Engineering Hydrology

References:

1- Applied Hydrology by Ven T. Chow

2- Engineering Hydrology by K. Subramanya

3- Engineering Hydrology by E. M. Wilson

Syllabus:

1- Introduction and hydrologic Cycle

2- Precipitations

3- Water Losses (Evaporation and Infiltration)

4- Runoff

5- Stream Flow measurement

6- Hydrograph

7- Groundwater Hydrology

8- Flood Routing

9- Probability in Hydrology

Introduction

- The science of hydrology deals with the occurrence and movement of water on and over the surface of the earth. It deals with the various forms of moisture that occur, the transformation between the liquid, solid and gaseous states in the atmosphere and in the surface layers of land masses.

- SCOPE OF HYDROLOGY

The study of hydrology helps us to know

- (i) the maximum probable flood that may occur at a given site and its frequency; this is required for the safe design of drains and culverts, dams and reservoirs, channels and other flood control structures.
- (ii) the water yield from a basin—its occurrence, quantity and frequency, etc; this is necessary for the design of dams, municipal water supply, water power, river navigation, etc.
- (iii) the ground water development for which a knowledge of the hydrogeology of the area, i.e., of the formation soil, recharge facilities like streams and reservoirs, rainfall pattern, climate, cropping pattern, etc. are required.
- (iv) the maximum intensity of storm and its frequency for the design of a drainage project in the area.

- The Hydrologic Cycle

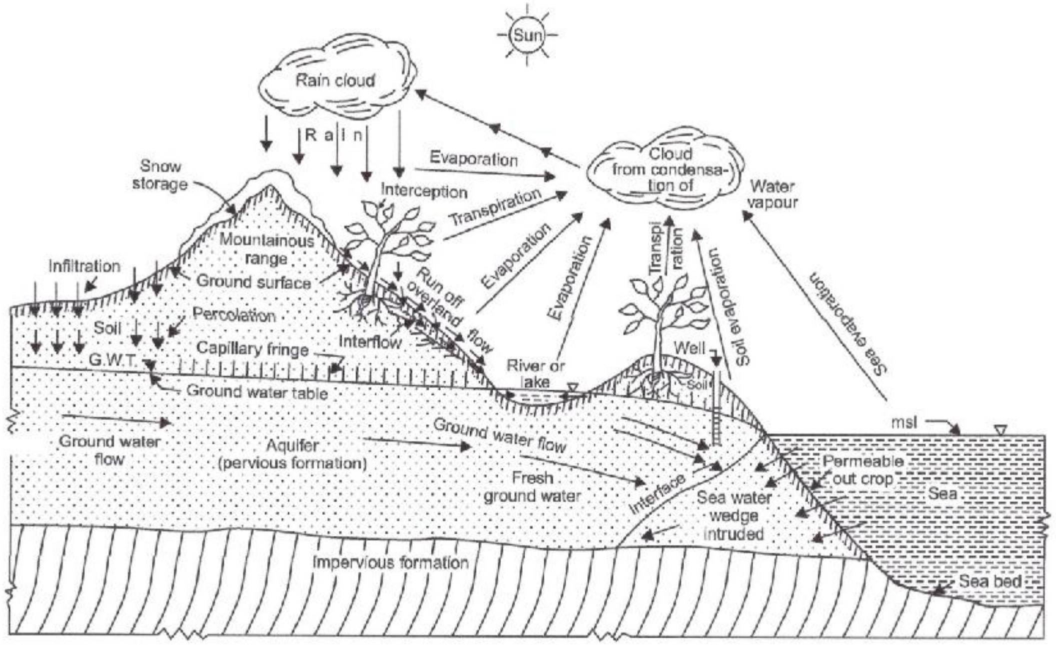
Hydrologic cycle is the water transfer cycle, which occurs continuously in nature. Evaporation from the surfaces of ponds, lakes, reservoirs and ocean surfaces, etc. and transpiration from plant leaves of cropped land and forests, etc. take place. These vapors rise to the sky and are condensed at higher altitudes by condensation nuclei and form clouds, resulting in droplet growth. The clouds melt and sometimes burst resulting in precipitation of different forms like rain, snow, hail, sleet, mist, dew and frost. A part of this precipitation flows over the land called runoff and part infiltrates into the soil which builds up the ground water table. The surface runoff joins the streams and the water is stored in reservoirs. A portion of surface runoff and ground water flows back to ocean. Again evaporation starts from the surfaces of lakes, reservoirs and ocean, and the cycle repeats.

Of these three phases of the hydrologic cycle, namely, evaporation, precipitation and runoff, it is the 'runoff phase', which is important to a civil engineer since he is concerned with the storage of surface runoff in tanks and reservoirs for the purposes of irrigation, municipal water supply hydroelectric power etc.

The three important phases of the hydrologic cycle are: (a) Evaporation and evapotranspiration, (b) precipitation and (c) runoff

See figure below

- The Hydrologic Cycle



The hydrologic cycle

- The main processes occurred in the H.C. are:

1- Evaporation E
2- Transpiration (from Plants) T
3- Precipitation (Rainfall) P
4- Runoff R
5- Infiltration I
6- Groundwater Flow G

- Water Budget Equation

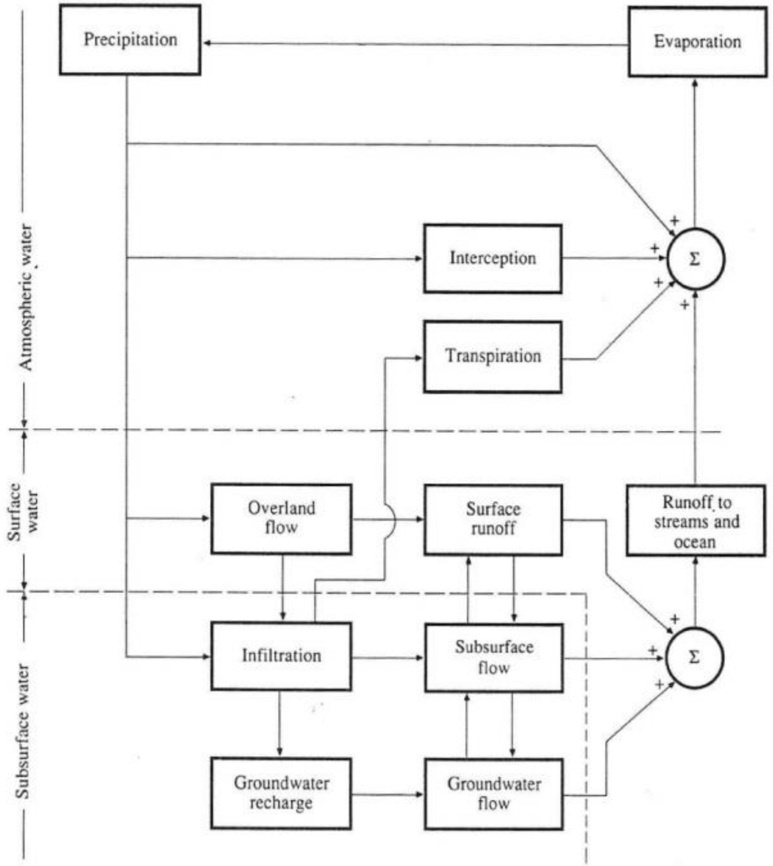
For any hydrologic system, a water budget can be developed to account for various flow pathways and storage components. The hydrologic continuity equation for any system is:

$Q_{inflow} - Q_{outflow} = \Delta Storage$

For Precipitation this eq. can be written as

$P\text{-}R\text{-}G\text{-}E\text{-}T\text{-}I = \Delta S$

This equation can be represented by figure below:

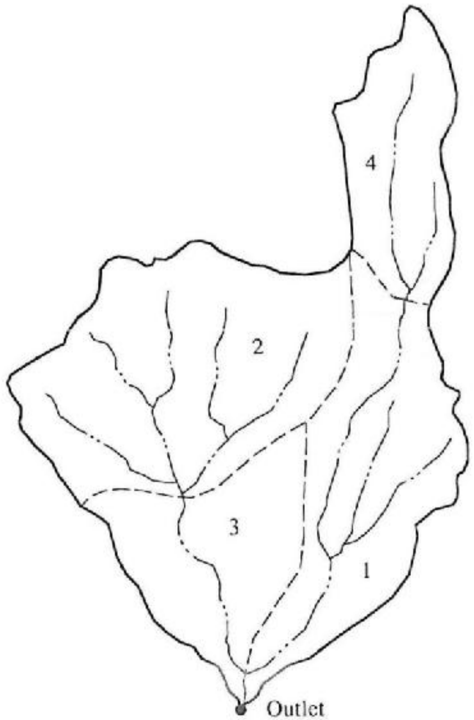


Block-diagram representation of the global hydrologic system.

- ## The Watershed

A watershed (also called catchment basin) is an area of land, often bounded by hills, where surface water from rain, melting snow, or ice drains to a single point at a lower elevation, usually the exit of the basin, where the waters join another waterbody, such as a river, lake, reservoir, estuary, wetland, sea, or ocean.

The figure below showed a catchment area



The Castro Valley watershed

Precipitation

precipitation denotes and includes all forms of water which falls from the atmosphere to the land surface.

Rainfall the most important form of precipitation because it inters directly in the H.C. components such as interception and runoff.

In Iraq forms of precipitation other than rainfall are little compared to rain.

Forms of precipitation

According the appearance , precipitation may take the forms:

(i) Drizzle: a light steady rain in fine drops (smaller than 0.5 mm).

(ii) Rain: the condensed water vapor of the atmosphere falling in drops (>0.5 mm, maximum size 6 mm).

(iii) Glaze: freezing of drizzle or rain when they come in contact with cold objects.

(iv) Sleet: frozen rain drops while falling through air at subfreezing temperature.

(v) Snow: ice crystals resulting from sublimation (i.e., water vapor condenses to ice)

(vi) Hail: small lumps of ice (>5 mm in diameter) formed by alternate freezing and melting, when they are carried up and down in highly turbulent air currents.

Types of precipitation

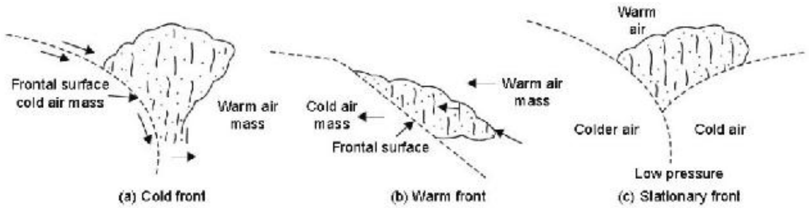
Precipitation is often typed according to the factor mainly responsible for the lifting which cause it.

Convection Prec.: due to the convection, the warm moist air rising and cooling to form clouds and subsequently to precipitate rain.

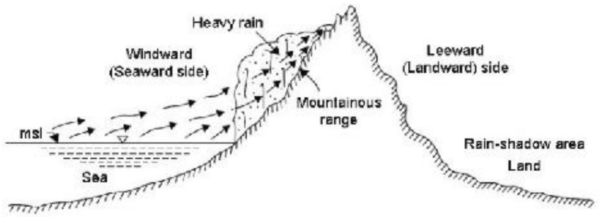
Orographic precipitation: results from ocean air streams passing over land and being deflected upward by coastal mountains, thus cooling below saturation temperature and spilling moisture.

Cyclonic and Frontal prec.: when low pressure areas exist, air tends to move into them from surrounding areas and in so doing displaced low pressure air upward, to cool and precipitate rain.

These types of rain may explained by the following figure.



Frontal precipitation



Orographic precipitation

•Rainfall intensity (or rainfall rate)

Rainfall during a year or season (or a number of years) consists of several storms. The characteristics of a rainstorm are (i) intensity (cm/hr), (ii) duration (min, hr, or days), (iii) frequency (once in 5 years or once in 10, 20, 40, 60 or 100 years), and (iv) areal extent (i.e., area over which it is distributed).

Intensity: Volume of rainfall per unit of time; usually expressed in inches per hour or in millimeters per hour. Obtained by dividing the total depth of rainfall by the duration under consideration.

Duration: The period of time elapsed by a single rain storm.

Frequency: This refers to the expectation that a given depth of rainfall will fall in a given time. Such an amount maybe equaled or exceeded in a given number of days or years.

Areal extent: This concern the area over which a point's rainfall can be held to apply.

•Intensity-duration relationship

The greater the density of rainfall, in general, the shorter length of time it continuous. A formula expressing the connection would be of the type:

$$i = \frac{a}{t+b}$$ for storms less than 2hr.

$$i = \frac{c}{t^n}$$ for storms more than 2hr.

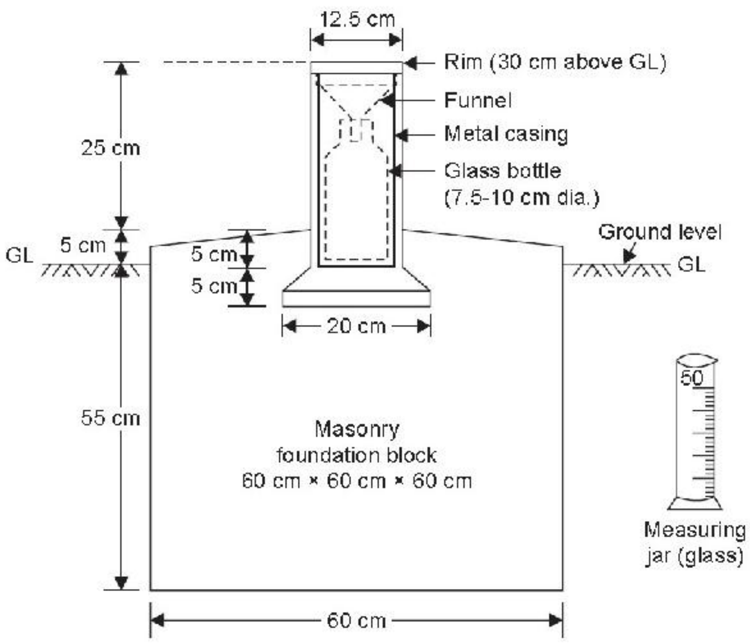
Where i= intensity, t=time, a ,b , c, and n are constants.

Measurement of Precipitation

Rainfall may be measured by a network of rain gauges which may either be of non-recording or recording type.

1- Non-recording rain gage: as shown in figure below a standard non-recording rain gage used in India.

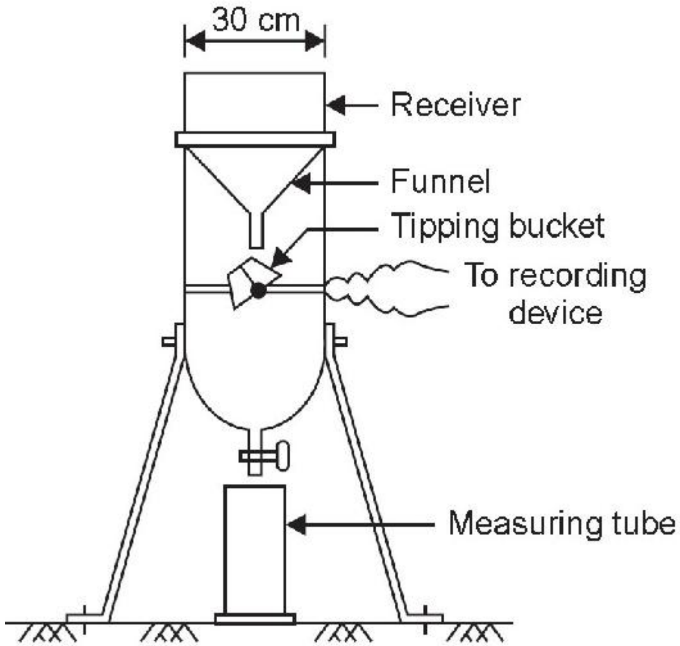
It consists of a circular funnel of 12.7 cm diameter and a glass bottle as a receiver. The cylindrical metal casing is fixed vertically to the masonry foundation with the funnel rim 30.5 cm above the ground surface. The rain falling into the funnel is collected in the receiver and is measured in a special measuring glass graduated in mm of rainfall.



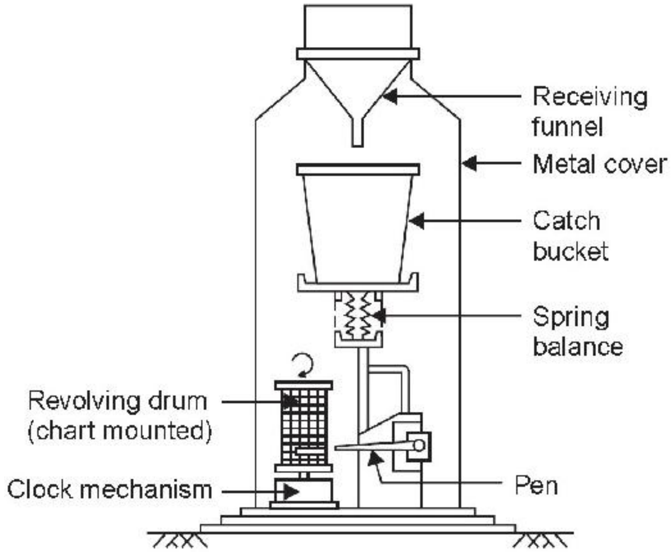
Symon's rain gauge

2- Recording Rain Gages:

Tipping bucket rain gauge. This consists of a cylindrical receiver 30 cm diameter with a funnel inside . Just below the funnel a pair of tipping buckets is pivoted such that when one of the bucket receives a rainfall of 0.25 mm it tips and empties into a tank below, while the other bucket takes its position and the process is repeated. The tipping of the bucket actuates on electric circuit which causes a pen to move on a chart wrapped round a drum which revolves by a clock mechanism. This type cannot record snow.



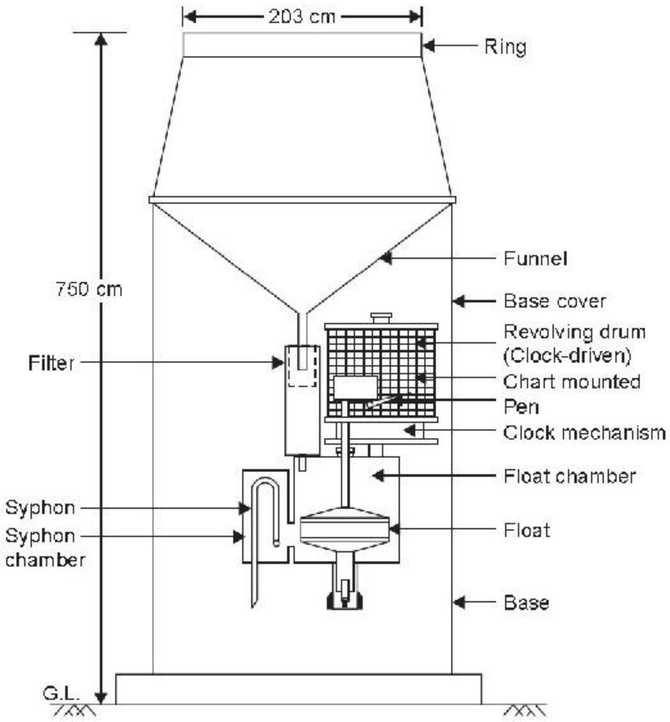
Tipping bucket gauge



Weighing type rain gauge

Weighing type rain gauge. In this type of rain-gauge, when a certain weight of rainfall is collected in a tank, which rests on a spring-lever balance, it makes a pen to move on a chart wrapped round a clock-driven drum . The rotation of the drum sets the time scale while the vertical motion of the pen records the cumulative precipitation.

Float type rain gauge. In this type, as the rain is collected in a float chamber, the float moves up which makes a pen to move on a chart wrapped round a clock driven drum. When the float chamber fills up, the water siphons out automatically through a siphon tube kept in an interconnected siphon chamber. The clockwork revolves the drum once in 24 hours. The clock mechanism needs rewinding once in a week when the chart wrapped round the drum is also replaced.



Float type rain gauge

Rain Gages Network

The following figures give a guideline as to the number of rain-gauges to be erected in a given area or what is termed as 'rain-gauge density'

Area	*Rain-gauge density*
Plains	1 in 520 Km^2
Elevated regions	1 in 260-390 Km^2
Hilly and very heavy rainfall areas	1 in 130 Km^2

In general, the suitable number of rain gages for a catchment could be determined depending upon previous rain record for the catchment, from the equation:

$$N = \left(\frac{Cv}{\epsilon}\right)^2$$

Where: N is the optimum number of rain gages.
ϵ = percentage of permissible error.
C_v = coefficient of variation.

$$Cv = \frac{\sigma_{m-1}}{P'} * 100\%$$

m = number of existing rain gages.
P' = average of prec. (existing rain gages)
σ = standard deviation

$$\sigma = \sqrt{\frac{\Sigma (P - P')^2}{m - 1}}$$

Example: A catchment has seven rain-gages. The annual rainfall recorded by these gages are:

Station:	1	2	3	4	5	6	7
Rain (cm):	130	142.1	118.2	108.5	165.2	102.1	146.9

For 5% error in the estimation of the mean rainfall, calculate the optimum No of rain-gauge stations in the catchment.

