objectives :

- The course will introduce fluid mechanics and establish its relevance in Petroleum engineering.
- Recognition of and develop the knowledge about the fundamental hydraulic definitions and the principle fluid properties underlying the subject.
- Establish how these definitions and properties are utilized to solve hydrostatical and hydro dynamical problems that may face the Petroleum engineer.

CONTENTS FOR THE 1-ST. SEMESTER

L1

Introduction

What we are meaning by Fluids?

Fluid may be defined as a substance which deforms continuously (flows) when subjected to shearing forces, or
A fluid is a substance which capable of flowing

A fluid has no definite shape unless it is supported (conforms to the shape of the containing vessel)

Mechanics is the field of science focused on the force, energy, motion, deformation interactions of material bodies based on their properties.

What are we meaning by Fluid Mechanics?

Fluid mechanics is the study of fluids, how they move, how they mix, how they interact with or how they effect on the bodies submerged within, and how they interact with and effect on the bodies that attached them and their reflections on human activities.

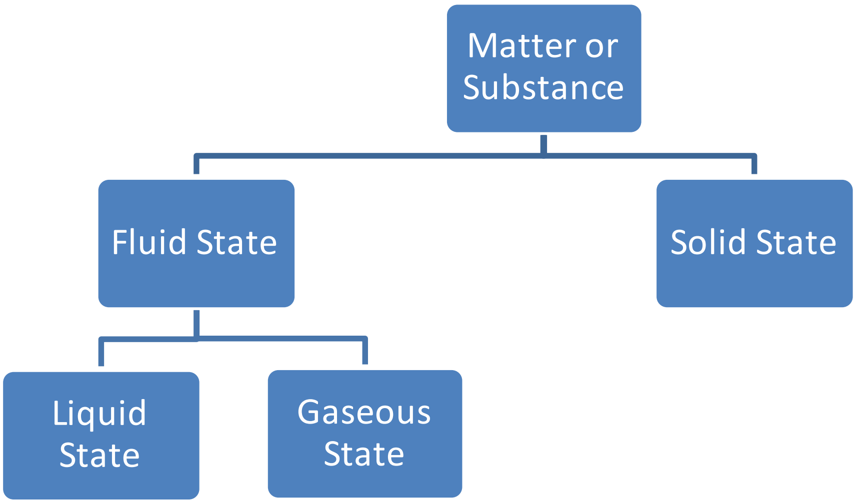
Fluid mechanics may be defined also as that branch of engineering science that deals with the behavior of fluid under the condition of rest and motion

Fluid mechanics may be divided into three parts: Statics, Kinematics, and Dynamics

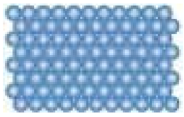
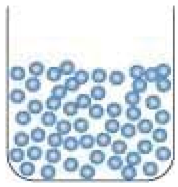
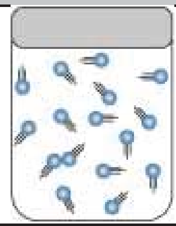
Statics Deals with fluid at rest in equilibrium state, no force no acceleration
Kinematics Deals With flow behaviors of fluid like velocity, acceleration and flow patterns.
Dynamics Deals with the effects of flow behaviors on fluid surroundings like forces and momentum exchange

The matter states

The matter or substance is classified on the bases of the spacing between the molecules of the matter as follows:



- In solids, the molecules are very closely spacing and then inter-molecules cohesive forces is quite large, and then possess compact and rigid form.
- Whereas in liquids these spacing are relatively large, and then less inter-molecules cohesive forces between them, and then can move freely, but it still has a definite volume (no definite shape, has free interface).
- While these forces is extremely small in gasses, and then have greater freedom of movement so that the gas fill the container completely in which they are placed (no definite volume, no definite shape, and no free interface).

Attribute	Solid	Liquid	Gas
Typical Visualization			
Macroscopic Description	Solids hold their shape; no need for a container	Liquids take the shape of the container and will stay in open container	Gases expand to fill a closed container
Mobility of Molecules	Molecules have low mobility because they are bound in a structure by strong intermolecular forces	Liquids typically flow easily even though there are strong intermolecular forces between molecules	Molecules move around freely with little interaction except during collisions; this is why gases expand to fill their container
Typical Density	Often high; e.g., density of steel is 7700 kg/m^3	Medium; e.g., density of water is 1000 kg/m^3	Small; e.g., density of air at sea level is 1.2 kg/m^3
Molecular Spacing	Small—molecules are close together	Small—molecules are held close together by intermolecular forces	Large—on average, molecules are far apart
Effect of Shear Stress	Produces deformation	Produces flow	Produces flow
Effect of Normal Stress	Produces deformation that may associate with volume change; can cause failure	Produces deformation associated with volume change	Produces deformation associated with volume change
Viscosity	NA	High; decreases as temperature increases	Low; increases as temperature increases
Compressibility	Difficult to compress; bulk modulus of steel is 160×10^9 Pa	Difficult to compress; bulk modulus of liquid water is 2.2×10^9 Pa	Easy to compress; bulk modulus of a gas at room conditions is about 1.0×10^5 Pa

System of units

MKS system of units

This is the system of units where the metre (m) is used for the unit of length, kilogram (kg) for the unit of mass, and second (s) for the unit of time as the base (primary) units.

CGS system of units

This is the system of units where the centimetre (cm) is used for length, gram (g) for mass, and second (s) for time as the base (primary) units.

International system of units (SI)

SI, the abbreviation of La Systeme International d'Unites, is the system developed from the **MKS** system of units. It is a consistent and reasonable system of units which makes it a rule to adopt only one unit for each of the various quantities used in such fields as science, education and industry. There are seven fundamental SI units, namely: metre (m) for length, kilogram (kg) for mass, second (s) for time, ampere (A) for electric current, kelvin (K) for thermodynamic temperature, mole (mol) for mass quantity and candela (cd) for intensity of light. Derived units consist of these units.

BASIC (PRIMARY) DIMENSIONS

Dimension	Symbol	Unit (SI)
Length	L	meter (m)
Mass	M	kilogram (kg)
Time	T	second (s)
Temperature	θ	kelvin (K)
Electric current	i	ampere (A)
Amount of light	C	candela (cd)
Amount of matter	N	mole (mol)

Quantity	SI Unit	
Velocity	m/s	
acceleration	m/s^2	
force	N kg m/s^2	
energy (or work)	Joule J N m, kg m^2/s^2	
power	Watt W N m/s kg m^2/s^3	
pressure (or stress)	Pascal P, N/m^2, kg/m/s^2	
density	kg/m^3	
specific weight	N/m^3 kg/m^2/s^2	
relative density	a ratio no units	
viscosity	N s/m^2 kg/m s	pa.s
surface tension	N/m kg /s^2	

Fluid properties

General fluid (liquid) properties:

There are many properties for fluids, but we will consider only main six characteristics:

1. Mass Density,
2. Weight Density,
3. Specific Volume,
4. Specific Gravity,
5. Viscosity and
6. surface tension

1. ***Mass Density***: the density (also known as ***specific mass*** or ***density***) of a liquid defined as the mass per unit volume at a standard temperature and pressure. It is usually denoted by Latin character ρ (rho). Its unit are Kg/m^3

$$\rho = \frac{m}{V}$$

ρ of water = 1000 kg/m^3 at 4^{o}C and 1 Atm.

$$\rho = f(P,T)$$

2. ***Weight Density:*** (also known as **specific weight**) is defined as the weight per unit volume at the standard temperature and pressure, it is usually denoted as γ. its unit ere N/m^3.

$$\gamma = \frac{W}{V} = \rho \times g$$

Where g gravitational acceleration=9.81 m/s^2

γ of water = 9810 N/m^3 at 4^{o}C and 1 Atm.

$$\gamma = f(P,T,g)$$

3. ***Specific Volume***: It is defined as a volume per unit mass of fluid, It is denoted by ***v***

$$v = \frac{V}{m} = \frac{1}{\rho}$$ Its unit are m^3/Kg.

4. ***Specific Gravity***: It is defined as the ratio of the **specific weight** of the fluid to the **specific weight** of a standard fluid

For liquids the standard fluid is pure water at the specified temperature, and denoted by ***Sg***

i.e. $$Sc = \left.\frac{\gamma_{liquid}}{\gamma_{water}}\right)_T$$

For Gasses the standard fluid is air

- As identical to specific gravity, ***Relative Density*** may come as the ratio of the **density** of the fluid to the **density** of a standard fluid

For liquids the standard fluid is pure water at the specified temperature, and denoted by ***rd***

i.e. $$\boldsymbol{rd} = \left.\frac{\rho_{liquid}}{\rho_{water}}\right)_T$$

Example (1):

Example: Calculate the Specific weight, specific mass, specific volume and specific gravity of a liquid having a volume of 6m^3 and weight of 44 kN at 4 °C.

:Solution

W=44 kN

V= 6 m^3

Specific weight, γ:

$$\gamma = \frac{W}{V} = \frac{44}{6} = 7.333\frac{kN}{m^3} = 7333\ {}^{N}/_{m^3}$$

Specific mass or density, ρ:

$$\rho = \frac{\gamma}{g} = \frac{7.333 \times 1000}{9.81} = 747.5\ kg/m^3$$

Specific volume, v:

$$v = \frac{1}{\rho} = \frac{1}{747.5} = 0.00134\ m^3/kg$$

Specific gravity, Sg:

$$Sg = \left.\frac{\gamma_{liquid}}{\gamma_{water}}\right)_{4^\circ C} = \frac{7333}{9810} = 0.747$$

Example2

A reservoir of glycerin (glyc) has a mass of 1200 kg and a volume of 0.952 m^3. Find the glycerin's weight (W), mass density (ρ), specific weight (γ), and specific gravity (s.g.).

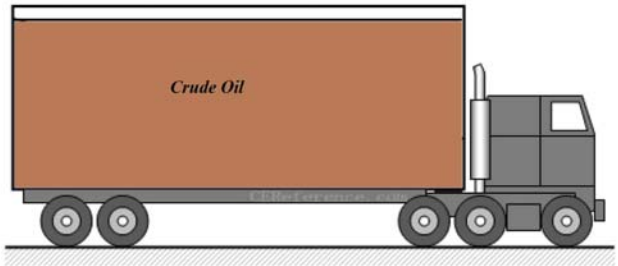
$$F = W = ma = (1200)(9.81) = 11\,770\ \text{N} \quad \text{or} \quad 11.77\ \text{kN}$$

$$\rho = m/V = 1200/0.952 = 1261\ \text{kg/m}^3$$

$$\gamma = W/V = 11.77/0.952 = 12.36\ \text{kN/m}^3$$

$$\text{s.g.} = \gamma_{glyc}/\gamma_{H_2O\ at\ 4^\circ C} = 12.36/9.81 = 1.26$$

Example 3: Calculate the Specific weight, specific mass, specific volum and the total weight of the crude oil truk having a volume of 36m^3 and S = 0.85 at 4 ^{o}C and the empty truck weigh = 16 T



Solution:

Given:

V=36m^3

S=0.85

Required : *Specific weight, Specific mass, Specific volume and Total Weight*

Solution:

Specific weight:

$$S = \frac{\gamma_{oil}}{\gamma_w} \rightarrow \gamma_{oil} = 0.85x9.81 = 8.33\ KN/m^3$$

Specific mass:

$$\rho = \frac{\gamma}{g} = \frac{8330}{9810} = 850\ Kg/m^3$$

Specific volume:

$$v = \frac{1}{\rho} = \frac{1}{850} = 0.00117\ m^3/Kg$$

Total Weight:

WT = W_{oil} + W_{trk}

$$\gamma_{oil} = \frac{W_{oil}}{V} \rightarrow W_{oil} = 8.33x36 = 299.88\ KN$$

WT = 299.88 + (16 x 1000 x 9.81)/1000 = 456.85 KN ≈ 45.7 T

Example 4

If 100 ml of oil weights 95 gr.

Calculate: Mass Density (ρ), Weight Density(γ), Specific Volume(V), Specific Gravity(S) and Relative Density(Rd) for this oil.

Solution: $m = 95gr = \frac{95}{1000} = 0.095\ kg$, $v = 100ml = \frac{100}{1000x1000} = 0.0001m^3$

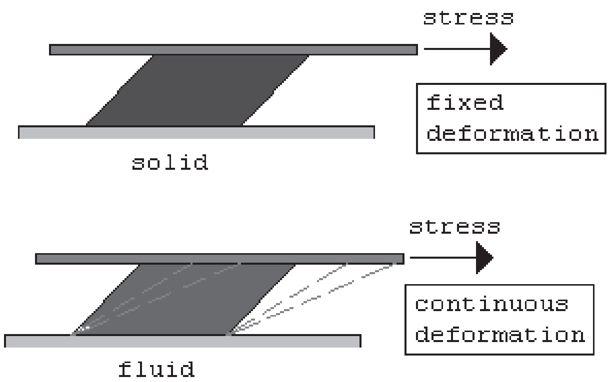
$\therefore \rho = \frac{m}{v} = \frac{0.095}{0.0001} = 950\ kg/m^3$, $\therefore \gamma = \rho.g = 950x9.81 = 9319.5\ N/m^3$

$\therefore V = \frac{v}{m} = \frac{0.0001}{0.095} = 1.05x10^{-3}\ m^3/\text{kg}$, $\therefore S = \frac{\gamma_{oil}}{\gamma_w} = \frac{9319.5}{9810} = 0.95$

$\therefore rd = \frac{\rho_{oil}}{\rho_w} = \frac{950}{1000} = 0.95$

L2

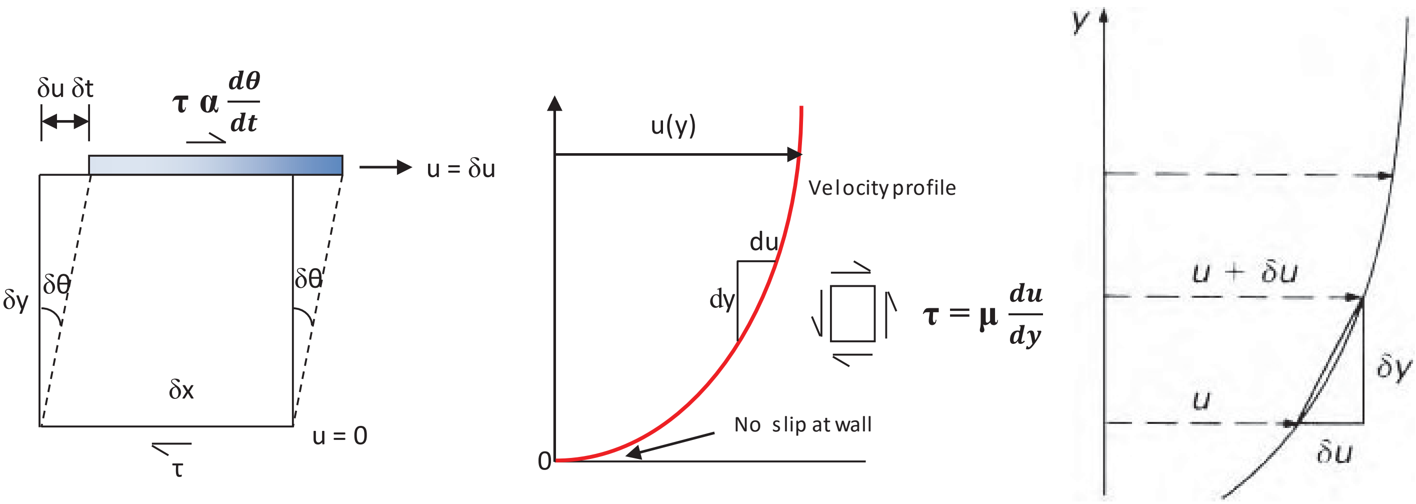
5. ***Viscosity***: it is a property of a real fluid (an ideal fluid has no viscosity) which determine its resistance to shearing stresses. It is primarily due to cohesion, adhesion and molecular momentum exchange between fluid layers.



1 - For solids, shear stress reflect on magnitude of angular deformation (τ ~ *angular deformation*, *θ*)

2 – For many fluids shear stress is proportional to the *time rate* of angular deformation (τ ~ *dθ/dt*)

When tow layer of fluid at the distance of δy apart, move one over the other at different velocities, say u and u+δu, the viscosity together with relative velocity



causes shear stress acting between layers. With respect to the distance between these two layers δy, the shear stress, τ, proportional to angular deformation

$$\tau \propto \frac{\delta\theta}{\delta t}$$

From the geometry of Fig. we see that

$$\tan\delta\theta = \frac{\delta u\ \delta t}{\delta y}$$

In the limit of infinitesimal changes, this becomes a relation between shear strain rate and velocity gradient:

$$\frac{d\theta}{dt} = \frac{du}{dy} \quad \Longrightarrow \quad \tau \propto \frac{\delta u}{\delta y}$$

Newton's law of viscosity: the shear stresses on a fluid element layers is directly proportional to the velocity gradient (rate of shear strain). The constant of proportionality is called the coefficient of viscosity (*absolute viscosity, dynamic viscosity,* or simply *viscosity*) and denoted as μ (mu).

i.e.

$$\boxed{\tau = \mu \frac{du}{dy}} \quad \boxed{= \mu \frac{\Delta V}{\Delta Y}}$$

Coefficient of Dynamic Viscosity: $\mu = \frac{\tau}{\frac{du}{dy}}$

Units: N s/m^2 or Pa s or kg/m s
The unit Poise (p) is also used where 10P = 1 Pa·s (1P = 0.1 Pa·s)
Water μ = 8.94 × 10^{-4} Pa s at 25 °C
Water μ = 1.00 × 10^{-3} Pa s at 20 °C
Mercury μ = 1.526 × 10^{-3} Pa s
Olive oil μ = .081 Pa s

Kinematic Viscosity, υ = the ratio of dynamic viscosity to mass density

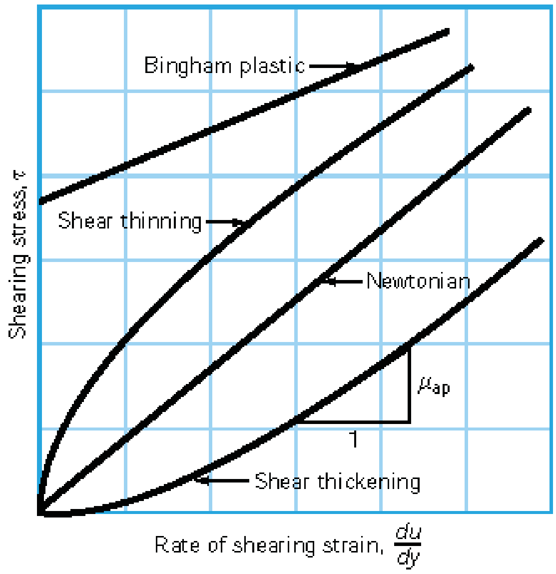
$$\upsilon = \frac{\mu}{\rho}$$

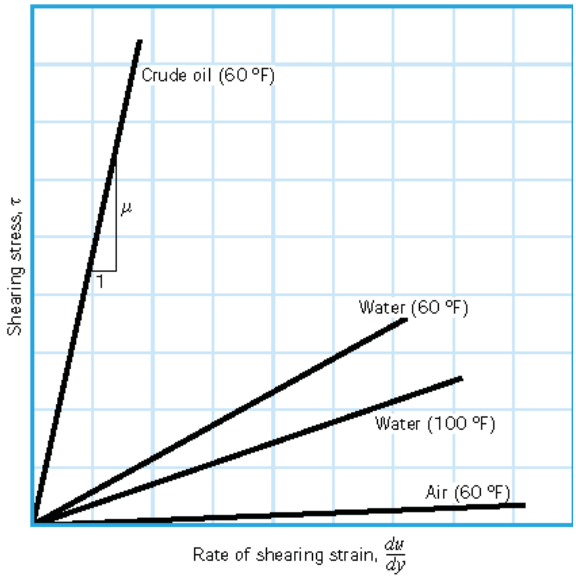
Units m^2/s and Called kinematic viscosity because it involves no force (dynamic) dimensions .
The unit Stoke (St) is also used where 1St = 10^{-4} m^2/s (1St=cm^2/s)

Water v = 1.7 × 10^{-6} m^2/s. at 0 °C

Water v = 1.00 × 10^{-6} m^2/s. at 20 °C

Air v = 1.5 × 10^{-5} m^2/s.

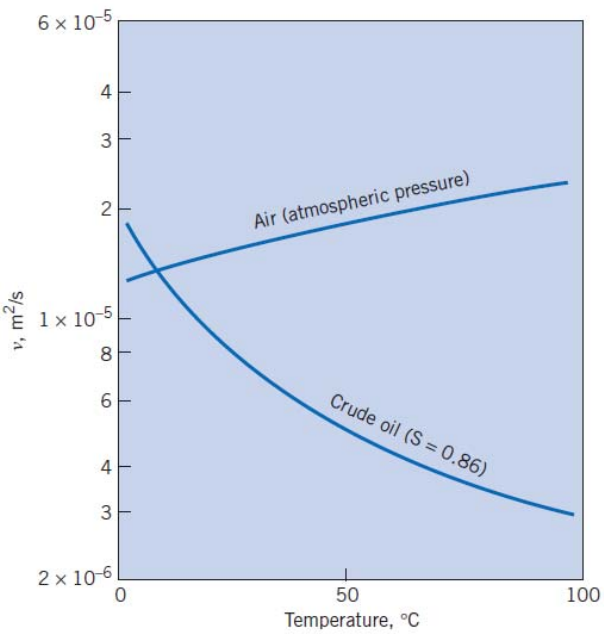




- The fluid is **non-Newtonian** if the relation between shear stress and shear strain rate is **non-linear**
- Typically, as temperature increases, the viscosity will decrease for a liquid, but will increase for a gas.

Figure 2.3

Kinematic viscosity for air and crude oil.



Note:

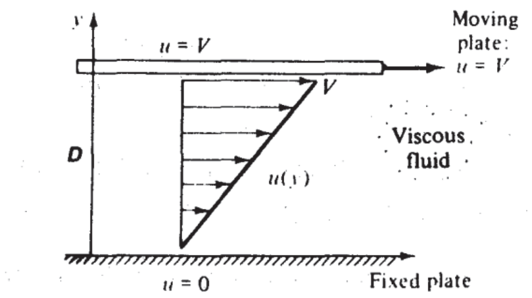
To convert from rotational (RPM) to linear velocity(v) : $\boxed{v\left(\frac{m}{s}\right) = RPM.\frac{2\pi.r}{60}}$

To convert from Torque stress (T) to shear stress (τ) : $\boxed{\tau = \frac{T}{2\pi.h.r^2}}$

Example 1

In figure if the fluid is oil at 20°C (μ = 0.44 Pa.s). What shear stress is required to move the upper plate at 3.5 m/s?

D=7mm



Solution:

$$\tau = \mu \frac{du}{dy} = 0.44\ Pa.s \times \frac{3.5\ m/s}{\frac{7}{1000}m} = 220\ Pa$$

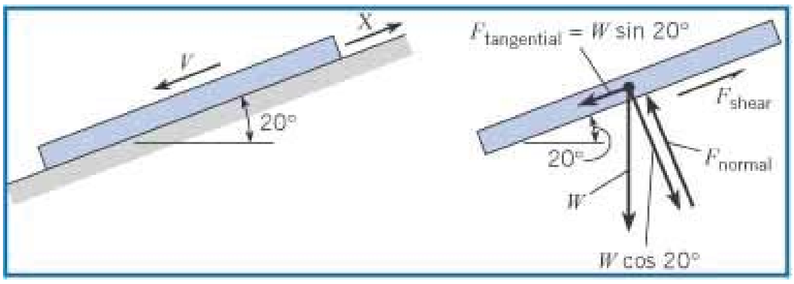
Example 2

A board 1 m by 1 m that weighs 25 N slides down an inclined ramp (slope = 20°) with a velocity of 2.0 cm/s. The board is separated from the ramp by a thin film of oil with a viscosity of 0.05 N.s/m^2. Neglecting edge effects, calculate the space between the board and the ramp.

Problem Definition

Situation: A board is sliding down a ramp, on a thin film of oil.
Find: Space (in m) between the board and the ramp.
Assumptions: A linear velocity distribution in the oil.

Properties: Oil, $\mu = 0.05$ N · s/m2.
Sketch:



Plan

1. Draw a free body diagram of the board, as shown in "sketch."

· For a constant sliding velocity, the resisting shear force is equal to the component of weight parallel to the inclined ramp (equilibrium condition must be exist).

· Relate shear force to viscosity and velocity distribution.

2. With a linear velocity distribution, dV/dy can everywhere be expressed as $\Delta V/\Delta y$, where ΔV is the velocity of the board, and Δy is the space between the board and the ramp.

3. Solve for Δy.

Solution

1. Free-body analysis

$$F_{tangential} = F_{shear}$$
$$W.\sin 20^o = \tau \times Area$$
$$W.\sin 20^o = \mu \frac{dV}{dy} A$$

2. Substitution of dV/dy as $\Delta V/\Delta y$

$$W.\sin 20^o = \mu \frac{\Delta V}{\Delta y} A$$

3. Solution for Δy

$$\Delta y = \mu \frac{\Delta V}{W.\sin 20^o} A$$
$$\Delta y = 0.05 \frac{0.02}{25.\sin 20^o} 1 = 0.000117\ m = 0.117\ mm$$

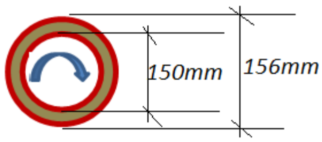
Example 3

Oil has dynamic viscosity ($\mu = 1.0 \times 10^{-3}$ Pa.s) filled the space between two concentric cylinders, where the inner one is movable and the outer is fixed. If the inner and outer cylinders has diameters 150mm and 156mm respectively and the height of both cylinders is 250mm, determine the value of the torque (T) that necessary to rotate the internal cylinder with 12 rpm?