Solution:

P' = ΣP/m =913/7= 130.43 cm

Gage:	1	2	3	4	5	6	7	: Σ
P(cm):	130	142.1	118.2	108.5	165.2	102.1	146.9	: 913
$(P-P')^2$:	0.185	136.2	142.3	480.9	1209	802.6	271.3	: 3042.4

□ $\sigma = \sqrt{3042.4/_{7-1}}$ = 22.52

$$Cv = \frac{22.52}{130.43} * 100\% = 17.26$$

$$N = \left(\frac{Cv}{\epsilon}\right)^2 = \left(\frac{17.26}{5}\right)^2 = 12$$

⇨ Optimum No of Rain gages are 12 and the required additional No is 5 stations.

Estimation of Missing Data:

Many precipitation stations have short breaks in their records because of the absence of the observer or because of instrumental failure. It is often necessary to estimate the missing record. The missing precipitation data are estimated from observations of (at least) three stations close to the inoperative station.

If the normal annual precipitation at each of the index stations is within 10% of that for the station with the missing record, a simple arithmetic average of the precipitation at the index stations provides the estimated amount.

If the normal annual precipitation at any of the index stations differs from that at the station in question by more than 10%, the normal-ratio method is used. In this method, the amounts at the index stations are weighted by the ratios of the normal-annual-precipitation values.

$$Px = \frac{Nx}{m} \left(\frac{P1}{N1} + \frac{P2}{N2} + \cdots + \frac{Pm}{Nm}\right)$$

Where: *Px* is the estimated rainfall.

P1,P2,.. Are the recoded precipitation at index stations.

m = No. of index stations.

N1, N2,…are the annual precipitation at the index stations.

<u>Example:</u> Station X was inoperative for a part of a month during which a storm was occurred. The respective storm totals at three surrounding stations, A, B and C, were 98, 80 and 110 mm. The normal precipitation amounts at stations X, A, B and C are 880, 1008, 842 and 1080 respectively. Estimate the storm precipitation for station X.

<u>Solution:</u>

$$\frac{1080 - 880}{880} * 100\% = 22.7\%$$

⇨ use method of normal ratio.

$$Px = \frac{880}{3} \left(\frac{98}{1008} + \frac{80}{842} + \frac{110}{1080}\right) = 86 \text{ mm.}$$

Double-Mass analysis:

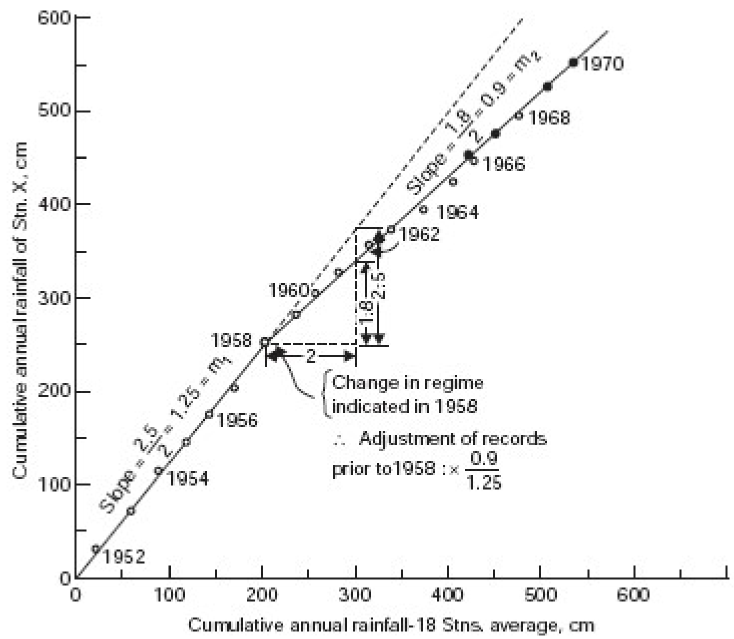
The trend of the rainfall records at a station may slightly change after some years due to a change in the environment of a station either due to coming of a new building, fence, planting of trees or cutting of forest nearby, which affect the catch of the gauge due to change in the wind pattern. The consistency of records at the station in question (say, X) is tested by a double mass curve by plotting the cumulative annual (or seasonal) rainfall at station X against the concurrent cumulative values of mean annual (or seasonal) rainfall for a group of surrounding stations, for the number of years of record. From the plot, the year in which a change in regime (or environment) has occurred is indicated by the change in slope of the straight line plot. The rainfall records of the station x are adjusted by multiplying the recorded values of rainfall by the ratio of slopes of the straight lines before and after change in environment.

<u>Example</u>: The annual rainfall at station X and the average annual rainfall at 18 surrounding stations are given below. Check the consistency of the record at station X and determine the year in which a change in regime has occurred. State how you are going to adjust the records for the change in regime. Determine the a.a.r. for the period 1952-1970 for the changed regime.

<u>Solution:</u>

Year	Annual Rain (cm)		Accumulative Prec. cm	
	Gage X	18 gages	Gage X	18 gages
1952	30.5	22.8	30.5	22.8
1953	38.9	35.0	69.4	57.8
1954	43.7	30.2	113.1	88.0
1955	32.2	27.4	145.3	115.4
1956	27.4	25.2	172.7	140.6
1957	32.0	28.2	204.7	168.8
1958	49.3	36.1	254.0	204.9
1959	28.4	18.4	282.4	233.3
1960	24.6	25.1	307.0	258.4
1961	21.8	23.6	328.8	282.0
1962	28.2	33.3	357.0	315.3
1963	17.3	23.4	374.3	338.7
1964	22.3	36.0	396.6	374.7
1965	28.4	31.2	425.0	405.9
1966	24.1	23.1	449.1	429.0
1967	26.9	23.4	476.0	452.4
1968	20.6	23.1	496.6	475.5
1969	29.5	33.2	526.1	508.7
1970	28.4	26.4	554.5	535.1

Now plot the accumulative precipitation of X against accumulative precipitation of the 18 surrounding gages.



⇨ Adjusted Prec. At X in 1957 = 28.4/(0.9/1.25)=39.4 cm

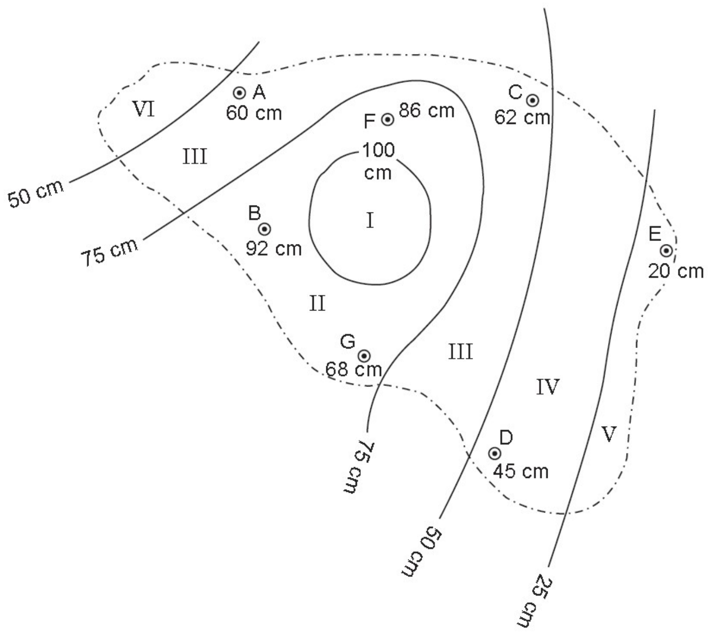
Representation of Rainfall Data

The variation of rainfall with respect to time may be shown graphically by (i) isohyetal maps, (ii) a hyetograph, and (iii) a mass curve.

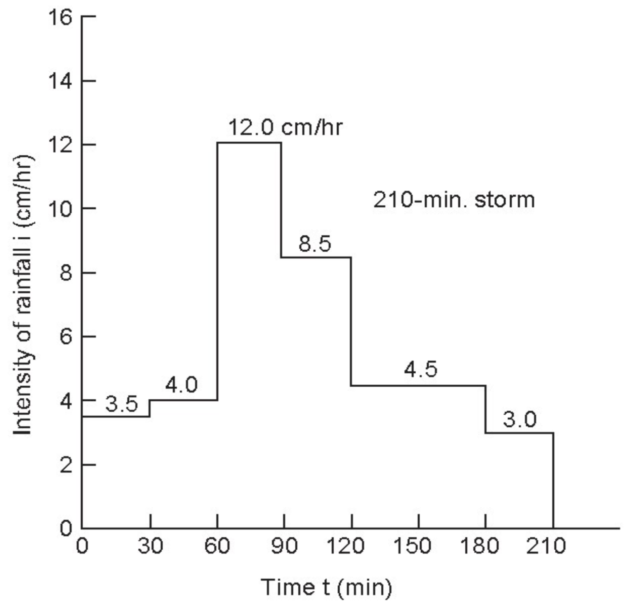
Isohyetal maps: isohyet is a contour of constant rainfall. Isohyetal maps are prepared by interpolating rainfall data recorded at gaged points.

A rainfall *hyetograph* is a plot of rainfall depth or intensity as a function of time, shown in form of histogram.

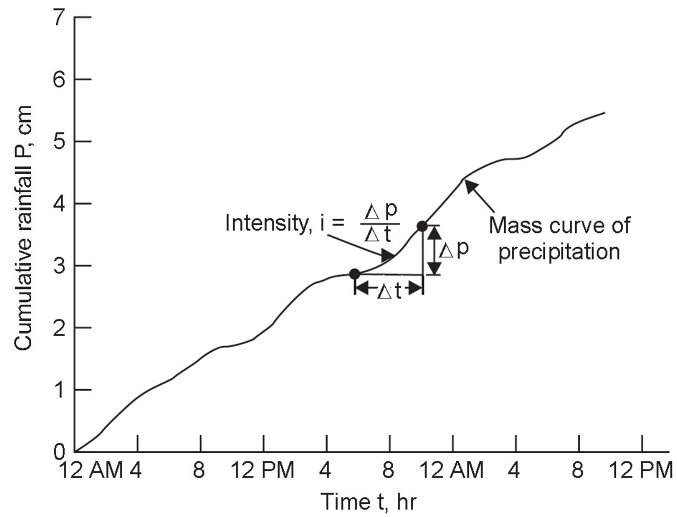
A *cumulative rain hyetograph* or *rainfall mass curve*, is a plot of cumulative depth of rainfall against time.



Isohyetal map



Hyetograph



Mass curve of rainfall

MEAN AND MEDIAN

The sum of all the items in a set divided by the number of items gives the mean value, i.e.,

$$\bar{x} = \frac{\sum x}{n}$$

Where: $\bar{x}$ = the mean value

$\sum x$ = sum of all the items

n = total number of items.

The magnitude of the item in a set such that half of the total number of items are larger and half are smaller is called the median. To find the median, the items are arranged in the ascending order; if the number of items is odd, the middle item gives the median; if the number of items is even, the average of the central two items gives the median.

Example:

The annual rainfall at a place for a period of 10 years from 1961 to 1970 are respectively 30.3, 41.0, 33.5, 34.0, 33.3, 36.2, 33.6, 30.2, 35.5, 36.3. Determine the mean and median values of annual rainfall for the place.

Solution:

(i) Mean x = ∑x /n

= (30.3 + 41.0 + 33.5 + 34.0 + 33.3 + 36.2 + 33.6 + 30.2 + 35.5 + 36.3)/10

= 343 9/10 = 34.39 cm

(ii) Median: Arrange the samples in the ascending order 30.2, 30.3 33.3, 33.5, 33.6, 34.0, 35.5, 36.2, 36.3, 41.0

No. of items = 10, i.e., even

 Median = (33.6 + 34.0)/2 = 33.8 cm

Example:

The following are the rain gauge observations during a storm. Construct: (a) mass curve of precipitation, (b) hyetograph.

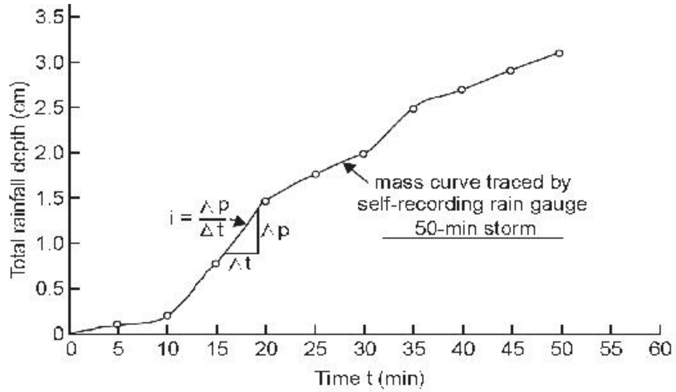
Time (min)	5	10	15	20	25	30	35	40	45	50
Acc. Rain (cm)	0.1	0.2	0.8	1.5	1.8	2.0	2.5	2.7	2.9	3.1

Solution:

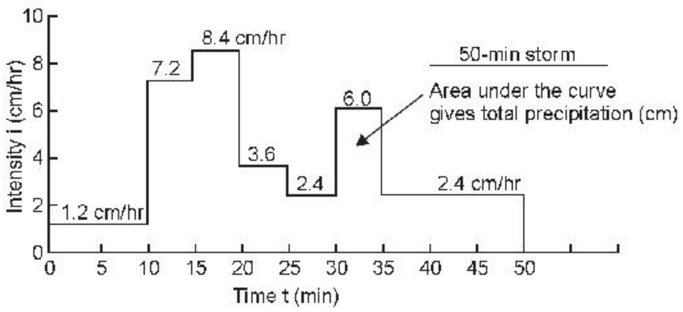
(a) Mass curve of precipitation. The plot of 'accumulated rainfall (cm) vs. time (min)' gives the 'mass curve of rainfall'.

(b) Hyetograph. The intensity of rainfall at successive 5 min interval is calculated and a bar-graph of 'i (cm/hr.) vs. t (min)' is constructed and is called the 'hyetograph'.

Time (min)	Acc. Rain (cm)	ΔP in time Δt Δt=5min (cm)	Intensity, $i = \frac{\Delta P}{\Delta t} * 60$ (cm/hr)
5	0.1	0.1	1.2
10	0.2	0.1	1.2
15	0.8	0.6	7.2
20	1.5	0.7	8.4
25	1.8	0.3	3.6
30	2.0	0.2	2.4
35	2.5	0.5	6.0
40	2.7	0.2	2.4
45	2.9	0.2	2.4
50	3.1	0.2	2.4



(a) Mass curve of precipitation



(b) Hyetograph

MEAN AREAL DEPTH OF PRECIPITATION (P_{ave})

The average depth of rainfall over a specific area, on a storm, seasonal, or annual basis, is required in many types of hydrologic problems.

The arithmetic-mean method is the simplest method of determining areal average rainfall. It involves averaging the rainfall depths recorded at a number of gages. This method is satisfactory if the gages are uniformly distributed over the area and the individual gage measurements do not vary greatly about the mean. It is also suitable for flat areas.

Another method, is Thiessen method, he defines the zone of influence of each gage by drawing lines between pairs of gages, bisecting the lines with perpendiculars, and assuming all the area enclosed within the boundary formed by these intersecting perpendiculars has had rainfall of the same amount as the enclosed gauge.

The third method is to draw isohyets, or contour of equal rainfall depth. The areas between successive isohyets are measured and assigned an average value of rainfall. The overall average for the area is thus derived from weighted averages. This method is the most accurate method among the three methods.

Example:

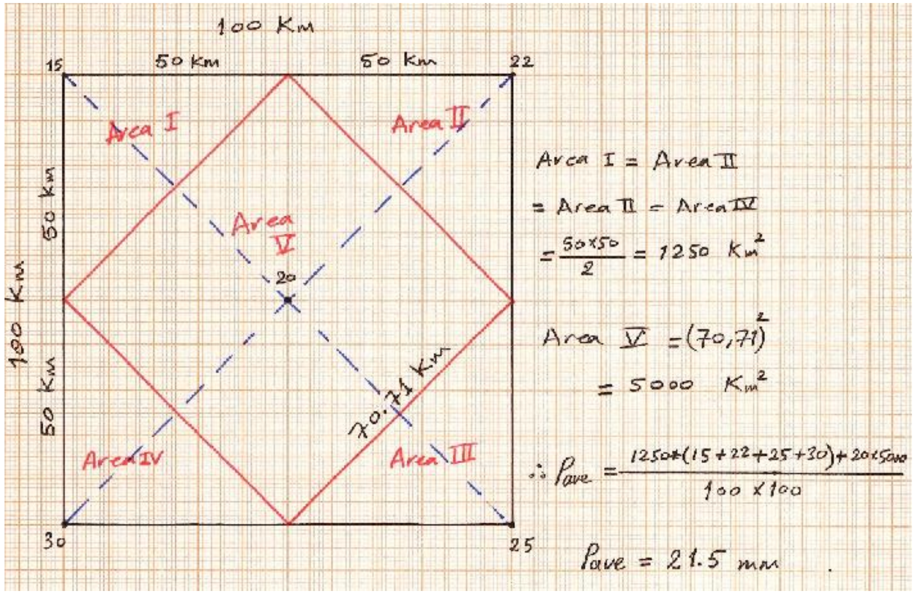
A square area of 100km side length, rain gages reading at corners are 15mm, 22mm, 25mm and 30mm, another gage at the center of the area reads 20mm of rain. Find the average rainfall over the area arithmetically and by Thiessen method.

Solution:

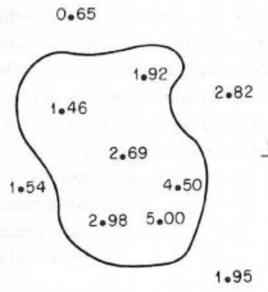
A) By arithmetic mean:

$\sum_{i=1}^{m} Pi/m$ ⇒ Pave = (15+22+25+30+20)/5=22.4mm

B) by Thiessen method:



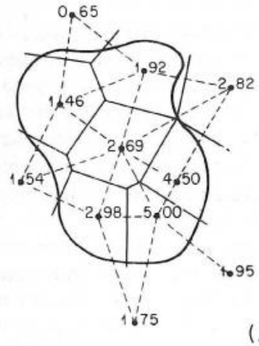
Another example:



Arithmetic mean:

$$\frac{1.46 + 1.92 + 2.69 + 4.50 + 2.98 + 5.00}{6} = 3.09 \text{ in.}$$

(a)

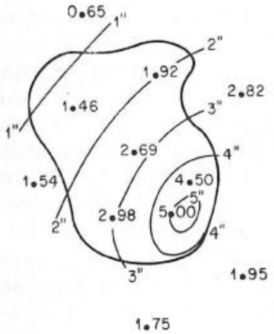


Thiessen method:

Observed precip (in.)	Area* (sq mi)	Per cent total area	Weighted precipitation (in.) (col.1 × col.3)
0.65	7	1	0.01
1.46	120	19	0.28
1.92	109	18	0.35
2.69	120	19	0.51
1.54	20	3	0.05
2.98	92	15	0.45
5.00	82	13	0.65
4.50	76	12	0.54
	626	100	2.84

Average = 2.84 in.

(b) *Area of corresponding polygon within basin boundary



Isohyetal method:

Isohyet (in.)	Area* enclosed (sq mi)	Net area (sq mi)	Avg. precip. (in.)	Precipitation volume (col 3 × col. 4)
5	13	13	5.3	69
4	90	77	4.6	354
3	206	116	3.5	406
2	402	196	2.5	490
1	595	193	1.5	290
<1	626	31	0.8	25
				1634

Average = 1634 ÷ 626 = 2.61 in.

*Within basin boundary

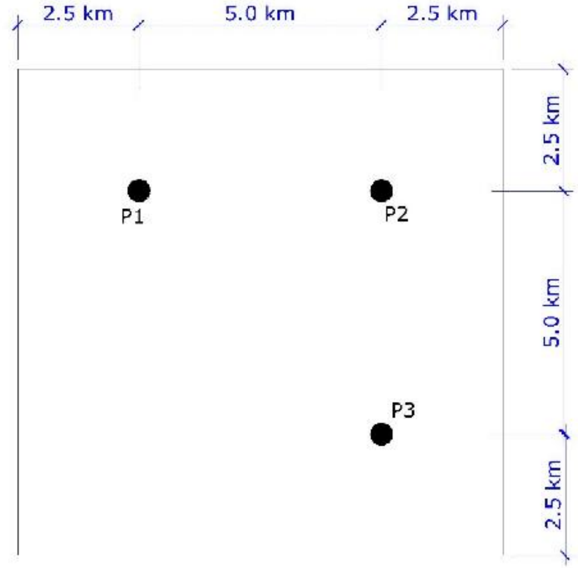
(c)

Another example:

A square area of 100km2 is gaged by three rain-gages as shown in figure. The data recorded is

Station	1	2	3
Rainfall (mm):	106	152	127

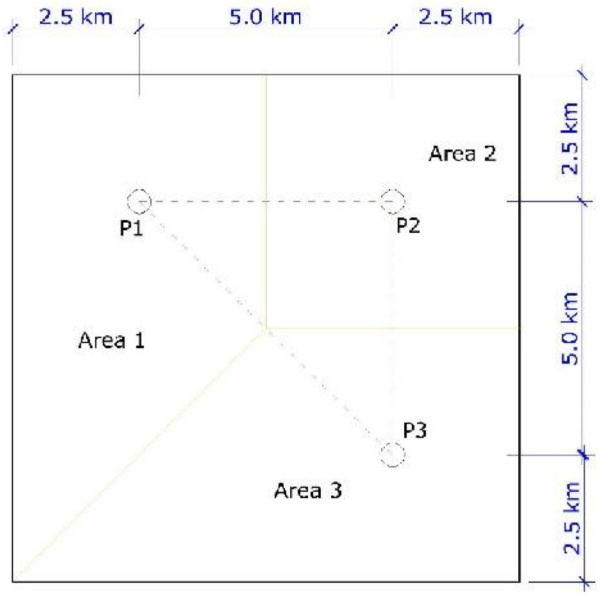
Find the average rainfall.



Solution:

By Arithmetic mean; Pave= (106+152+127) / 3 = 128.3 mm

By Thiessen polygon:

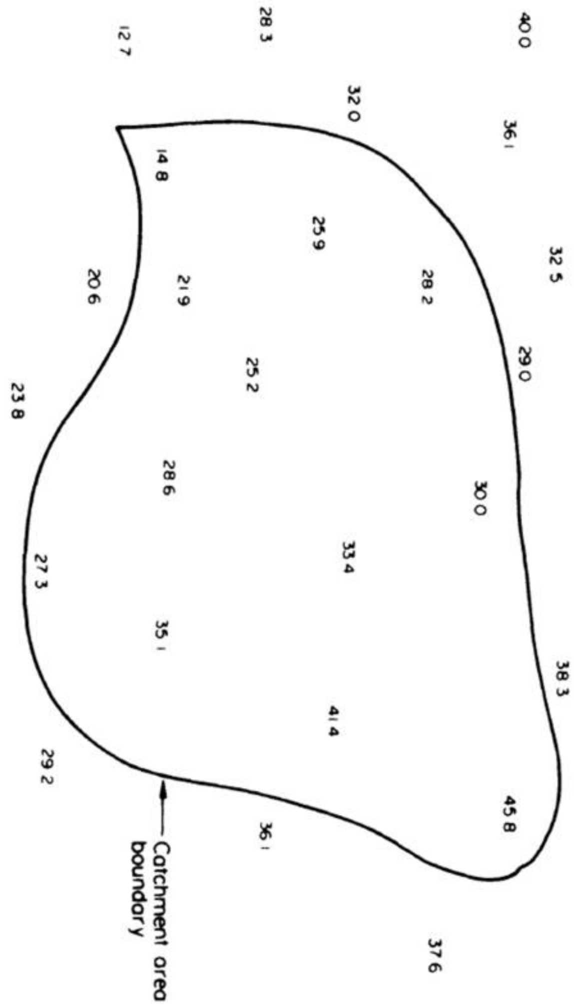


A1 = 0.375 x 10 x 10 = 37.5 km^2
A2 = 5 x 5 = 25 km^2
A3 = 0.375 x 10 x 10 = 37.5 km^2

$$\Rightarrow Pave = \frac{106\times37.5+152\times25+127\times37.5}{37.5+25+37.5} = 125.4 \text{ mm}$$

Example:

For the catchment shown below, draw the isohyetal map.



Frequency of Point Rainfall

In many hydraulic-engineering applications such as those concerned with floods, the probability of occurrence of a particular extreme rainfall, e.g. a 24-hr maximum rainfall, will be of importance. Such information obtained by the frequency analysis of the point-rainfall data. The rainfall at a place is a random hydrologic process and a sequence of rain data at a place in chronological order constitute a time series. One of the commonly used data series is the annual series composed of annual values such as annual rainfall. If the extreme values of a rainfall event occurring in each year is listed, it also constitutes an annual series. Thus for example, one may list the maximum 24-h rainfall occurring in a year at a station to prepare an annual series of maximum 24-h rainfall values. The probability of occurrence of an event in this series is studied by frequency analysis of this annual data series.

The probability of occurrence of an event of a random variable (e.g. rainfall) whose magnitude is equal to in excess of a specified magnitude X is denoted by P.

The recurrence interval (return period) is denoted as

$$T = \frac{1}{P}$$

This represents the average interval between the occurrence of a rainfall of magnitude equal to or greater than X . Thus if it is stated that the return period of rainfall of 20 cm in 24 h is 10 years at a certain station A, it implies that on an average rainfall magnitudes equal to or exceeds 20 cm in 24 h occurs once in 10 years. The probability of a rainfall of 20 cm in 24 h occurring in anyone year at station A is 1/T = 1/10 = 0.1.

If the probability of an event occurring is P , the probability of the event not occurring in a given year is q = (1-P). The binomial distribution can be used to find the probability of occurrence of the event r times in n successive years. Thus:

$$P_{r,n} = {}^nC_r\, P_r\, q^{n-1} = \frac{n!}{(n-r)!\,r!} P^r q^{n-r}$$

Where: $P_{r,n}$ = probability of random hydrologic event (rainfall) of given magnitude and exceedance probability $\boldsymbol{P}$ occurring r times in n successive years. Thus, for example,

a) The probability of an event of exceedance probability $\boldsymbol{P}$ occurring $\boldsymbol{2}$ times in $\boldsymbol{n}$ successive years is

$$P_{2,n} = \frac{n!}{(n-2)!2!} P^2 q^{n-2}$$

b) The probability of the event not occurring at all in n successive years is

$$P_{0,n} = q^0 = (1-P)^n$$

c) The probability of the event occurring at least once in n successive years is

$$P_1 = 1 - q^n = 1 - (1-P)^n$$

<u>Example:</u>

Analysis of data on maximum one-day rainfall depth at a certain station indicated that a depth of 280 mm had a return period of 50 years. Determine the probability of a one-day rainfall depth equal to or greater than 280 mm, (a) once in 20 successive years, (b) two times in 15 successive years, and (c) at least once in 20 successive years.

<u>Solution:</u>

Hence, $P = \frac{1}{T} = \frac{1}{50} = 0.02$

(a) n = 20, r = 1 ➡ $P_{1,20} = \frac{20!}{(19)!1!} \times 0.02^1 \times 0.98^{19} = 0.272$

(b) n = 15, r = 2 ➡ $P_{2,15} = \frac{15!}{(13)!2!} \times 0.02^2 \times 0.98^{13} = 0.323$

(c) $P_1 = 1 - (1-0.02)^{20} = 0.332$

Plotting Position

The purpose of the frequency analysis of an annual series is to obtain a relation between the magnitude of the event and its probability of exceedance. The probability analysis may be made by empirical or by analytical methods.

A simple empirical technique is to arrange the given annual extreme series in descending order of magnitude and to assign an order number m. Thus for the first entry m=1, for the second entry m=2, and so on, till the last event for which m=N=Number of years of record. The probability P of an event equaled to or exceeded is given by the *Weibull formula*

$$P = \left(\frac{m}{N+1}\right) \Rightarrow T = \frac{1}{P} = \left(\frac{N+1}{m}\right)$$

Weibull formula is an empirical formula and there are several empirical formulae available to calculate P . The probability obtained by the use of an empirical formula is called plotting position.

<u>Example:</u>

The record of annual rainfall at station A covering a period of 22 years is given below, (a) Estimate the annual rainfall with return period of 10 years and 50 years, (b) What would be the probability of an annual rainfall of magnitude equal to or exceeds 100cm occurring in station A, and (c) What is the 75% dependable annual rainfall at station A?

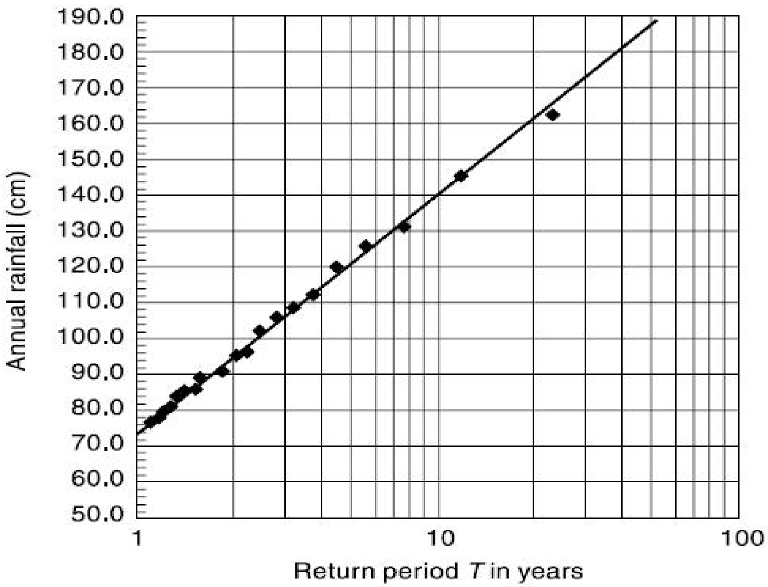
Year	Annual Rain (cm)	Year	Annual Rain (cm)	Year	Annual Rain (cm)
1960	130.0	1968	143.0	1976	120.0
1961	84.0	1969	89.0	1977	160.0
1962	76.0	1970	78.0	1978	85.0
1963	89.0	1971	90.0	1979	106.0
1964	112.0	1972	102.0	1980	83.0
1965	96.0	1973	108.0	1981	95.0
1966	80.0	1974	60.0		
1967	125.0	1975	75.0		

<u>Solution:</u>

The data are arranged in descending order as in table below:

Order (m)	Annual Rain (cm)	Probability = m/(N+1)	Return Period T=1/P (years)	Order (m)	Annual Rain (cm)	Probability = m/(N+1)	Return Period T=1/P (years)
1	160.0	0.043	23.000	12	90.0	0.522	1.917
2	143.0	0.087	11.500	13	89.0	0.565	---
3	130.0	0.130	7.667	14	89.0	0.609	1.643
4	125.0	0.174	5.750	15	85.0	0.652	1.533
5	120.0	0.217	4.600	16	84.0	0.696	1.438
6	112.0	0.261	3.833	17	83.0	0.739	1.353
7	108.0	0.304	3.286	18	80.0	0.783	1.278
8	106.0	0.348	2.875	19	78.0	0.826	1.211
9	102.0	0.391	2.556	20	76.0	0.870	1.150
10	96.0	0.435	2.300	21	75.0	0.913	1.095
11	95.0	0.478	2.091	22	60.0	0.957	1.045

A graph is plotted between the annual rainfall as the ordinate (on arithmetic scale) and the return period T as the abscissa (on logarithmic scale) as shown



(a) For T=10 years, the corresponding rainfall magnitude is obtained by interpolation between T=11.5 and T=7.667 years , rainfall will be= 137.9cm.

For T=50 years, the corresponding rainfall magnitude is obtained by extrapolation , rainfall will be= 180.0cm.

(b) Return period of an annual rainfall equal or exceeding 100cm, by interpolation is 2.4 years and thus P=1/2.4=0.417

(c) 75% dependable annual rainfall at station A = Annual rainfall with probability P=0.75, thus, T=1/0.75= 1.33 years. By interpolation between T=1.28 and T=1.35 years, the 75% dependable annual rainfall at station A = 82.3cm.

Water Losses

The various water losses that occur in nature are:

(i) Interception loss-due to surface vegetation, i.e., held by plant leaves.

(ii) Evaporation:

(a) from water surface, i.e., reservoirs, lakes, ponds, river channels, etc.

(b) from soil surface, appreciably when the ground water table is very near the soil surface.

(iii) Transpiration—from plant leaves.

(iv) Evapotranspiration for consumptive use—from irrigated or cropped land.

(v) Infiltration—into the soil at the ground surface.

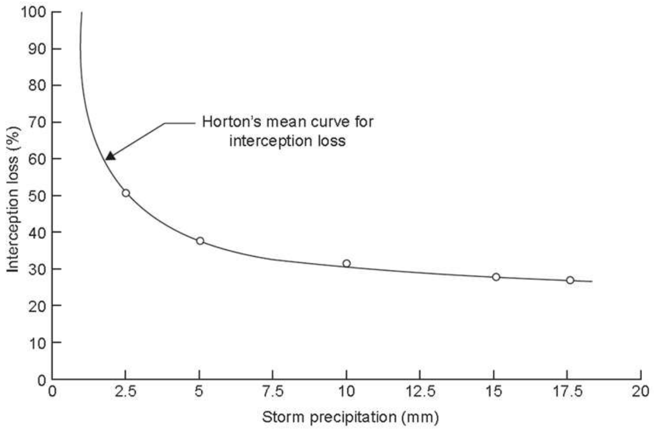
(vi) Watershed leakage—ground water movement from one basin to another or into the sea.

The various water losses are discussed below:

Interception loss:

The precipitation intercepted by plant leaves and buildings and returned to atmosphere (by evaporation) without reaching the ground surface is called interception loss. Interception loss is high in the beginning of storms and gradually decreases; the loss is of the order of 0.5 to 2 mm per shower and it is greater in the case of light showers than when rain is continuous. Figure below shows the *Horton's* mean curve of interception loss for different showers.

Effective rain = Rainfall – Interception loss



Interception loss (Horton)

Evaporation:

Evaporation is the rate of liquid water transformation to vapor from open water, bare soil, or covered soil with vegetation. Evaporation usually stated by millimeters of evaporated water per day.

Evaporation from free water surfaces and soil are of great importance in water resources studies. It affects the yield of river basins, the necessary capacity of reservoirs, the size of pumping plant, the consumptive use of water by crops …etc.

Potential evaporation is defined as the quantity of water evaporated per unit area, per unit time from an idealized, extensive free water surface under exiting atmospheric conditions.

Factors affecting evaporation are:

Solar radiation. The change of state of water from liquid to gas requires an energy input (known as the latent heat of vaporization), this energy comes from the sun.

Wind. As the water vaporize into the boundary layer between earth and air becomes saturated and it must be removed and continually replaced by dryer air if evaporation to be proceed. This movement of air in the boundary layer depends on wind and so the wind speed is important.

Relative humidity. As the air humidity rises, its ability to absorb more water vapor decreases and the rate of evaporation slows.

Temperature. An energy input is necessary for evaporation to proceed. If the temperature of air and ground is high, evaporation will proceed more rapidly than if they are cool, also the air capacity to absorb water vapor increases as its temperature rises.

Transpiration and evapo-transpiration.

Growing vegetation of all kind needs water to sustain life, only a small fraction of the water needed by plant is retained in the plant structure. Most of it passes through the roots to the stem and is transpired into the atmosphere through the leafy part of the plant.

In field condition it is practically impossible to differentiate between evaporation and transpiration if the ground is covered with vegetation. The two processes are commonly linked together and referred to as *evapo-transpiration* (E_{pt}) .

Evapotranspiration (E_t) or consumptive use is the total water lost from a cropped (or irrigated) land due to evaporation from the soil and transpiration by the plants or used by the plants in building up of plant tissue.

Potential Evapotranspiration is the evapotranspiration that would occur from a well vegetated surface when moisture supply is not limited, and this is calculated in a way similar to that for open water evaporation. Actual Evapotranspiration drops below its potential level as the soil dries out.

Evaporation from water surfaces (Lake Evaporation)

The factors affecting evaporation are air and water temperature, relative humidity, wind velocity, surface area (exposed), barometric pressure and salinity of the water, the last two having a minor effect. The rate of evaporation is a function of the differences in vapour pressure at the water surface and in the atmosphere, and the Dalton's law of evaporation is given by (mass transfer)

$$E = K\ (e_s - e_a)$$

Where: E = daily evaporation
e_s = saturated vapour pressure at the temperature of water
e_a = vapour pressure of the air (about 2 m above)
K = a constant.

the Dalton's law states that the evaporation is proportional to the difference in vapour pressures e_s and e_a. A more general form of the above Eq. is given by

$$E = K\ (e_s - e_a)(a + bV)$$

Where: K', a, b = constants and V = wind velocity.

Higher the temperature and wind velocity, greater is the evaporation, while greater the humidity and dissolved salts, smaller is the evaporation.

Definitions:

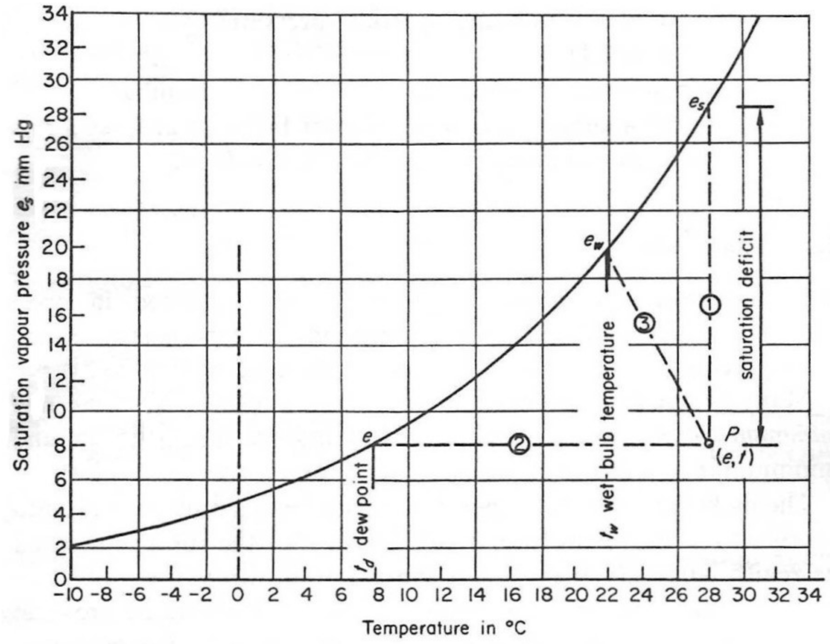
Air easily absorbs moisture in the form of water vapor. The amount of water vapour absorbed by air depends on the temperature of the air and of the water. The greater the temperature the more vapour the air can contain. The water vapor exerts a *potential pressure* usually measured in bars (1bar=100000N/m2) or mm height of a mercury column (Hg), (1mm Hg=1.36 mbar).

Suppose an evaporating surface of water is in a closed system and enveloped in air. If a source of heat energy is available to the system, evaporation of the water into the air will take place until a state of equilibrium is reached when the air is saturated with vapor and can absorbed no more. The molecules of water vapour will then exert a pressure which is known as *saturation vapour pressure* (e_s), for the particular temperature of the system.

The value of e_s changes with temperature as indicated in figure below. Referring to this figure, consider what can happen to a mass of atmospheric air *P*, whose temp is *t* and whose vapor pressure is e_a. Since *P* is lies below the saturation curve, it is clear that the air mass could absorb more water vapor and that is it did so while its temperature remained constant, then the position of *P* would move vertically up dashed line 1 until the air was saturated. The corresponding vapor pressure of *P* in the new position would be e_s. The increase (e_s- e_a) is known as the *saturation deficit*.

Alternatively, if no change were to take place in the humidity of the air while it was cooled, then P would move horizontally to the left along line 2 until the saturation line was intersected again. At this point *P* would be saturated, at a new temperature t_d, the *dewpoint*. Cooling of the air beyond this point would result in condensation or mist being formed.

If the water is allowed to evaporate freely into the air mass, neither of the above two possibilities occurs. This is because the evaporation requires heat which is withdrawn from the air itself. This heat called the latent heat of evaporation, h_r, and given by:



Saturation vapour pressure of water in air

$$h_r = 606.5 - 0.695t \qquad cal/g$$

⇨ the latent heat is the amount of heat absorbed by a unit mass of water, without changing in temperature , while passing from the liquid to the vapor state.

So, as the humidity and vapor pressure rise, the temperature of the air falls and the point *P* moves diagonally along line 3 until saturation vapour pressure is reached at the point defined by e_w and t_w. This temperature t_w is called the *wet bulb temperature* and is the temperature to which the original air can be cooled by evaporating water into it. This is the temperature found by a wet bulb thermometer.

The relative humidity is now given as:

$$h = \frac{e_a}{e_s} \text{ , or as percentage, } h = \frac{e_a}{e_s} \times 100$$

Example:
An air mass is at temperature of 28 ^{0}C with relative humidity of 70%. Determine:
a. Saturation vapour pressure.
b. Saturation deficit.
c. Actual vapor pressure.
d. Dewpoint.
e. Wet bulb temperature.

Solution:
T= 28 ^{0}C ⇨ from the vapour pressure curve ⇨ es =28.3 mmHg.

H=70% ⇨ ea =28.3x0.7 = 19.81 mmHg.

$\therefore e_s - e_a$=28.3-19.81=8.50 mmHg

t_d ⇨ from same curve ⇨ t_d=22 ^{0}C

From the same curve ⇨ wet bulb temp=24.7^{0}C

Methods of Estimating Lake Evaporation

Evaporation from water surfaces can be determined from the following methods :

(i) The storage equation (water budget equation)

$$P + I \pm G = E + O \pm \Delta S$$

Where:
P = Precipitation
I = surface inflow
G = subsurface inflow or outflow
E = evaporation
O = surface outflow
ΔS = change in surface water storage

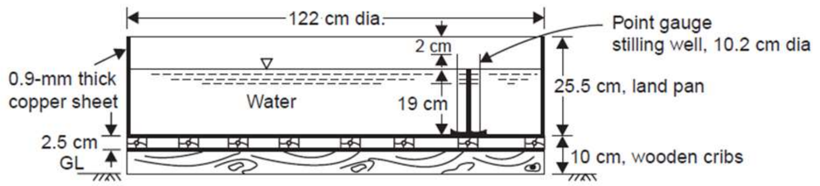
(ii) Auxiliary pans like land pans, floating pans, Colorado sunken pans, etc.
(iii) Evaporation formula like that of Dalton's law.
(iv) Humidity and wind velocity gradients.
(v) The energy budget, this method involves too many hydro-meteorological factors (variables) with too much sophisticated instrumentation and hence it is a specialist approach
(vi) Combination of aerodynamic and energy balance equations, Penman's equation (involves too many variables)

EVAPORATION PANS:

(i) *Floating pans* (made of GI) of 90 cm square and 45 cm deep are mounted on a raft floating in water. The volume of water lost due to evaporation in the pan is determined by knowing the volume of water required to bring the level of water up to the original mark daily and after making allowance for rainfall, if there has been any.