Solution:

$$\tau = \mu \frac{\Delta V}{\Delta Y}$$

$$v = \frac{rpm}{60} \times 2\pi r = \frac{12}{60} \times 2\pi \times 0.075 = 0.09425\ \text{m/s}$$



$$T = \tau Chr$$

$$\tau = T(2\pi r \times h \times r)^{-1} = T(2\pi hr^2)^{-1}$$

$$\tau = \mu \frac{dv}{dy} = \mu \frac{\Delta v}{\Delta y}$$

$$\tau = 10^{-3} \times \frac{0.09425}{0.003} = 31.41667 \times 10^{-2} \frac{N}{m^2}$$

$$T = 31.41667 \times 10^{-2} \times 2 \times .075 \times \pi \times .25 \times .075 = 2.7 \times 10^{-4}\ N.m$$

Example 4

Example 1.17. *In the Fig. 1.14 is shown a central plate of area 6 m^2 being pulled with a force of 160 N. If the dynamic viscosities of the two oils are in the ratio of 1:3 and the viscosity of top oil is 0.12 N.s/m^2 determine the velocity at which the central plate will move.*

Solution: Area of the plate, $A = 6$ m^2

Force applied to the plate, $F = 160$ N

Viscosity of top oil, $\mu = 0.12$ N.s/m^2

Velocity of the plate, *u*:

Let F_1 = Shear force in the upper side of *thin* (assumed) plate,

F_2 = Shear force on the lower side of the thin plate, and

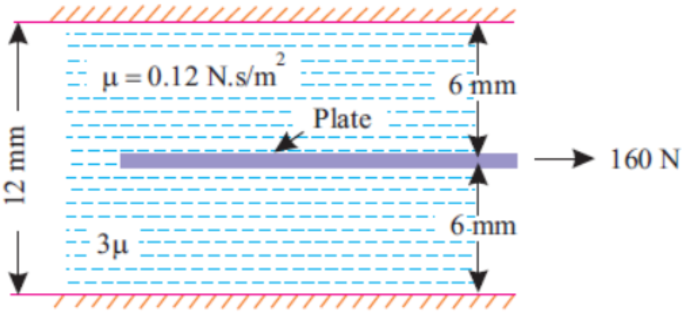


Fig. 1.14

F = Total force required to drag the plate $(= F_1 + F_2)$

Then, $F = F_1 + F_2 = \tau_1 \times A + \tau_2 \times A$

$$= \mu\left(\frac{\partial u}{\partial y}\right)_1 \times A + 3\mu\left(\frac{du}{dy}\right)_2 \times A$$

(where τ_1 and τ_2 are the shear stresses on the two sides of the plate)

$$160 = 0.12 \times \frac{u}{6 \times 10^{-3}} \times 6 + 3 \times 0.12 \times \frac{u}{6 \times 10^{-3}} \times 6$$

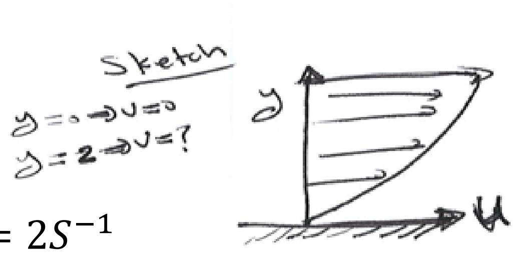
or $\quad 160 = 120u + 360u = 480u \quad$ or $\quad u = \frac{160}{480}$ = **0.333 m/s (Ans.)**

Example 5

The velocity distribution for flow over a plate is given by u= $2y+y^2$ where u is the velocity in m/s at a distance y meters above the plate surface. Determine the velocity gradient and shear stresses at the boundary and 1.5m from it. Take dynamic viscosity of fluid as 0.9 N.s/m^2

Solution

$$u = 2y + y^2 \quad \rightarrow \frac{du}{dy} = 2 + 2y$$



$$velosity\ gradiant\ \left(\frac{du}{dy}\right) at\ (y = 0) = 2S^{-1}$$

$$velosity\ gradiant\ \left(\frac{du}{dy}\right) at\ (y = 1.5) = 2 + 2 * 1.5 = 5S^{-1}$$

$$\tau = \mu.\left(\frac{du}{dy}\right) \rightarrow at\ (y = 0) \rightarrow \tau = 2x0.9 = 1.8\ Pa$$

$$\tau = \mu.\left(\frac{du}{dy}\right) \rightarrow at\ (y = 1.5) \rightarrow \tau = 5x0.9 = 4.5\ Pa$$

1.80 Example 6

A square block weighing 1.1 kN and 250 mm on an edge slides down an incline on a film of oil 6.0 μm thick (see Fig. 1-6). Assuming a linear velocity profile in the oil, what is the terminal speed of the block? The viscosity of the oil is $7x10^{-3}$ Pa.S

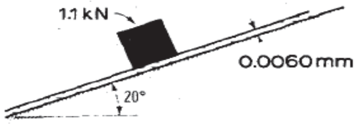


Fig. 1-6(a)

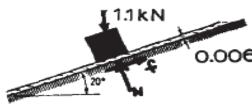


Fig. 1-6(b)

Solution:

Given:

$\mu = 7x10^{-3}$ *pa.s*, *y=6x10^-6^*, *F= 1.1 KN*

$$\tau = \mu\frac{dv}{dy} \rightarrow \tau = \frac{Ft}{A} = 1100 \sin 20/0.25^2 = 6019.5\ Pa$$

$$\rightarrow 6019.5 = 7x10^{-3}\frac{v}{6x10^{-6}} \rightarrow v = 5.16\ m/s$$

Example 7

1.81 A shaft 70.0 mm in diameter is being pushed at a speed of 400 mm/s through a bearing sleeve 70.2 mm in diameter and 250 mm long. The clearance, assumed uniform, is filled with oil at 20 °C with $\nu = 0.005\ m^2/s$ and s.g. = 0.9. Find the force exerted by the oil on the shaft.

Solution:

Given:

V= 400mm/s, l= 250 mm, υ= 0.005 m²/s, S=0.9

$$\tau = \mu\frac{dv}{dr}$$

$$\mu = \rho x \upsilon \ \ and \ \ \rho_{oil} = \frac{\gamma_{oil}}{g} \ \ and \ \ \gamma_{oil} = Sx\gamma_w = 0.9x9810 = \frac{8829N}{m^3}$$

$$\rightarrow \rho_{oil} = \frac{8829}{9.81} = 900\frac{Kg}{m^3} \rightarrow \mu = 900x0.005 = 4.5\ Pa.s$$

r=(70.2-70)/2= 0.1x10⁻³ m

$$\therefore \tau = 4.5x\frac{400x10^{-3}}{0.1x10^{-3}} = 17960\ Pa \ \ then\ F = \tau x A s$$
$$= 17960 x\pi x 70x10^{-3}x0.25 = 987.4\ N$$

Example 8

1.82 If the shaft in Prob. 1.81 is fixed axially and rotated inside the sleeve at 2000 rpm, determine the resisting torque exerted by the oil and the power required to rotate the shaft.

Solution:

Given: *V= 2000 rpm, l= 250 mm, υ= 0.005 m²/s, S=0.9*

$\tau = \mu\frac{dv}{dr}$ and $\mu = 4.5\ Pa.s$

r=(70.2-70)/2= 0.1x10⁻³ m

$$V = \frac{2\pi r}{60}x(rpm) \rightarrow V = \frac{2\pi x35x10^{-3}x2000}{60} = 7.33\frac{m}{s}.$$

$$\therefore \tau = 4.5x\frac{7.33}{0.1x10^{-3}} = 329.1x\ 10^3\ Pa$$

$$then\ T = \tau x A s x r = \ 329.1x\ 10^3 x70x\ 10^{-3}x\pi x0.25\ x\frac{70}{2}x10^{-3}$$
$$= 633\ N.m$$

H.W

1.97 Determine the viscosity of fluid between shaft and sleeve in Fig. 1-18.

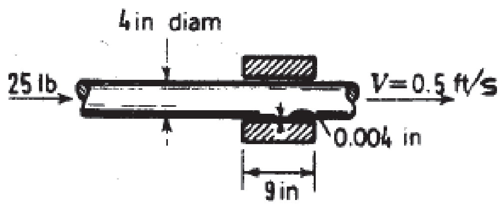
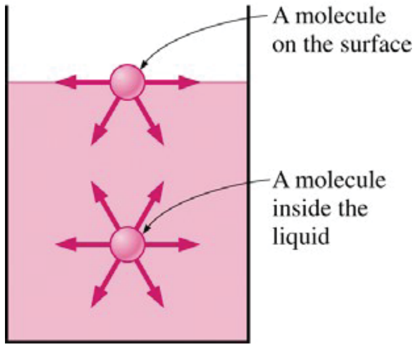


Fig. 1-18 ans 1.015 pa.s $= 0.0212\ lb \cdot s/ft^2$

6. ***Surface Tension***: Surface tension is a property of **liquids** which is making what is like a thin tensioned membrane at the interface between the liquid and another fluid (typically a gas). Surface tension has dimensions of force per unit length and denoted as, σ (Sigma), and its unit is N/m.

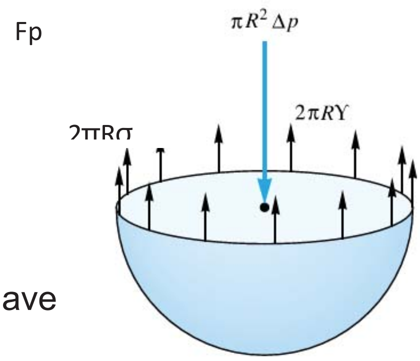


- ***It is a fluid (liquid)*-fluid (gas) interface property**

Surface tension is a properties of certain **fluid-fluid interface**
water-air ….. 0.075 N/m at 20°C   Water-air …. 0.056 N/m at 100°C
mercury-air … 0.1 N/m

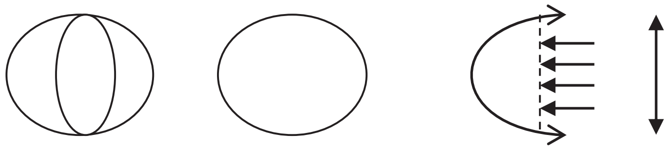
Pressure inside water droplet:

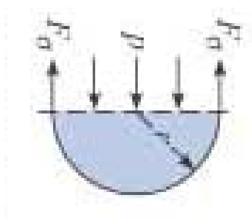
let P= The pressure inside the drop



d= Diameter of droplet
σ= Surface tension of the liquid (water-air interface)

From sectional free body diagram of water droplet we have



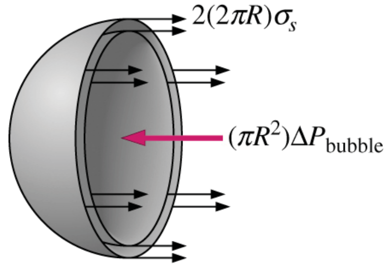


1. ΔP between inside and outside = P-0 =P
2. Pressure force $= P \times \frac{\pi}{4} d^2, and$
3. Surface tension force acting around the circumference= $\sigma \times \pi d$,
   under equilibrium condition these two forces will be equal and opposite, i.e.
   $P \times \frac{\pi}{4} d^2 = \sigma \times \pi d$

$$P = \frac{\sigma \times \pi d}{\frac{\pi}{4} d^2} = \frac{4\sigma}{d}$$

From this equation we show that (with an increase in size of droplet the pressure intensity is decreases)

- ***Derive P for air bubble with the help of figure below***



(b) Half a bubble

Table 1 Surface Tensions of Common Liquids

Liquid	Surface Tension γ (N/m)
Benzene (20 °C)	0.029
Blood (37 °C)	0.058
Glycerin (20 °C)	0.063
Mercury (20 °C)	0.47
Water (20 °C)	0.073
Water (100 °C)	0.059

***Example* 1:**

If the surface tension of water-air interface is 0.069 N/m, what is the pressure inside the water droplet of diameter 0.009 mm?

Solution:

Given d= 0.009 mm; σ= 0.069 N/m

The water droplet has only one surface, hence,

$$P = \frac{4\sigma}{d} = \frac{4 \times 0.069}{0.009 \times 10^{-3}} = 30667\frac{N}{m^2} = \mathbf{30.667}\frac{\boldsymbol{kN}}{\boldsymbol{m^2}}\,\boldsymbol{or\ kPa}$$

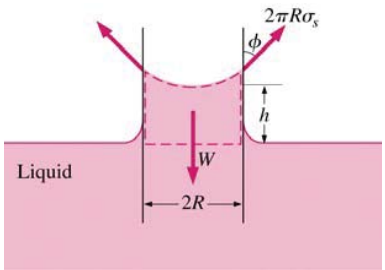
Surface Tension - Capillarity

- Property of exerting forces on fluids by fine tubes and porous media, due to both cohesion and adhesion (**surface tension**)
- Cohesion < adhesion, liquid wets solid, rises at point of contact
- Cohesion > adhesion, liquid surface depresses at point of contact, non-wetting fluid
- ***The contact angle*** is defined as the angle between the liquid and solid surface.
- ***Capillarity is a fluid (liquid)-surface property***
- **Meniscus**: curved liquid surface that develops in a tube

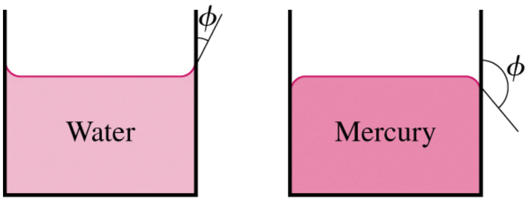
weight of fluid column = surface tension pulling force

$$\rho g(\pi R^2 h) = 2\pi R\sigma\, cos\emptyset$$

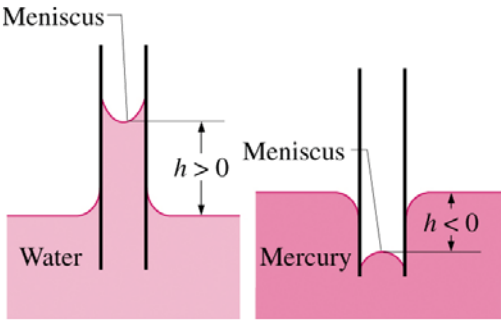
$$\boldsymbol{h} = \frac{\boldsymbol{2\sigma\, cos\emptyset}}{\boldsymbol{\rho g R}}$$



- Expression above calculates the *approximate* capillary rise in a small tube
- The meniscus lifts a small amount of liquid near the tube walls, as *r* increases this amount may become insignificant
- Thus, the equation developed *overestimates* the amount of capillary rise or depression, particularly for large *r*.
- For a clean tube, $\emptyset = 0_o$ for water, $\emptyset$ = 140_o for mercury
- For *r* > *¼ in* (*6 mm*), capillarity is negligible . Its effects are negligible in most engineering situations.
- Important in problems involving capillary rise, e.g., soil water zone, water supply to plants
- When small tubes are used for measuring properties, e.g., pressure, account must be made for capillarity



(*a*) Wetting fluid (*b*) Nonwetting fluid



Example 2:

To what height above the reservoir level will water (at 20°C) rise in a glass tube, such as that shown in Figure below, if the inside diameter of the tube is 1.6 mm?

Properties: Water (20 °C), σ = 0.073 N/m; γ = 9790 N/m^3

Solution

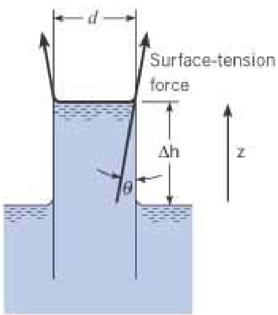
1. Force balance: Weight of water (down) is balanced by surface tension force (up).

$$F_{\sigma,z} - W = 0$$

$$\sigma\pi d\cos\theta - \gamma(\Delta h)(\pi d^2/4) = 0$$

; Because the contact angle θ for water against glass is so small, it can be assumed to be 0° :therefore *cos θ* ≈ 1. Therefore

$$\sigma\pi d - \gamma(\Delta h)\left(\frac{\pi d^2}{4}\right) = 0$$



2. Solve for Δ*h*

$$\Delta h = \frac{4\sigma}{\gamma d} = \frac{4 \times 0.073\ \text{N/m}}{9790\ \text{N/m}^3 \times 1.6 \times 10^{-3}\ \text{m}} = \boxed{18.6\ \text{mm}}$$

Example 3

Find the capillary rise in the tube shown in Fig. 1-26 for a water–air–glass interface ($\theta = 0°$) if the tube radius is 1 mm and the temperature is 20 °C.

$$h = \frac{2\sigma \cos\theta}{\rho g r} = \frac{(2)(0.0728)(\cos 0°)}{(1000)(9.81)(\frac{1}{1000})} = 0.0148 \text{ m} \quad \text{or} \quad 14.8 \text{ mm}$$

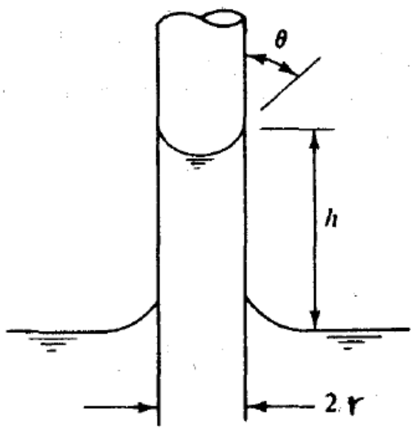


Fig. 1-26

Example 4

A clean tube of diameter 2.5 mm is immersed in a liquid with a coefficient of surface tension = 0.4 N/m. the angle of contact of the liquid with the clean glass can be assumed to be 135°. The density of the liquid= 13600 kg/m³. What would be the level of the liquid in tube relative to free surface of the liquid inside the tube?

Solution:

Given d= 2.5 mm, σ= 4 N/m, $\emptyset$ = 135°; ρ = 13600 kg/m³

Level of the liquid in the tube, h:

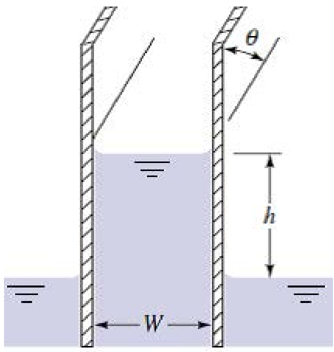
$$\boldsymbol{h = \frac{2\sigma\, cos\emptyset}{\rho g R}}$$

$$h = \frac{4 \times 0.4 \times cos135}{(9.81 \times 13600) \times 2.5 \times 10^{-3}}$$

$$= -3.3910^{-3}m \quad or - 3.39mm$$

Negative sign indicates that there is a capillary depression (fall) of 3.39 mm.

Example 5: Derive an expression for the capillary height change *h*, as shown, for a fluid of surface tension σ and contact angle between two parallel plates W apart. .Evaluate *h* for water at 20°C (σ=0.0728 N/m) if W = 0.5 mm



Solution: With b the width of the plates into the paper, the capillary forces on each wall together balance the weight of water held above the reservoir free surface:

$$hb\rho gW = 2(\sigma b cos\emptyset)$$
$$\boldsymbol{h = \frac{2(\sigma cos\emptyset)}{\rho g W}}$$

for water at 20°C (σ=0.0728 N/m, $\gamma = 9790\ N/m^3$) and W = 0.5 mm.

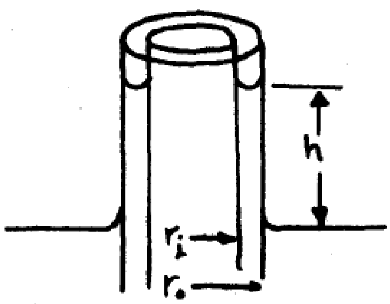
$$\boldsymbol{h = \frac{2 \times (0.0728 \times cos(0))}{9790 \times 0.0005} = 0.03m = 30mm}$$

H.W

1.104 Two clean, parallel glass plates, separated by a distance $d = 1.5$ mm, are dipped in a bath of water. How far does the water rise due to capillary action, if $\sigma = 0.0730$ N/m?

$h = 0.00994$ m

1.124 Develop a formula for capillary rise between two concentric glass tubes of radii r_o and r_i and contact angle θ.



$$\frac{2\sigma \cos\theta}{\gamma(r_o - r_i)}$$

L3

The (fluid at rest), fluid statics, or hydrostatics

Pressure is defined as the ratio of normal force to area at a point***, or*** may be defined as the normal force that's applied toward the unit area, and denoted by ***P***. Its units are ***N/m²*** or what is called ***Pascal, Pa.***

Highlights

- Fluid exerted, in general, both normal (due to their weights) and shearing forces (primary due to their viscosity) on surfaces (any plane) that are contacted with (or submerged in) it.

- The normal forces that are exerted by **fluid weights** is called the fluid pressure force and fluid pressure or intensity of fluid pressure. So the pressure can be defined also as **the weight of fluid column intensity above a certain area**.

- The source of pressure and its effects and its variation of a ***fluid at rest*** is due only to the weight of the fluid.

- Pressure is a scalar quantity; that is, it has magnitude only.

- Pressure is not a force; rather it is a scalar that produces a resultant force by its action on an area.

- The resultant force is normal to the area and acts in a direction toward the surface (compressive).

- **Fluids at rest cannot resist a shear stress**; in other words, when a shear stress is applied to a fluid at rest, the fluid will not remain at rest, but will move (flow) because of the shear stress.

- **Hydrostatics** is the study of pressures throughout a fluid at rest
- The controlling laws are relatively simple, and analysis is based on a straight forward application of the mechanical principles of force and moment.

Pressure Units

- Some units for pressure give a ratio of force to area. **Newtons per square meter of area, or pascals (Pa), is the SI unit**. The traditional units include psi, which is pounds-force per square inch, and psf, which is pounds-force per square foot.
- Other units for pressure give the height of a column of liquid. Engineers state that the pressure in the balloon is 20 cm of water: When pressure is given in units of "height of a fluid column," the pressure value can be directly converted to other units using Table below.

Pressure Units

	Pascal **(Pa)**	bar **(bar)**	technical atmosphere **(at)**	**atmosphere (atm)**	torr **(Torr)** mmHg	**pound-force /in^2 (psi)**	**m of water**
1 Pa	≡1 N/m^2	10^{-5}	1.0197×10^{-5}	9.8692×10^{-6}	7.5006×10^{-3}	145.04×10^{-6}	10.19×10^{-5}
1 bar	100,000	≡ 10^6 dyn/cm^2	1.0197	0.98692	750.06	14.5037744	10.1936
1 at	98,066.5	0.980665	≡1 kgf/cm^2	0.96784	735.56	14.223	9.9966
1 atm	101,325	1.01325	1.0332	≡1 **atm**	760	14.696	10.33
1 torr	133.322	1.3332×10^{-3}	1.3595×10^{-3}	1.3158×10^{-3}	≡1 Torr; ≈ 1 mmHg	19.337×10^{-3}	13.59×10^{-3}
1 psi	6.894×10^3	68.948×10^{-3}	70.307×10^{-3}	68.046×10^{-3}	51.715	≡1 lbf/in^2	0.703
1 m water	9813.54	0.0981	0.10003	0.0968	73.584	1.4225	≡1 m water

Example 1: 1 Pa = 1 N/m^2 = 10^{-5} bar = 10.197×10^{-6} at = 9.8692×10^{-6} atm , etc.

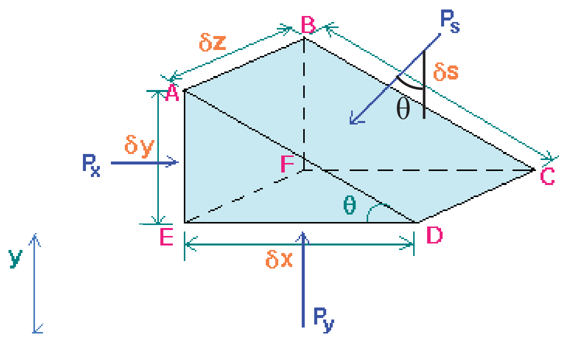
A pressure of 1 atm (Standard atmospheric pressure) can also be stated as:

≡ 1.013 25 bar
≡ 1013.25 hectopascal (hPa)
≡ 1013.25 millibars (mbar, also mb)
≡ 760 torr [B]
≈ 760.001 mm-Hg, 0 °C≈ 1.033 227 452 799 886 kgf/cm²
≈ 1.033 227 452 799 886 technical atmosphere
≈ 1033.227 452 799 886 cm–H_2O, 4 °C

Pressure at Point

- **At a point, fluid at rest has the same pressure in all direction.**

To prove this, a small wedge-shaped free body element is taken at the point (x,y,z) in a fluid at rest.



$$\sum f_x = P_x \cdot \delta y \delta z - P_s \cdot \delta s \delta z \cdot sin\theta = 0 \quad \ldots\ldots\ldots 1$$

$$\sum f_y = P_y \cdot \delta x \delta z - P_s \cdot \delta s \delta z \cdot cos\theta - \frac{1}{2}\delta x \delta y \delta z \cdot \gamma = 0 \ldots\ldots\ldots\ldots 2$$

For unit width of element in z direction, and from the geometry of wedge we have the follows:

$$\delta s \cdot sin\theta = \delta y \,, \quad and \quad \delta s \cdot cos\theta = \delta x \ldots\ldots\ldots\ldots 3$$

Substitute of eq.3 in eqs. 1 and 2 and rearrange the terms yields:

$$P_x = P_s$$

$$P_y \cdot \delta x = P_s \cdot \delta x + \frac{1}{2}\delta y \delta x \cdot \gamma$$

At a point the element limits to have an infinitesimal dimensions and then we can eliminate the term ($\frac{1}{2}\delta y \delta x \cdot \gamma$) from the above equation because of it's a higher order of differential values. Thus we have at final that:

$$P_x = P_s = P_y$$

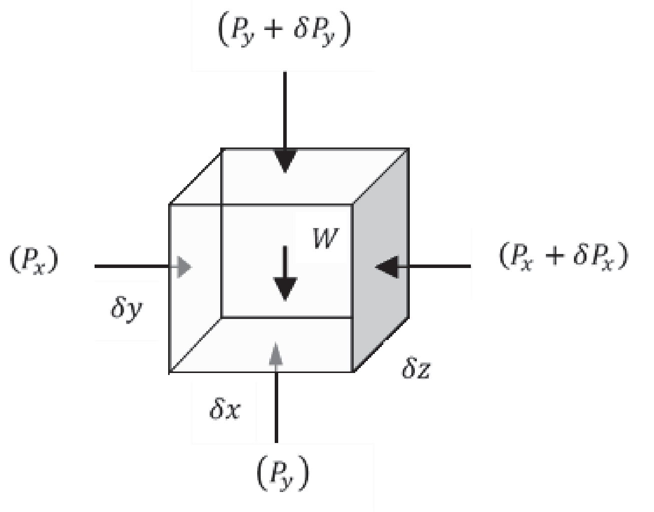
Where θ is an arbitrary angle, these results gives an important first principle of hydrostatics:

At a point, fluid at rest has the same pressure in all direction.

Pressure variation:

- **For static fluid, pressure varies only with elevation (depth) change within fluid.**

To prove this real, we take a cubic fluid element as shown



While fluid at rest, applying the equations of equilibrium on the element. That's yield:

1. **In vertical direction-y:**

$$\sum f_y = P_y \cdot \delta x \delta z - (P_y + \delta P_y) \cdot \delta x \delta z - \delta x \delta y \delta z \cdot \gamma = 0$$

$$P_y \cdot \delta x \delta z - P_y \cdot \delta x \delta z - \delta P_y \cdot \delta x \delta z - \delta x \delta y \delta z \cdot \gamma = 0$$

$$\rightarrow \quad \delta P_y \cdot \delta x \delta z = -\delta x \delta y \delta z \cdot \gamma$$

$$\rightarrow \quad \delta P_y = -\gamma \cdot \delta y$$

For certain fluid surface elevation, when the direction of δy downward away from surface (means in the negative direction of *y*), this called the depth difference and denoted as δh, so the last above equation become:

$$\delta P_y = \gamma \cdot \delta h$$

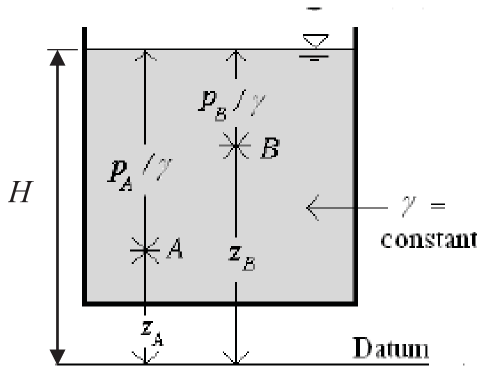
these results gives an important second principle of hydrostatics:

- **For static fluid, pressure varies only with elevation (depth) change within fluid by rate equal to specific weight γ of that fluid.**
- In a fluid, pressure decreases linearly with increase in elevation (height, y or z) and versa visa.
- In most textbooks and reference applications, they are use z-coordinate instead of y-coordinate as vertical direction axis so:
  $\Delta P_y = -\gamma \cdot \Delta y$ becomes $\Delta P_z = -\gamma \cdot \Delta z$
- Second principle of hydrostatics means that for any two point in a same continuous fluid A and B:

$$\Delta P_{A-B} = -\gamma \cdot \Delta z_{A-B}$$

$$P_B - P_A = -\gamma \cdot (z_B - z_A)$$

$$\boxed{\frac{P_B}{\gamma} + z_B = \frac{P_A}{\gamma} + z_A = H}$$



- This is ***the hydrostatics equation*** and ***H*** called ***the hydrostatics***

pressure head or what is called ***piezometric head***. With liquids we normally measure from the surface.