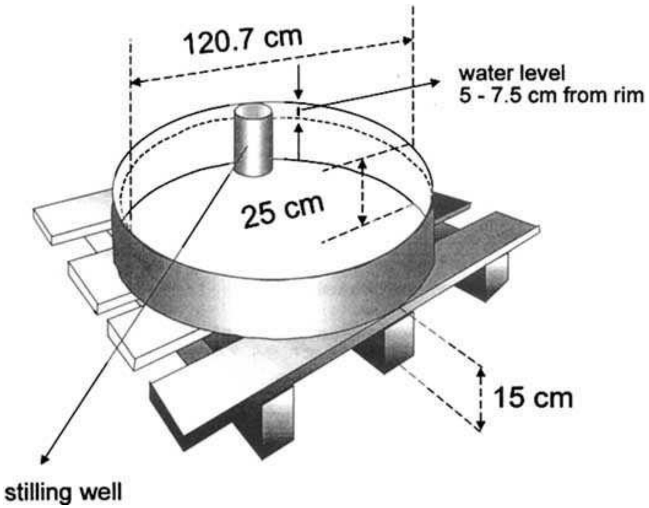
(ii) *Colorado sunken pan*. This is 92 cm square and 42-92 cm deep and is sunk in the ground such that only 5-15 cm depth projects above the ground surface and thus the water level is maintained almost at the ground level. The evaporation is measured by a point gauge

(iii) *Land pan*. Evaporation pans are installed in the vicinity of the reservoir or lake to determine the lake evaporation. The IMD Land pan shown in Figure below is 122 cm diameter and 25.5 cm deep made of unpainted GI; and set on wood grillage 10 cm above ground to permit circulation of air under the pan. The pan has a stilling well, Vernier point gauge, a thermometer with clip and may be covered with a wire screen. The amount of water lost by evaporation from the pan can be directly measured by the point gauge. The air temperature is determined by reading a dry bulb thermometer. An anemometer is normally mounted at the level of the instrument to provide the wind speed information required. Allowance has to be made for rainfall, if there has been any. Water is added to the pan from a graduated cylinder to bring the water level to the original mark (5 cm below the top of the pan).



IMD land pan

In the USA the standard or class A pan is (very similar to the IMD land pan) circular 1.22m in diameter and 254mm deep, filled to a depth of 180mm, set on a timber grillage with the pan bottom 150mm above ground level.



Pan coefficient: Evaporation pan data cannot be applied to free water surfaces directly but must be adjusted for the differences in physical and climatological factors. For example, a lake is larger and deeper and may be exposed to different wind speed, as compared to a pan. The small volume of water in the metallic pan is greatly affected by temperature fluctuations in the air or by solar radiations in contrast with large bodies of water (in the reservoir) with little temperature fluctuations. Thus the pan evaporation data have to be corrected to obtain the actual evaporation from water surfaces of lakes and reservoirs, i.e., by multiplying by a coefficient called pan coefficient and is defined as:

$$pan\ coefficient = \frac{Lake\ Evaporation}{Pan\ Evaporation}$$

The experimental values for pan coefficients range from 0.67 to 0.82 with an average of 0.7.

Example:

Compute the daily evaporation from a Class A pan if the amounts of water added to bring the level to the fixed point are as follows:

Day:	1	2	3	4	5	6	7
Rainfall (mm):	14	6	12	8	0	5	6
Water (added or removed):	-5	3	0	0	7	4	3

What is the evaporation loss of water in this week from a lake (surface area = 640 ha) in the vicinity, assuming a pan coefficient of 0.75?

Solution:

Pan evaporation, Ep, mm = Rainfall + water added or water removed

Day:	1	2	3	4	5	6	7
Ep(mm):	14-5=9	6+3=9	12	8	7	5+4=9	6+3=9

⇨ pan evaporation in the week = $\sum_1^7 Ep$ = 63 mm

Pan coefficient $= \frac{E_{Lake}}{E_{pan}} = 0.75$

⇨ Lake evaporation in the week= 63x0.75= 47.25 mm

Water lost from the lake = 640 x (47.25/1000) =30.24 ha.m

$\simeq$0.3 Mm^3

Penman Equation:

Penman derived the following eq.

$$E = \frac{\Delta}{\Delta + \gamma} Q_n + \frac{\gamma}{\Delta + \gamma} E_a$$

Where:

Δ is slope of the saturation-vapor-pressure versus temperature curve at the air temperature T_a,

E_a is the evaporation given by Dalton assuming the water surface T_o=T_a, $E_a = (e_s - e_a)(a + bv)$.

Q_n is the net radiation absorbed by water body expressed in the same unit as E.

$\gamma = 0.00066\, p$

$\Delta = (0.00815 T_a + 0.8912)^7$

p is the atmospheric pressure, p=1013 mbar.

Ta is the air temperature ^{0}C.

Knowing that : $\frac{\Delta}{\Delta+\gamma} + \frac{\gamma}{\Delta+\gamma} = 1$

$$Q_n = 7.14 \times 10^{-3} Q_s + 5.26 \times 10^{-6} Q_s (T_a + 17.8)^{1.87} + 3.94 \times 10^{-6} Q_s{}^2 - 2.39 \times 10^{-9} Q_s{}^2 (T_a - 7.2)^2 - 1.02$$

Q_n is the net radiation, expressed in equivalent millimeters of evaporation per day.

Q_s is the daily solar radiation in calories per square centimeter per day.

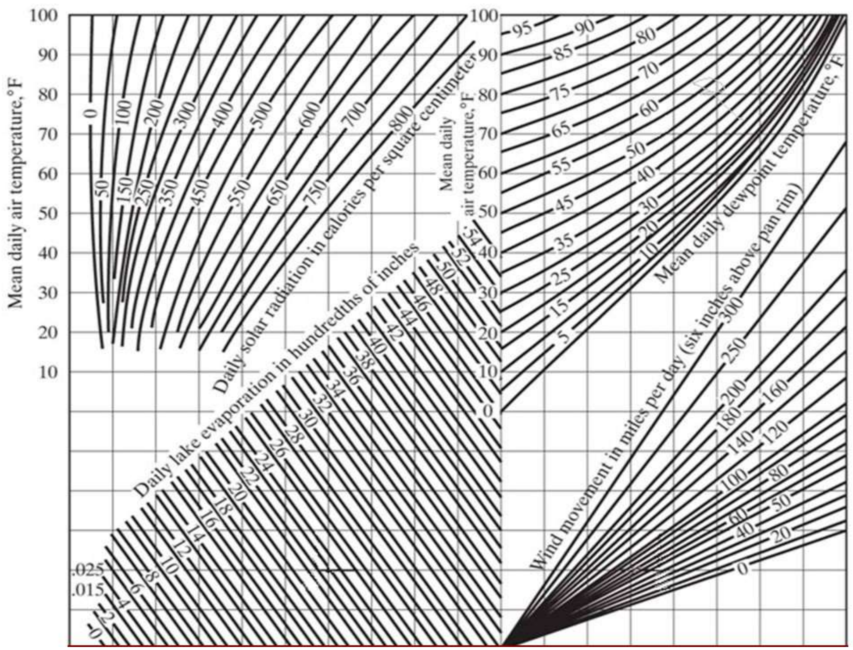
$$e_s - e_a = 33.86[(0.00738 T_a + 0.8072)^8 - (0.00738 T_d + 0.8072)^8]$$

Where; T_d is dewpoint, ^{0}C.

$$E_a = (e_s - e_a)^{0.88}(0.42 + 0.0029 v_p)$$

v_p is the wind movement 150mm above the pan rim (km/day)

Figure below is a graphical solution of the Penman equation for the estimation of lake evaporation as a function of solar radiation, air temperature, dewpoint and wind movement.



Shallow-lake evaporation as a function of solar radiation, air temperature, dewpoint, and wind speed according to Penman's Eq.

Example:

Calculate lake evaporation for the following data using Penman's equation.

v_p=130 km/day, T_a=18 ^{0}C, T_d=8 ^{0}C, Qs=450 cal./cm^2/day.

Solution:

$$e_s - e_a = 33.86[(0.00738T_a + 0.8072)^8 - (0.00738T_d + 0.8072)^8]$$
$$= 33.86[(0.00738 \times 18 + 0.8072)^8 - (0.00738 \times 8 + 0.8072)^8]$$
$$= 9.91$$

$$E_a = (e_s - e_a)^{0.88}(0.42 + 0.0029v_p)$$
$$\Rightarrow E_a = (9.91)^{0.88}(0.42 + 0.0029 \times 130) = 6.00$$

$$\gamma = 0.00066\, p = 0.00066 \times 1013 = 0.668$$
$$\Delta = (0.00815T_a + 0.8912)^7 = (0.00815 \times 18 + 0.8912)^7 = 1.038$$

$$\Rightarrow \frac{\Delta}{\Delta+\gamma} = 0.608 \quad \Rightarrow \frac{\gamma}{\Delta+\gamma} = 0.392$$

$$Q_n = 4.84 \text{ equivalent mm/day}$$

⇨ E = 0.608x4.84+0.392x6.00= 5.30 mm/day =0.208″/day

Estimation of Evapotranspiration

The following are some of the methods of estimating evapotranspiration:

(i) Tanks and lysimeter experiments.
(ii) Installation of sunken (Colorado) tanks.
(iii) Evapotranspiration equations as developed by Lowry-Johnson, Penman, Thornthwaite, Blaney Criddle, etc.
(iv) Evaporation index method, i.e., from pan evaporation data as developed by Hargreaves and Christiansen.

Thornthwaite's Formula:

Thornthwaite carried out many experiments using lysimeters. He suggested a method for the estimation of evapo-transpiration in the latitudes of USA.

If t_n=average monthly temperature of the consecutive months of the year in ^{0}C (n=1, 2, 3, ….,12), and j=monthly heat index.

Then $j = 0.09{t_n}^{1.5}$

$J = \sum_1^{12} j$ (for 12 months)

The potential evapotranspiration for any month with average temperature $t\,^0$C is given, as P_{ex} , by:

$$PE_x = 16\left(\frac{10\,t}{J}\right)^a$$

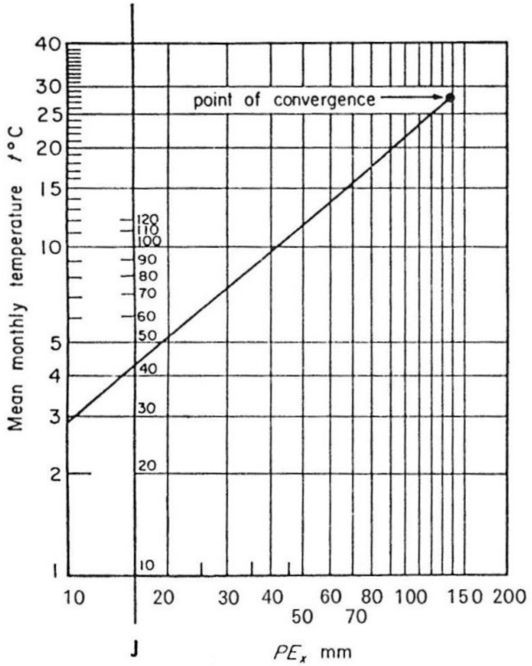
Where : $a = 0.016J + 0.5$

PE_x is a theoretical standard monthly value based on 30 days and 12hr sunrise per day. The actual PE for a certain month is given by:

$$PE = PE_x \frac{DT}{360}$$

Where: D= number of days in the month.
T= average number of hours between sunrise and sunset in the month.

Thornthwaite published a nomogram and table for the solution of his formula, the nomogram shown in figure below.



t °C	PE$_x$
26·5	135·0
27·0	139·5
27·5	143·7
28·0	147·8
28·5	151·7
29·0	155·4
29·5	158·9
30·0	162·1
30·5	165·2
31·0	168·0
31·5	170·7
32·0	173·1
32·5	175·3
33·0	177·2
33·5	179·0
34·0	180·5
34·5	181·8
35·0	182·9
35·5	183·7
36·0	184·3
36·5	184·7
37·0	184·9
37·5	185·0
38·0	185·0

Nomogram and table for finding potential evapo-transpiration

Example:

Compute the potential evapo-transpiration for April and November according to Thornthwaite for the following data:

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
tn °C	-5	0	5	9	13	17	19	17	13	9	5	0
t °C	--	--	--	10	--	--	--	--	--	--	3	--
Day hours	--	--	--	13	--	--	--	--	--	--	9	--

Solution:

Month	tn °C	$j = 0.09t_n^{1.5}$	PEx mm/month	PE mm/month
Jan	-5	--	--	--
Feb	0	0.00	--	--
Mar	5	1.01	--	--
Apr	9	2.43	48.43	52.47
May	13	4.22	--	--
Jun	17	6.31	--	--
Jul	19	7.45	--	--
Aug	17	6.31	--	--
Sep	13	4.22	--	--
Oct	9	2.43	--	--
Nov	5	1.01	13.42	10.1
Dec	0	0.00	--	--
	Σ	35.38		

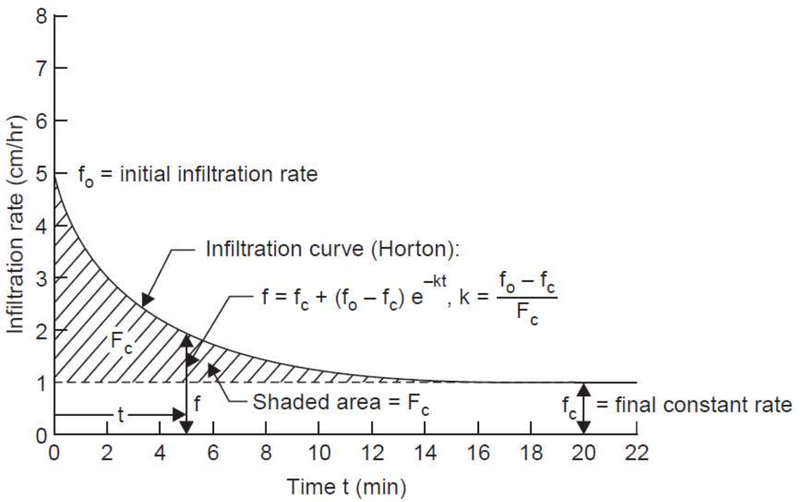
$$a = 0.016J + 0.5 = 0.016 \times 35.38 + 0.5 = 1.066$$

Infiltration:

Water entering the soil at the ground surface is called infiltration. It complements the soil moisture deficiency and the excess moves downward by the force of gravity called deep seepage or percolation and builds up the ground water table. The maximum rate at which the soil in any given condition is capable of absorbing water is called its infiltration capacity *(f_p)*. Infiltration rate *(f)* often begins at a high rate and decreases to a fairly steady state rate *(f_c)* as the rain continues, figure below. The infiltration rate *(f)* at any time t is given by Horton's equation.

$$f = f_c + (f_o - f_c)e^{-kt}$$

f_0 = initial rate of infiltration capacity
f_c = final constant rate of infiltration at saturation
k = a constant depending primarily upon soil and vegetation
F_c = shaded area in Fig.
t = time from beginning of the storm



Infiltration Curve (Horton)

The infiltration takes place at capacity rates only when the intensity of rainfall equals or exceeds fp;
$f = fp$ when $i \geq fp$
But $f = i$ when $i < fp$

The infiltration depends upon the intensity and duration of rainfall, weather (temperature), soil characteristics, vegetal cover, land use, initial soil moisture content (initial wetness), entrapped air and depth of the ground water table. The vegetal cover provides protection against rain drop impact and helps to increase infiltration.

Methods of Determining Infiltration

The methods of determining infiltration are:
(i) Infiltrometers.
(ii) Observation in pits and ponds.
(iii) Placing a catch basin below a laboratory sample.
(iv) Artificial rain simulators.
(v) Hydrograph analysis.

Double-ring infiltrometer. A double ring infiltrometer is shown in Figure (A) below. The two rings (22.5 to 90 cm diameter) are driven into the ground by a driving plate and hammer, to penetrate into the soil uniformly without tilt to a depth of 15 cm. After driving is over, any disturbed soil adjacent to the sides tamped with a metal tamper. Point gauges are fixed in the center of the ring. Water is poured into the rings to maintain the desired depth (2.5 to 15 cm with a minimum of 5 mm) and the water added to maintain the original constant depth at regular time intervals (after the starting of the experiment) of 5, 10, 15, 20, 30, 40, 60 min, etc. up to a period of at least 6 hours is noted and the results are plotted as infiltration rate in cm/hr versus time in minutes as shown in Figure (B) below. The purpose of the outer tube is to eliminate to some extent the edge effect of the surrounding drier soil and to prevent the water within the inner space from spreading over a larger area after penetrating below the bottom of the ring.

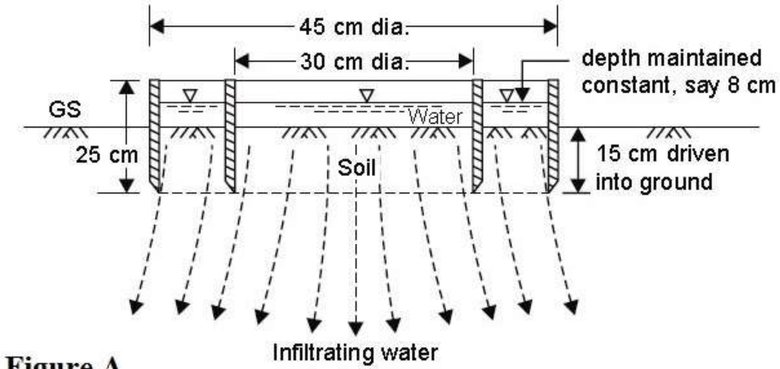


Figure A

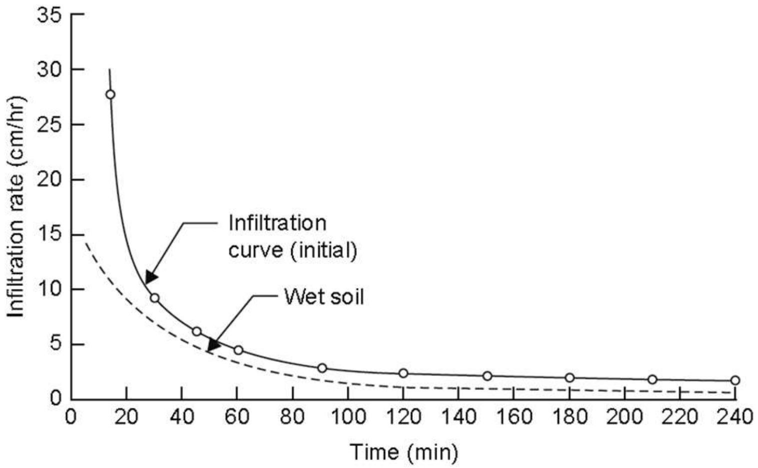


Figure B Typical infiltration curve

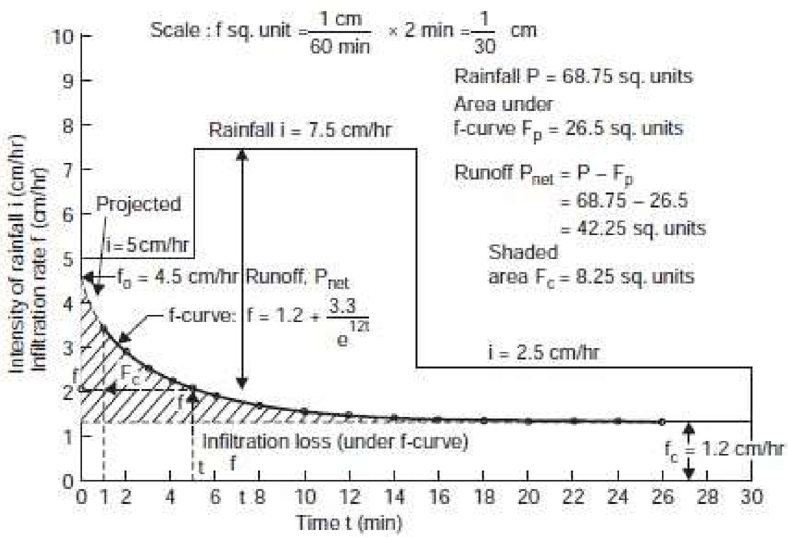
Example:

For a given basin, the following are the infiltration capacity rates at various time intervals after the beginning of the storm. Make a plot of the f-curve and establish an equation of the form developed by Horton. Also determine the total rain and the excess rain (runoff).

Time (min)	Rainfall (cm/hr)	f (cm/hr)
1	5.0	3.90
2	5.0	3.40
3	5.0	3.10
4	5.0	2.70
5	5.0	2.50
6	7.5	2.30
8	7.5	2.00
10	7.5	1.80
12	7.5	1.54
14	7.5	1.43
16	2.5	136
18	2.5	131
20	2.5	1.28
22	2.5	1.25
24	2.5	1.23
26	2.5	1.22
28	2.5	1.20
30	2.5	1.20

Solution:

The precipitation and infiltration rates versus time are plotted as shown in Figure.