- Open free surface pressure in liquids mostly is atmospheric, $P_{atmospheric}$.
- For constant density fluids, and if taking the free surface pressure (atmospheric pressure, $P_{atmospheric}$) as zero, the pressure at any depth h becomes:

$$\boxed{P_h = \gamma \cdot h}$$

- Thus $h = \frac{P_h}{\gamma}$
- Pressure related to the depth, h, of a fluid column referred to as the pressure head, *h.*

2. **In horizontal direction**-x:

$$\sum f_x = P_x \cdot \delta y \delta z - (P_x + \delta P_x) \cdot \delta y \delta z = 0$$

$$P_x \cdot \delta y \delta z - P_x \cdot \delta y \delta z - \delta P_x \cdot \delta y \delta z = 0$$

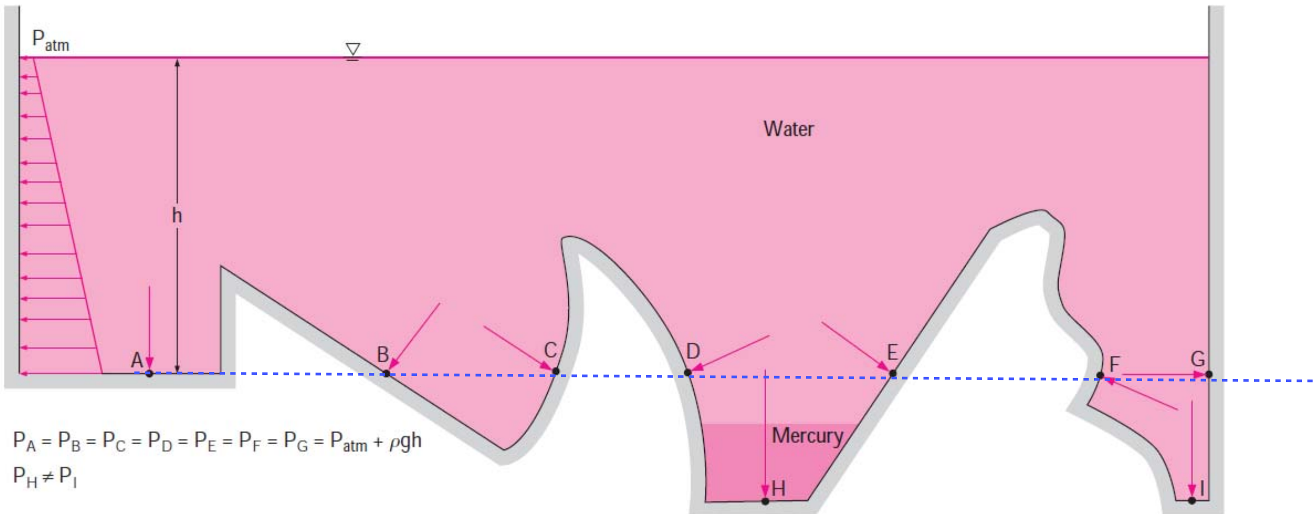
$$\rightarrow \ \delta P_x = 0$$

This equation means there is no change in horizontal pressure with horizontal direction.

These results gives an important third principle of hydrostatics:

- **For certain continuous static fluid, there is no pressure change in horizontal direction (explain!)**

The above mentioned principles is called Pascal principles.



Example 2:

A freshwater lake, has a maximum depth of 60m, and the mean atmospheric pressure is 91 kPa. Estimate the absolute pressure in kPa at this maximum depth.

Solution

Take γ = 9790 N/m^3. With pa = 91 kPa and h = 60 m, the pressure at this depth will be

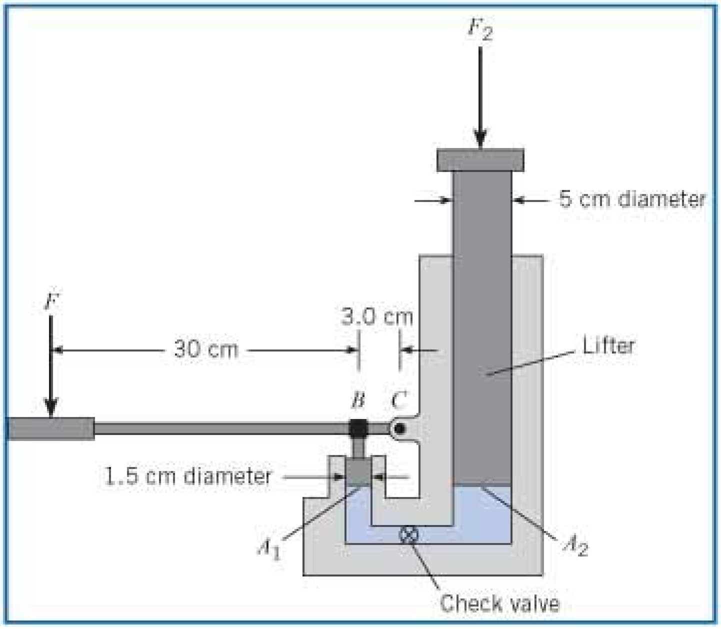
p = 91 kN/m^2 + (9790 N/m^3)(60 m) 1kN/1000N

= 91 kPa + 587 kN/m^2 = 678 kPa   Ans.

By omitting P_{atm} we could state the result as p = 587 kPa (gage).

Example 3: (EXAMPLE 3.1 LOAD LIFTED BY A HYDRAULIC JACK)

A hydraulic jack has the dimensions shown. If one exerts a force F of 100 N on the handle of the jack, what load, F_2, can the jack support? Neglect lifter weight.



Problem Definition

Situation: A force of F = 100 N is applied to the handle of a jack.

Find: Load F_2 in kN that the jack can lift.

Assumptions: Weight of the lifter component (see sketch) is negligible.

Plan

1. Calculate force acting on the small piston by applying moment equilibrium.
2. Calculate pressure p_1 in the hydraulic fluid by applying force equilibrium.
3. Calculate the load F_2 by applying force equilibrium.

Solution

1. Moment equilibrium

$$\sum M_C = 0$$

$$(0.33\ \text{m}) \times (100\ \text{N}) - (0.03\ \text{m})F_1 = 0$$

$$F_1 = \frac{0.33\ \text{m} \times 100\ \text{N}}{0.03\ \text{m}} = 1100\ \text{N}$$

2. Force equilibrium (small piston)

$$\sum F_{\text{small piston}} = p_1 A_1 - F_1 = 0$$

$$p_1 A_1 = F_1 = 1100\ \text{N}$$

Thus

$$p_1 = \frac{F_1}{A_1} = \frac{1100\ \mathrm{N}}{\pi d^2/4} = 6.22 \times 10^6\ \mathrm{N/m^2}$$

3. Force equilibrium (lifter)

· Note that $p_1 = p_2$ because they are at the same elevation.

· Apply force equilibrium:

$$\sum F_{\text{lifter}} = F_2 - p_1 A_2 = 0$$

$$F_2 = p_1 A_2 = \left(6.22 \times 10^6\ \frac{\mathrm{N}}{\mathrm{m}^2}\right)\left(\frac{\pi}{4} \times (0.05\ \mathrm{m})^2\right) = \boxed{12.2\ \mathrm{kN}}$$

- The jack in this example, which combines a lever and a hydraulic machine, provides an output force of 12,200 N from an input force of 100 N. Thus, this jack provides a mechanical advantage of 122 to 1 !

Example 4

2.2 **For the vessel containing glycerin under pressure as shown in Fig. 2-2, find the pressure at the bottom of the tank.**

Solution $p = 50 + \gamma h = 50 + (12.34)(2.0) = 74.68\ \mathrm{kN/m^2}$ or 74.68 kPa

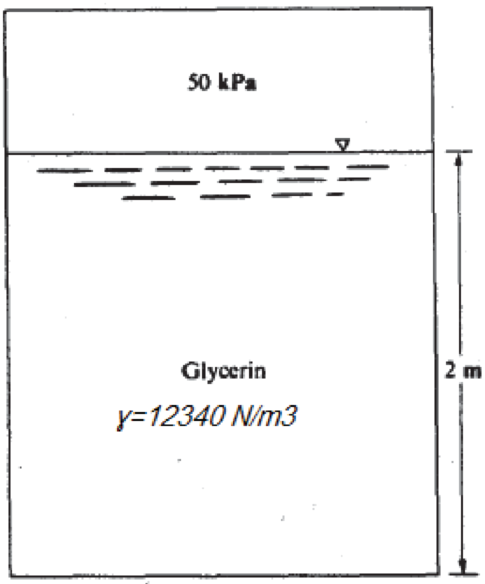
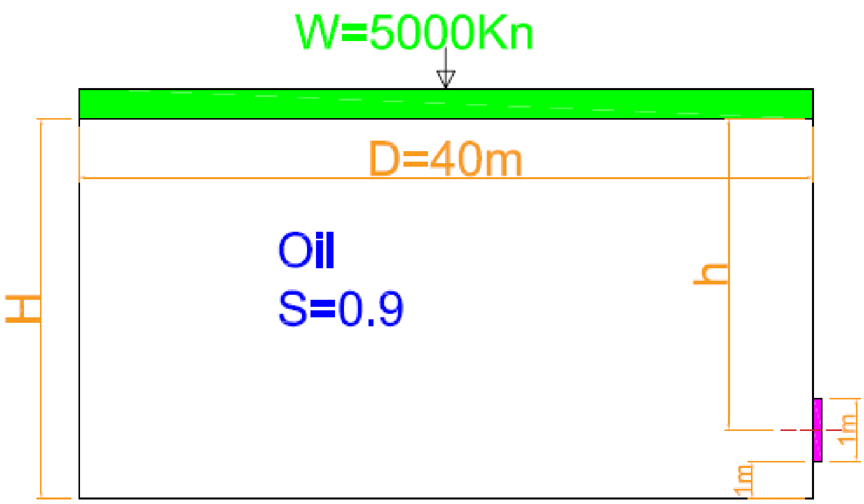


Fig. 2-2

Example 5

For the Cruid Oil Storage tank of 40 m Dia. shown with Floating steel cover of weight about 5000Kn what would be the oil height from the tank base if maximum presure at the center of the valve not exeed 1 Bar



Solution

Given:
D= 40 m , S=0.90 , W_{cover} = 5000KN
P_{max}. = 1 par = 100000 Pa.
Required (H).

$$P = \gamma . h + P_{cover}$$

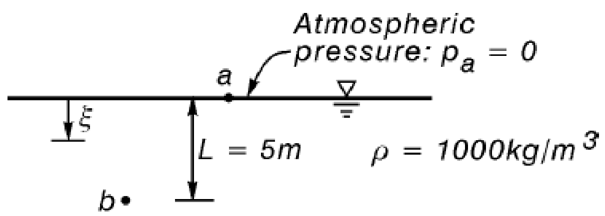
$$\rightarrow P = (0.9x9.81)x\, h + P_{cover} = 100\ Kpa.$$

$$P_{cover} = \frac{5000}{\frac{\pi}{4}(40)^2} = 3.97\ Kpa.$$

$$\therefore h = \frac{100 - 3.97}{(0.9x9.81)} = 10.87$$

→ Total height from base (H)=10.87+0.5+1=12.37≈ 12.3 m

Example 3.1



Given: ρ and L.

Calculate: p at point b in gage pressure.

Example 6.1 WATER PRESSURE IN A TANK

What is the water pressure at a depth of 35 ft in the tank shown?

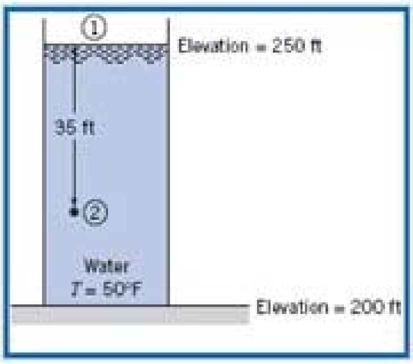
Problem Definition

Situation: Water is contained in a tank that is 50 ft deep.

Find: Water pressure (psi) at a depth of 35 ft.

Properties: Water (50°F), $\gamma = 62.4$ lbf/ft^3.

Sketch:



Solution

$$P2 = P1 + \gamma h$$

$$P2 = 0 + \gamma h = \gamma h$$

P2 = 35x62.4 = 2184 psf = $\boxed{15.2psi}$

Example 7

The system in Fig. 2-5 is at 20 °C. If atmospheric pressure is 101.03 kPa and the absolute pressure at the bottom of the tank is 231.3 kPa, what is the specific gravity of olive oil? g=9.79

101.03 + (0.89)(9.79)(1.5) + (9.79)(2.5) + (s.g.)(9.79)(2.9) + (13.6)(9.79)(0.4) = 231.3 s.g. = 1.39

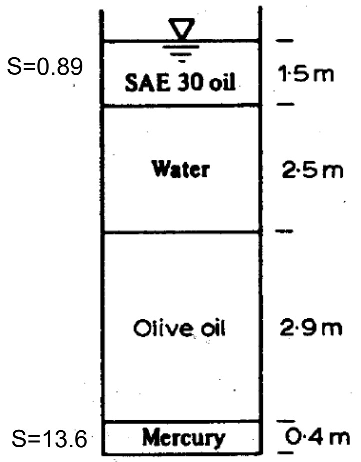


Fig. 2-5

H.W

2.6 A pressure gage 7.0 m above the bottom of a tank containing a liquid reads 64.94 kPa; another gage at height 4.0 m reads 87.53 kPa. Compute the specific weight and mass density of the fluid.

ans ρ =767.58 kg/m^3

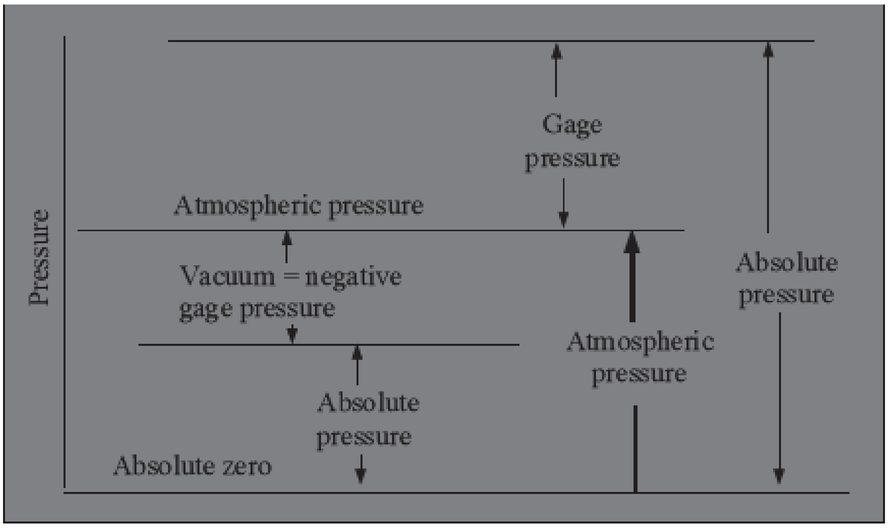
L4

Pressure

- Pressure measurement reads as follows:
  1. Relative to absolute zero (perfect vacuum): called ***absolute pressure***
  2. Relative to atmospheric pressure: called ***gage (gauge) pressure***

- If $P < P_{atm}$, we call it a vacuum (or negative or suction) pressure , its gage value = how much below atmospheric
- Absolute pressure values are all positive
- While gage pressures may be either:

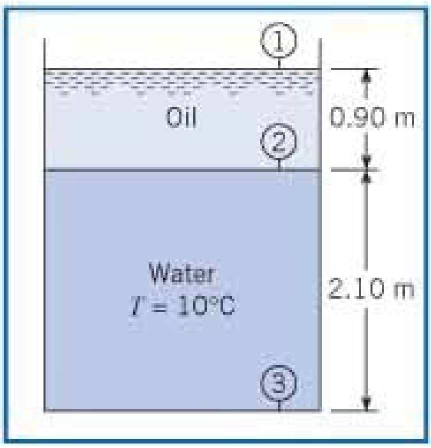
– Positive: if above atmospheric, or
– Negative (vacuum, suction): if below atmospheric



- Relationship between absolute, gage and atmospheric pressure reading:

$$\mathbf{P_{abs} = P_{atm} + P_{gage}}$$

Example 1:
Oil with a specific gravity of 0.80 forms a layer 0.90 m deep in an open tank that is otherwise filled with water. The total depth of water and oil is 3 m. What is the gage pressure at the bottom of the tank?



<u>Problem Definition</u>
Situation: Oil and water are contained in a tank.
Find: Pressure (kPa gage) at the bottom of the tank.
Properties:
1. Oil (10°C), S = 0.8.
2. Water (10°C), Table A.5: γ = 9810 N/m3.

Solution:

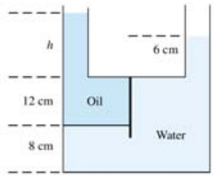
$$P_3 = P_1 + (\gamma.h)_{oil} + (\gamma.h)_w$$

$$\rightarrow P_3 = 0 + (0.8x9810)x\,0.9 + (9810)x2.1 = 27664.2Pa$$

$$\approx 27.7\ Kpa.$$

Example 2

In Fig. the tank contains water and immiscible oil at 20°C. What is h in cm if the density of the oil is 898 kg/m^3?

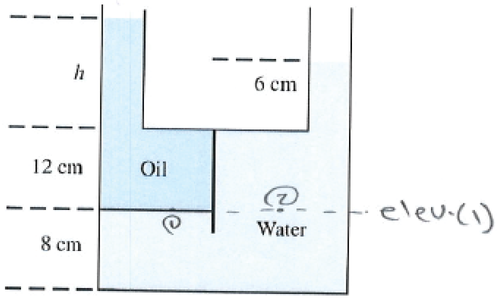


Solution: For water take the density = 998 kg/m^3. Apply the hydrostatic relation from the oil surface to the water surface, skipping the 8-cm part:

at elev 1 P1=P2 at water

$$p_{atm} + (898)(g)(h + 0.12) = (998)(g)(0.06 + 0.12) + p_{atm},$$

Solve for $h \approx 0.08 \text{ m} \approx \mathbf{8.0\ cm}$ *Ans.*



Example 3

2.15 **The closed tank in Fig. 2-3 is at 20 °C. If the pressure at point *A* is 98 kPa abs, what is the absolute pressure at point *B*? What percent error results from neglecting the specific weight of the air?**

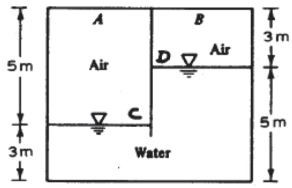


Fig. 2-3

Solution:

at elev.(1): P1=P2 (water)

PA=PB+(5-3). γ_w

→ 98 = PB+2x(9.81)

∴ PB=78.38Kpa.

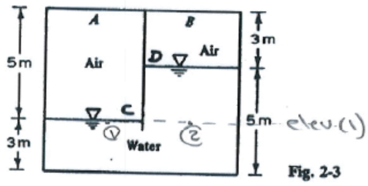


Fig. 2-3

Example 4

2.20 **Calculate the pressure, in kPa, at *A*, *B*, *C*, and *D* in Fig. 2-8.**

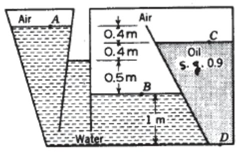


Fig. 2-8

Solution:

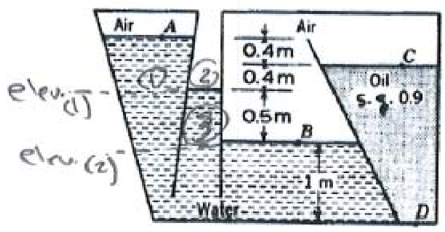
$at\ elev.(1){:}p1 = P2 \quad (water)$

$\therefore PA + 0.8\gamma_w = 0 \rightarrow PA = -0.8x9.81 = -7.85\ KPa$

$at\ elev.(2){:}p3 = PB \quad (water)$

$\therefore 0 + 0.5x9.81 = PB \rightarrow PB = 4.9\ KPa$

$PC = PB\ (air) \rightarrow PC = 4.9\ KPa$



$PD = PC + \gamma h_{oil} = 4.9 + (0.9x9.81)(1.9) = 21.27\ KPa$

Example 5

2.19 The tube shown in Fig. 2-7 is filled with oil. Determine the pressure heads at A and B in meters of water.

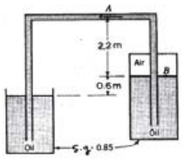


Fig. 2-7

Solution:

$at\ elev.(1): p1 = P2 \quad (oil)$

$\therefore 0 = PB + \gamma h_{oil} \rightarrow PB = -0.6(0.85x9.81) = -5\ KPa$

$at\ elev.(1): p1 = P3 \quad (oil)$

$\therefore 0 = PA + \gamma h_{oil} \rightarrow PA = -2.8(0.85x9.81) = -23.34\ KPa$

$for\ convert\ the\ (P) to\ water\ head\ (h): \rightarrow P = hx\gamma_w \rightarrow h = {}^{P}/_{\gamma_w}$

$\therefore hA = {}^{PA}/_{\gamma_w} = {}^{-23.34}/_{9.81} = -2.38\ m$

$\therefore hB = {}^{PB}/_{\gamma_w} = {}^{-5}/_{9.81} = -0.51\ m$

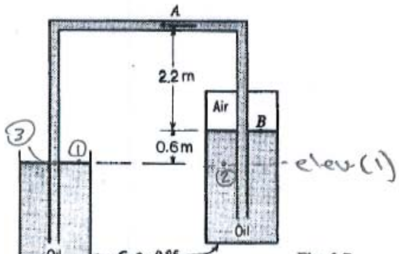


Fig. 2-7

Example 6

2.53 For the configuration shown in Fig. 2-36, calculate the weight of the piston if the gage pressure reading is **70.0 kPa.**

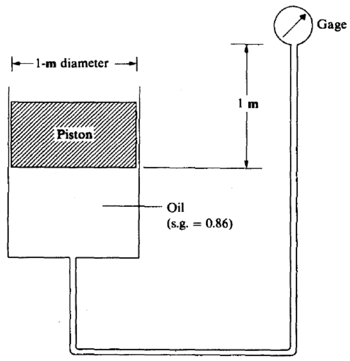


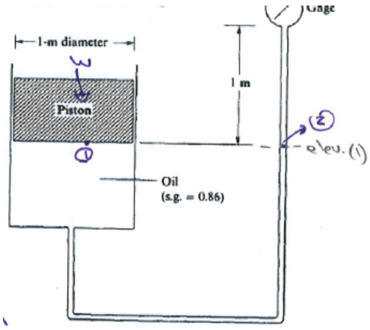
Fig. 2-36

Solution:

Let W = weight of the piston.

$at\ elev.(1): p1 = P2 \quad (oil)$

$$\therefore \frac{W}{({}^{\pi}/_{4})x1^2} = 1x(0.86x9.81) + 70 \rightarrow W = 61.6\ KN$$

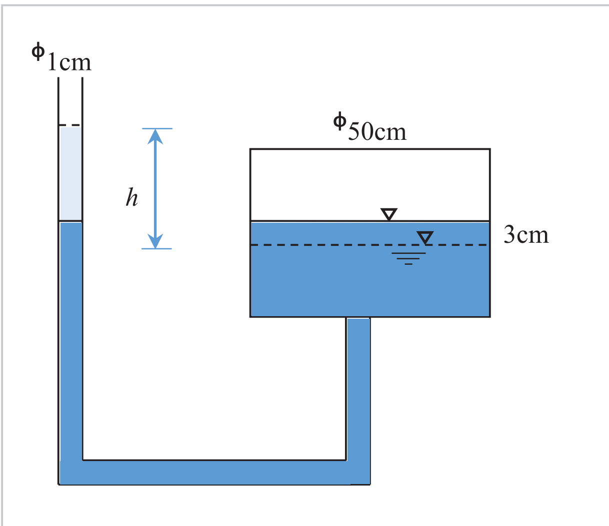


Example 7 H.W

A closed circular tank filled with water and connected by a U-piezometric tube as shown in figure. At the beginning the pressure above the water table in the tank is atmospheric, then the gauge that connected with tank read an increasing in pressure that caused falling in the water level in the tank by 3 cm. a) calculate the deference in height that accrued between water levels inside the tank and in the external tube leg. b) Determine the final pressure that was .reading by the gauge

h=75m

P=735.75 KPa



L5

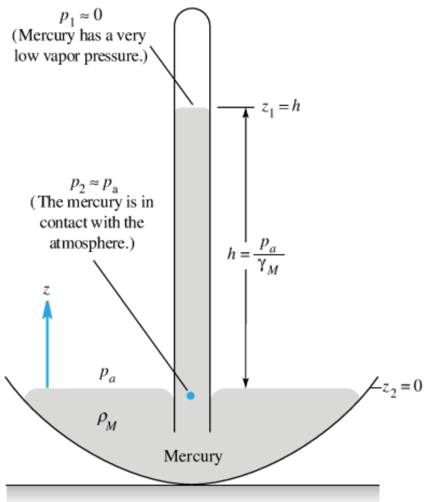
Pressure measurement devices

Absolute pressure measurement

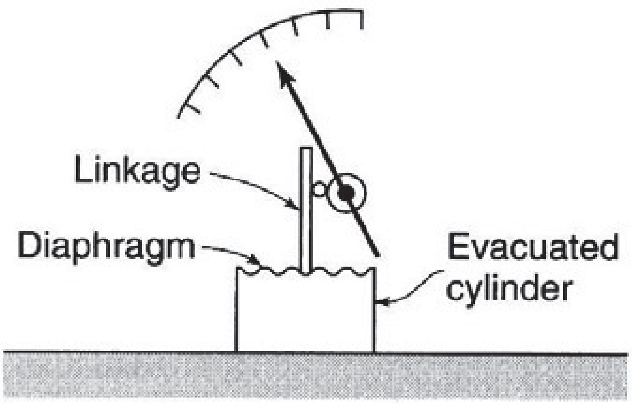
Barometers: The instrument used to measure atmospheric pressure is called barometer

1. **Mercury Barometer**: which is illustrated in figure below, which consist of a one meter length tube filled with mercury and inverted into a pan that's filled partially with mercury. The height difference of mercury in inverted tube respect to outside them reads the atmospheric pressure value (first was invented by *E. Torricelli*, 1643).

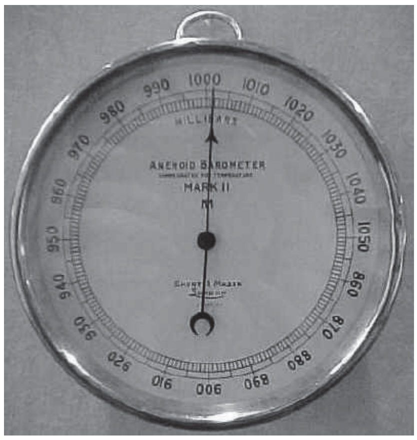
Values of standard sea-level atmospheric pressure=*101.325 kPa abs*
=1013.25 mbar abs
= 760 mm Hg, Torr
=10.34 m H₂0



2. **Aneroid Barometer**: uses elastic diaphragm to measure atmospheric pressure



Aneroid barometer

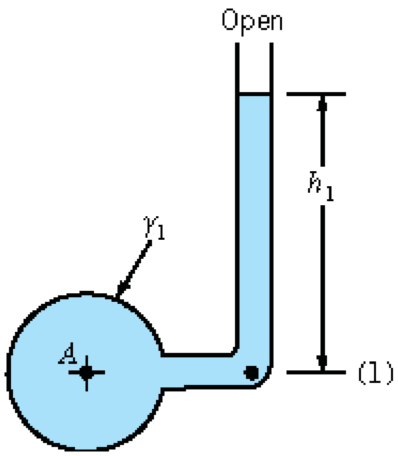


Gage pressure measurement:

1. Manometry

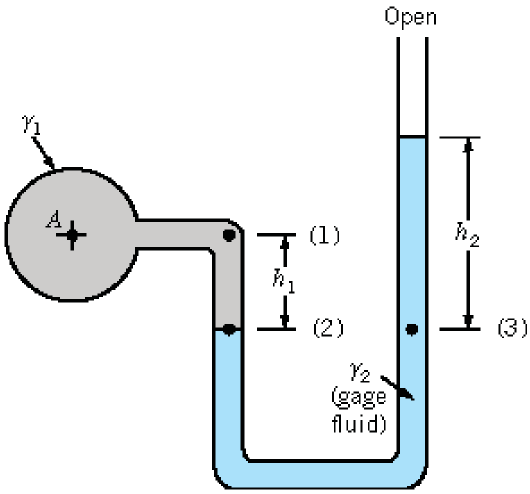
1.1. Piezometer

For measuring pressure inside a vessel or pipe in which liquid is there, a tube open at the top to atmosphere may be attached, tapped, to the walls of the container (or pipe or vessel) containing liquid at a pressure (higher than atmospheric) to be measured, so liquid can rise in the tube. By determining the height to which liquid rises and using the relation P1 = ρgh, gauge pressure of the liquid can be determined. Such a device is known as piezometer. To avoid capillary effects, a piezometer's tube should be about 12mm or greater.



1.2. Manometers

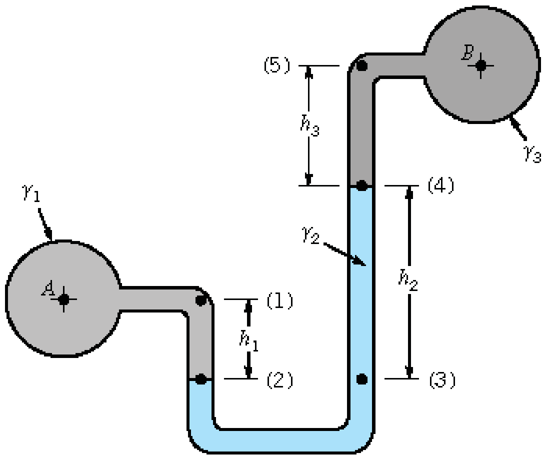
The using of piezometers for **high pressures** measurement become impractical and it is **useless for pressure measurement in gases** and **negative pressure**. The manometers in its various forms is an extremely useful type of pressure measuring instrument for these cases.



at any level for same fluids contacting any two points
use the following formula

$$P_1 = P_2$$

$$P_{down} = P^{up} + \gamma h$$



When liquids and gases are both involved in a manometer problem, it is well within engineering accuracy to neglect the pressure changes due to the columns of gas. This is because $\gamma_{liquid} \gg \gamma_{gas}$

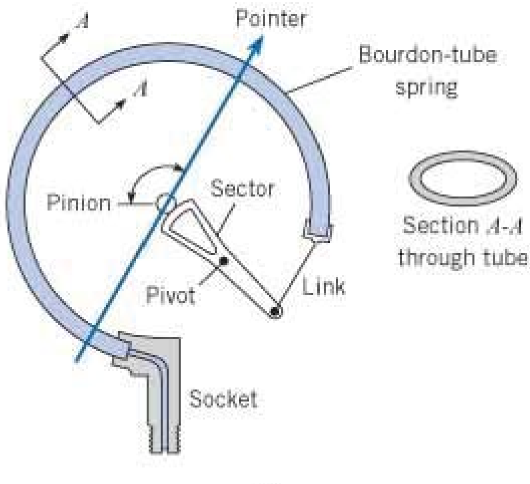
Manometers limitations: manometers suffers from a number of limitations.

1. While it can be adapted to measure very small pressure differences, it cannot be used conveniently for large pressure differences - although it is possible to connect a number of manometers in series and to use mercury as the manometric fluid to improve the range. (limitation)
2. A manometer does not have to be calibrated against any standard; the pressure difference can be calculated from second and third principles in hydrostatics. (Advantage)
3. Some liquids are unsuitable for use because they do not form well-defined interface. Surface tension can also cause errors due to capillary rise; this can be avoided if the diameters of the tubes are sufficiently large - preferably not less than 12 mm diameter. (limitation)
4. A major disadvantage of the manometer is its slow response, which makes it unsuitable for measuring fluctuating pressures
5. It is essential that the pipes connecting the manometer to the pipe or vessel containing the liquid under pressure should be filled with this liquid and there should be no air bubbles in the liquid.(limitation).

2. Bourdon gage

Curved tube of elliptical cross-section changes curvature with changes in pressure. Moving end of tube rotates a hand on a dial through a linkage system. Pressure indicated by gage graduated in bar , kPa or kg/cm^2 (=98.0665 kPa) or psi or other pressure units.

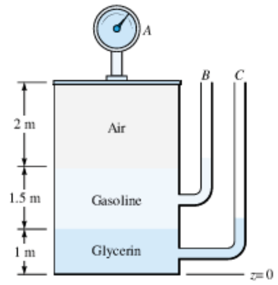
(a)



(b)

Example1: (**Piezometers**)

In figure pressure gage A reads 1.5 kPa. The fluids are at 20°C. Determine the elevations z, in meters, of the liquid levels in the open piezometer tubes B and C.



Solution:

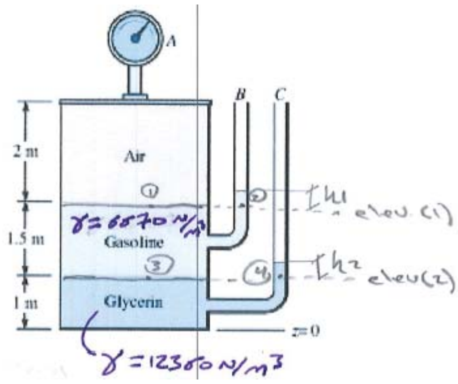
$let\ hB\ above\ gas\ level = h1\ and\ let\ hC\ above\ clyc.level = h2$

$at\ elev.(1):\ \ p1 = P2 \qquad (gasoline)$

$\therefore PA = \gamma_{ga}.h1 \ \rightarrow 1.5 = 6.67h1 + 0 \ \ \rightarrow h1 = 0.23\ m$

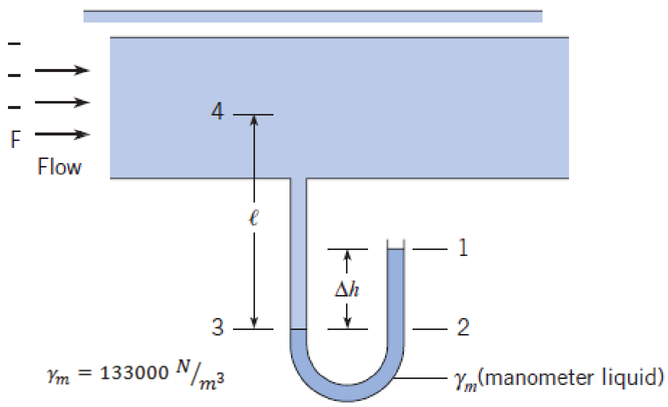
$at\ elev.(2):\ \ p3 = P4 \qquad (glyc.)$

$\therefore 1.5 + 1.5x6.67 = 12.36xh2 + 0 \ \rightarrow h2 = 0.93\ m$



Example 2: (U-manometers)

Water at 10°C is the fluid in the pipe of Fig. 3.11, and mercury is the manometer fluid. If the deflection Δh is 60 cm and ℓ is 180 cm, what is the gage pressure at the center of the pipe?

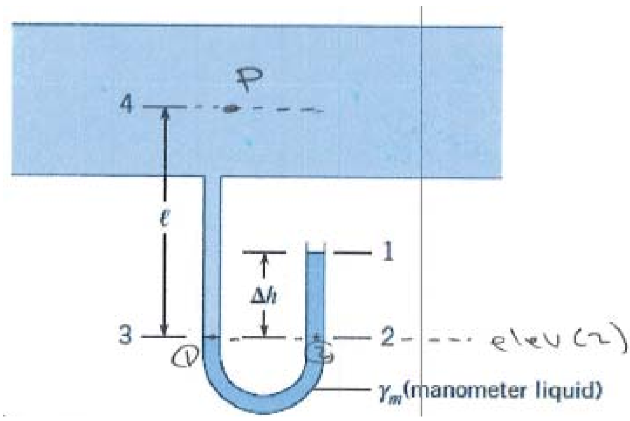


Solution:

$let\ presure\ at\ pipe\ center = P$

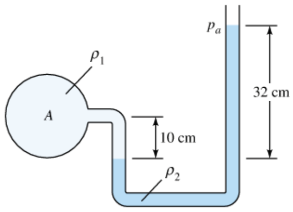
$at\ elev.(2):\ \ p1 = P2 \qquad (mercury)$

$$\therefore P + \gamma_w.l = \Delta h.\gamma_m + 0$$
$$\rightarrow P + 9.81x1.8 = 0.6x133$$
$$\rightarrow P\ = 62.14\ KPa$$



Example 3: (**U-manometers**)

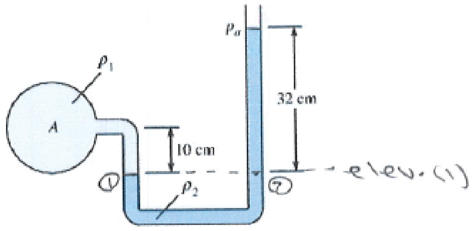
In Figure fluid 1 is oil (Sg=0.87) and fluid 2 is glycerin at 20°C (γ=12360 N/m^3). If P_{atm}=98 kPa, determine the absolute pressure at point A



Solution:

$$at\ elev.(1):\ p1 = P2 \qquad (glyc.)$$

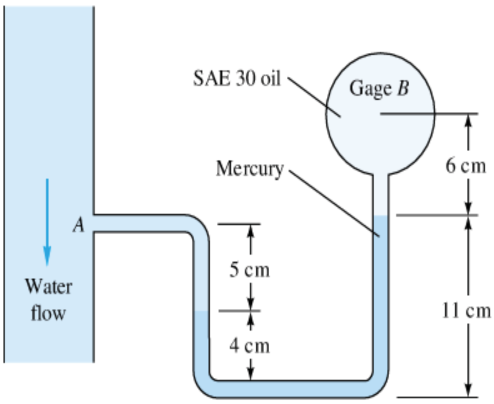
$$\therefore PA + 0.1\gamma_1 = 0.32\,\gamma_2 + P(atm)$$
$$\rightarrow PA + 0.1x0.87x9.81 = 0.32x12.36 + 98$$
$$\rightarrow PA = 101.1\,KPa$$



Example 4:

(**Differential-Manometers**)

Pressure gage B in figure is to measure the pressure at point A in a water flow. If the pressure at B is 87 kPa estimate the pressure at A, in kPa. Assume all fluids at 20oC.



$\gamma_{oil} = 8720\ {}^{N}/_{m^3}$

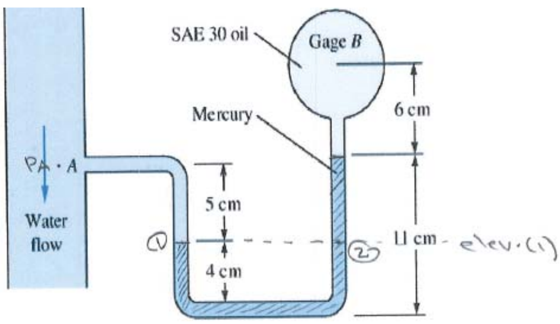
$\gamma_m = 133100\ {}^{N}/_{m^3}$

Solution:

$$at\ elev.(1):\ p1 = P2 \qquad (merc.)$$

$$\therefore PA + 0.05\gamma_w = PB + 0.06.\gamma_{oil} + \frac{(11-4)}{100}.\gamma_m$$

$$\rightarrow PA + 0.05x9.81 = 87 + 0.06x8.72 + 0.07x133.1$$
$$\rightarrow PA = 96.35\,KPa$$



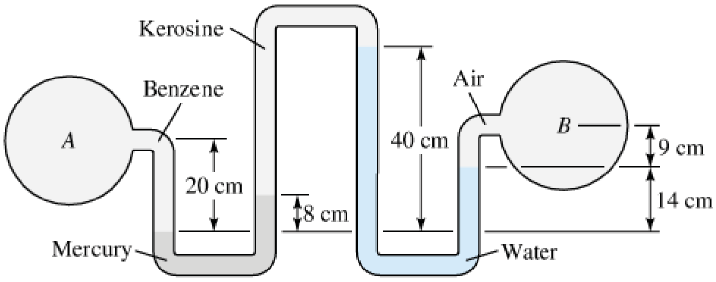
Example 5: (**Differential-Manometers**)

In figure all fluids are at 20°C. Determine the pressure difference (Pa) between points A and B.

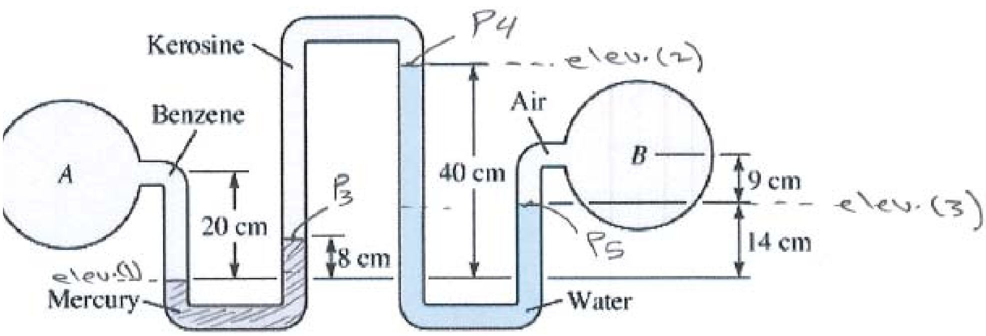
$\gamma_B = 8640\ {}^{N}/_{m^3}$

$\gamma_m = 133100\ {}^{N}/_{m^3}$

$\gamma_K = 7885\ {}^{N}/_{m^3}$



Solution:



$$at\ elev.(1):\ p1 = P2 \qquad (merc.)$$

$$\therefore PA + 0.2\gamma_B = 0.08.\gamma_m + P3 \ ....(1)$$

$$at\ elev.(2):\ p1 = P2 \qquad (Ker.)$$

$$\therefore P3 - 0.32\gamma_K = P4 \ ....(2)$$

$$at\ elev.(3):\ p1 = P2 \qquad (water)\ \ and\ P5 = PB\ (air)$$

$$\therefore P4 + 0.26\gamma_w = P5 = PB \ ....(3)$$

$$from\ (1) \rightarrow P3 = (PA - 8.92)$$

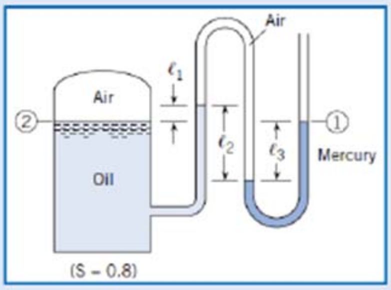
$$from\ (3) \rightarrow P4 = (PB - 2.55)$$

$$subs\ P3\ \&\ P4\ at\ eq.(2)$$
$$\rightarrow PA - 8.92 - 2.52 = PB - 2.55$$
$$Re\ arrange{:}\rightarrow PA - PB = 8.92 + 2.52 - 2.55$$
$$= 8.89\,Kpa.$$

Example 6: (Successive Differential-Manometers)

EXAMPLE 3.7 MANOMETER ANALYSIS

Sketch: What is the pressure of the air in the tank if $\ell_1 = 40$ cm, $\ell_2 = 100$ cm, and $\ell_3 = 80$ cm?



Solution:

$at\ elev.(3):\ p1 = P2 \quad (merc.)$

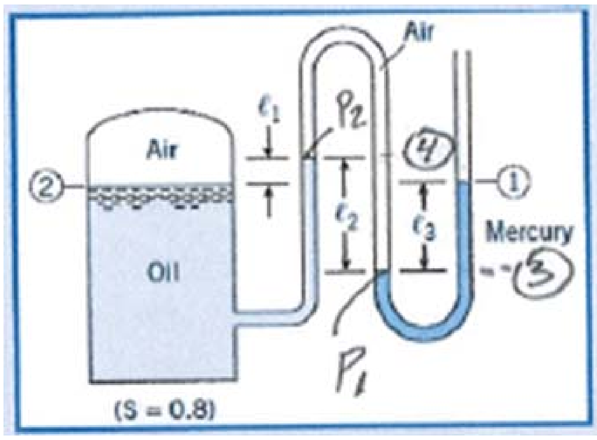
$\therefore P1 = l_3.\gamma_m + 0 \ \rightarrow P1 = 106.4\ KPa.$

$at\ elev.(4):\ p1 = P2 \quad (air.)$

$\therefore P2 = 106.4\ KPa.$

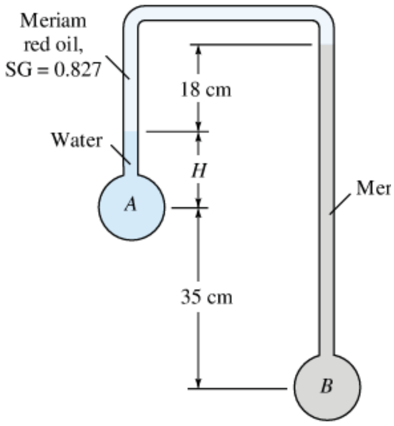
$at\ elev.(2):\ p2 + l_1.\gamma_{oil} = P(air) \quad (oil.)$

$\therefore 106.4 + 7.85x0.4 = P(air) \quad \rightarrow P(air) = 109.6\ KPa.$



Example 7: Inverted-Manometers

,For inverted manometer of figure ,all fluids are at 20°C. If $p_B - p_A = 97$ kPa what must the height H be in cm



$\gamma_{mrm} = 8096\ {}^{N}/_{m^3} \qquad \gamma_m = 133100\ {}^{N}/_{m^3}$

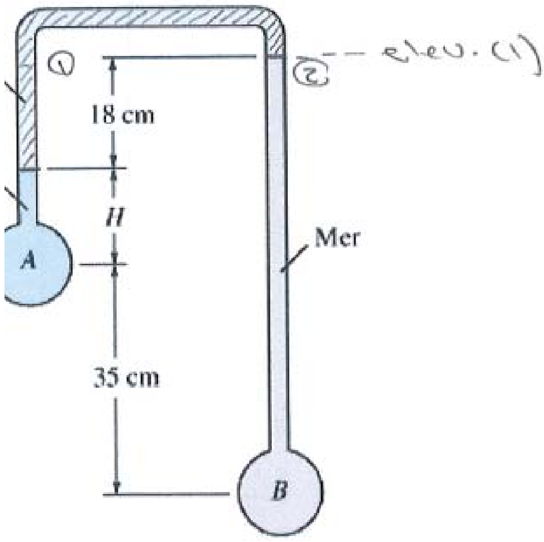
Solution:

$at\ elev.(1):\ p1 = P2 \quad (meriam)$

$\therefore PA - H.\gamma_w - 0.18\gamma_{mrm} = PB - (0.35 + H + 0.18)\gamma_{mrm}.$

$\therefore PB - PA = 70.54 + 133.1.H - 9.81.H - 1.457$

$\rightarrow H = 0.226\ m \ \approx 22.6\ cm$



Example 8

2.94 An open tube is attached to a tank, as shown in Fig. 2-66. If the water rises to a height of 800 mm in the tube, what are the pressures p_A and p_B of the air above the water? Neglect capillary effects in the tube.

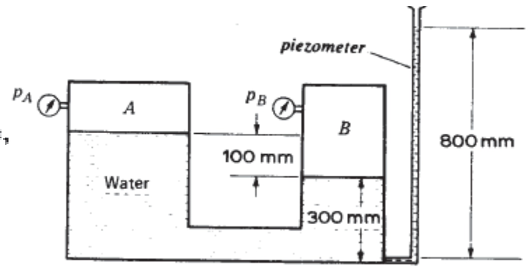


Fig. 2-66

Solution:

$$at\ elev.(1):\ p1 = PB \qquad (water)$$

$$\therefore (0.8 - 0.3).\gamma_w = PB \rightarrow PB = 0.5x9.81 = 4.9KPa.$$

$$\because PB = P2 \ \rightarrow 4.9 = PA + 0.1x9.81$$
$$\rightarrow PA = 3.92\ KPa.$$

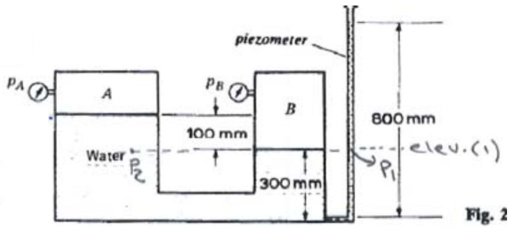


Fig. 2

Example 9

2.92 Find the difference in pressure between tanks A and B in Fig. 2-64 if $d_1 = 330$ mm, $d_2 = 160$ mm, $d_3 = 480$ mm, and $d_4 = 230$ mm.

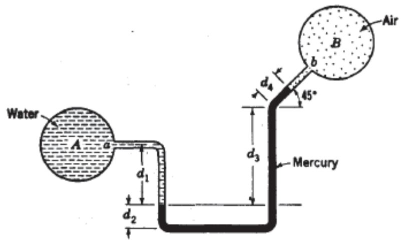


Fig. 2-64

Solution:

$$at\ elev.(1):\ p1 = P2 \qquad (Hg)$$

$$\therefore PA + 9.79x0.33 = 13.6x9.79x(0.48 + 0.23\sin(45) + PB$$
$$\rightarrow PA - PB = 82.33\ \ KPa.$$

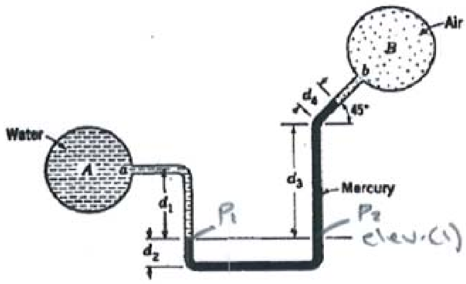


Fig. 2-64

Example 10

2.96 For the setup shown in Fig. 2-68, calculate the absolute pressure at a. Assume standard atmospheric pressure, 101.3 kPa. $\gamma_m = 133000\ {}^{N}/_{m^3}$

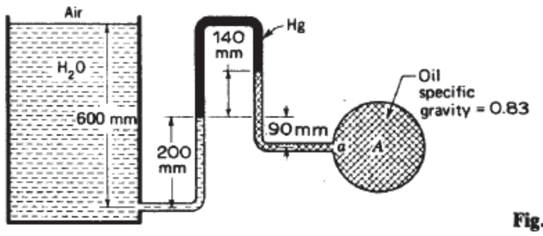


Fig. 2-68

Solution:

$$at\ elev.(1):\ p1 = P2 \qquad (water)$$

$$\therefore 101.3 + (0.6 - 0.2).\gamma_w \rightarrow P2 = 105.22KPa.$$

$$at\ elev.(2):\ p3 = P4 \qquad (Hg)$$

$$\therefore P2 - 0.14x133 = P4 \rightarrow P4 = 86.6KPa.$$

$$\therefore PA = P4 + (0.14 + 0.09)(0.83x9.81) = 88.47KPa.$$

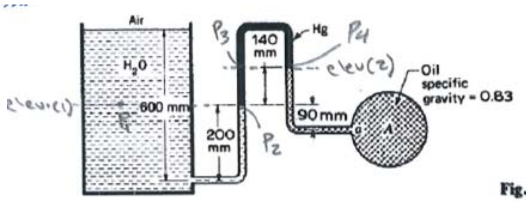


Fig. 2-68

L6

Pressure Forces and Pressure Distributions on Surface

Hydrostatic Force **(Force due to the pressure of a fluid at rest)**

(e.g Force exerted on the wall of storage tanks, dams, and

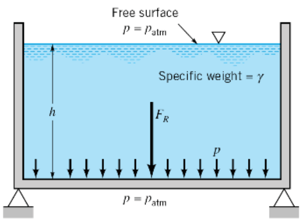
(ships

Q. How is ***Hydrostatic Force*** on the vertical or inclined planes determined?

Basic conditions for a Plane surface submerged in a fluid

- Force on the surface: ***Perpendicular*** to the surface (No τ)
- Pressure: Linearly dependent only to the vertical depth

1. On a **Horizontal surface** (e.g. the bottom of a tank)



Pressure at the bottom, $p = \gamma h$

: Uniform on the entire plane

$\therefore$ Resultant force $\boxed{F_P = pA = \gamma hA}$

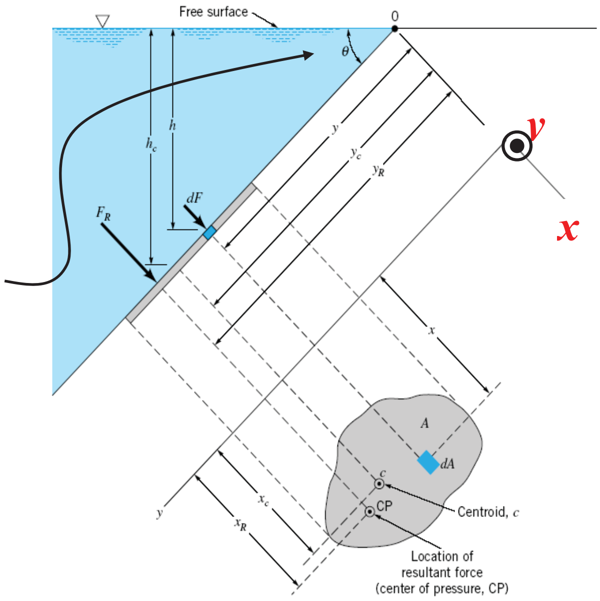
(A: the bottom area of container)

2. On an **Inclined surface**

Consider a plane shown

- At surface: $p = p_{atm}$

- Angle θ between free surface & the inclined plane



y axis: Along the surface

x axis: Out of the plane

- Along the vertical depth *h*

☞ Pressure linearly changes ☞ Hydrostatic force changes

Differential Force acting on the differential area *dA* of plane,

$$dF = (\text{Pressure}) \cdot (\text{Area}) = (\gamma h) \cdot (dA) \qquad \text{(Perpendicular to plane)}$$

Then, Magnitude of ***total*** resultant force F_P

$$F_P = \int_A \gamma h dA = \int_A \gamma (y \sin\theta) dA \qquad \text{where } h = y\sin\theta$$

$$= \gamma \sin\theta \boxed{\int_A y dA} \leftarrow \boxed{\text{1st moment of the area about x-axis}}$$

$$y_c = \frac{\int_A y dA}{A}$$

where y_c: *y* coordinate of the center of area (Centroid)

※ $\int_A y dA = y_c A$

Then,

$$F_P = \gamma A y_c \sin\theta = (\gamma\, h_c) A$$ where $\gamma\, h_c$: Pressure at the centroid

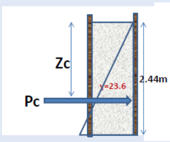
= (Pressure at the centroid) × Area

Magnitude of a force on an INCLINED plane

- Dependent on γ , *Area*, and *Depth of centroid*
- Perpendicular to the surface (Direction)

Example1: HYDROSTATIC FORCE DUE TO CONCRETE

Determine the force acting on one side of a concrete form 2.44 m high and 1.22 m wide (8 ft by 4 ft) that is used for pouring a basement wall. The specific weight of concrete is 23.6 kN/m^3 (150 lbf/ft^3).