From figure above, shaded area

$$F_c = 4.25\ sq\ units = 8.25\left(\frac{1cm}{60min} \times 2min\right) = 8.25 \times \frac{1}{30}$$

$$= 0.275\ cm$$

$$k = \frac{(4.5 - 1.2)cm/hr}{0.275\ cm} = 12\ hr^{-1}$$

The Horton's equation is:

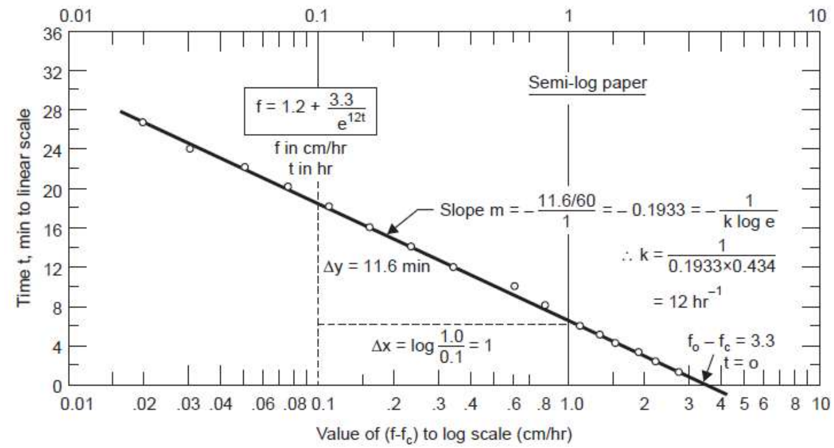
$$f = f_c + (f_o - f_c)e^{-kt} = 1.2 + (4.5 - 1.2)e^{-12t} = 1.2 + \frac{3.3}{e^{-12t}}$$

The total infiltration loss F_p can also be determined by integrating the Horton's equation for the duration of the storm.

$$F_P = \int_0^t f dt = \int_0^{30/60}\left(1.2 + \frac{3.3}{e^{-12t}}\right)dt = 1.2t + \frac{3.3}{-12e^{-12t}}\Big]_0^{30/60} = 0.88\ cm$$

To determine the Horton's constant by drawing a semi-log plot of *t* vs. *(f – f_c)*:

The Horton's equation is



Semi-log plot for infiltration constants

From the graph, when *t=0*,

$f - f_c = 3.3 = f_o - f_c$, (Since $f = f_0$ $when\ t = 0$)

$\therefore f_o$= 3.3+1.2 =4.5 cm/hr

Hence the Horton's equation is of the form

$$f = 1.2 + (4.5 - 1.2)e^{-12t} = 1.2 + \frac{3.3}{e^{-12t}}$$

Total rain $P = 5 \times \frac{5}{60} + 7.5 \times \frac{10}{60} + 2.5 \times \frac{15}{60} = 2.29\ cm$

Infiltration loss $F_P = 0.88$ cm

$\therefore P_{net} = excess\ rain = Runoff = P - F_P = 2.29 - 0.88 = 1.41cm$

Example:

In a double ring infiltrometer test, a constant depth of 100 mm was restored at every time interval the level dropped as given below:

Time (min): 0 5 10 15 25 45 60 75 90 110 130

Depth of water (mm): 100 83 87 90 85 78 85 85 85 80 80

Establish the infiltration equation of the form developed by Horton.

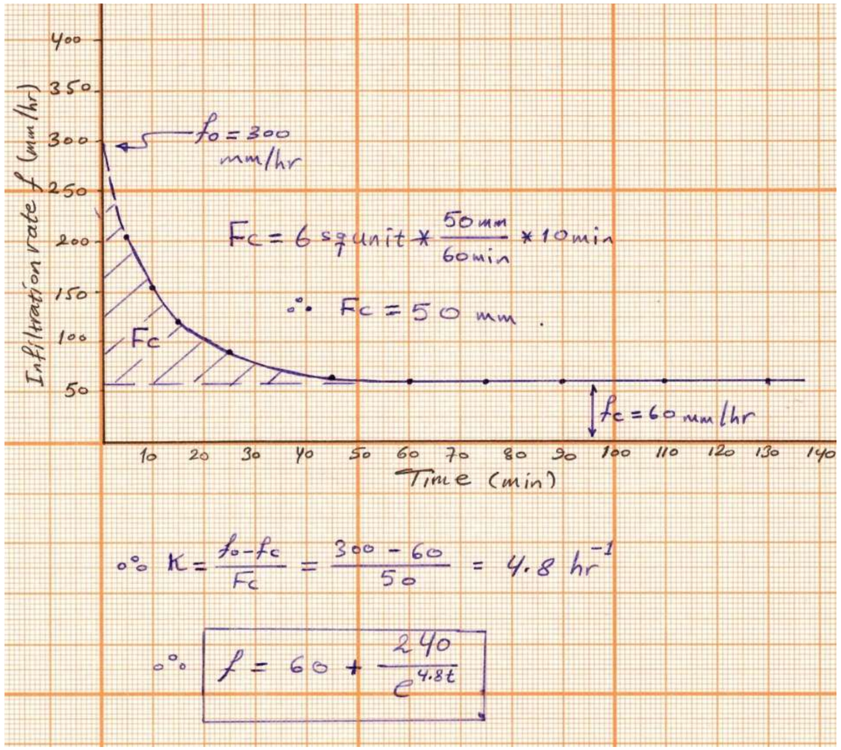
Solution:

Time (min)	Depth to water Surface (mm)		Depth of infiltration (mm)	Infiltration rate $f = \frac{d}{\Delta t} \times 60$ (mm/hr)	$f - f_c$ (mm/hr) $f_c = 60$ (mm/hr)
	Before filling	After filling			
0	100	--	0	--	--
5	83	100	17	204	144
10	87	100	13	156	96
15	90	100	10	120	60
25	85	100	15	90	30
45	78	100	22	66	6
60	85	100	15	60	0
75	85	100	15	60	0
90	85	100	15	60	0
110	80	100	20	60	0
130	80	100	20	60	0

Now plot on millimetric paper, t vs f, figure below.

Solution:

The precipitation and infiltration rates versus time are plotted as shown in Figure.



INFILTRATION INDICES

The infiltration curve expresses the rate of infiltration (cm/hr) as a function of time. The area between the rainfall graph and the infiltration curve represents the rainfall excess, while the area under the infiltration curve gives the loss of rainfall due to infiltration. The rate of loss is greatest in the early part of the storm, but it may be rather uniform particularly with wet soil conditions from rainfall.

Estimates of runoff volume from large areas are sometimes made by the use of infiltration indices, which assume a constant average infiltration rate during a storm, although in actual practice the infiltration will be varying with time. This is also due to different states of wetness of the soil after the start of the rainfall. There are three types of infiltration indices:

(i) ϕ-index (ii) *W*-index (iii) f_{ave}-index.

(i) ϕ-index : The ϕ-index is defined as that rate of rainfall above which the rainfall volume equals the runoff volume. The ϕ -index is relatively simple and all losses due to infiltration, interception and depression storage (storage in pits and ponds) are accounted for; hence,

$$\emptyset = \frac{F_P}{t_R} = \frac{P-R}{t_R}$$

Provided $i > \phi$ throughout the storm. The bar graph showing the time distribution of rainfall, storm loss and rainfall excess (net rain or storm runoff) is called a hyetograph, Figure below. Thus, the ϕ-index divides the rainfall into net rain and storm loss.

(ii) W-index: The W-index is the average infiltration rate during the time rainfall intensity exceeds the infiltration capacity rate, i.e.,

$$W = \frac{P-R-S}{t_R} = \emptyset - \frac{S}{t_R}$$

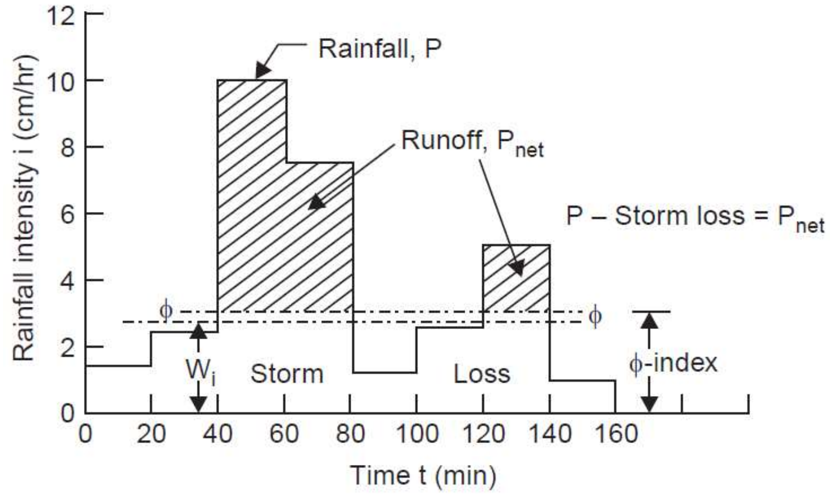
Where P = total rainfall

R = surface runoff

S = effective surface retention

t_R = duration of storm during which $i > fp$

F_p = total infiltration



Infiltration loss by ϕ-index

The W-index attempts to allow for depression storage, short rainless periods during a storm and eliminates all rain periods during which $i < f$. Thus, the W-index is essentially equal to the ϕ-index minus the average rate of retention by interception and depression storage, i.e., $W < \phi$.

Information on infiltration can be used to estimate the runoff coefficient C in computing the surface runoff as a percentage of rainfall i.e.,

$$R = C \times P$$

$$C = \frac{i - W}{i}$$

(iii) *fave*-index: In this method, an average infiltration loss is assumed throughout the storm, for the period *i* > *f*.

<u>Example</u>

The rates of rainfall for the successive 30 min period of a 3-hour storm are: 1.6, 3.6, 5.0, 2.8, 2.2, 1.0 cm/hr.
The corresponding surface runoff is estimated to be 3.6 cm.
Establish the ϕ-index. Also determine the *W*-index.

<u>Solution</u>

Construct the hyetograph as shown in Figure A.

$\Sigma(i - \phi)t = P_{net}$, and thus it follows

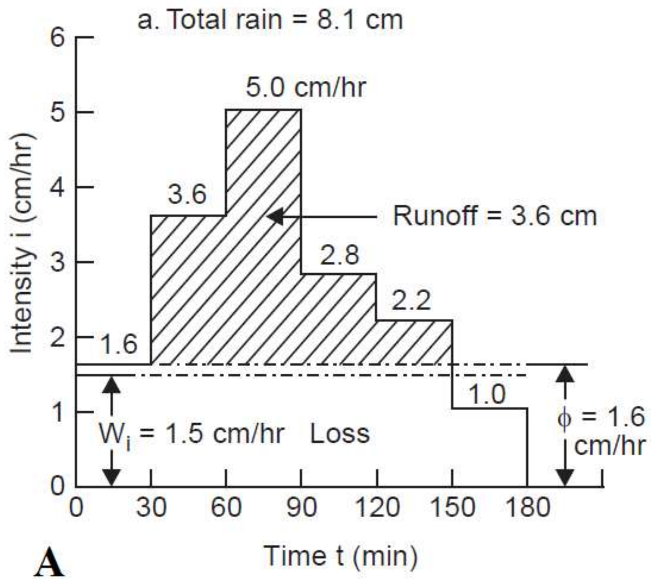
$$[(3.6 - \emptyset) + (5.0 - \emptyset) + (2.8 - \emptyset) + (2.2 - \emptyset)] \times \frac{30}{60} = 3.6$$

$$\Rightarrow \emptyset = 1.6\ cm/hr$$

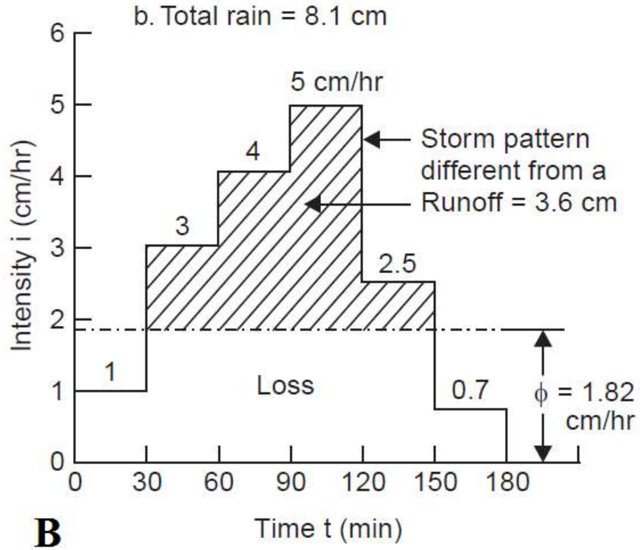
$$P = (1.6 + 3.6 + 5.0 + 2.8 + 2.2 + 1.0)\frac{30}{60} = 8.1 \text{ cm}$$

$$\Rightarrow W = \frac{P - R}{t_R} = \frac{8.1 - 3.6}{3} = 1.5\ cm/hr$$

Suppose the same 3-hour storm had a different pattern as shown in Figure B producing the same total rainfall of 8.1 cm. To obtain the same runoff of 3.6 cm (shaded area), the ϕ -index can be worked out as 1.82 cm/hr. Hence, it may be seen that a single determination of ϕ -index is of limited value and many such determinations have to be made and averaged, before the index is used. The determination of ϕ -index for a catchment is a trial and error procedure.



A

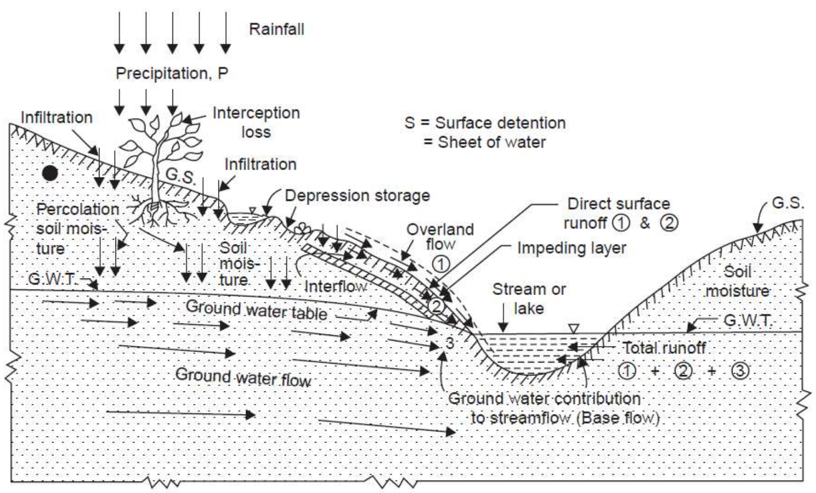


B

Runoff

When a storm occurs, a portion of rainfall infiltrates into the ground and some portion may evaporate. The rest flows as a thin sheet of water over the land surface which is termed as *overland flow.* If there is a relatively impermeable stratum in the subsoil, the infiltrating water moves laterally in the surface soil and joins the stream flow, which is termed as *underflow* (*subsurface flow*) or *interflow*, Figure below. If there is no impeding layer in the subsoil the infiltrating water percolates into the ground as *deep seepage* and builds up the ground water table (*GWT or phreatic surface*). The ground water may also contribute to the stream flow, if the GWT is higher than the water surface level of the stream, creating a hydraulic gradient towards the stream.

All the three types of flow contribute to the stream flow, it is the overland flow, which reaches first the stream channel, the interflow being slower reaches after a few hours and the ground water flow being the slowest reaches the stream channel after some days. The term *direct runoff* is used to include the overland flow and the interflow. If the snow melt contributes to the stream flow it can be included with the direct runoff (from rainfall).



Disposal of rain water

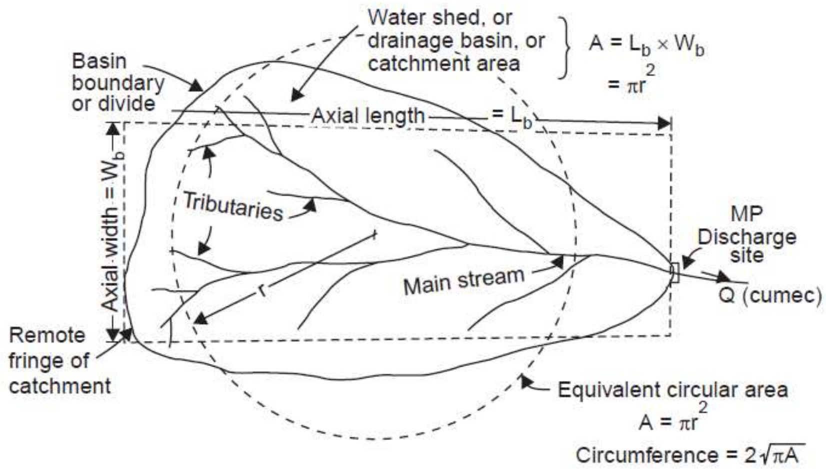
The ground water flow into the stream would have continued even if there had been no storm immediately preceding. It is for this reason it is termed as *base flow* in hydrograph analysis.

When the overland flow starts (due to a storm) some flowing water is held in puddles, pits and small ponds; this water stored is called *depression storage*. The volume of water in transit in the overland flow which has not yet reached the stream channel is called *surface detention* or *detention storage*. The portion of runoff in a rising flood in a stream, which is absorbed by the permeable boundaries of the stream above the normal phreatic surface, is called *bank storage*.

CATCHMENT CHARACTERISTICS

The entire area of a river basin whose surface runoff (due to a storm) drains into the river in the basin is considered as a hydrologic unit and is called *drainage basin*, watershed or catchment area of the river flowing Figure below. The boundary line, along a topographic ridge, separating two adjacent drainage basins is called *drainage divide*. The single point or location at which all surface drainage from a basin comes together or concentrates as outflow from the basin in the stream channel is called *concentration point* or *measuring point*, since the stream outflow is usually measured at this point.

The time required for the rain falling at the most distant point in a drainage area (i.e., on the fringe of the catchment) to reach the concentration point is called the *concentration time*. This is a very significant variable since only such storms of duration greater than the time of concentration will be able to produce runoff from the entire catchment area and cause high intensity floods.



Drainage basin characteristics

The characteristics of the drainage net may be physically described by:

(*i*) The number of streams (*ii*) The length of streams

(*iii*) Stream density (*iv*) Drainage density

✓ The stream density of a drainage basin is expressed as the number of streams per square kilometer.

$$Stream\ Density, D_s = \frac{N_s}{A}$$

Where: N_s=number of streams A=area of basin

✓ Drainage density is expressed as the total length of all stream channels (perennial and intermittent) per unit area of the basin and serves as an index of the areal channel development of the basin.

$$Drainage\ Density, D_d = \frac{L_s}{A}$$

Where: L_s=total length of all stream channels in the basin.

FACTORS AFFECTING RUNOFF

The various factors, which affect the runoff from a drainage basin, depend upon the following characteristics:

(i) Storm characteristics	1)	Intensity
	2)	Duration
	3)	Areal extent (distribution)
	4)	Frequency
	5)	Antecedent precipitation
(ii) Metrological characteristics	1)	Temperature
	2)	Humidity
	3)	Wind velocity
	4)	Pressure variation
(iii) Basin characteristics	1)	Size
	2)	Shape
	3)	Slope
	4)	Altitude (elevation)
	5)	Topography
	6)	Geology (type of soil)
	7)	Land use/vegetation
	8)	Type of drainage net
	9)	Proximity to ocean
	10)	Proximity to mountain ranges
(iv) Storage characteristics	1)	Depressions
	2)	Pools and ponds/lakes
	3)	Streams
	4)	Channels
	5)	Check dams (in gullies)
	6)	Upstream reservoir/or tanks
	7)	Flood plains, swamps
	8)	Ground water storage

✓ Low intensity storms over longer spells contribute to ground water storage and produce relatively less runoff. A high intensity storm or smaller area covered by a storm increases the runoff since the losses like infiltration and evaporation are less. If there is a succession of storms, the runoff will increase due to initial wetness of the soil due to antecedent rainfall. Rain during summer season will produce less runoff, while that during winter will produce more.

✓ Greater humidity decreases evaporation. The pressure distribution in the atmosphere helps the movement of storms. Snow storage and specially the frozen ground greatly increase the runoff.

✓ Peak runoff (if expressed as m^3/km^2) decreases as the catchment area increases due to higher time of concentration. A fan-shaped catchment produces greater flood intensity than a fern-shaped catchment. (figure below).