Problem Definition

Situation:

1. Concrete in a liquid state acts on a vertical surface.
2. Vertical wall is 2.44 m high and 1.22 m wide

Find: The resultant force (kN) acting on the wall.

Assumptions: Freshly poured concrete can be represented as a liquid.

Properties: Concrete: γ = 23.6 kN/m^3.

Plan

Apply the panel equation given in Eq. (3.23).

Solution

1. Panel equation

$$F = \bar{p}A$$

2. Term-by-term analysis
   - $\bar{p}$ = pressure at depth of the centroid

$$\bar{p} = (\gamma_{\text{concrete}})(z_{\text{centroid}}) = (23.6\ \text{kN/m}^3)(2.44/2\ \text{m}) = 28.79\ \text{kPa}$$

   - A = area of panel

$$A = (2.44\ \text{m})(1.22\ \text{m}) = 2.977\ \text{m}^2$$

3. Resultant force

$$F = \bar{p}A = (28.79\ \text{kPa})(2.977\ \text{m}^2) = \boxed{85.7\ \text{kN}}$$

Example 2

3.37 **Water in a tank is pressurized to 85 cmHg (Fig. 3-29). Determine the hydrostatic force per meter width on panel *AB*.**

Solution **On panel *AB*, $p_{cg} = [(13.6)(9.79)](0.85) + (9.79)(4 + \frac{3}{2}) = 167.0$ kPa, $F_{AB} = (167.0)[(3)(1)] = 501$ kN.**

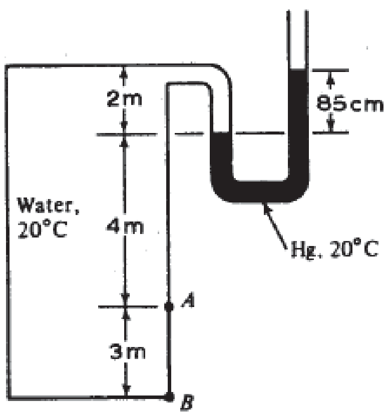


Fig. 3-29

The location of point of action of F_P (Center of pressure, CP)

- ***Not passing though Centroid!!*** **(*Why?*)**
- Related with the ***balance of torques*** due to of F_P

i) Position of F_P on *y*-axis

y_p: *y* coordinate of the point of action of F_P

Moment about x axis:

The moment of resultant force = The moment of its components

$$F_P y_P = \int_A y dF$$

$$\Rightarrow \quad (\cancel{\gamma} A y_c \cancel{\sin}\theta) y_p = \int_A \gamma \sin\theta y^2 dA = \cancel{\gamma} \cancel{\sin}\theta \int_A y^2 dA$$

$$\Rightarrow \quad \boxed{\therefore\ y_P = \frac{\int_A y^2 dA}{y_c A} = \frac{I_x}{y_c A}}$$

where $I_x = \int_A y^2 dA$: *2nd moment of area (*Moment of inertia, +ve always *)*

or, by using the parallel-axis theorem, $I_x = I_{xc} + A{y_c}^2$

$$\Rightarrow \quad \boxed{\therefore\ y_P = \frac{I_{xc}}{y_c A} + y_c}$$

(***Always below the centroid** !*)

ii) Position of F_P on x-axis

x_p: x coordinate of the point of action of F_P

By the similar manner,

The moment of resultant force = The moment of its components

$$F_P x_P = \int_A x dF$$

$$\Rightarrow \quad (\cancel{\gamma} A y_c \cancel{\sin\theta}) x_P = \int_A \gamma \sin\theta xydA = \cancel{\gamma} \cancel{\sin\theta} \int_A xydA$$

$$\Rightarrow \quad \boxed{\therefore \; x_P = \frac{\int_A xydA}{y_c A} = \frac{I_{xy}}{y_c A}}$$

where $\int_A xydA = I_{xy}$: **Area product of inertia (+ve or –ve)**

or, by using the perpendicular-axis theorem, $I_{xy} = I_{xyc} + Ax_c y_c$

$$\Rightarrow \quad \boxed{\therefore \; x_P = \frac{I_{xyc}}{y_c A} + x_c}$$

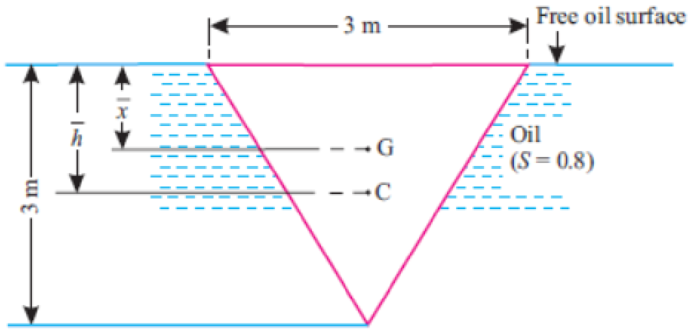
CP For a ***symmetric*** submerged area, $x_P = x_c$ ($I_{xyc} = 0$)

Example-6 *. An isosceles triangular plate of base 3 m and altitude 3 m is immersed vertically in an oil of specific gravity 0.8. The base of the plate coincides with the free surface of oil. Determine:*

(i) Total pressure on the plate; (ii) Centre of pressure.

Solution. Base of the plate, $b = 3$ m

Height of the plate, $h = 3$ m



Area, $A = \frac{b \times h}{2} = \frac{3 \times 3}{2} = 4.5 \text{ m}^2$

Specific gravity of oil, $S = 0.8$

The distance of C.G. from the free surface of oil,

$$\bar{x} = \frac{1}{3}h = \frac{1}{3} \times 3 = 1 \text{ m}$$

(i) Total pressure on the plate, P:

We know that, $P = wA\bar{x}$

$= (0.8 \times 9.81) \times 4.5 \times 1$

$P = $ **35.3 kN (Ans.)**

(ii) Centre of pressure, $\bar{h}$:

Centre of pressure is given by the relation:

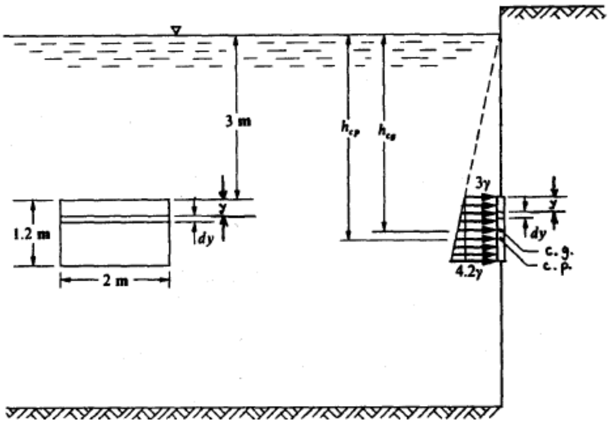
$$\bar{h} = \frac{I_G}{A\bar{x}} + \bar{x} = \frac{(bh^3/36)}{A\bar{x}} + \bar{x}$$

$$= \frac{(3 \times 3^3/36)}{4.5 \times 1} + 1$$

$\bar{h} = $ **1.5 m (Ans.)**

Example 7

A vertical, rectangular gate with water on one side is shown in Fig. 3-7. Determine the total resultant force acting on the gate and the location of the center of pressure.



Solution:

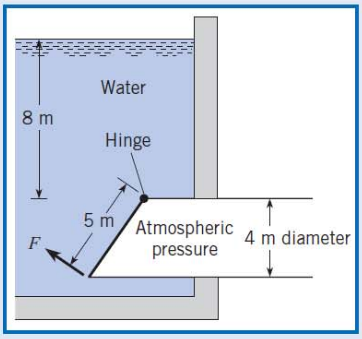
$$F = \gamma h_{cg} A = (9.79)(3 + 1.2/2)[(2)(1.2)] = 84.59 \text{ kN}$$

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = \left(3 + \frac{1.2}{2}\right) + \frac{(2)(1.2)^3/12}{(3 + 1.2/2)[(2)(1.2)]} = 3.633 \text{ m}$$

Example 4: **FORCE TO OPEN AN ELLIPTICAL GATE**

An elliptical gate covers the end of a pipe 4 m in diameter. If the gate is hinged at the top, what normal force F is required to open the gate when water is 8 m deep above the top of the pipe and the pipe is open to the atmosphere on the other side? Neglect the weight of the gate.

Sketch:



Problem Definition

Situation: Water pressure is acting on an elliptical gate.

Find: Normal force (in newtons) required to open gate.

Properties: Water (10°C), Table A.5: $\gamma = 9810\ \text{N/m}^3$.

Assumptions:

1. Neglect the weight of the gate.
2. Neglect friction between the bottom on the gate and the pipe wall.

Plan

1. Calculate resultant hydrostatic force using $F = \bar{p}A$.
2. Find the location of the center of pressure using Eq. (3.28).
3. Draw an FBD of the gate.
4. Apply moment equilibrium about the hinge.

Solution

1. Hydrostatic (resultant) force
   - $\bar{p}$ = pressure at depth of the centroid

$$\bar{p} = (\gamma_{\text{water}})(z_{\text{centroid}}) = (9810\ \text{N/m}^3)(10\ \text{m}) = 98.1\ \text{kPa}$$

   - A = area of elliptical panel (using Fig. A.1 to find formula)

$$A = \pi ab$$
$$= \pi(2.5\ \text{m})(2\ \text{m}) = 15.71\text{m}^2$$

   - Calculate resultant force

$$F_p = \bar{p}A = (98.1\ \text{kPa})(15.71\ \text{m}^2) = \boxed{1.54\ \text{MN}}$$

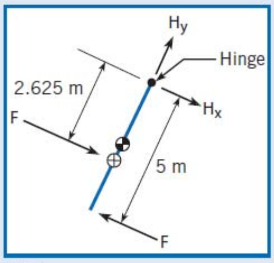
2. Center of pressure
   - $\bar{y} = 12.5$ m, where $\bar{y}$ is the slant distance from the water surface to the centroid.
   - Area moment of inertia $\bar{I}$ of an elliptical panel using a formula from Fig. A.1

$$\bar{I} = \frac{\pi a^3 b}{4} = \frac{\pi(2.5\ \text{m})^3(2\ \text{m})}{4} = 24.54\ \text{m}^4$$

   - Finding center of pressure

$$y_{cp} - \bar{y} = \frac{\bar{I}}{\bar{y}A} = \frac{25.54\ \text{m}^4}{(12.5\ \text{m})(15.71\text{m}^2)} = 0.125\ \text{m}$$

3. FBD of the gate:



4. Moment equilibrium

$$\sum M_{\text{hinge}} = 0$$

$$1.541 \times 10^6\ \text{N} \times 2.625\ \text{m} - F \times 5\ \text{m} = 0$$

$$F = \boxed{809\ \text{kN}}$$

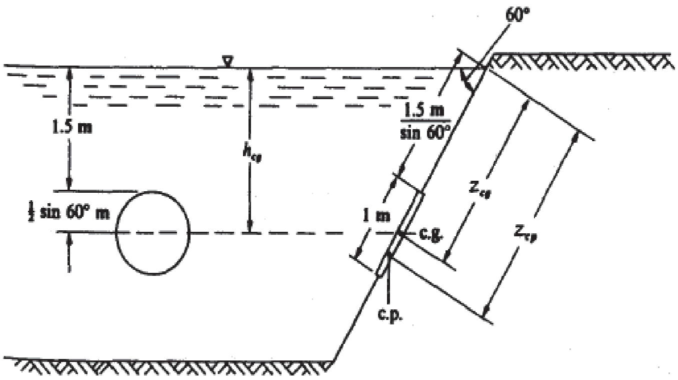
Example5

An inclined, circular gate with water on one side as shown in figure. Determine the total resultant force acting on the gate and the location of the centre of pressure.

Solution:

$$F = \gamma h_{cg} A = (9.79)[1.5 + \tfrac{1}{2}(1.0 \sin 60°)][\pi(1.0)^2/4] = 14.86\ \text{kN}$$

$$z_{cp} = z_{cg} + \frac{I_{cg}}{z_{cg}A} = \left[\frac{1.5}{\sin 60°} + \frac{1}{2}(1.0)\right] + \frac{\pi(1.0)^4/64}{[1.5/\sin 60° + \frac{1}{2}(1.0)][\pi(1.0)^2/4]} = 2.260\ \text{m}$$



Example 6

3.44 Circular gate *ABC* in Fig. 3-35 is 4 m in diameter and is hinged at *B*. Compute the force *P* just sufficient to keep the gate from opening when *h* is 8 m.

Solution

$$F = \gamma h_{cg} A = (9.79)(8)[\pi(4)^2/4] = 984.2 \text{ kN} \qquad I_{xx} = \pi d^4/64 = \pi(4)^4/64 = 12.57 \text{ m}^4$$

$$y_{cp} = \frac{-I_{xx} \sin\theta}{h_{cg} A} = \frac{-(12.57)(\sin 90°)}{(8)[(\pi)(2)^2]} = -0.125 \text{ m}$$

$$\sum M_B = 0 \qquad (P)(2) - (984.2)(0.125) = 0 \qquad P = 61.5 \text{ kN}$$

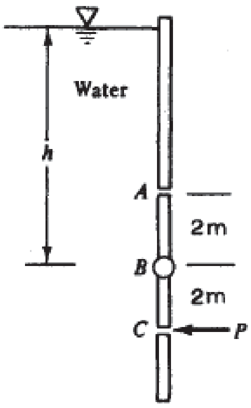


Fig. 3-35(*a*)

$T = 88.5$ kN

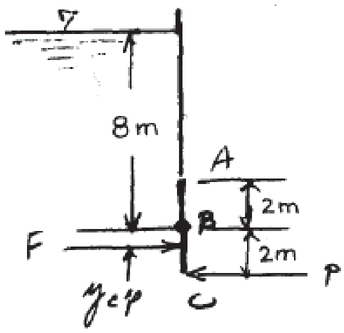
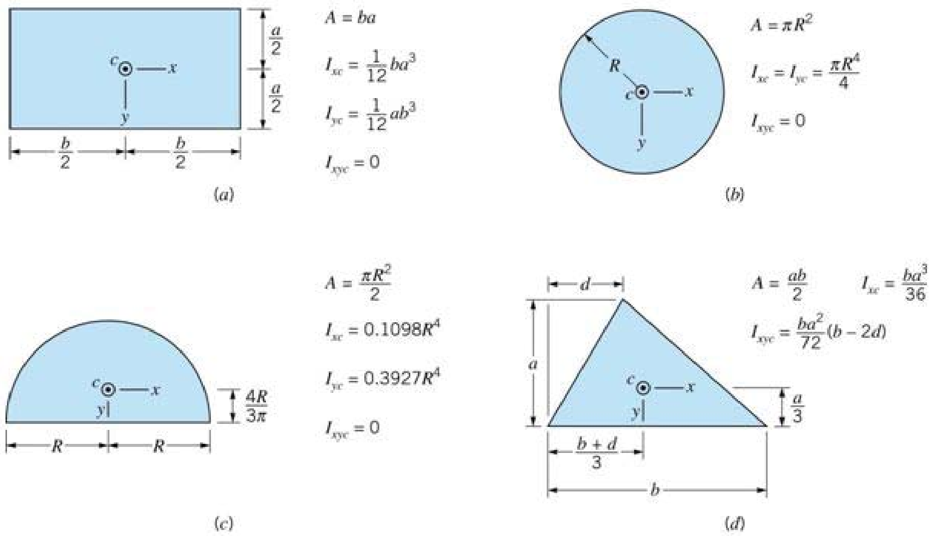


Fig. 3-35(*b*)

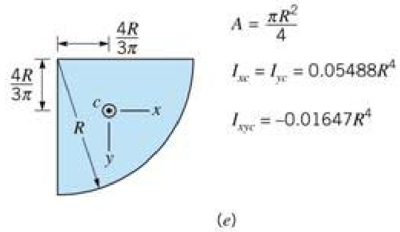
Hydrostatic Force on a Plane Surface: Geometric Properties



Centroid Coordinates

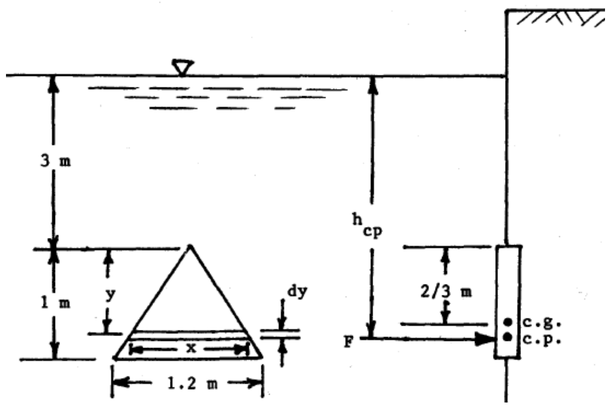
Areas

Moments of Inertia



Example 8

A vertical, triangular gate with water on one side is shown in Fig. 3-11. Determine the total resultant force acting on the gate and the location of the center of pressure.



Solution:

$$F = \gamma h_{cg} A = (9.79)[3 + \tfrac{2}{3}(1)][(1.2)(1)/2] = 21.54\ \text{kN}$$

$$h_{cp} = h_{cg} + \frac{I_{cg}}{h_{cg}A} = [3 + (\tfrac{2}{3})(1)] + \frac{(1.2)(1)^3/36}{[3 + \tfrac{2}{3}(1)][(1.2)(1)/2]} = 3.68\ \text{m}$$

H.W

3.40 The triangular trough in Fig. 3-32 is hinged at A and held together by cable BC at the top. If cable spacing is 1 m into the paper, what is the cable tension?

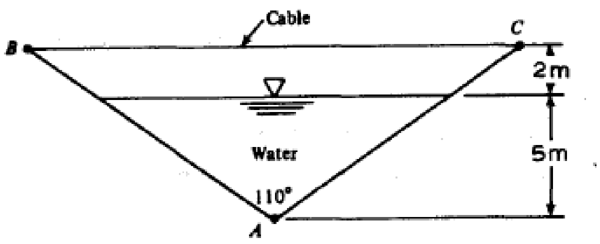


Fig. 3-32(a)

ans $T = 88.5\ \text{kN}$

H.W 2

Gate AB in Fig. 3-25 is semicircular, hinged at B. What horizontal force P is required at A for equilibrium?

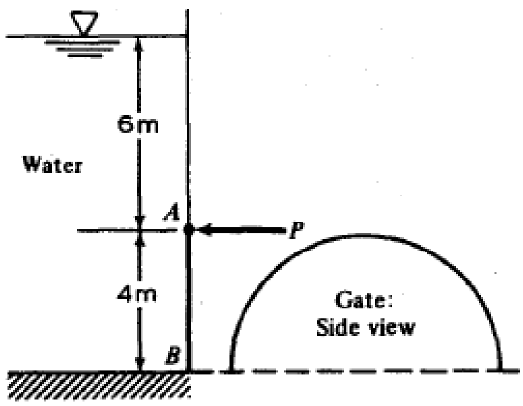


Fig. 3-25(a)

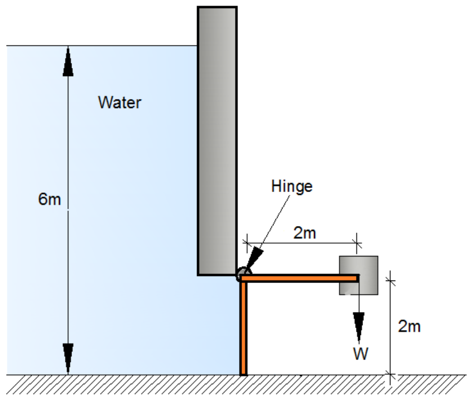
Anc. $P = 798\ \text{kN}$

H.W 3

:Q2)) A rectangular gate, 3m width , hinged as shown in figure (1). Determine

A - The total hydrostatic force on the gate when the storage depth is 6m. and ***Anc. 294.3kN***

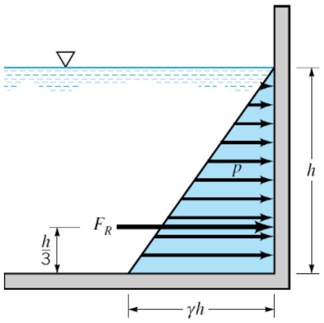
B- For the gate to be stable, what value of the weight W?. ***Anc. W= 156.96kN***

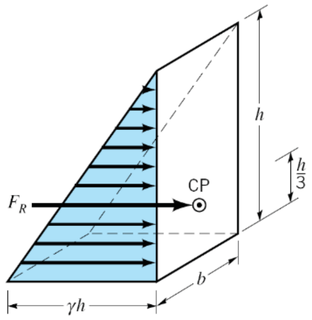


L7 **Pressure Prism** Graphical interpretation of pressure distribution

- Especially applied for a rectangular surfaces (areas)
- Simple method for ***finding the force*** and ***the point of action***

Consider the situation shown





※ Information from the diagram

- Vertical wall of width *b* and height *h*
- Contained liquid with specific weight γ
- Pressure: $p_{top} = 0$ & $p_{bottom} = \gamma h$

From the last section,

$F_P = (\gamma h_c) \cdot (A) = p_{av}$ (at the centroid)×area = $\gamma\left(\frac{h}{2}\right)A$

Let's define a pressure-area space. (See the right figure above]

1. Horizontal axis: Magnitude of the pressure
2. Vertical axis: Height of the area
3. Axis toward the plane: Width of the area

: Resultant volume (***Pressure prism***)

- How to find the resultant force F_R from the pressure prism

$$F_P = \gamma\left(\frac{h}{2}\right)A = \frac{1}{2}(\gamma h)(bh) = \textbf{\textit{Volume of the pressure prism}}$$

- How to find the point of action of F_R (the point of action)

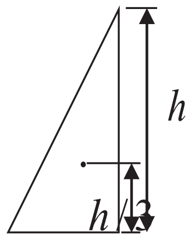
From the last section,

$$y_P = \frac{I_{xc}}{y_c A} + y_c = \frac{I_{xc}}{\left(\frac{1}{2}h\right)(bh)} + \frac{1}{2}h$$

(In case of rectangular plate, $I_{xc} = \frac{1}{12}Ah^2 = \frac{1}{12}bh^3$)

$$y_P = \frac{\frac{1}{12}bh^3}{\frac{1}{2}h\cdot(bh)} + \frac{1}{2}h = \frac{1}{6}h + \frac{1}{2}h = \frac{2}{3}h \text{ (from the top)}$$

From the pressure prism



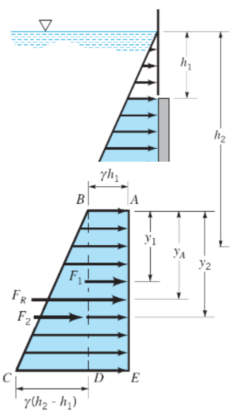
Y_P = ***Centroid of the pressure prism***

$= \frac{2}{3}h$ (from the top) $= \frac{1}{3}h$ (above the base)

X_P = Horizontal center

※ Special case of a plane surface not extending up to the fluid surface

- ***Completely submerged plane*** **(See Figure)**



Consider the situation shown

Pressure prism

- Trapezoidal cross section

(1) Resultant force F_P

= Volume of the shadow region

F_P = Volume of hexahedron + Volume of prism

$$= F_{1(ABDE)} + F_{2(BCD)}$$
$$= (\gamma h_1)A + \frac{1}{2}[\gamma(h_2 - h_1)]A$$

(2) The location of $F_R(y_A)$: Consider the moments again

Moment by F_P acting at y_A
= Moment by F_1 at y_1 + Moment by F_2 at y_2

$$F_P y_P = F_1 y_1 + F_2 y_2$$

where $y_1 = \frac{h}{2}$ for rectangle

$y_2 = \frac{2h}{3}$ for triangle (From the top)

- The effect of the atmospheric pressure p_{atm}
  : Increasing Volume of hexahedron (F_1), NOT the prism (F_2)

Example 1

3.6 A dam 20 m long retains 7 m of water, as shown in Fig. 3-6. Find the total resultant force acting on the dam and the location of the center of pressure.

Solution $F = \gamma hA = (9.79)[(0+7)/2][(20)(7/\sin 60°)] = 5339$ kN. The center of pressure is located at two-thirds the total water depth of 7 m, or 4.667 m below the water surface (i.e., $h_{cp} = 4.667$ m in Fig. 3-6).