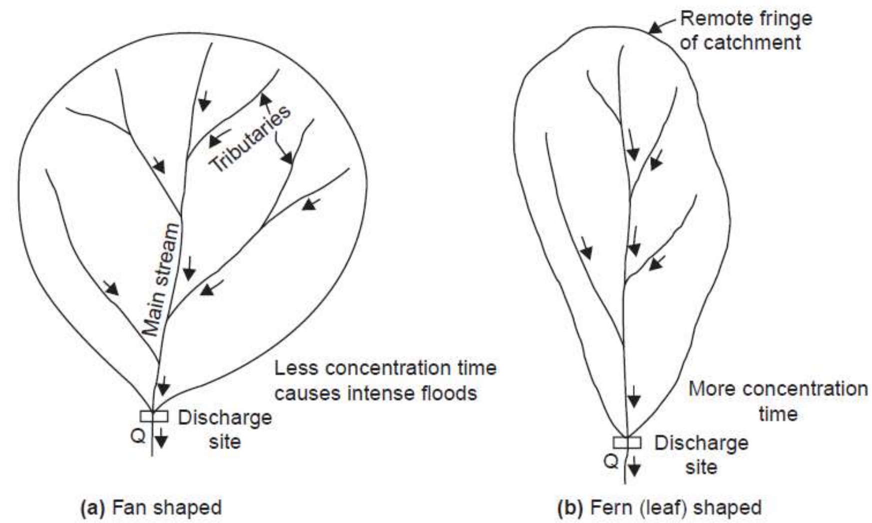
✓ Steep rocky catchments with less vegetation will produce more runoff compared to flat tracts with more vegetation. If the vegetation is thick greater is the absorption of water, so less runoff.

✓ If the direction of the storm producing rain is down the stream receiving the surface flow, it will produce greater flood discharge than when it is up the stream.

✓ If the catchment is located on the orographic side (windward side) of the mountains, it receives greater precipitation and hence gives a greater runoff. If it is on the leeward side, it gets less precipitation and so less runoff. Similarly, catchments located at higher altitude will receive more precipitation and yield greater runoff.

✓ The storage in channels and depressions (valley storage) will reduce the flood magnitude.

✓ Upstream reservoirs, lakes and tanks will moderate the flood magnitudes due to their storage effects. For drainage basins having previous deposits, large ground water storage may be created, which may also contribute to the stream flow in the form of delayed runoff.



Fan-and fern-shaped catchments

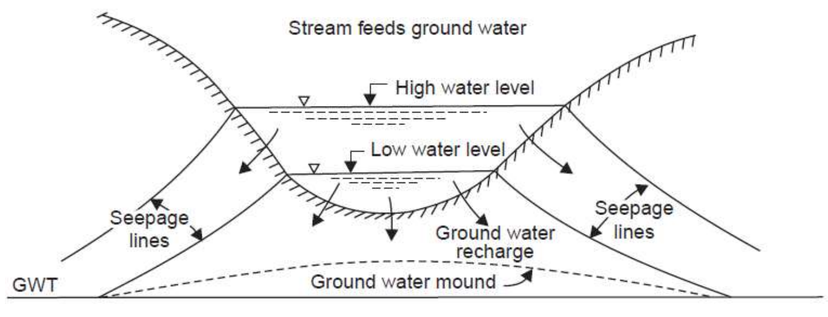
CLASSIFICATION OF STREAMS

Streams may be classified as:
(*i*) Influent and Effluent streams.
(*ii*) Intermittent and perennial streams.

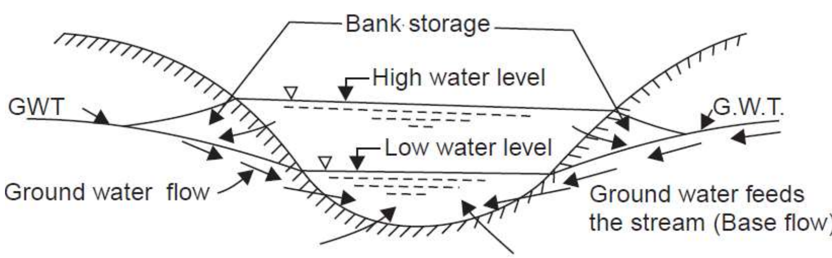
(*i*) Influent and Effluent streams. If the GWT is below the bed of the stream, the seepage from the stream feeds the ground-water resulting in the buildup of water mound (Figures below).

Such streams are called influent streams. Irrigation channels function as influent streams and many rivers which cross desert areas do so. Such streams will dry up completely in rainless period and are called ephemeral streams. The ephemeral streams, generally seen in arid regions, which flow only for a few hours after the rainfall.

When the GWT is above water surface elevation in the stream, the ground water feeds the stream. Such streams are called effluent streams. The base flow of surface streams is the effluent seepage from the drainage basin. Most of the perennial streams are mainly effluent streams.



Influent streams



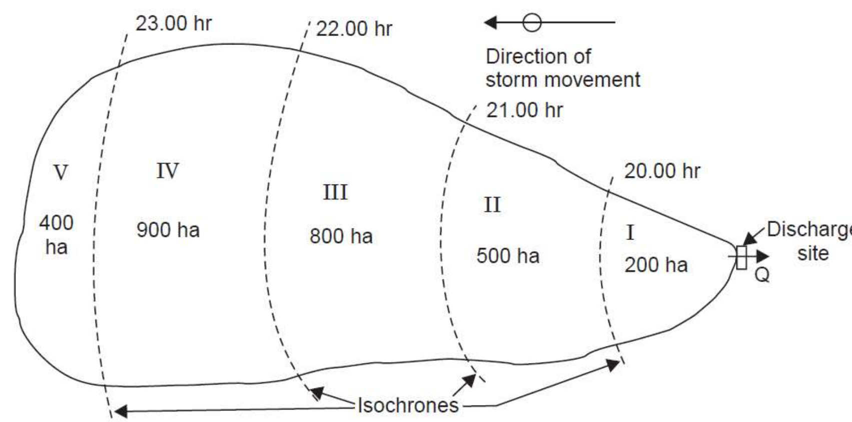
Effluent streams

(*ii*) Intermittent and perennial streams. If the GWT lies above the bed of the stream during the wet season but drops below the bed during the dry season, the stream flows during wet season (due to surface runoff and ground water contribution) but becomes dry during dry seasons. Such streams are called intermittent streams.

While in the case of perennial streams, even in the most severe droughts, the GWT never drops below the bed of the stream and therefore they flow throughout the year. For power development a perennial stream is the best; power can also be generated from intermittent streams by providing adequate storage facilities.

ISOCHRONES

The lines joining all points in a basin of some key time elements in a storm, (such as beginning of precipitation, time of concentration, etc.) are called isochrones. They are the time contours and represent lines of equal travel time and they are helpful in deriving hydrographs.



Isochrones

ESTIMATION OF RUNOFF

Runoff is that balance of rain water, which flows or runs over the natural ground surface after losses by evaporation, interception and infiltration.

The yield of a catchment (usually means annual yield) is the net quantity of water available for storage, after all losses, for the purposes of water resources utilization and planning, like irrigation, water supply, etc.

Maximum flood discharge. It is the discharge in times of flooding of the catchment area,

i.e., when the intensity of rainfall is greatest and the condition of the catchment regarding humidity is also favorable for an appreciable runoff.

Runoff Estimation

The runoff from rainfall may be estimated by the following methods:

(i) Empirical formulae, curves and tables

(ii) Infiltration method

(iii) Rational method

(iv) Overland flow hydrograph

(v) Unit hydrograph method

(i) Empirical formulae, curves and tables: Several empirical formulae, curves and tables relating to the rainfall and runoff have been developed as follows:

Usually, $R = a\,P + b$ ....A

Sometimes, $R = a\,P^n$ ......B

Where $R =$ runoff, $P =$ rainfall, a, b, and n, are constants. Eq. (A) gives a straight line plot on natural graph paper while Eq. (B) gives an exponential curve on natural graph paper and a straight line when plotted on log-log paper.

(ii) Infiltration Method. By deducting the infiltration loss, i.e., the area under the infiltration curve, from the total precipitation or by the use of infiltration indices. These methods are largely empirical and the derived values are applicable only when the rainfall characteristics and the initial soil moisture conditions are identical to those for which these are derived.

(iii) Rational Method. A rational approach is to obtain the yield of a catchment by assuming a suitable runoff coefficient.

$Yield = C\,A\,P$

Where $A =$ area of catchment

$P =$ precipitation

$C =$ runoff coefficient

The value of the runoff coefficient C varies depending upon the soil type, vegetation geology etc. and the following Table given by Richards may be taken as a guide.

Runoff coefficients for various types of catchments

Type of catchment	Value of C
Rocky and impermeable	0.8–1.0
Slightly permeable, bare	0.6–0.8
Cultivated or covered with vegetation	0.4–0.6
Cultivated absorbent soil	0.3–0.4
Sandy soil	0.2–0.3
Heavy forest	0.1–0.2

In the rational method, the drainage area is divided into a number of sub-areas and with the known times of concentration for different subareas the runoff contribution from each area is determined. The choice of the value of the runoff coefficient C for the different sub-areas is an important factor in the runoff computation by this method.

Empirical Formulas:

1- Rainfall-Runoff Correlation

A correlation between rainfall and runoff may take the form:

$R = a\,P + b$

Where R= runoff, P= rainfall, a and b are constants.

$$a = \frac{N(\sum PR)-(\sum P)(\sum R)}{N(\sum P^2)-(\sum P)^2} \quad , \qquad b = \frac{\sum R - a\sum P}{N}$$

$$r = \frac{N(\sum PR)-(\sum P)(\sum R)}{\sqrt{[N(\sum P^2)-(\sum P)^2]\times[N(\sum R^2)-(\sum R)^2]}}$$

r= correlation factor , $0 \le r \le 1$, and the formula accepted if $r \ge 0.6$.

Example:

For the following data, find a correlation between rainfall and runoff in the form of $R = a\,P + b$.

month	P (cm)	R (cm)
1	5	0.5
2	35	10.0
3	40	13.8
4	30	8.2
5	15	3.1
6	10	3.2
7	5	0.1
8	31	12.0
9	36	16.0

Solution:

Complete the table of solution as follows;

month	P (cm)	R (cm)	PR	P^2	R^2
1	5	0.5	2.5	25	0.25
2	35	10.0	350	1225	100
3	40	13.8	552	1600	190.4
4	30	8.2	246	900	67.3
5	15	3.1	46.5	225	9.61
6	10	3.2	32	100	10.24
7	5	0.1	0.5	25	0.01
8	31	12.0	372	961	144
9	36	16.0	576	1296	256
Σ	207	66.9	2177.5	6357	777.8

N=9

$$a = \frac{N(\sum PR)-(\sum P)(\sum R)}{N(\sum P^2)-(\sum P)^2} = \frac{9(2177.5)-(207\times 66.9)}{9(6357)\ \ (6357^2)} = 0.40$$

$$b = \frac{\sum R - a\sum P}{N} = \frac{66.9-0.4\times 207}{9} = -1.77$$

$$\therefore R = 0.4P - 1.77$$

R=0.95 ⇨ the correlation is accepted.

2- Khosla's Formula

Khosla analyzed the rainfall, runoff, and temperature of several catchments in India and USA, and he suggested the following formula for the monthly runoff:

$R_m = P_m - L_m$,

$L_m = 0.48\, T_m$, When $T_m > 4.5°C$

- When $T_m \leq 4.5°C$ ⇨

T_m	4.5	-1	-6.5
L_m	2.77	1.78	1.52

- R_m = monthly runoff (cm)>0
- P_m = monthly average temp. ^{0}C.
- L_m = monthly losses(cm).

Example:

The following data were collected for a catchment in India, find the annual runoff and calculate the runoff coefficient using Khosla's formula.

Month	T_m	P (cm)
Jan	24	0.7
Feb	27	0.9
Mar	32	1.1
Apr	33	4.5
May	31	10.7
Jun	26	7.1
Jul	24	11.1
Aug	24	13.7
Sep	23	16.4
Oct	21	15.3
Nov	20	6.1
Dec	21	1.3

Solution:

Complete the table of solution as follows:

Month	T_m	P (cm)	$R_m = P_m - L_m$ (cm)
Jan	24	0.7	--
Feb	27	0.9	--
Mar	32	1.1	--
Apr	33	4.5	--
May	31	10.7	--
Jun	26	7.1	--
Jul	24	11.1	--
Aug	24	13.7	2.18
Sep	23	16.4	5.36
Oct	21	15.3	5.22
Nov	20	6.1	--
Dec	21	1.3	--
Σ		88.9	12.76 cm

∴ annual runoff = 12.76 cm

⇨Annual Runoff Coefficient $= \frac{Ann.Runoff}{Ann\ Rain} = \frac{12.76}{88.9} = 0.143$

HYDROGRAPHS

A hydrograph is a graph or table showing flow rate (stream discharge) as a function of time at a given location on the stream (concentration point). Figure No.1 shows an annual hydrograph for Hit station.

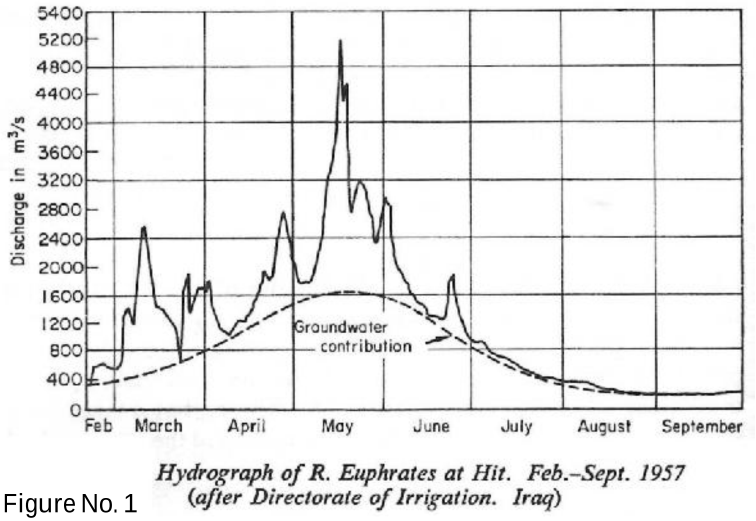


Figure No. 1

HYDROGRAPH COMPONENTS

The various components of a natural hydrograph (storm hydrograph) are shown in Fig. No.2. At the beginning, there is only base flow (the ground water contribution to the stream) gradually depleting in an exponential form. After the storm starts, the initial losses like interception and infiltration are met and then the surface flow begins. The hydrograph gradually rises (rising limb) and reaches its peak value after a time t_p (called lag time or basin lag) measured from the centroid of the hyetograph of net rain. Thereafter it declines and there is a change of slope at the inflection point, i.e., there has been, inflow of the rain up to this point and after this there is gradual withdrawal of catchment storage.

By this time the ground water table has been built up by the infiltrating and percolating water, and now the ground water contributes more into the stream flow than at the beginning of storm, but thereafter the GWT declines and the hydrograph again goes on depleting in the exponential form called the ground water depletion curve or the recession curve. If a second storm occurs now, again the hydrograph starts rising till it reaches the new peak and then falls and the ground water recession begins, Fig.3.

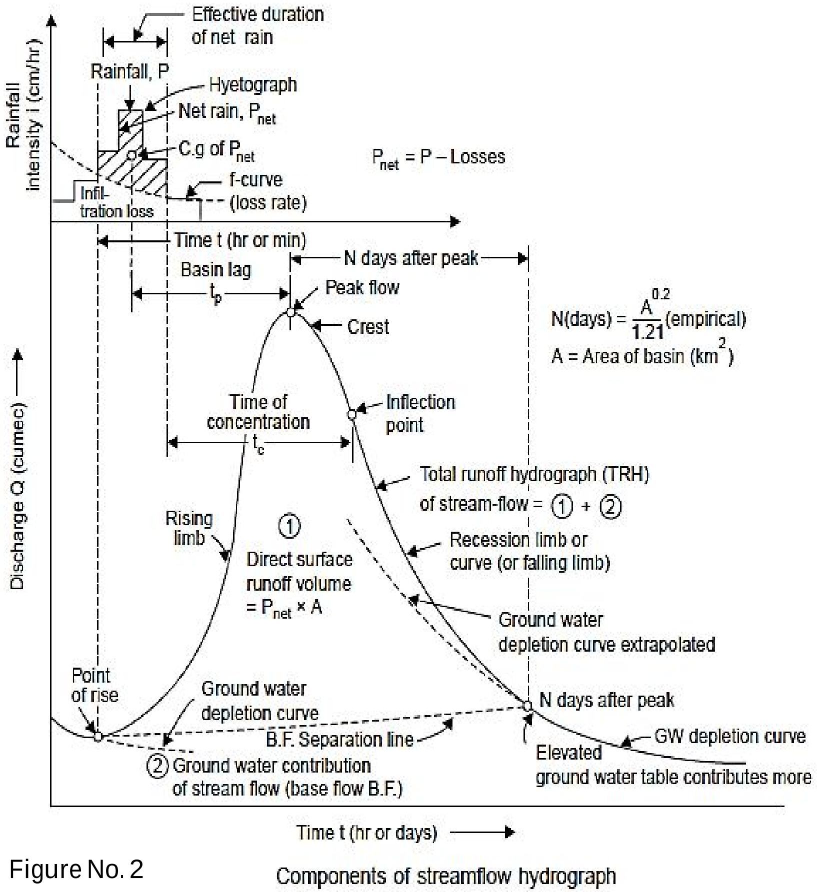


Figure No. 2

Thus, in actual streams gauged, the hydrograph may have a single peak or multiple peaks according to the complexity of storms. For flood analysis and derivation of unit hydrograph, a single peaked hydrograph is preferred. A complex hydrograph, however, can be resolved into simple hydrographs by drawing hypothetical recession lines as shown in Fig.3.

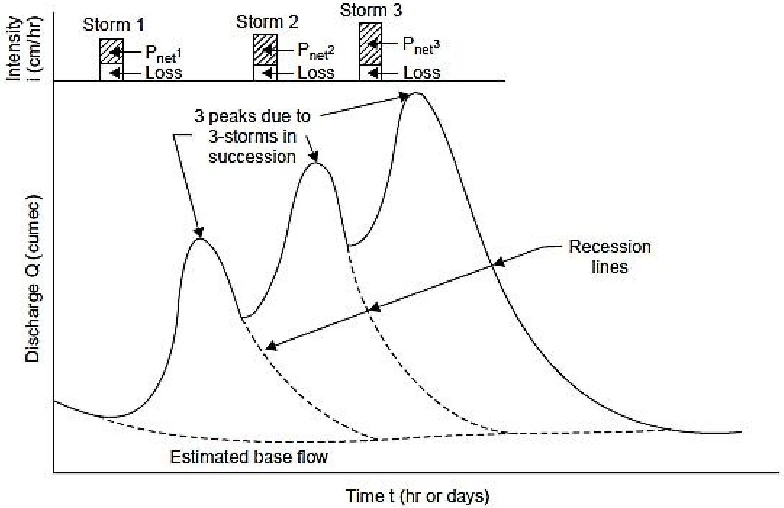


Figure No. 3

Hydrograph with multiple peaks

Factors Affecting Hydrograph:

Factors mainly affecting hydrograph are:

Storm and Metrological characteristics which affects the rising limb of hydrograph.

Basin characteristics which affects recession limb of hydrograph.

(i) Shape of basin, it affects the time of concentration. Thus, it affects the shape of the hydrograph and the location of peak point. Figure No.4.

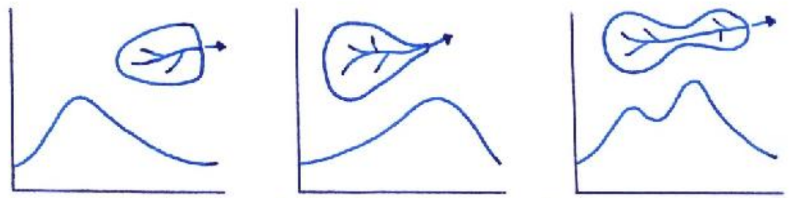
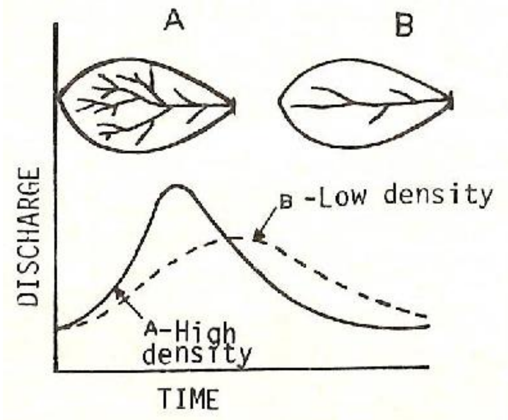


Figure No. 4 تأثير شكل الجابيه على الهيدروغراف

(ii) Slope of basin, it affects the Slope of the recession curve. Thus, it affects the time of base of hydrograph.

(iii) Drainage density, it affects the peak of the hydrograph. Figure No.5.



تأثير كثافة البزل على الهيدروغراف Figure No. 5

HYDROGRAPH SEPARATION

For the derivation of unit hydrograph, the base flow has to be separated from the total runoff hydrograph (i.e., from the hydrograph of the gauged stream flow). Some of the well-known base flow separation procedures are given below, Fig.6.

(i) Simply by drawing a line AC tangential to both the limbs at their lower portion.

This method is very simple but is approximate and can be used only for preliminary estimates.

(ii) Extending the recession curve existing prior to the occurrence of the storm up to the point D directly under the peak of the hydrograph and then drawing a straight line DE, where E is a point on the hydrograph N days after the peak, and N (in days) is given by

$N = 0.83\ A^{0.2}$

(iii) Simply by drawing a straight line AE, from the point of rise to the point E, on the hydrograph, N days after the peak.