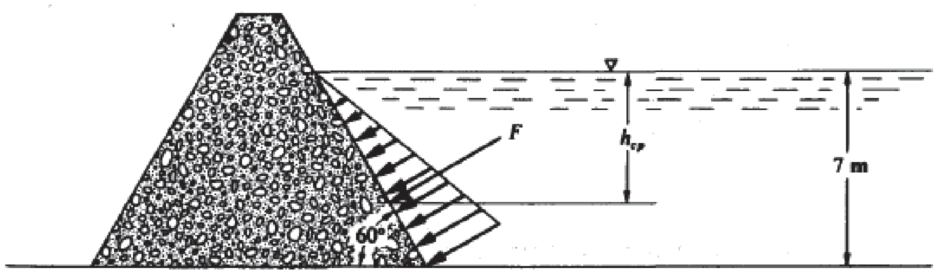


Fig. 3-6

Example 2

3.60 Determine the pivot location y of the square gate in Fig. 3-49 so that it will rotate open when the liquid surface is as shown.

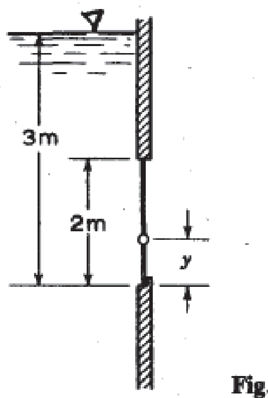


Fig. 3-49

Solution

$$F = [1\gamma x2 + 0.5(3-1)\gamma x2]x2 = 8\gamma = 78.48KN$$

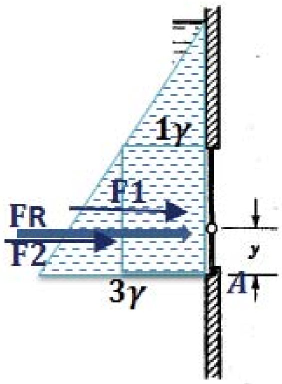
The location of pivot must be at the resultant action (FR)
So that $\sum M$ *about Point A* $= 0$

$$F1x1 + F2x\left(\frac{2}{3}\right) = FR.y$$

$$F1 = 2\gamma x2 = 39.24\ KN$$
$$F2 = 0.5x2\gamma x2x2 = 39.24KN$$

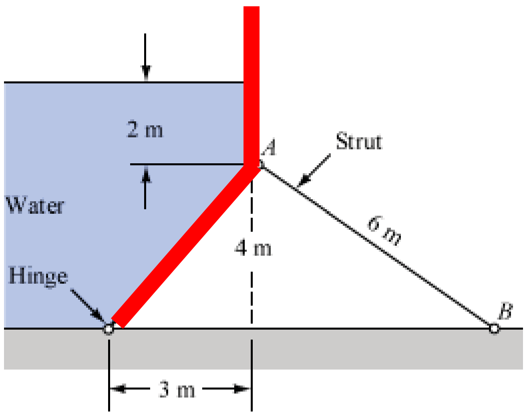
$$\therefore 39.24x1 + 39.24x\left(\frac{2}{3}\right) = 78.48.y$$

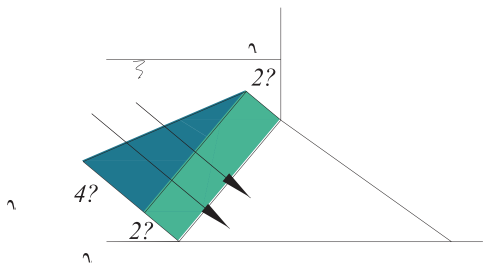
$$\rightarrow y = 0.834\ m$$



Example 3

The dam of figure has a strut AB every 6 m. Determine the compressive force in the strut, neglecting the weight of the dam.





Solution: If the struts are doing their job, the moment about the hinge should be 0! Every six meters there is a strut, so the width each strut supports is 6 m.

Use any method to determine the forces on the dam due to the water. Using the pressure prism method:

(1) the force on the upper 2 m portion of the dam is

$$F_1 = \frac{1}{2}(2\gamma)(2\text{ m})(6\text{ m wide}) = 12\,\gamma$$

(2) the force on the lower portion of the dam is split into 2 pieces

$$F_2 = (2\gamma)(5\text{ m})(6\text{ m wide}) = 60\,\gamma$$

and

$$F_3 = \frac{1}{2}(4\gamma)(5\text{ m})(6\text{ m wide}) = 60\,\gamma$$

The compressive force on the strut AB is in a direction along the strut, and so to determine the moment about the hinge, the force is split into x- and y-components, determined by the geometry shown in Fig. 2.61,

$$F_{AB_x} = \frac{\sqrt{20}}{6} F_{AB} \qquad F_{AB_y} = \frac{4}{6} F_{AB}$$

Now writing the moment equation about the hinge, (moment = force × lever arm)

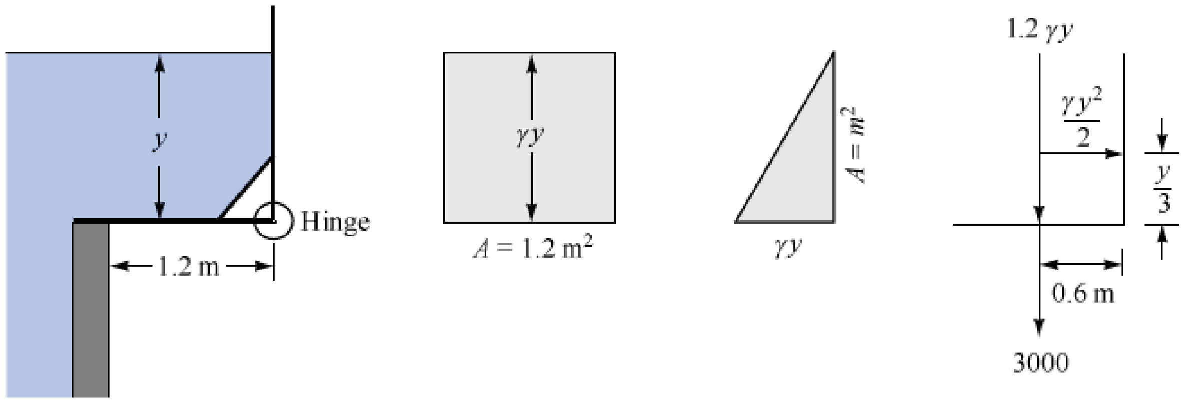
$$+\circlearrowleft \Sigma M_H = 0: \quad -F_1\left[4+\frac{1}{3}(2)\right] - F_2(2.5) - F_3\left[\frac{1}{3}(5)\right] + \frac{\sqrt{20}}{6}F_{AB}(4) + \frac{4}{6}F_{AB}(3) = 0$$

or, after rearranging

$$4.9814\, F_{AB} = 549{,}136 \text{ N·m} + 1{,}470{,}900 \text{ N·m} + 980{,}600 \text{ N·m}$$
$$F_{AB} = 602{,}368 \text{ N} = 602.4 \text{ kN}$$

:Example 4

A structure is so arranged along a channel that it will spill the water out if a certain height y (Fig. 2.18a) is reached. The gate is made of steel plate weighing 2500 N/m^2. Determine the height of y.



(a) (b) (c) (d)

Solution Using pressure-prism concepts, for unit width normal to the page the force on the horizontal leaf (Fig. 2.18b) is given by the volume of a pressure prism of base 1.2 m^2 and constant altitude γy N/m^2, which yields $F_y = 1.2\gamma y$ N acting through the center of the base. The pressure prism for the vertical face (Fig. 2.18c) is a wedge of base y m^2 and altitude varying from 0 to γy N/m^2. The average altitude is $\gamma y/2$, so $F_x = \gamma y^2/2$ N. The centroid of the wedge prism is $y/3$ from the hinge. The weight of the gate floor exerts a force of 3000 N at its center. Figure 2.18d shows all the forces and moment arms. For equilibrium, that is, the value of y for tipping, moments about the hinge must be zero.

$$M = (3000 \text{ N})(0.6 \text{ m}) + (1.2\gamma y \text{ N})(0.6 \text{ m}) - \left(\frac{\gamma y^2}{2} \text{ N}\right)\left(\frac{y}{3} \text{ m}\right) = 0$$

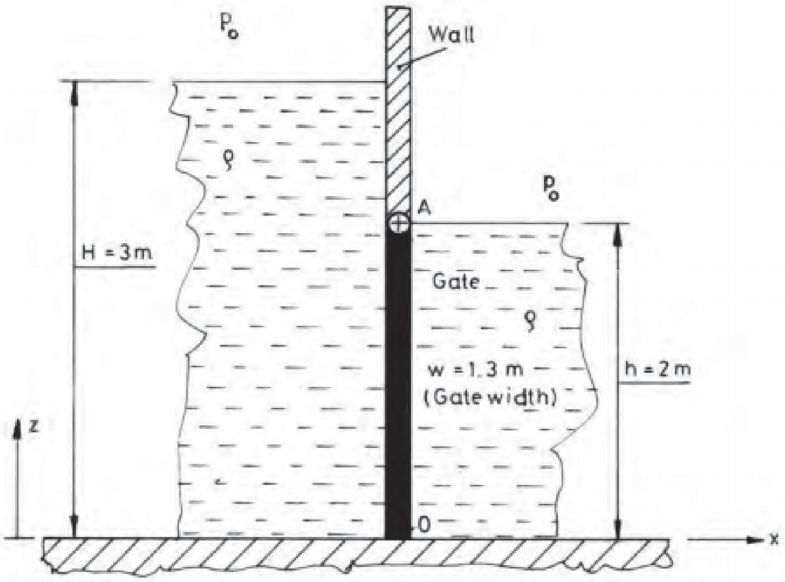
or

$$M = y^3 - 4.32y - 1.1014 = 0$$

By using try and error technique (or other techniques like Newton Raphson method) we find that y=2.196m

Example 5:

1.11 Find the resultant force due to water on both sides of the gate including its line of action.



Solution

$$F1 = 1\gamma x 2x1.3 = 25.5\ KN$$

$$F2 = 0.5x2\gamma x2x1.3 = 25.5\ KN$$

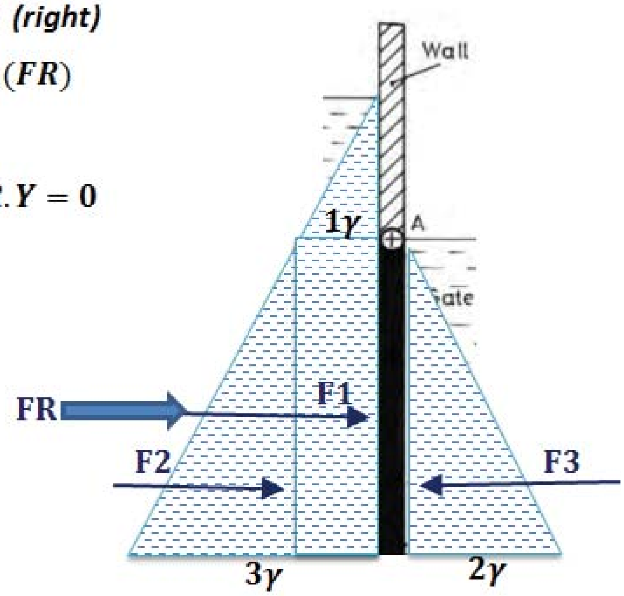
$$F3 = 0.5x2\gamma x2x1.3 = 25.5\ KN$$

$$\therefore FR = 25.5 + 25.5 - 25.5 = 25.5\ KN \ \textit{(right)}$$

The location of the resultant action (FR)
by $\sum M$ ***about Point A = 0***

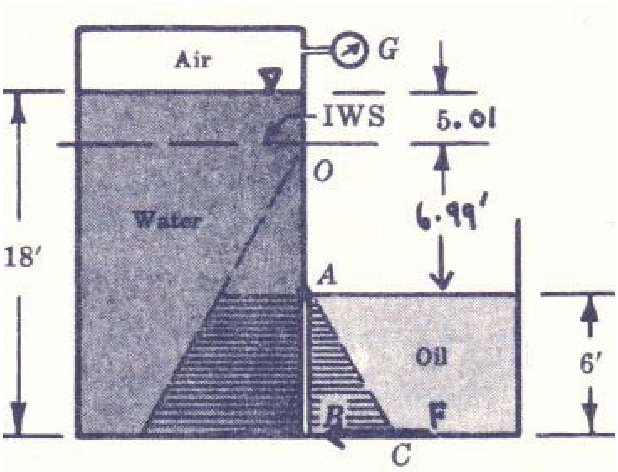
$$F1x1 + F2x\left(2x\frac{2}{3}\right) - F3x\left(2x\frac{2}{3}\right) + FR.Y = 0$$

$$\rightarrow Y = 1\ m$$



F2 canceld F3
So that FR is same of F1

Example 6: Gate AB in Fig. is 4 ft wide and hinged at A. Gage G reads −2.17 psi, while oil (s.g. = 0.75) is in the right tank. What horizontal force must be applied at B for equilibrium of gate AB?



Solution

$F = \gamma h_{cg} A \qquad F_{oil} = [(0.75)(62.4)](\frac{6}{2})[(6)(4)] = 3370 \text{ lb}$

F_{oil} acts $(\frac{2}{3})(6)$, or 4.0 ft from A. For the left side, the negative pressure due to the air can be converted to its equivalent head in feet of water. $h = p/\gamma = (-2.17)(144)/62.4 = -5.01$ ft. This negative pressure head is equivalent to having 5.01 ft less water above A. Hence, $F_{H_2O} = (62.4)(6.99 + \frac{6}{2})[(6)(4)] = 14\,960$ lb.

$$y_{cp} = \frac{-I_{xx} \sin\theta}{h_{cg} A} = \frac{-[(4)(6)^3/12](\sin 90°)}{(6.99 + \frac{6}{2})[(6)(4)]} = -0.30 \text{ ft}$$

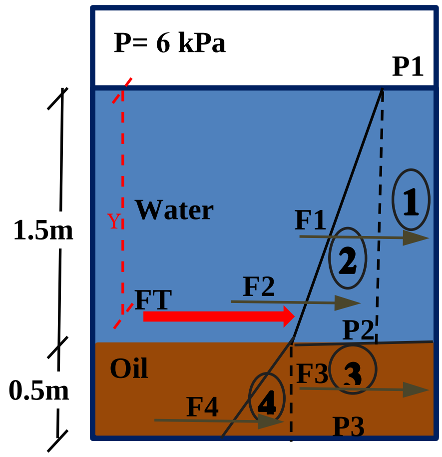
F_{H_2O} acts at $(0.30 + \frac{6}{2})$, or 3.30 ft below A. $\sum M_A = 0$; $(3370)(4.0) + 6F - (14\,960)(3.30) = 0$, $F = 5980$ lb (acting leftward).

Example 7:

EXAMPLE (8):

A tank 2 m deep and 1 m wide is layered with 0.5 m of oil (S=1.2), and 1.5 m of water, and with air pressurized at top with 6 kPa. Compute (a) the total hydrostatic force and (b) the resultant center of pressure of the fluid on the right-hand side of the tank.

Solution: (a)



$P_1 = 6\ kPa.$

$P_2 = 1.5x9.81 + 6 = 20.715\ kPa$

$P_3 = 20.715 + 0.5x1.2x9.81 = 26.6\ kPa$

$\therefore F_1 = 6x1.5x1 = 9\ kN$

$\therefore F_2 = 0.5x(20.715 - 6)x1.5x1 = 11.03\ kN$

$\therefore F_3 = 20.715x0.5x1 = 10.35\ kN$

$\therefore F_4 = 0.5x(26.6 - 20.715)x0.5x1 = 1.47\ kN$

$\therefore F_T = \sum F = 9 + 11.03 + 10.35 + 1.47 = 31.85\ kN$

(b): The center of pressure from the water surface:

Take Moment to the forces from the water surface:

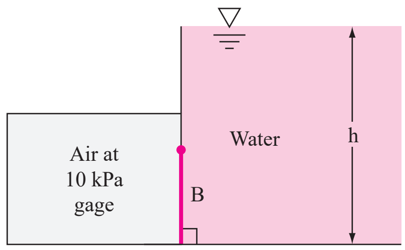
$$FTxY = \sum F.y$$

$$\therefore 31.85xY = 9x0.75 + 11.03x\left(\frac{2}{3}\right)x1.5 + 10.35x(1.5 + 0.25) + 1.47x\left[1.5 + \left(\frac{2}{3}\right)x0.5\right]$$

$\rightarrow Y$ =1.21 m

H.W

P2.72 Gate B in Fig. P2.72 is 30 cm high, 60 cm wide into the paper, and hinged at the top. What water depth h will firs cause the gate to open?

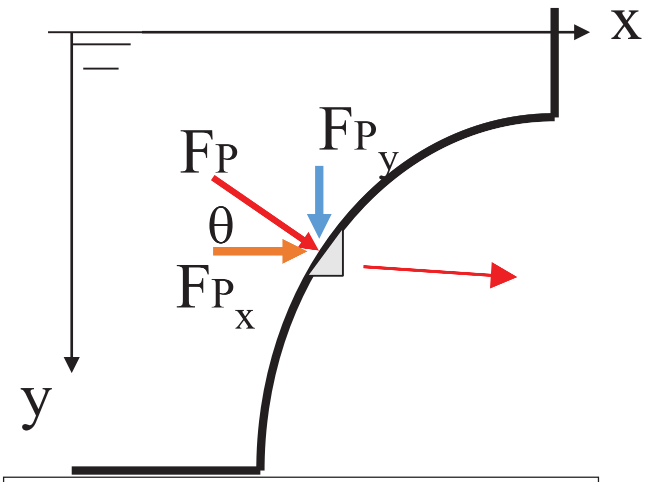


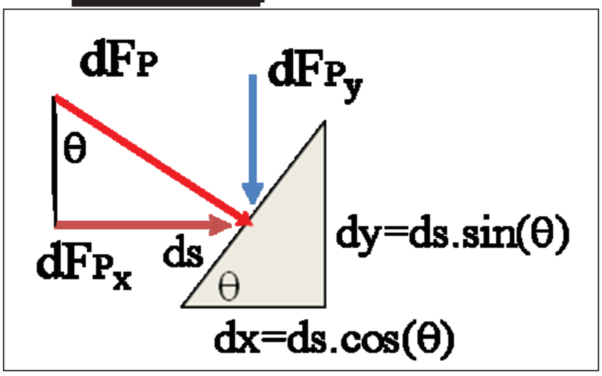
P2.72

anc $h \approx 1.12$ m

L8

Forces on Curved Surfaces - Non-planar surfaces





$$\vec{F}_P = \vec{F}_{py} + \vec{F}_{px}$$

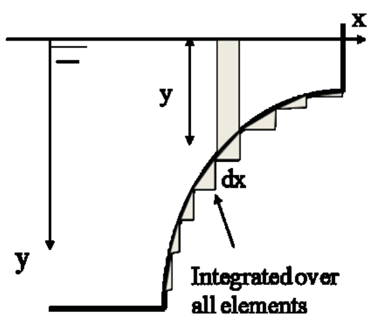
For unit width of surface

$$dF_{py} = P \cdot ds.\cos\theta$$

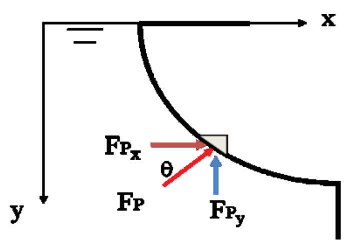
$$P = \gamma \cdot y \qquad dx = ds.\cos\theta$$

$$dF_{py} = \gamma \cdot y \cdot dx$$

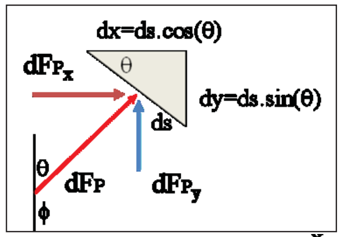
$$\therefore \downarrow F_{py} = \int \gamma \cdot y \cdot dx = \gamma \cdot \int y dx = \gamma \cdot \forall$$



Where ∀ the volume of liquid above the surface to the zero pressure surface



- By the same way we find the vertical component of pressure force if the liquid exist under the surface by taking the sign of ∀ as –ve to represent the upward direction of this force

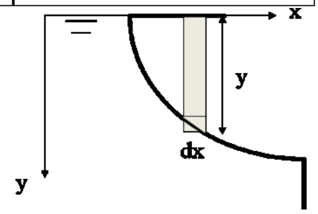


$$dF_{py} = P \cdot ds.\cos\phi = -P \cdot ds.\cos\theta$$

$$P = \gamma \cdot y \qquad dx = ds.\cos\theta$$

$$dF_{py} = -\gamma \cdot y \cdot dx$$

$$\therefore \uparrow F_{py} = -\int \gamma \cdot y \cdot dx = -\gamma \cdot \int ydx = -\gamma \cdot \forall$$



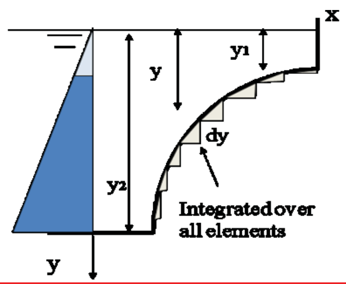
$$dF_{px} = P \cdot ds.\sin\theta$$

$$P = \gamma \cdot y \qquad dy = ds.\sin\theta$$

$$dF_{px} = \gamma \cdot y \cdot dy$$

$$\therefore \rightarrow F_{px} = \int_{y_1}^{y_2} \gamma \cdot y \cdot dy = \gamma \cdot \int_{y_1}^{y_2} y \cdot dy = \gamma \cdot \frac{y^2}{2}\Bigg]_{y_1}^{y_2}$$

$$\therefore \rightarrow F_{px} = \gamma \cdot \frac{y_2^{\,2}}{2} - \gamma \cdot \frac{y_1^{\,2}}{2}$$



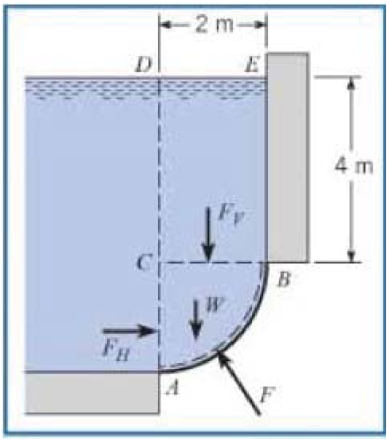
Where $(\gamma \cdot \frac{y^2}{2})$ is the volume of pressure prism on the surface projection on vertical plan

$$\therefore \; F_P = \sqrt{(F_{px})^2 + (F_{py})^2} \text{ : Magnitude} \quad \& \; \tan\theta = \frac{F_{py}}{F_{px}} \text{ : Direction \&}$$

the line of action can be find from the concept of:

Moment of resultant force = The summation of the moments of its components, i.e. $MF_P = \sum(MF_{px}, MF_{py})$

Example 1: A curved surface AB is a circular arc in its section with radius of 2m and width of 1m into the paper. The distance EB is 4m. The fluid above surface AB is water, and atmospheric pressure applied on free surface of water and on the bottom side of surface AB. Find the magnitude and line of action of the hydrostatic force acting on surface AB.



Solution

1. Equilibrium in the horizontal direction

$$F_x = F_H = \bar{p}A = (5\ \text{m})(9810\ \text{N/m}^3)(2\times 1\ \text{m}^2)$$
$$= 98.1\ \text{kN}$$

2. Equilibrium in the horizontal direction

- Vertical force on side *CB*

$$F_v = \bar{p}_0 A = 9.81\ \text{kN/m}^3 \times 4\ \text{m} \times 2\ \text{m} \times 1\ \text{m} = 78.5\ \text{kN}$$

- Weight of the water in volume *ABC*

$$W = \gamma V_{ABC} = (\gamma)(\tfrac{1}{4}\pi r^2)(w)$$
$$= (9.81\ \text{kN/m}^3) \times (0.25 \times \pi \times 4\ \text{m}^2)(1\ \text{m}) = 30.8\ \text{kN}$$

- Summing forces

$$F_y = W + F_V = 109.3\ \text{kN}$$

3. Line of action (horizontal force)

$$y_{cp} = \bar{y} + \frac{\bar{I}}{\bar{y}A} = (5\ \text{m}) + \left(\frac{1\times 2^3/12}{5\times 2\times 1}\text{m}\right)$$
$$y_{cp} = 5.067\ \text{m}$$

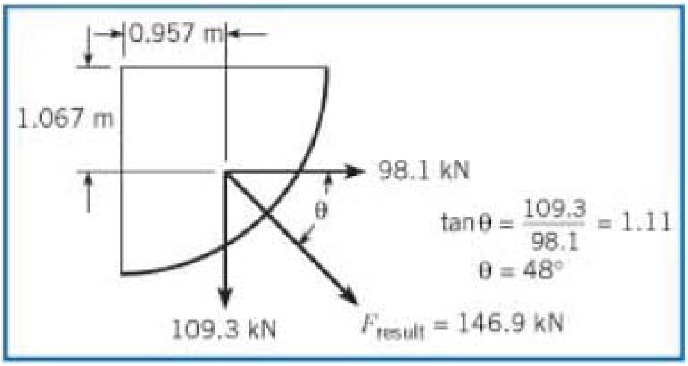
4. The line of action (x_{cp}) for the vertical force is found by summing moments about point *C*:

$$x_{cp}F_y = F_V \times 1\ \text{m} + W \times \bar{x}_W$$

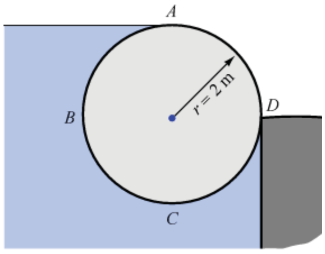
The horizontal distance from point *C* to the centroid of the area *ABC* is found using Fig. A.1: $\bar{x}_W = 4r/3\square = 0.849$ m. Thus,

$$x_{cp} = \frac{78.5\ \text{kN} \times 1\ \text{m} + 30.8\ \text{kN} \times 0.849\ \text{m}}{109.3\ \text{kN}} = 0.957\ \text{m}$$

5. The resultant force that acts on the curved surface is shown in the following figure.



Example 2: A cylindrical barrier in Fig. holds water as shown. The contact between the cylinder and wall is smooth. Consider a 1-m length of cylinder; determine (a) its weight, and (b) the force exerted against the wall.



SOLUTION (*a*) For equilibrium the weight of the cylinder must equal the vertical component of force exerted on it by the water. (The imaginary free surface for CD is at elevation A). The vertical force on BCD is

$$F_{v_{BCD}} = \left(\frac{\pi r^2}{2} + 2r^2\right)\gamma = (2\pi + 8)\ \gamma$$

The vertical force on AB is

$$F_{v_{AB}} = -\left(r^2 - \frac{\pi r^2}{4}\right)\gamma = -(4 - \pi)\ \gamma$$

Hence, the weight per meter of length is

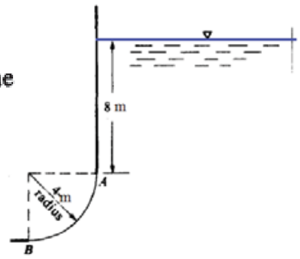
$$F_{v_{BCD}} + F_{v_{AB}} = (3\pi + 4)\gamma = 0.132\ \text{MN}$$

(*b*) The force exerted against the wall is the horizontal force on ABC minus the horizontal force on CD. The horizontal components of force on BC and CD cancel; the projection of BCD on a vertical plane is zero. Hence,

$$F_H = F_{H_{AB}} = 2\gamma = 19.6\ \text{kN}$$

since the projected area is 2 m^2 and the pressure at the centroid of the projected area is 9806 Pa.

Example 3



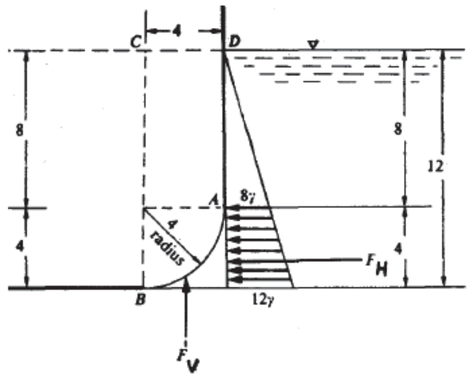
The submerged, curved surface *AB* in Fig. Find the horizontal and vertical components of the total resultant force acting on the curved surface and their locations.

Solution:

$$FH = \frac{8\gamma + 12\gamma}{2}\, x4x1 = 40\gamma = 392.4\, KN$$

$$FV = \left[(4x8) + \frac{\pi 4^2}{4}\right] x1x\gamma = 437.24KN$$

Location of FV& FH



Moment about B for Fh

$$\rightarrow 8\gamma x4x2 + 0.5x4\gamma x4x\left(\frac{4}{3}\right) = 40\gamma xY$$

$$\rightarrow Y = 1.87\, m\, from\, B$$

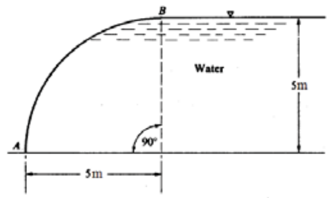
Moment about B for Fv

$$\rightarrow 32\gamma x2 + \left(\frac{\pi 4^2}{4}\right)\gamma x\left(\frac{4x4}{3\pi}\right) = 437.24xX$$

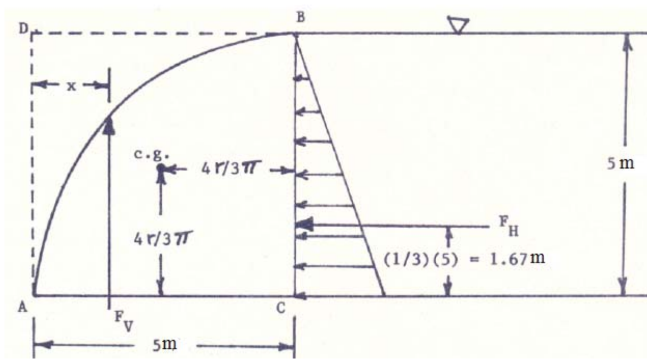
$$\rightarrow X = 1.91\, m\, from\, B$$

Example 4

5.4 **The curved surface *AB* shown in Fig. 5-4*a* is a quarter of a circle of radius 5 ft. Determine, for an 8-ft length perpendicular to the paper, the amount and location of the horizontal and vertical components of the total resultant force acting on surface *AB*.**



Solution:



$$FH = \frac{5\gamma}{2}\, x5x8 = 100\gamma = 981\, KN$$

$$FV = \left[(5x5) - \frac{\pi 5^2}{4}\right] x8x\gamma = 421.83\, KN \uparrow$$

Location of FV& FH

FH act at (5/3) m from C

Moment about C for FV

$$\rightarrow (5x5)\gamma x8x2.5 - \left(\frac{\pi 5^2}{4}\right)\gamma x8x\left(\frac{4x5}{3\pi}\right) = 421.83xX$$

$$\rightarrow X = 0.258\, m\, from\, C$$

Example 5

5.22 A 3-m-diameter water tank consists of two half-cylinders, each weighing 3.5 kN/m, bolted together as shown in Fig. 5-22*a*. If support of the end caps is neglected, determine the force induced in each bolt.

Solution: See Fig. 5-22*b*. Assuming the bottom half is properly supported, only the top half affects the bolt force. $p_1 = (9.79)(1.5+1) = 24.48\ \text{kN/m}^2$; $\sum F_y = p_1 A_1 - 2F_{\text{bolt}} - W_{H_2O} - W_{\text{tank half}} = 0$, $24.48[(3)(\frac{25}{100})] - 2F_{\text{bolt}} - 9.79[(\frac{25}{100})(\pi)(1.5)^2/2] - 3.5/4 = 0$, $F_{\text{bolt}} = 4.42\ \text{kN}$.

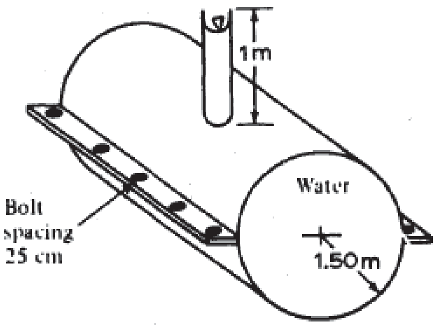


Fig. 5-22(*a*)

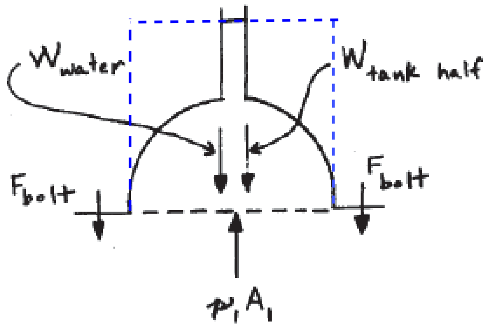


Fig. 5-22(*b*)

or $\left(2.5x3 - \frac{\pi}{2}.1.5^2\right)x0.25x9.79 = 2b + 0.25x3.5 \rightarrow b = 4.42\ kN$

Example 6

5.31 The cylindrical tank in Fig. 5-31 has a hemispherical end cap ABC. Compute the total horizontal forces exerted on ABC by the oil and water.

Solution:

$$F = \gamma h_{cg} A \qquad (F_H)_1 = [(0.9)(9.79)](3 + \tfrac{4}{3})[(\pi)(2)^2/2] = 221\ \text{kN} \quad \text{(left)}$$

$$(F_H)_2 = \{[(0.9)(9.79)](3+2) + (9.79)(\tfrac{4}{3})\}[(\pi)(2)^2]/2 = 338\ \text{kN} \quad \text{(left)}$$

$$(F_H)_{\text{total}} = 221 + 338 = 559\ \text{kN} \quad \text{(left)}$$

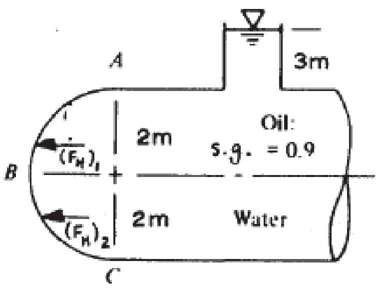
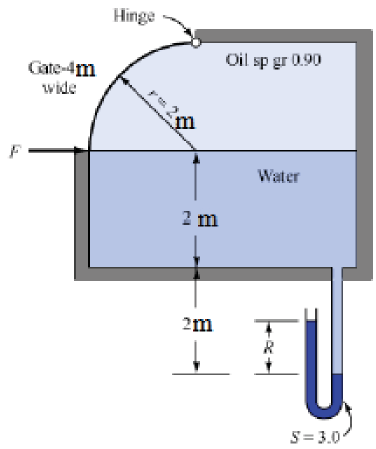


Fig. 5-31

Example 7 **2.84** Calculate the force F required to hold the gate of Fig. 2.74 in a closed position when $R = 2$ m

H.W **2.85** Calculate the force F required to open or hold closed the gate of Fig. 2.74 when $R = 1.5$ m

2.86 What is R of Fig. 2.74 if no force F is required to hold the gate closed or open?



L9

The Buoyant Force

A *buoyant force* is defined as the upward force that is produced on a body that is totally or partially submerged in a fluid. Buoyant forces are significant for most problems as surface ships. In Fig. 3.10a shown, consider a body *ABCD* submerged in a liquid of specific weight γ. The pressures acting on the lower portion of the body create an upward force equal to the weight of liquid needed to fill the volume above surface *ADC*.

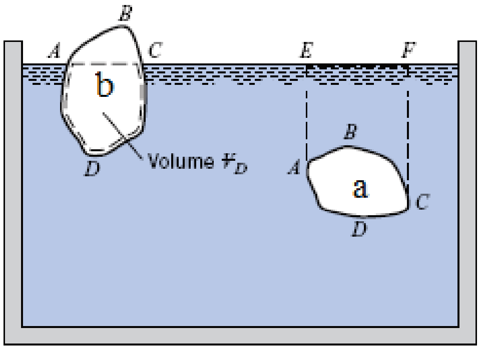


Fig. 3.10

The upward force is

$$F_{up} = \gamma(\forall_b + \forall_a)$$

where $\forall_b$ is the volume of the body (i.e., volume *ABCD*) and $\forall_a$ is the volume of liquid above the body (i.e., volume *ABCFE*).

As shown by Fig. 3.10a, pressures acting on the top surface of the body create a downward force equal to the weight of the liquid above the body:

$$F_{down} = \gamma \forall_a$$

Subtracting the downward force from the upward force gives the **buoyant force F_B** acting on the body:

$$\boxed{F_B = F_{up} - F_{down} = \gamma \forall_b \qquad 3.18}$$

Hence, the buoyant force ($\mathbf{F_B}$) equals the weight of liquid that would be needed to occupy the volume of the body.

The body that is floating as shown in Fig. 3.10b. Pressure acts on curved surface *ADC* causing an upward force equal to the weight of liquid that would be needed to fill volume V_D (displaced volume). The buoyant force is given by

$$F_B = F_{up} = \gamma \,V_D \qquad 3.19$$

Hence, the buoyant force equals the weight of liquid that would be needed to occupy the volume V_D. We can write a single equation for the buoyant force:

$$F_B = \gamma \,V_D \qquad 3.20$$

Stability of Immersed Bodies

When a body is completely immersed in a liquid, its stability depends on the relative positions of the center of gravity of the body and the centroid of the displaced volume of fluid, which is called the *center of buoyancy*.

- If the center of buoyancy is above the center of gravity (Fig. 3.11a), any tipping of the body produces a righting couple, and consequently, the body is stable.
- If the center of gravity is above the center of buoyancy (Fig. 3.11c), any tipping produces an increasing overturning moment, thus causing the body to turn through 180°.
- Finally, if the center of buoyancy and center of gravity are coincident, the body is neutrally stable—that is, it lacks a tendency for righting or for overturning, as shown in Fig. 3.11*b*.

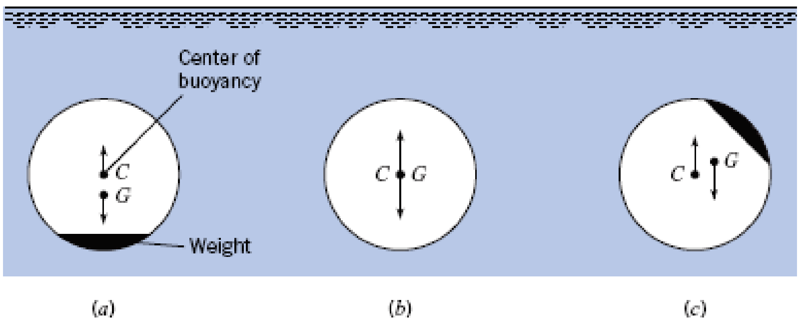
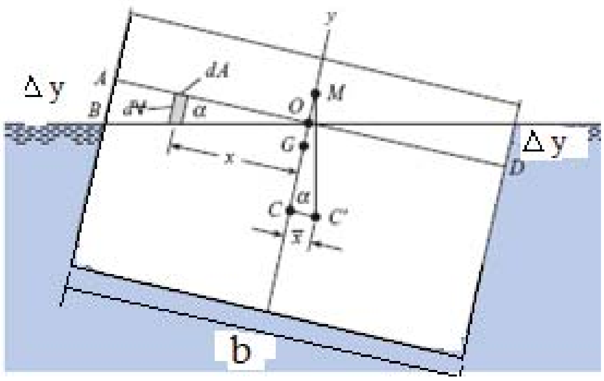


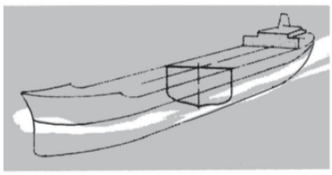
Fig. 3.11

Stability Floating Bodies

The stability for floating bodies than for immersed bodies is very important because the center of buoyancy may take different positions with respect to the center of gravity, depending on the shape of the body and the position in which it is floating. When the center of gravity *G* is above the center of buoyancy *C* (center of displaced volume) for floating body, the body will be stable and equilibrium. The reason for the change in the center of buoyancy for the ship is that part of the original buoyant volume, as shown in Fig.3.12 by the wedge shape *AOB*, is transferred to a new buoyant volume *EOD*. Because the buoyant center is at the centroid of the displaced volume, it follows that for this case the buoyant center must move laterally to the right. The point of intersection of the lines of action of the buoyant force before and after heel is called the ***metacenter M***, and the distance ***GM*** is called the ***metacentric height***.

- If ***GM*** is positive—that is, if ***M*** is above ***G***, the body is stable
- If ***GM*** is negative, the body is unstable.





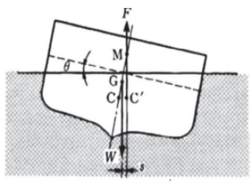


Fig.3.12

Consider the prismatic body shown in Fig. 3.12, which has taken a small angle of heel α. First evaluate the lateral displacement of the center of buoyancy **CC'**, then it will be easy by simple trigonometry to solve for the metacentric height *GM* or to evaluate the righting moment.
The righting couple $= WMG \sin\alpha$
Where : W is weight of body and α angle of heel.
The ***metacenter M*** distance from center of bouncy (C) or ***MC***
Can be found from:

$$\boxed{MC = \frac{I}{Vd} \quad \text{and then} \quad GM = MC - GC}$$

Where:
I is the Moment of inertia for the shortest submersed bed about the centroid (m^4).
Vd is the submersed volume (m^3).
GC is the distance from center of bouncy C to center of gravity G (m).

Example(1):

EXAMPLE 3.12 BUOYANT FORCE ON A METAL PART

A metal part (object 2) is hanging by a thin cord from a floating wood block (object 1). The wood block has a specific gravity $S_1 = 0.3$ and dimensions of $50 \times 50 \times 10$ mm. The metal part has a volume of 6600 mm^3. Find the mass m_2 of the metal part and the tension T in the cord.

Problem Definition

Situation: A metal part is suspended from a floating block of wood.

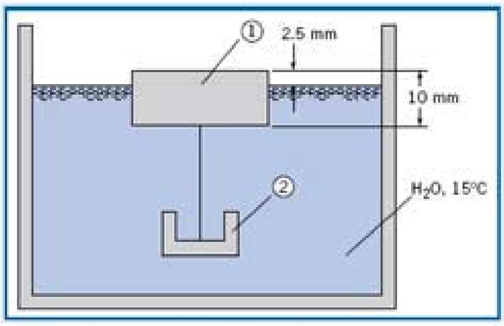
Find:

1. Mass (in grams) of the metal part.
2. Tension (in newtons) in the cord.

Properties:

1. Water (15°C), $\gamma = 9800$ N/m^3.
2. Wood: $S_1 = 0.3$.

Sketch:

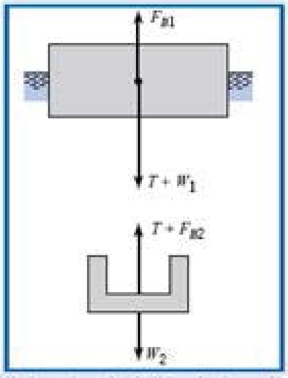


Plan

1. Draw FBDs of the block and the part.
2. Apply equilibrium to the block to find the tension.
3. Apply equilibrium to the part to find the weight of the part.
4. Calculate the mass of the metal part using $W = mg$.

Solution

1. FBDs



2. Force equilibrium (vertical direction) applied to block

$$T = F_{B1} - W_1$$

- Buoyant force $F_{B1} = \gamma V_{D1}$, where V_{D1} is the submerged volume

$$F_{B1} = \gamma V_{D1}$$
$$= (9800\ \text{N/m}^3)(50 \times 50 \times 7.5\ \text{mm}^3)(10^{-9}\ \text{m}^3/\text{mm}^3)$$
$$= 0.184\ \text{N}$$

- Weight of the block

$$W_1 = \gamma S_1 V_1$$
$$= (9800\ \text{N/m}^3)(0.3)(50 \times 50 \times 10\ \text{mm}^3)(10^{-9}\ \text{m}^3/\text{mm}^3)$$
$$= 0.0735\ \text{N}$$

- Tension in the cord

$$T = (0.184 - 0.0735) = \boxed{0.110\ \text{N}}$$

3. Force equilibrium (vertical direction) applied to metal part
   - Buoyant force

$$F_{B2} = \gamma V_2 = (9800\ \text{N/m}^3)(6600\ \text{mm}^3)(10^{-9}) = 0.0647\ \text{N}$$

- Equilibrium equation

$$W_2 = T + F_{B2} = (0.110\ \text{N}) + (0.0647\ \text{N})$$

4. Mass of metal part

$$m_2 = W_2/g = \boxed{17.8\ \text{g}}$$

Review

Notice that tension in the cord (0.11 N) is less than the weight of the metal part (0.18 N). This result is consistent with the common observation that an object will "weigh less in water than in air."

Example(2):

6.6 A barge is loaded with 150 tons of coal. The weight of the empty barge in air is 35 tons. If the barge is 18 ft wide, 52 ft long, and 9 ft high, what is its draft (i.e., its depth below the water surface)?

Solution:

$F_b = W \qquad 62.4[(18)(52)(D)] = (150 + 35)(2000) \qquad D = 6.33$ ft

$$L = 52ft = 15.85m\,, \qquad w = 18ft = 5.5m$$

$$\rightarrow 9810(5.5x15.85xD) = (150 + 35)x1000x9.81 \ \rightarrow D = 2.12m \ \approx 6.4\ ft$$

Example(3):

6.36 A wooden beam (s.g. = 0.64) is 140 mm by 140 mm by 5 m and is hinged at A, as shown in Fig. 6-18. At what angle θ will the beam float in water?

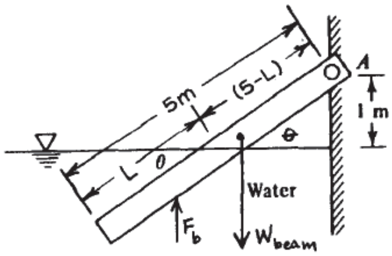


Fig. 6-18

Solution:

The forces acting on the beam are shown in Fig. 6-18. $W_{beam} = [(0.64)(9.79)][(0.140)(0.140)(5)] = 0.6140$ kN and $F_b = 9.79[(0.140)(0.140)(L)] = 0.1919L$. $\sum M_A = 0$; $(0.1919L)[(5 - L/2)(\cos\theta)] - (0.6140)[(\frac{5}{2})(\cos\theta)] = 0$, $-0.0960L^2 + 0.9595L - 1.535 = 0$, $L = 2.000$ m; $\sin\theta = 1/(5 - 2.000) = 0.33333$, $\theta = 19.5°$.

Example(4):

6.56 What is the weight of the loaded barge in Fig. 6-25? The barge is 7 m in width.

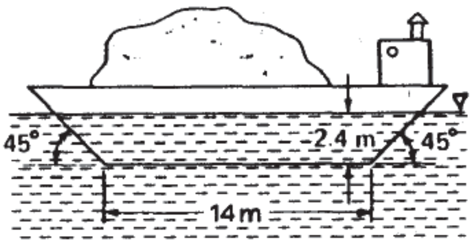


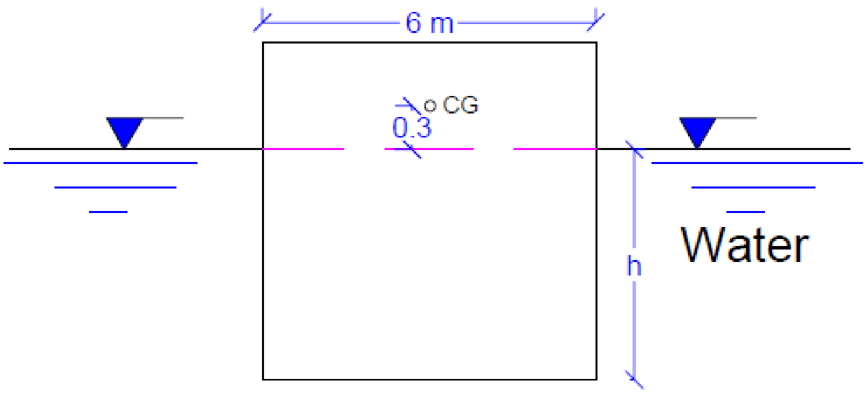
Fig. 6-25

Solution:

$F_b = W \qquad 9.79\{(7)[(14)(2.4) + (2)(2.4)(2.4)/2]\} = W \qquad W = 2359$ kN

Example(5):

In figure shown: a scow 6m wide and 18m long with weight 225T have G at 0.3m above water surface ; check its stability in water:



Solution:

$$L = 18m\,, \qquad w = 6m\,, \qquad W = 225x1000x9.81 = 2207250N$$

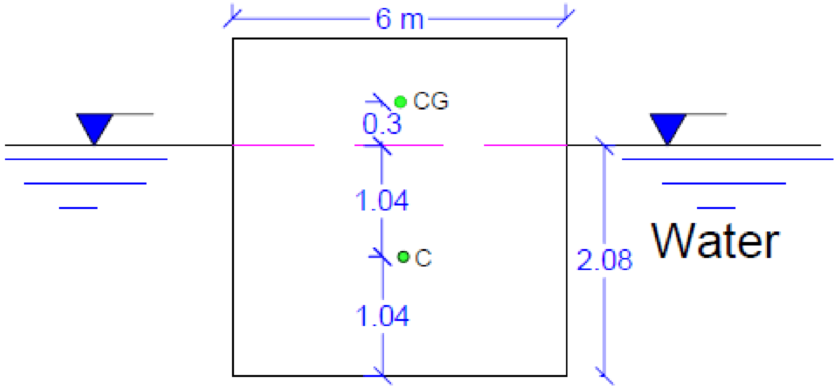
$$\rightarrow 9810(18x6xD) = 2207250 \ \rightarrow D = 2.08m$$

$$\therefore C = 1.04\, m\ from\ the\ water\ surface$$

$$CG = 1.04 + 0.3 = 1.34\,,$$

$$CM = \frac{I}{vb} = \frac{\frac{18x6^3}{12}}{18x6x2.08} = 1.44m \ \rightarrow GM = CM - CG = 1.44 - 1.34 = 0.1m$$

$$\therefore +value \ \rightarrow stable\ body$$



Example(6):

6.75 The barge shown in Fig. 6-42 has the form of a parallelopiped having dimensions 10 m by 26.7 m by 3 m. The barge weighs 4450 kN when loaded and has a center of gravity 4 m from the bottom. Find the metacentric height for a rotation about its longest centerline, and determine whether or not the barge is stable.

Solution:

First, find the center of buoyancy of the barge. $F_b = W$, $9.79[(10)(26.7)(D)] = 4450$, $D = 1.702$ m. Hence, the center of buoyancy (CB) is at a distance 1.702/2, or 0.851 m above the bottom of the barge. $\overline{MB} = I/V_d = [(26.7)(10)^3/12]/[(10)(26.7)(1.702)] = 4.896$. The distance from CB to CG is $4 - 0.851$, or 3.149 m. Therefore, the metacenter is located $4.896 - 3.149$, or 1.747 m above the CG, and the barge is stable.

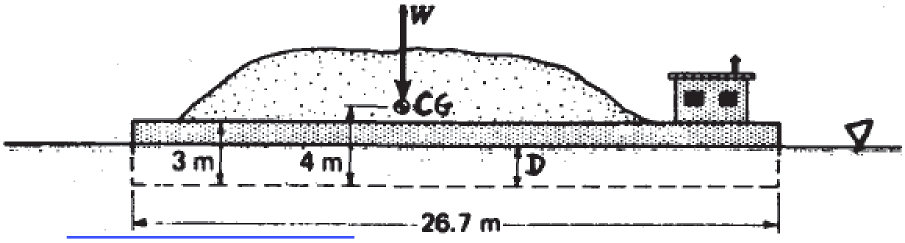
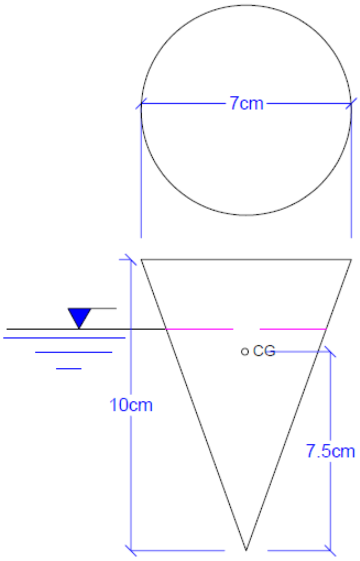


Fig. 6-42

Example(7):

6.69 A wood cone floats in water in the position shown in Fig. 6-36*a*. The specific gravity of the wood is 0.60. Would it be stable?



$$G = 7.5\ cm\ from\ the\ tip,\quad W = 0.6x9810\left(\frac{\pi x7^2x10}{12}\right)x10^{-6} = 0.756N$$

$$let\ submerged\ depth = h\ \rightarrow \frac{7}{10} = \frac{D}{h} \rightarrow D = 0.7h$$

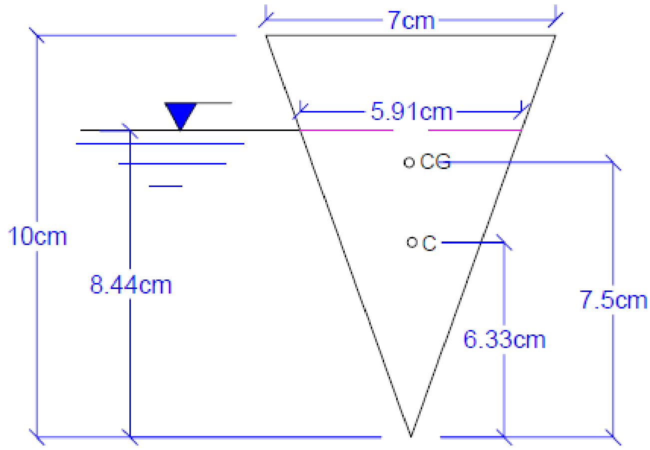
$$\therefore Vb = \frac{h.\pi.D^2}{12} = 0.1283h^3\ \rightarrow Fb = w\ \rightarrow 9810x0.1283\ h^3 = 0.756N$$

$$\rightarrow h = 0.0844\ m = 8.44cm\ \&\ D{=}0.7x8.44{=}5.91cm\ \&\ Vb{=}\frac{8.44.\pi.5.91^2}{12} = 77.1\ cm^3$$

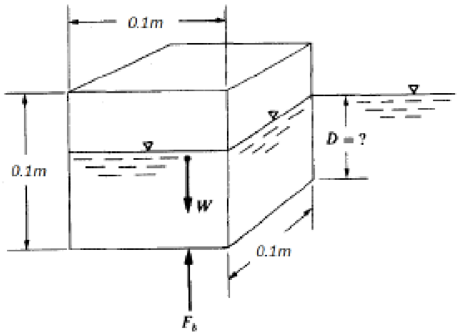
$$\therefore C = 0.75x8.44 = 6.33\ cm\ from\ the\ tib.\ \rightarrow CG = 7.5 - 6.33 = 1.17\ cm$$

$$CM = \frac{I}{vb} = \frac{\frac{\pi x5.91^4}{64}}{77.1} = 0.78\ cm\ \rightarrow GM = CM - CG = 0.784 - 1.17 = -0.39\ cm$$

$\therefore -value\ \rightarrow unstable\ body$



:Example 8



A cube of timber 100 mm on each side weight 650 gr:

Estimate its draft in water and in oil with (s=0.9):

Estimate its draft with 200gr additional weight in water and oil:

Solution:

$w = 0.65x9.81 = 6.376N\ ,\qquad \rho_t = \frac{m}{v} = \frac{0.65}{0.01^3} = 650\frac{kg}{m^3} \rightarrow \gamma = 650x9.81 = 6376.5N/m^3$

$in\ water : Fb = w \ \rightarrow 6.376 = 0.1x0.1xDx9810 \ \rightarrow D = 0.065m$

$in\ oil : Fb = w \ \rightarrow 6.376 = 0.1x0.1xDx0.9x9810 \ \rightarrow D = 0.072m$

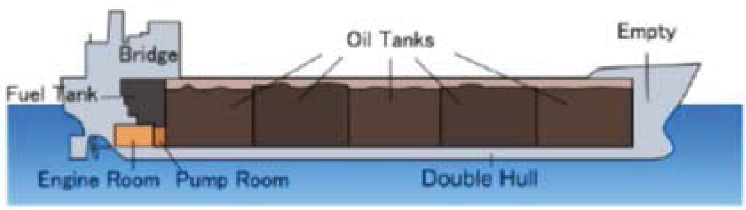
with 200gr additional weight in water:

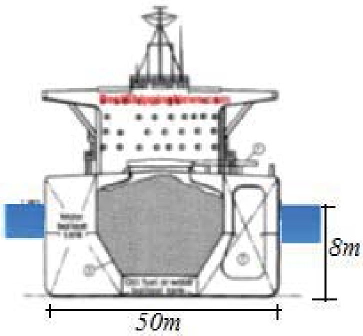
$in\ water : Fb = w \ \rightarrow (0.2 + 0.65)x9.81 = 0.1x0.1xDx9810 \ \rightarrow D = 0.085m$

:. Example(9) H.W

For the crude oil ship shown: if the empty ship weight =10000T and its length of 200m.