(iv) Construct a line AFG by projecting backwards the ground water recession curve after the storm, to a point F directly under the inflection point of the falling limb and sketch an arbitrary rising line from the point of rise of the hydrograph to connect with the projected base flow recession. This type of separation is preferred where the ground water storage is relatively large and reaches the stream fairly rapidly, as in lime-stone terrains.

Many a time a straight line AE meets the requirements for practical purposes. Location of the point E is where the slope of the recession curve changes abruptly, and as a rough guide E is N days after the peak.

.

In all the above four separation procedures, the area below the line constructed represents the base flow, i.e., the ground water contribution to stream flow. Any further refinement in the base flow separation procedure may not be needed, since the base flow forms a very insignificant part of high floods. In fact, very often, a constant value of base flow is assumed

After separation of base flow, the result hydrograph is the direct runoff hydrograph or direct flow hydrograph (DRH or DFH). The area under DRH represents the direct surface runoff, and by dividing it by the catchment area we get the excess rain or effective rain.

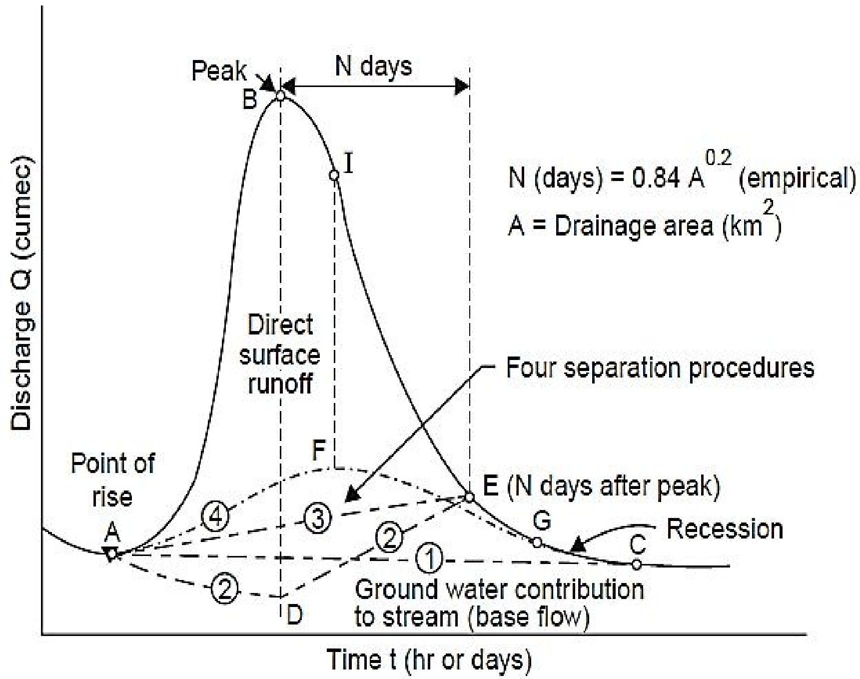


Figure No. 6 Hydrograph separation

Example:

The hydrograph tabulated below was observed for a river draining a 103.6km² catchment, following a storm lasting 3hr. Separate base flow from runoff and calculate total runoff volume. What was the net rainfall in mm/hr?. If the total rainfall was 20cm, find the ϕ index for the storm.

Time (hr)	Flow (m^3/sec)	Time (hr)	Flow (m^3/sec)	Time (hr)	Flow (m^3/sec)
0	12.7	24	99.1	48	30.3
3	155.7	27	85.0	51	26.9
6	254.9	30	73.6	54	23.8
9	212.4	33	62.6	57	21.2
12	184.1	36	53.6	60	18.7
15	158.6	39	45.9	63	16.7
18	135.9	42	39.6	66	15.3
21	116.1	45	34.5	---	---

Solution:

By plotting time versus discharge on natural paper we get the total hydrograph, from which we found the base flow as shown.

N days $=0.83\,A^{0.2} = 0.83\,(103.6)^{0.2} = 2.1$ days ≅ 51 hr.

Base Flow Increment = 3((21.2-12.7)/57) = 0.45 m^3/sec each 3hrs.

DRH ordinates = TH ordinates – Base Flow

The result tabulated in the following Table.

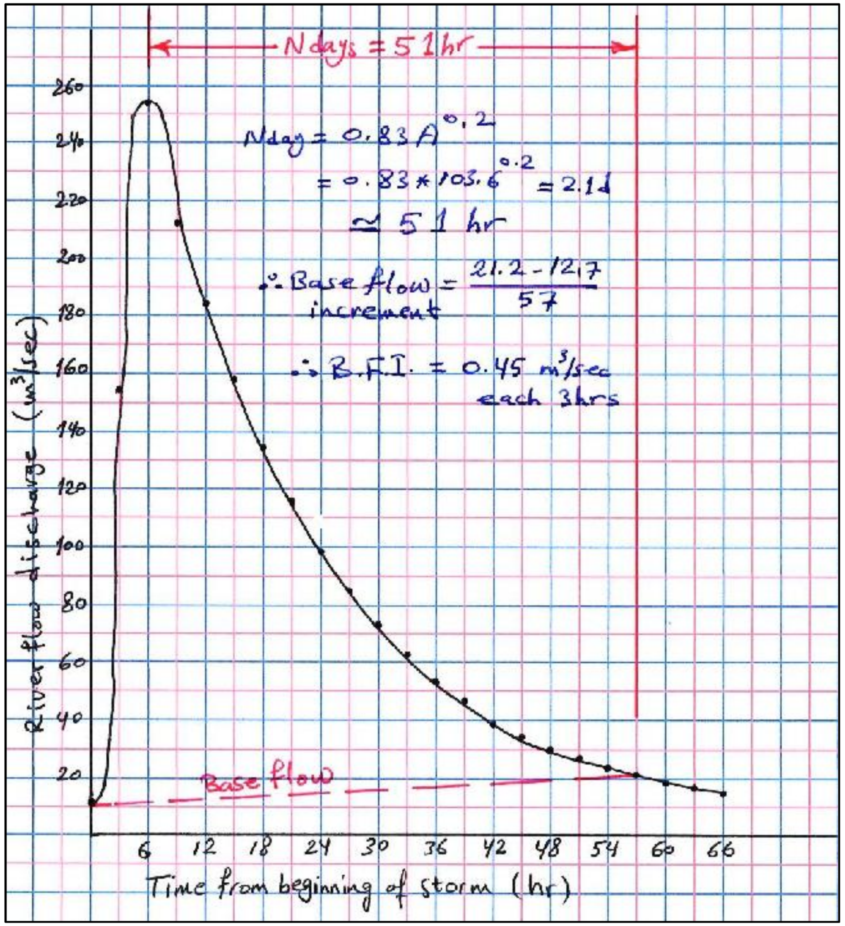


Figure No. 7

Hr	Q	BF	DRH	Hr	Q	BF	DRH
0	12.7	12.7	0	36	53.6	18.10	35.5
3	155.7	13.15	142.55	39	45.9	18.55	27.35
6	254.9	13.60	241.3	42	39.6	19.00	20.6
9	212.4	14.05	198.35	45	34.5	19.45	15.05
12	184.1	14.50	169.6	48	30.3	19.90	10.4
15	158.6	14.95	143.65	51	26.9	20.35	6.55
18	135.9	15.40	120.5	54	23.8	20.80	3
21	116.1	15.85	100.25	57	21.2	21.2	0
24	99.1	16.30	82.8	60	18.7	--	--
27	85.0	16.75	68.25	63	16.7	--	--
30	73.6	17.20	56.4	66	15.3	--	--
33	62.6	17.65	44.95	--	--	--	--

Now we can plot the direct runoff hydrograph as in figure No.8.

Total Surface Runoff = Area Under D.R.H.

$$TSR = \left(\frac{3 \times 60 \times 60}{2}\right)(0 + 2(142.55 + 241.3 + 198.35 + 169.6 + 143.65 + 120.5 + 100.25 + 82.8 + 68.25 + 56.4 + 44.95 + 35.5 + 27.35 + 20.6 + 15.05 + 10.4 + 6.55 + 3) + 0) = \frac{2 \times 1487.05 \times 3 \times 3600}{2} = 16059600\ m^3$$

$$= 16.06 Mm^3$$

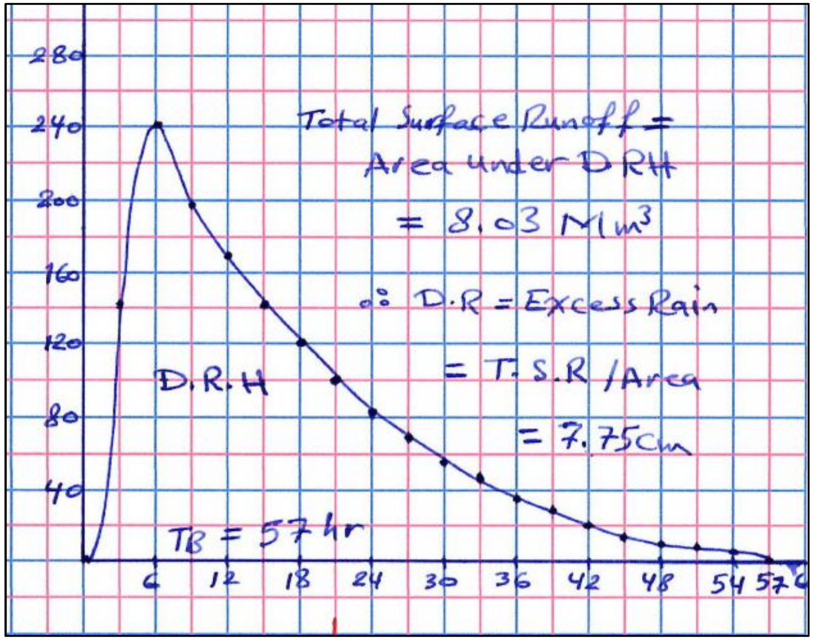


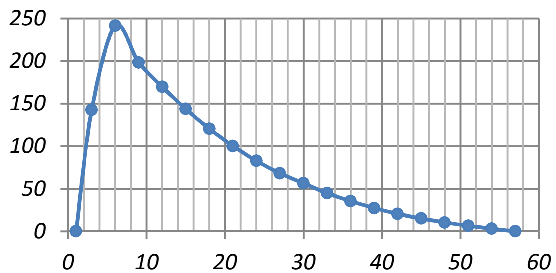
Figure No. 8

∴ Net Rainfall (Excess Rain) = T.S.R. / Area of Catchment

∴Net Rain= 16059600 / (103.6×1000×1000)= 15.5 cm

Or; Net Rain= 16.06/103.6 = 15.5 cm = 15.5/3= 52 mm/hr

∴ ϕindex = (P – R)/t_R = (20-15.5)/3 = 1.5 cm/hr



UNIT HYDROGRAPH

The unit hydrograph (unit-graph) is defined as the hydrograph of storm runoff resulting from an isolated rainfall of some unit duration occurring uniformly over the entire area of the catchment, produces a unit volume (i.e., 1 cm) of runoff.

Derivation of the unit hydrograph: The following steps are adopted to derive a unit hydrograph from an observed flood hydrograph (Fig. No.9).

(i) Select from the records isolated (single-peaked) intense storms, which occurring uniformly over the catchment have produced flood hydrographs with appreciable runoff.

(iii) Separate the base flow from the total runoff.

(iv) From the ordinates of the total runoff hydrograph (at regular time intervals) deduct the corresponding ordinates of base flow, to obtain the ordinates of direct runoff.

(iv) Divide the volume of direct runoff by the area of the drainage basin to obtain the net precipitation depth over the basin.

(v) Divide each of the ordinates of direct runoff by the net precipitation depth to obtain the ordinates of the unit hydrograph.

(vi) Plot the ordinates of the unit hydrograph against time since the beginning of direct runoff. This will give the unit hydrograph for the basin, for the duration of the unit storm (producing the flood hydrograph) selected in item (i) above.

In unit hydrograph derivation, such storms should be selected for which reliable rainfall and runoff data are available. The net rain graph (hyetograph of excess rain) should be determined by deducting the storm loss and adjusting such that the total volume of net storm rain is equal to the total volume of direct surface runoff. The unit hydrograph derived, which, when applied to the known net rain data, should yield the known direct runoff hydrograph.

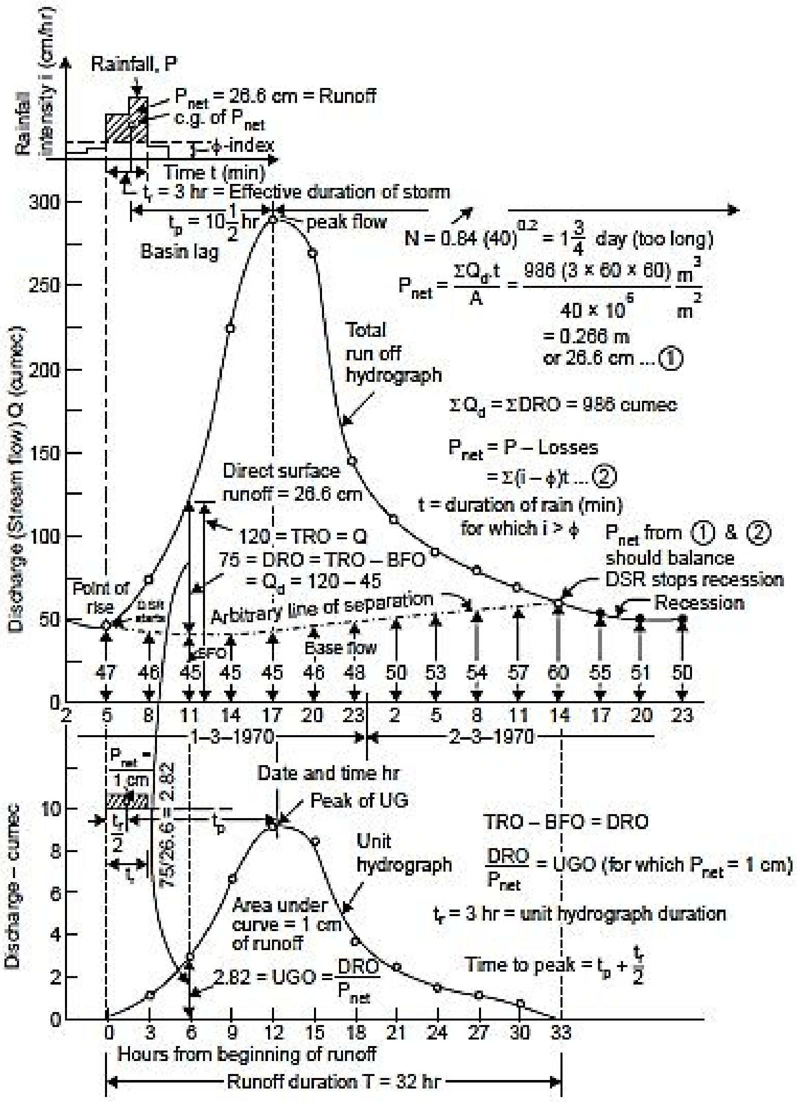


Figure No. 9 Derivation of a unit hydrograph (Example 5.1)

Example:

Data below represent the discharge results from a storm of 6-hr duration falls on a catchment of 500km². Assume base flow equal to zero, drive the 6-hr unit hydrograph.

Time (hr)	Q m^3/sec	Time (hr)	Q m^3/sec
0	**0**	**42**	**50**
6	**100**	**48**	**35**
12	**250**	**54**	**25**
18	**200**	**60**	**15**
24	**150**	**66**	**5**
30	**100**	**72**	**0**
36	**70**	**--**	**---**

Solution:

للحصول على الهيدروغراف القياسي نتبع الخطوات التالية:

١- نقوم بفصل الجريان القاعدي عن السيح السطحي للحصول على احداثيات هيدروغراف الجريان السطحي DRH(اي طرح الجريان القاعدي من احداثيات هيدروغراف الجريان الكلي) وفي مثالنا هذا لا يوجد جريان قاعدي.

٢- نقوم بحساب السيح السطحي المكافئ (المطر الفعال) والذي يمثل المساحة تحت هيدروغراف السيح السطحي DRH.

٣- احداثيات الهيروغراف القياسي UH هي عباره عن احداثيات السيح المباشر مقسومه على كمية المطر الفعال.

الجدول والشكل اللاحقين هما جدول الحسابات للحل وشكل هيدروغراف الجريان المباشر والهيدروغراف القياسي ونلاحظ ما يلي:

- يلاحظ بان الزمن القاعدي (TB) لـ UH هو نفسة زمن القاعده لـ DRH.
- الاستدامة المطرية لـ UH هي نفس الاستدامة للهيدروغراف الكلي TH للعاصفة المطرية التي اشتق منها الهيدروغراف القياسي.

Time (hr)	Q m^3/sec	UH Ordinate m^3/sec	Time (hr)	Q m^3/sec	UH Ordinate m^3/sec
0	0	0	42	50	**11.6**
6	100	23.1	48	35	**8.1**
12	250	57.9	54	25	**5.8**
18	200	46.3	60	15	**3.5**
24	150	34.7	66	5	**1.2**
30	100	23.1	72	0	**0**
36	70	16.2	--	---	**---**
			Σ	1000	**----**

Total Runoff=1000*6*60*60 = 21600000 m³
Equivalent Depth of Runoff= Excess Rain =
21600000/(500*1000*1000) = 0.0432 m = 4.32 cm

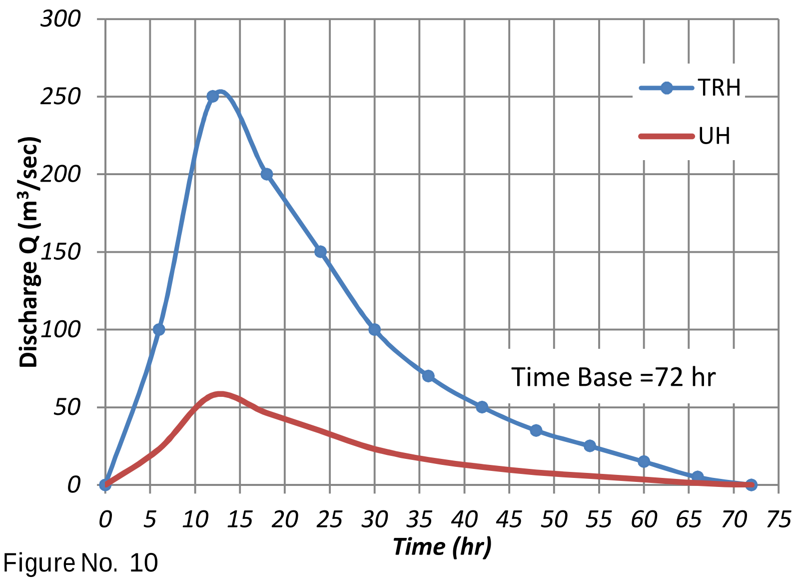


Figure No. 10

هذا عندما يكون المطلوب هو انشاء UH ولكن بعد انشاء الـــ UH للعاصفه الممطره لجابية معينه فاننا يمكن ان نستفيد منه في حساب حجم السيح السطحي للامطار المستقبلية الساقطه على نفس الجابية وذلك بضرب المطر الفعال (المؤثر) باحداثيات الهيدروغراف القياسي للحصول على احداثيات الـ DRH (والذي تمثل احداثياته تصريف النهر بعد اضافة الجريان القاعدي اليها) والمساحه تحته تمثل حجم السيح السطحي الكلي.

Example:

Peak flow of total hydrograph due to 6-hr storm was 470 m^3/sec and average depth of rain=8cm. Assume average infiltration losses =0.25 cm/hr and base flow is constant and equal to 15 m^3/sec. Estimate the peak flow of a 6-hr unit-graph.

Solution:

Direct flow = total flow – base flow = 470-15 = 455 m^3/sec

Infiltration losses = average losses * duration = 0.25*6 = 1.5 cm

 Effective Rain = average depth of rain – infiltration losses
= 8-1.5 = 6.5 cm

 Peak flow of UH = peak of DRH / ER = 455/6.5 = 70 m^3/sec.

Example:

For the following 6-hr unit hydrograph, find the total flow hydrograph due to a 3.5 cm excess rain. Suppose base flow increases linearly from 60 m^3/sec at starting of hydrograph to 126m^3/sec at the end.

Tim110e 60(hr)	UH (Q) m^3/sec	Time (hr)	UH (Q) m^3/sec
0	0	30	110
3	25	36	60
6	50	42	36
9	85	48	25
12	125	54	16
15	160	60	8
18	185	66	0
24	160	---	---

Solution:

Time	UH	BF	DRH	TH
0	0	60	0	60
3	25	63	88	151
6	50	66	175	241
9	85	69	298	367
12	125	72	438	510
15	160	75	560	635
18	185	78	648	726
24	160	84	560	644
30	110	90	385	475
36	60	96	210	306
42	36	102	126	228
48	25	108	88	196
54	16	114	56	170
60	8	120	28	148
66	0	126	0	126

في الجدول السابق تم الحصول على الاحداثيات العمودية لـ DRH بضرب المطر المؤثر والذي مقداره (3.5 cm) باحداثيات الهيدروغراف القياسي. ثم تم الحصول على الاحداثيات العمودية لهيدروغراف النهر او الهيدروغراف الكلي Total Hydrograph باضافة مقدار الجريان القاعدي الى احداثيات هيروغراف الجريان السطحى المباشر Direct Runoff Hydrograph .

والشكل التالي يمثل حل المثال السابق.