Find the total oil volume of S=0.85 that can be transmitted by the ship in (barrel) and check the ship stability if SG. at 2 m above water surface.

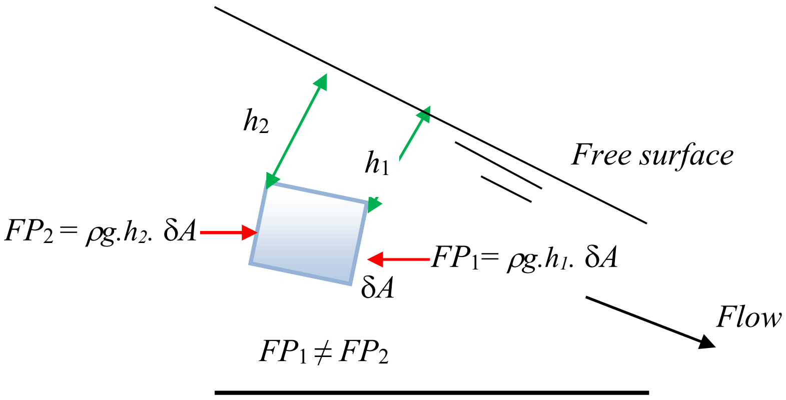




Kinematic of Fluid Motion

Lec:1

Fluid flow



Motion (flowing) of a fluid mass accrues when it is subjected to unbalanced forces that reveal if the fluid mass was subjected to hydraulic gradient (e.g. tilting of free surface by certain angle or connect two containers have different levels). This means that fluid mass lies under an acceleration toward its flow direction. This motion continues as long as unbalanced forces are applied.

Flow is defined as the quantity (mass or volume) of fluid (gas, liquid or vapour) that passes a point (section) per unit time. A simple equation to represent this is:

$$\boldsymbol{Flow} = \frac{\boldsymbol{Quantity}}{\boldsymbol{time}}$$

Flow Classification (Flow pattern)

Having introduced the general concepts of flow patterns, it is convenient to make distinctions between different types of flows. These concepts can be best introduced by expressing the velocity of the fluid in the form:

$$\boldsymbol{V} = \boldsymbol{V}(\boldsymbol{s}, \boldsymbol{t})$$

where *s* is the distance traveled by a fluid particle along a path, and *t* is the time.

➢ Uniform or Non-uniform

- **A uniform flow** is a flow in which the velocity does not change along a streamline, i.e. $\frac{\partial v}{\partial s} = 0$

  In uniform flows the streamlines are straight and parallel.

- **A non-uniform flow** is a flow in which the velocity changes along a streamline, i.e. $\frac{\partial v}{\partial s} \neq 0$

➢ Steady or Unsteady

- In **a steady flow** the velocity at a given point on a streamline does not change with time: $\frac{\partial v}{\partial t} = 0$

- **An unsteady flow** exists if: $\dfrac{\partial v}{\partial t} \neq 0$

➢ **Combining the above we can classify any flow into one of four types:**

- ***Steady uniform flow***. Conditions do not change with position in the stream or with time. **An example** is the flow of water in a pipe of constant diameter at constant velocity or discharge (flow rate).
- ***Steady non-uniform flow***. Conditions change from point to point in the stream but do not change with time. **An example** is flow in a tapering pipe with constant velocity at the inlet - velocity will change as you move along the length of the pipe toward the exit.
- ***Unsteady uniform flow***. At a given instant in time the conditions at every point are the same, but will change with time. **An example** is a pipe of constant diameter connected to a pump pumping at a constant rate which is then switched off or in open/close valves.
- ***Unsteady non-uniform flow***. Every condition of the flow may change from point to point and with time at every point. **An example** is surface waves in an open channel.

Flow rate

$$\textit{Weight flowrate} = \frac{\textit{weight}}{\textit{time taken to accumulate this mass}} = \rho g.Q$$

$$\textit{mass flowrate} = \frac{\textit{mass}}{\textit{time taken to accumulate this mass}}$$

$$\textit{mass flow rate} = \rho \cdot \textit{volum flow rate}$$

- Volume flow rate - Discharge.

$$\textit{volume flowrate} = \frac{\textit{Volume}}{\textit{time taken to accumulate this volume}} = \textit{Discharge}\,(Q)$$

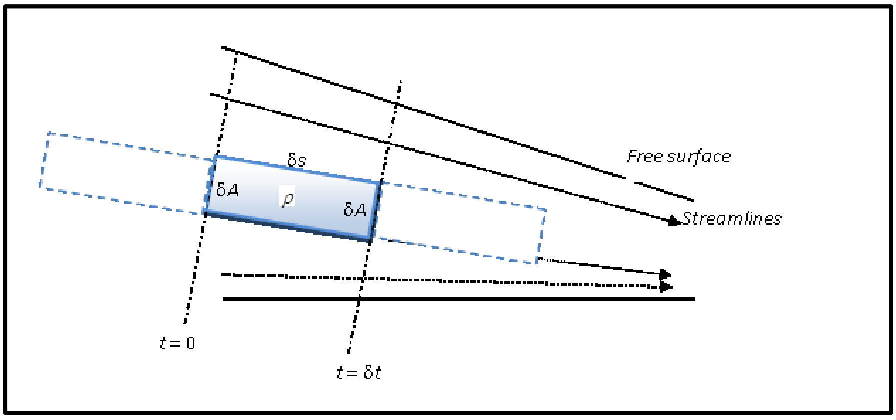
- More commonly we use volume flow rate
- Also known as discharge.
- The symbol normally used for discharge is Q.

Discharge and mean velocity

Cross sectional area of a pipe is *A*
Mean velocity is v_m.

$$Q = A \cdot v_m$$

We usually drop the “m” and imply mean velocity



Flow Equations

Equation of Continuity

The application of the principle of conservation of mass to fluid flow in a stream tube results in the "equation of continuity' expressing the continuity of the flow from point to point along the stream tube. If the cross-sectional areas and average velocities at sections 1 and 2 in the stream tube of Fig. 31 are designated by A_1, A_2, V_1 and V_2, respectively, the .quantity of fluid passing ***section 1*** per unit of time will be expressed by A_1V_1, and the mass of fluid passing ***section 1*** per unit of time will be $A_1V_1\rho_1$. Similarly, the mass of fluid passing ***section 2*** will be $A_2V_2\rho_2$, Obviously, no fluid mass is being created or destroyed between sections 1 and 2, and therefore

$$A_1 V_1 \rho_1 = A_2 V_2 \rho_2 \qquad \ldots\ldots 4.1$$

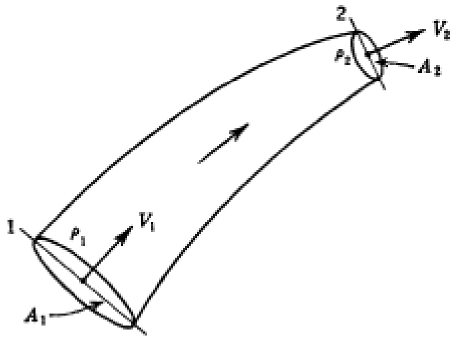


Fig. 4.6

Thus the mass of fluid passing any point in a streamtube per unit of time is the same.

- If this equation is multiplied by ***g***, the acceleration due to gravity, there results giving the equation of continuity in terms of weight.

$$A_1 V_1 W_1 = A_2 V_2 W_2 \qquad \ldots\ldots 4.2$$

The product will be found to have units of ***N/s*** and is termed the "weight rate of flow" or "weight flow."

- For liquids, and for gases when pressure and temperature changes are negligible, $W_1 = W_2$, resulting in

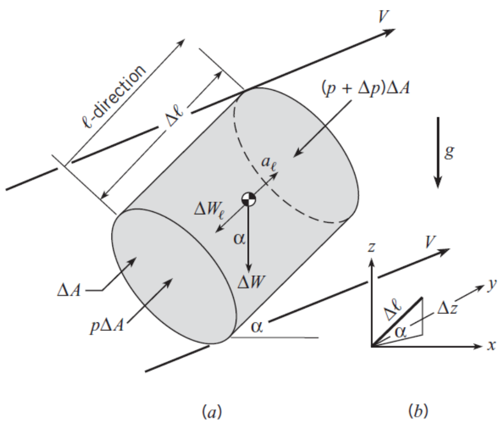
$$\boxed{A_1 V_1 = A_2 V_2 = Q} \qquad \ldots\ldots 4.3$$

$$\boxed{A_1 V_1 = A_2 V_2, \text{ or } Q_1 = Q_2}$$ Continuity equation

where $Q = VA$ is the volume flow rate (m^3/s)

Energy Equation (Bernoulli's Equation):

Consider a small element of ideal fluid (non-viscous and incompressible fluid) ossraligned along a streamline. It has a c sectional area ΔA, pressure is assumed launiform across its ends ΔA, and the loc velocity is defined v and subject to ontal)zacceleration in both directions x (hori .(and z (vertical instead of y



1. First from previous lectures, recall to the pressure difference due to pressure variation in both directions (x, z)

$$\Delta p = \frac{\partial p}{\partial x}\Delta x + \frac{\partial p}{\partial z}\Delta z$$

or
$$dp = \frac{\partial p}{\partial x} dx + \frac{\partial p}{\partial z} dz$$

Also, we know that $\frac{\partial p}{\partial x} = -\rho a_x$, and $\frac{\partial p}{\partial z} = -\rho(g + a_z)$

So

(Hydrostatic eq. extension due to accelerations)

$$\Delta p = -\rho \cdot a_x \,\Delta x + (-\rho \cdot (a_x + g))\,\Delta z$$

or
$$dp = -\rho a_x dx - \rho(g + a_z) dz$$

$$\Rightarrow dp = -\rho a_x dx - \rho g dz - \rho a_z dz$$

$$\Rightarrow dp + \rho a_x dx + \rho g dz + \rho a_z dz = 0 \qquad (1)$$

2. We look at the acceleration of the fluid element.
   - Ignoring the possibility that the flow might be steady, $\frac{\partial v}{\partial t} \neq 0$
   - ***v*** can change with time ***t***, and also with position ***s*** along the direction of motion.

     i.e. ***v* = *f* (*t*, *s*)**.
   - Hence, if the element moves a distance ***δs*** in time ***δt***, then the total change in velocity ***δv*** is given by:

$$\delta v = \frac{\partial v}{\partial s}\delta s + \frac{\partial v}{\partial t}\delta t$$

and in the limit as ***δt*** tends to zero, the "substantive" derivative represent the acceleration in that direction and is given as:

$$a_s = \frac{dv}{dt} = \mathrm{Lim}_{\delta t \to 0}\frac{\delta v}{\delta t} = \frac{\partial v}{\partial s}\frac{ds}{dt} + \frac{\partial v}{\partial t} = \overbrace{v\frac{\partial v}{\partial s}}^{spatially} + \overbrace{\frac{\partial v}{\partial t}}^{temporarily}$$

- For a steady flow the local velocity at a point does not vary with time, so the last term under such conditions ($\frac{\partial v}{\partial t}$) will be zero. And the acceleration remain as: $a_s = \frac{dv}{dt} = v\frac{dv}{ds}$ (i.e. $a_x = \frac{dv_x}{dt} = v_x\frac{dv_x}{dx}$, and $a_z = \frac{dv_z}{dt} = v_z\frac{dv_z}{dz}$)

3. Now substitute the form of horizontal and vertical acceleration in equ's. (1) we get;

$$dp + \rho . v_x \frac{dv_x}{dx} dx + \rho g dz + \rho . v_z \frac{dv_z}{dz} dz = 0$$

$$\Rightarrow dp + \rho . v_x \, dv_x + \rho g dz + \rho . v_z \, dv_z = 0$$

$$\frac{dp}{\rho} + v_x dv_x + v_z dv_z + g dz = 0$$

Euler's equation (for ideal, steady flow)

This is a form of ***Euler's equation***, and relates *p*, *v*, and *z* in flow field.

- it then becomes possible to integrate it - giving:

$$\frac{p}{\rho} + \frac{1}{2}(v_x^2 + v_z^2) + g\,z = C$$

$$\frac{p}{\rho} + \frac{1}{2} v^2 + g\,z = C$$

$$p + \frac{1}{2} \rho\, v^2 + \rho\, g\, z = C$$

$$\frac{p}{\rho\, g} + \frac{v^2}{2\, g} + z = C$$

Bernoulli's equation (for ideal, steady flow)

The three equations above are valid for incompressible, frictionless steady flow, and what they state is that total energy is conserved along a streamline.

The first of these forms of the ***Bernoulli equation*** is a measure of energy per unit mass, the second of energy per unit volume, and the third of "head", equivalent to energy per unit weight.

In the second equation, the term ***p*** is the static pressure**, {½ρv²}** is the dynamic pressure, ***ρgz*** is the elevation term, and the **SUM** of all three is known as the **stagnation (or total) pressure, *p₀***

In the third equation:

- ***p/ρg*** is known as **the pressure head (or flow work head or flow energy head)**, which is the work done to move fluid against pressure,
- ***z*** is the potential head (elevation head),
- the summation of two terms ***(p/ρg + z)*** is called **piezometric head** or **hydraulic head**,
- ***v²/2g*** as the **kinetic head (dynamic energy head or velocity head)**, and
- the sum of the three terms as the **Total Head *H***. The sum of first and third tem of 3rd equation is called the piezometric head respect to piezometer's tube.

where **C** is a constant along a streamline.

- For the special case of **irrotational flow**, the constant **C** is the same everywhere in the flow field.
- Therefore, the Bernoulli equation can be applied **between any two points in the flow field** if the flow is [1]**ideal**, [2]**steady**, [3]**incompressible**, and [4]**irrotational**.
- i.e. for two points 1 and 2 in the flow field:

$$\boxed{\frac{p_1}{\rho g}+\frac{v_1^2}{2g}+z_1=\frac{p_2}{\rho g}+\frac{v_2^2}{2g}+z_2}$$

Equation above is called Bernoulli's equation (for frictionless, steady flow). All of terms of Bernoulli's equation having dimension of length (L) or dimension of energy times dimension of weight(FL/F). The elevation head ,represent the potential energy per unit weight as below

$$\therefore \qquad \boxed{elevation\ head = z}$$

The velocity head represent the kinetic energy per unit weight as below,

$$\therefore \boxed{velocity\ head = \frac{V^2}{2g}}$$

The pressure head represent the pressure energy per unit weight as below,

$$\therefore \qquad \boxed{presure\ head = \frac{p}{\gamma}}$$

The sum of elevation, velocity and pressure heads for ideal steady incompressible flow is constant for all point in stream line,

$$\boxed{\frac{P}{\gamma}+z+\frac{V^2}{2g}=H=total\ head}$$

the sum of elevation and pressure heads called piezometric head which represent the manometric height of liquid from datum,

$$\boxed{\frac{P}{\gamma}+z=h=piezometric\ head}$$

Hydraulic and Energy Grade Lines

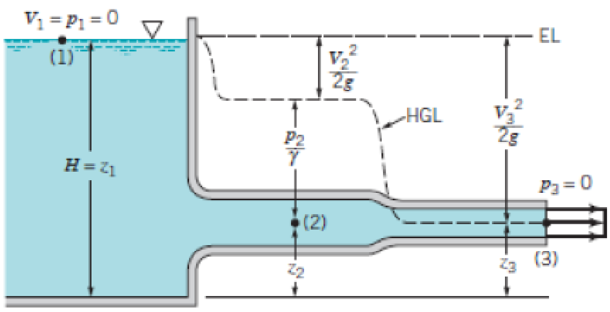
The *energy grade line* (EGL) shows the height of the total Bernoulli constant

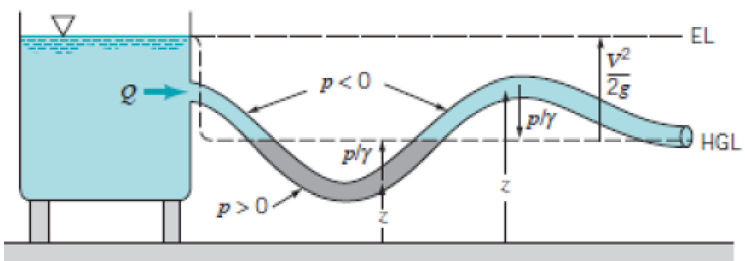
$\frac{P}{\gamma} + z + \frac{V^2}{2g} = H = total\ head$. The EGL has constant height.

The *hydraulic grade line* (HGL) shows the height corresponding to elevation and pressure head $\boxed{\frac{P}{\gamma} + z} = h = piezometric\ head$, that is, the EGL minus the velocity head $V^2/2g$. The HGL is the height to which liquid would rise in a piezometer tube

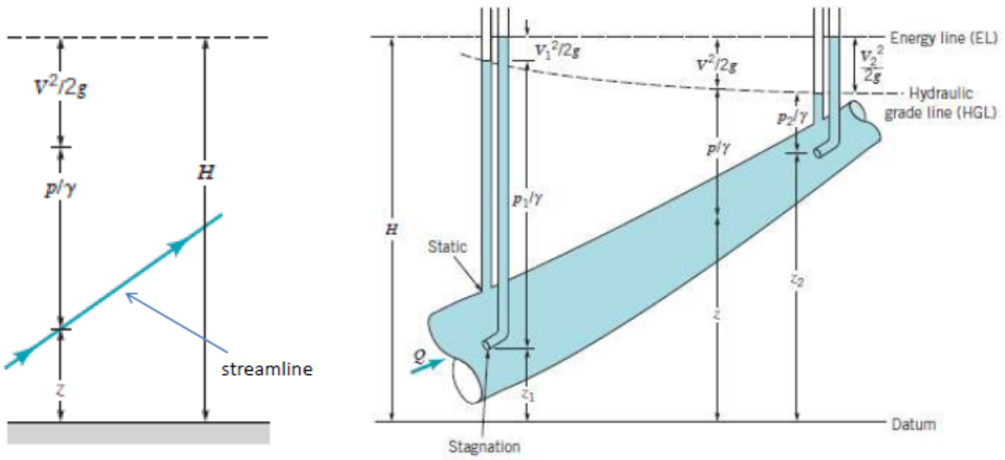
- In an open-channel flow the HGL is identical to the free surface of the water.
- The EGL will drop slowly due to friction losses and will drop sharply due to a substantial loss (a valve or obstruction) or due to work extraction (to a turbine).
- The EGL can rise only if there is work addition (as from a pump or propeller).
- The HGL generally follows the behavior of the EGL with respect to losses or work transfer, and it rises and/or falls if the velocity decreases and/or increases.

The energy line and hydraulic grade line for flow from a tank.

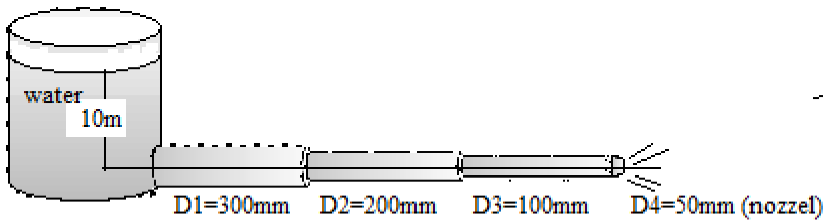




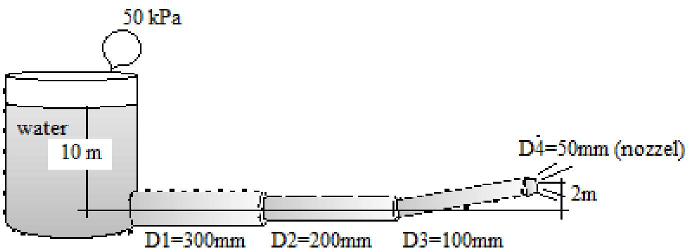
- Under the assumptions of the Bernoulli equation, the energy line is horizontal.
- If the fluid velocity changes along the streamline, the hydraulic grade line will not be horizontal.
- If viscous effects are important the total head does not remain constant due to a loss in energy as the fluid flows along its streamline ⇨ the energy line is no longer horizontal



Example 1: A flow of water from a reservoir to a pipe of different diameters shown in Figure below. Calculate 1) the discharge and velocity at each pipe, 2) the pressure in each pipe and 3) the energy and hydraulic grade lines.



Example 2: A flow of water from a closed reservoir with interior pressure of 50 kPa to a pipe of different diameters shown in figure below. Calculate 1) the discharge and velocity at each pipe, 2) the pressure in each pipe and 3) the .energy and hydraulic grade lines



Example 3: A pipe gradually tapers from 0.6m at A to 0.9m at point B. the elevation difference between A and B is 3m. Find pressure head and pressure at point B if the pressure head at A is 15m and velocity at A is 2m/s. Assume the frictionless flow.

Example.4

3.33 In some wind tunnels the test section is perforated to suck out fluid and provide a thin viscous boundary layer. The test section wall in Fig. P3.33 contains 1200 holes of 5-mm diameter each per square meter of wall area. The suction velocity through each hole is V_r = 8 m/s, and the test-section entrance velocity is V_1 = 35 m/s. Assuming incompressible steady flow of air at 20°C, compute (a) V_o, (b) V_2, and (c) V_f, in m/s.

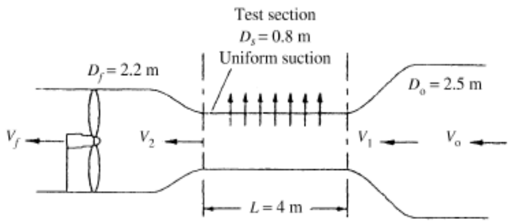


Fig. P3.33

Solution: The test section wall area is (π)(0.8 m)(4 m) = 10.053 m^2, hence the total number of holes is (1200)(10.053) = 12064 holes. The total suction flow leaving is

$$Q_{suction} = NQ_{hole} = (12064)(\pi/4)(0.005\ m)^2(8\ m/s) \approx 1.895\ m^3/s$$

$$\text{(a)}\quad \text{Find } V_o\text{:}\quad Q_o = Q_1 \quad \text{or} \quad V_o\frac{\pi}{4}(2.5)^2 = (35)\frac{\pi}{4}(0.8)^2,$$

$$\text{solve for} \quad V_o \approx \mathbf{3.58\ \frac{m}{s}} \quad \textit{Ans. (a)}$$

$$\text{(b)}\quad Q_2 = Q_1 - Q_{suction} = (35)\frac{\pi}{4}(0.8)^2 - 1.895 = V_2\frac{\pi}{4}(0.8)^2,$$

$$\text{or:}\quad V_2 \approx \mathbf{31.2\ \frac{m}{s}} \quad \textit{Ans. (b)}$$

$$\text{(c)}\quad \text{Find } V_f\text{:}\quad Q_f = Q_2 \quad \text{or} \quad V_f\frac{\pi}{4}(2.2)^2 = (31.2)\frac{\pi}{4}(0.8)^2,$$

$$\text{solve for} \quad V_f \approx \mathbf{4.13\ \frac{m}{s}} \quad \textit{Ans. (c)}$$

Example. 5

3.167 In Fig. P3.167 the fluid is gasoline at 20°C at a weight flux of 120 N/s. Assuming no losses, estimate the gage pressure at section 1.

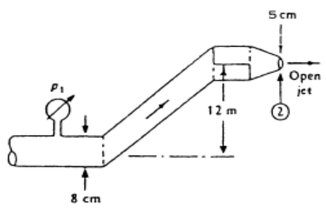


Fig. P3.167

Solution: For gasoline, ρ = 680 kg/m^3. Compute the velocities from the given flow rate:

$$Q = \frac{\dot{W}}{\rho g} = \frac{120\ N/s}{680(9.81)} = 0.018\ \frac{m^3}{s},$$

$$V_1 = \frac{0.018}{\pi(0.04)^2} = 3.58\ \frac{m}{s};\quad V_2 = \frac{0.018}{\pi(0.025)^2} = 9.16\ \frac{m}{s}$$

Now apply Bernoulli between 1 and 2:

$$\frac{p_1}{\rho} + \frac{V_1^2}{2} + gz_1 \approx \frac{p_2}{\rho} + \frac{V_2^2}{2} + gz_2, \quad \text{or:}\quad \frac{p_1}{\rho} + \frac{(3.58)^2}{2} + 0 \approx \frac{0(gage)}{680} + \frac{(9.16)^2}{2} + 9.81(12)$$

$$\text{Solve for} \quad p_1 \approx \mathbf{104{,}000\ Pa\ (gage)} \quad \textit{Ans.}$$