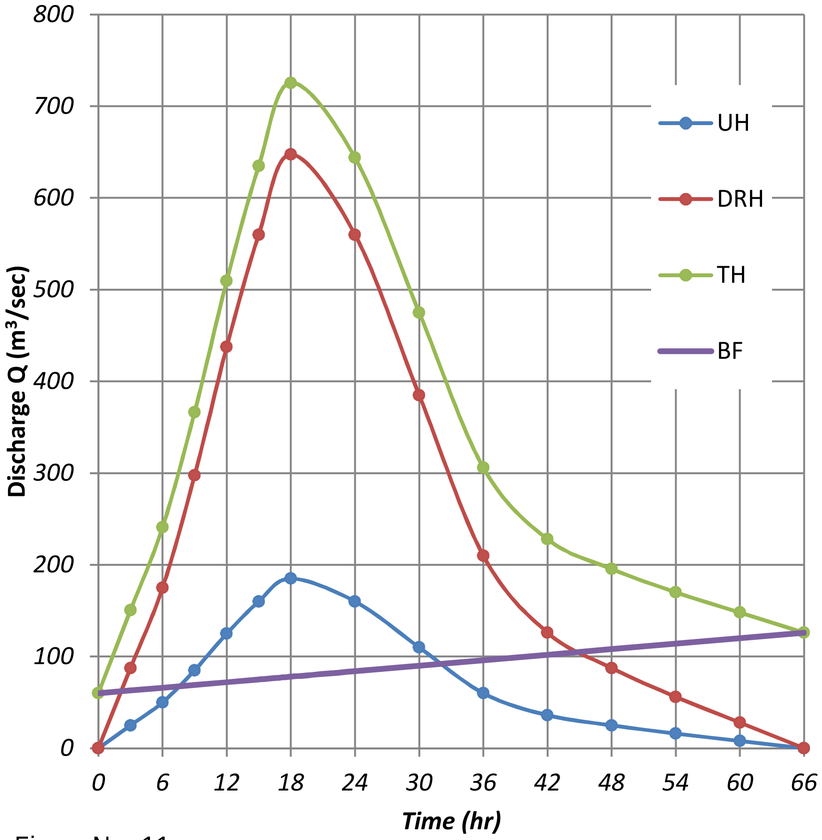


Figure No. 11

Example:

For the same 6-hr unit hydrograph in previous example, if tow storms (6-hr duration) occur successively, the first have an excess rain of 3cm and the second of 2cm. Find and draw the resulted direct runoff hydrograph (direct flow hydrograph, DRH or DFH) from these storms.

Solution:

The solution tabulated in table below.

Time	UH	DFH 1	DFH 2	TDFH
0	0	0	---	0
3	25	75	---	75
6	50	150	0	150
9	85	255	50	305
12	125	375	100	475
15	160	480	170	650
18	185	555	250	805
24	160	480	370	850
30	110	330	320	650
36	60	180	220	400
42	36	108	120	228
48	25	75	72	147
54	16	48	50	98
60	8	24	32	56
66	0	0	16	16
72	0	0	0	0

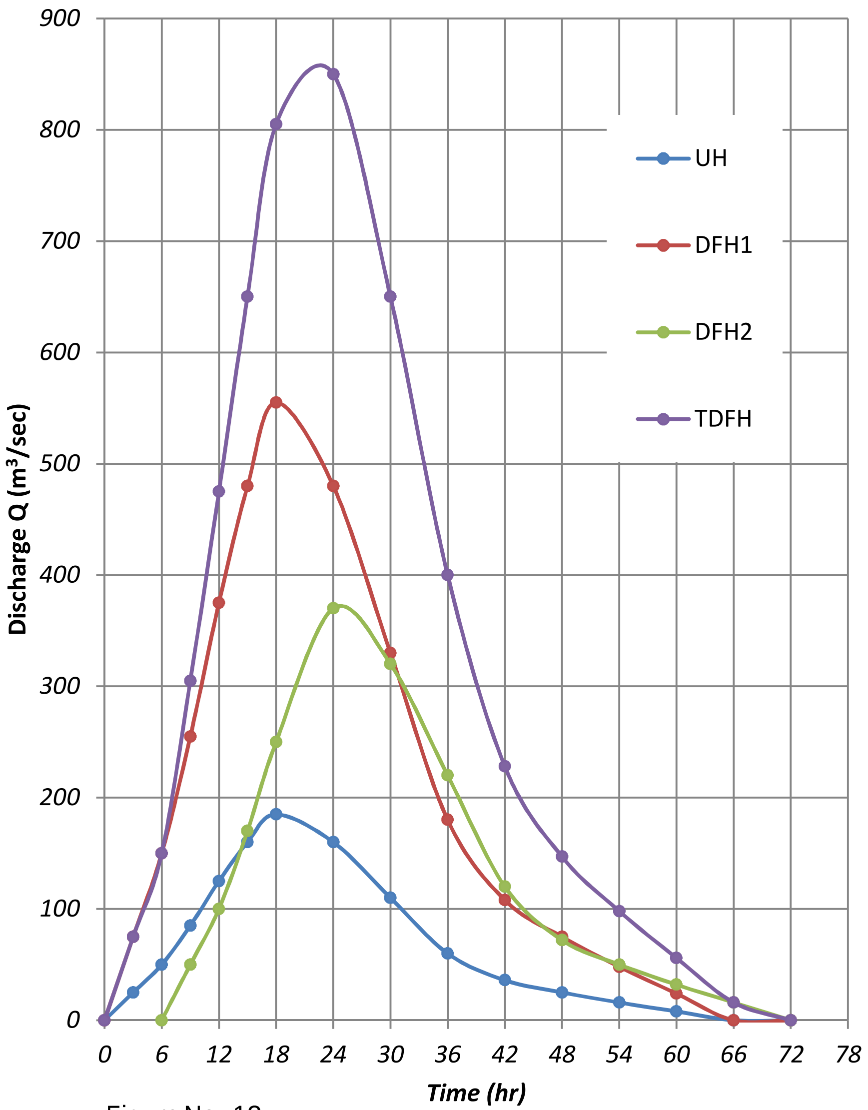


Figure No. 12

Example:

The vertical ordinates of a 6-hr U-H is

Time hr	**0**	**3**	**6**	**9**	**12**	**18**	**24**
U-H m^3/s	0	150	250	450	600	800	700
Time hr	30	36	42	48	54	60	66
U-H m^3/s	600	450	320	200	100	50	0

A storm of three successive intervals of 6-h duration was occurs. Assume it's total rain is 3 , 5 , and 4 cm respectively, and ϕ-index=0.2 cm/hr and base flow is constant and equals $150 m^3/sec$. determine and draw the total hydrograph for this storm.

Solution:

The effective rain for:

1^{st} interval = 3 – (0.2×6) = 1.8 cm

2^{nd} interval = 5 – (0.2×6) = 3.8 cm

3^{rd} interval = 4 – (0.2×6) = 2.8 cm

Ordinates of DRH = Ordinates of UH × Effective rain of interval

Ordinates of TRH = Sum of ordinates of DR-Hydrographs

Ordinates of T-H = Ordinates of TRH + Base flow

The solution was tabulated in table below.

1	2	3	4	5	6	7	8
Time hr	UH m^3/s	DF int1	DF int2	DF int3	TDF	BF	T-H m^3/s
0	0	0	-	-	0	150	250
3	150	270	-	-	270	150	520
6	250	450	0	-	450	150	700
9	450	810	570	-	1380	150	1630
12	600	1080	950	0	2030	150	2280
18	800	1440	2280	700	4420	150	4670
24	700	1260	3040	1680	5980	150	6230
30	600	1080	2660	2240	5980	150	6230
36	450	810	2280	1960	5050	150	5300
42	320	576	1710	1680	3966	150	4216
48	200	360	1216	1260	2836	150	3086
54	100	180	760	896	1836	150	2086
60	50	90	380	560	1030	150	1280
66	0	0	190	280	470	150	720
72			0	140	140	150	390
78				0	0	150	250

The following drawing is the total hydrograph.

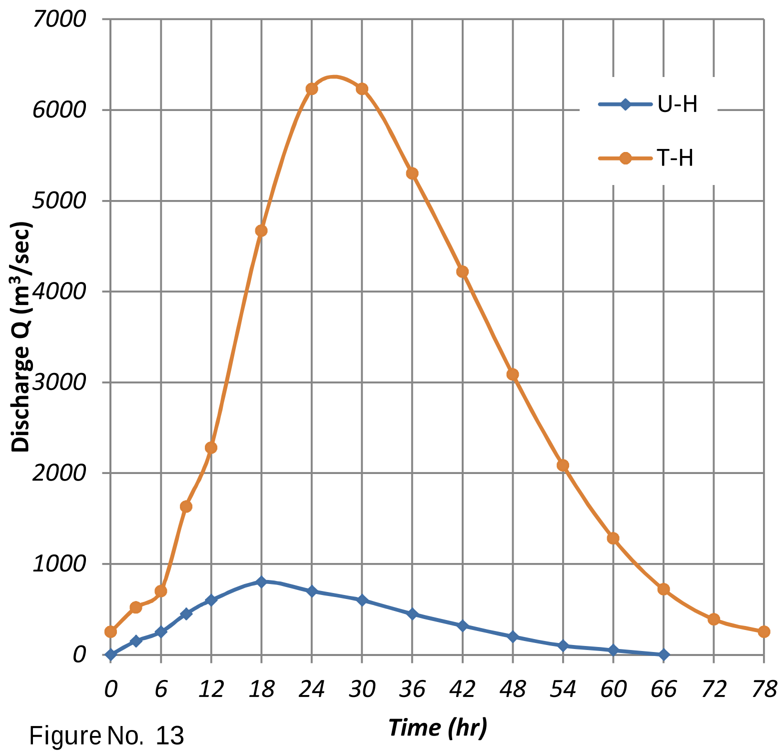


Figure No. 13

The Average Unit-Hydrograph

A unit hydrograph derived from a single storm may not be representative, and it is therefore desirable to average unit hydrographs from several storms of about the same duration. This should not be an arithmetic average of superimposed ordinates. The proper procedure is to compute average peak flow and time to peak. The average unit hydrograph is then sketched to conform to the shape of other graphs, passing through the computed average peak, and having the required unit volume.

Example:

Given below are three unit hydrographs derived from separate storms on a small catchment, all are considered to have resulted from 3hr rains. Derive the average unit hydrograph for this catchment.

Time hr	UH1 ft³/s	UH2 ft³/s	UH3 ft³/s	Time hr	UH1 ft³/s	UH2 ft³/s	UH3 ft³/s
0	0	0	0	8	195	255	322
1	165	37	25	9	143	195	248
2	547	187	87	10	97	135	183
3	750	537	260	11	60	90	135
4	585	697	505	12	33	52	90
5	465	608	660	13	15	30	53
6	352	457	600	14	7	12	24
7	262	330	427	15	0	0	0

Solution:

The peaks of unit hydrographs are 750 , 697, and 660 resp.

 Average peak flow = (750+697+660) / 3 = 702 ft^3/s

Time of average peak = (3+4+5) / 3 = 4 hrs

Now we can plot the hydrographs and the average hydrograph.

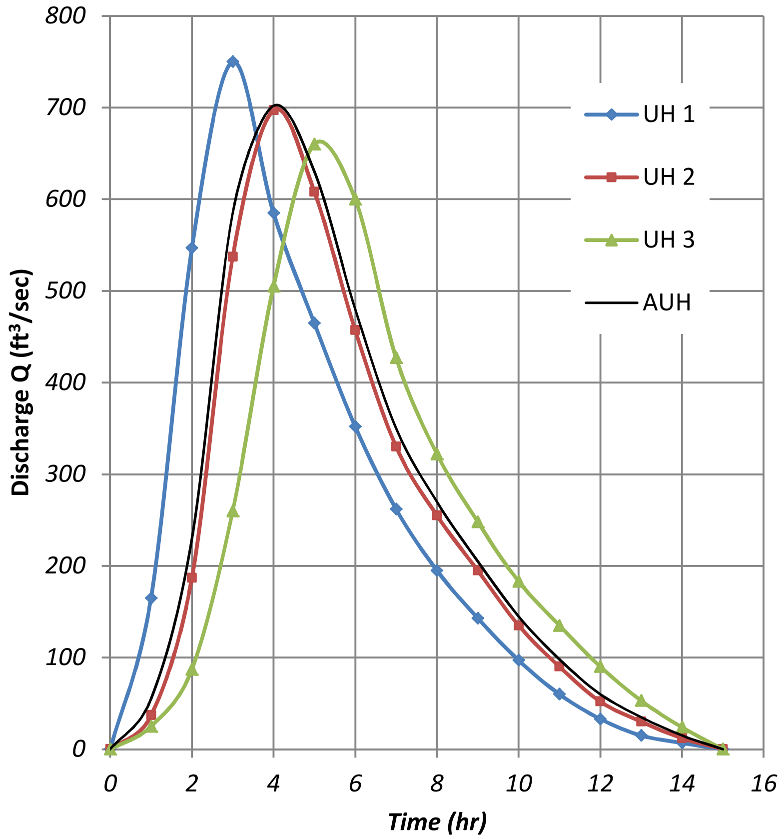


Figure No. 14

The Unit-Hydrograph from Complex or Multi-period Storms:

When suitable simple isolated storms are not available, data from complex storms of long duration will have to be used in unit hydrograph derivation. The problem is to decompose a measured composite flood hydrograph into its component DRHs and base flow. A common unit hydrograph of appropriate duration is assumed to exist. Consider a rainfall excess made up of three consecutive durations of D-hr and Equivalent-Rain values of i_1, i_2, and i_3. Figure 15 shows the ERH. By base flow separation of the resulting composite flood hydrograph, a composite DRH is obtained (fig. 15). Let the ordinates of the composite DRH be drawn at a time interval of D-hr. At various time interval 1D, 2D, 3D,... from the start of the ERH, let the ordinates of the unit hydrograph be u_1, u_2, u_3, ... and the ordinates of the composite DRH be Q_1, Q_2, Q_3, ...,

Then:

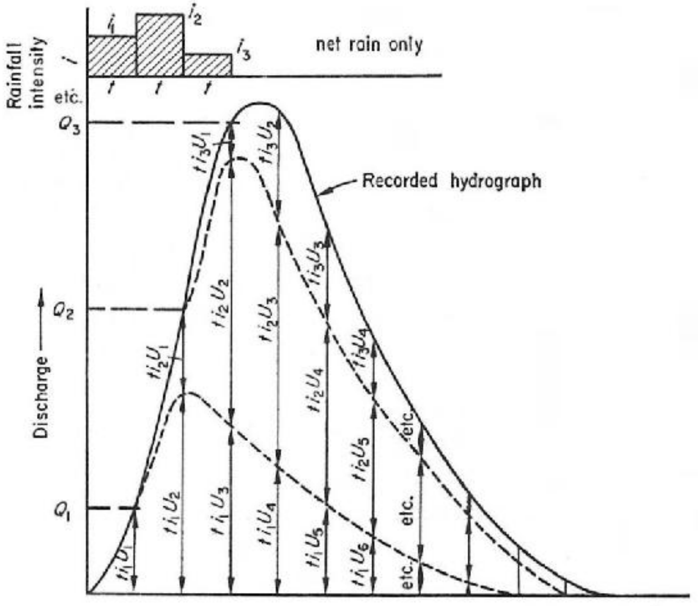
$Q_1 = i_1 u_1$

$Q_2 = i_1 u_2 + i_2 u_1$

$Q_3 = i_1 u_3 + i_2 u_2 + i_3 u_1$

$Q_4 = i_1 u_4 + i_2 u_3 + i_3 u_2$ .... and so on

From eq. above the values of u_1, u_2, u_3, ... can be determined.



Derivation of unit hydrograph from a multi-period storm hydrograph

Figure No. 15

Example:

The hydrograph tabulated below resulted from three consecutive 6-hr periods of rainfall having runoffs estimated as 0.6, 1.2, 0.9 cm respectively. Find the 6-hr unit hydrograph for the basin. Drainage area was 58 sq. km. Base flow have been subtracted.

Time, hr	0	3	6	9	12	15	18
Flow m^3/s	0	75	280	283	662	432	645
Time, hr	21	24	27	30	33	36	
Flow m^3/s	314	195	93	31	9	0	

Solution:

Time, hr	Flow m^3/s	
0	0	
3	75	
6	280	← Q_1
9	283	
12	662	← Q_2
15	432	
18	645	← Q_3
21	314	
24	195	← Q_4
27	93	
30	31	← Q_5
33	9	
36	0	

Then:

$Q_1 = i_1 u_1$ □ $280 = 0.6 \times u_1$ □ $u_1 = 467\ m^3/sec.$

$Q_2 = i_1 u_2 + i_2 u_1$ □ $662 = 0.6 \times u_2 + 1.2 \times u_1$

□ $662 = 0.6\ u_2 + 1.2 \times 467$ □ $u_2 = 170\ m^3/sec.$

$Q_3 = i_1 u_3 + i_2 u_2 + i_3 u_1$ □ $645 = 0.6\ u_3 + 1.2\ u_2 + 0.9\ u_1$

□ $645 = 0.6\ u_3 + 1.2 \times 170 + 0.9 \times 467$ □ $u_3 = 35\ m^3/sec.$

$Q_4 = i_1 u_4 + i_2 u_3 + i_3 u_2 = 195$ □ u_4 □ 0

Now, we can establish the table of the unit hydrograph as follows:

Time, hr	0	6	12	18	24
Flow m^3/s	0	467	170	35	0

The Unit-Hydrograph can be plotted as in figure 16

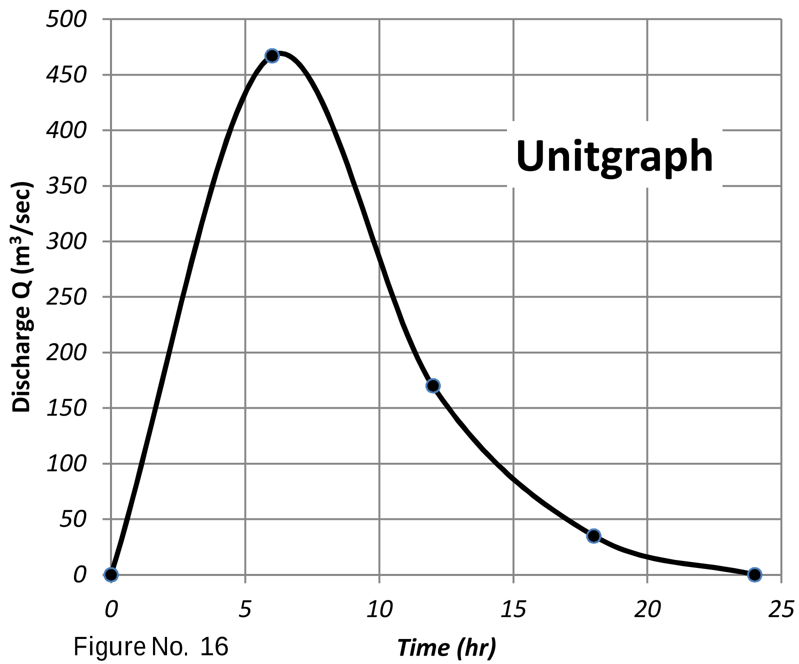


Figure No. 16

The Conversion of U-H Duration:

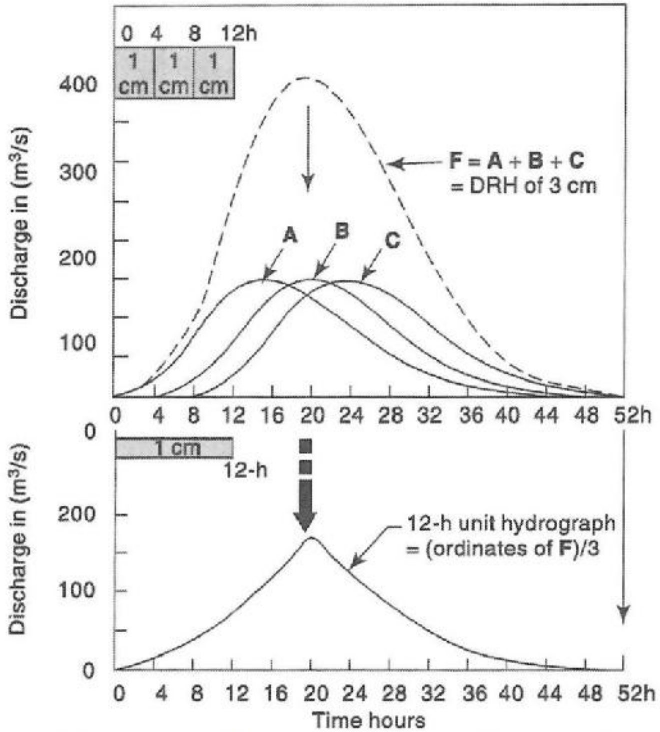
Ideally, unit hydrographs are derived from simple isolated storms and if the duration of the various storms do not differ very much they would all be grouped under one average duration of D-hr. If in practical applications unit hydrographs of different durations are needed they are best derived from field data. Lack of adequate data normally prevents development of unitgraphs covering a wide range of durations for a given catchment. Under such conditions a D-hr unitgraph is used to develop unitgraphs of differing durations nD-hr.

Two methods are available for this purpose.

- Method of superposition
- The S-curve

Method of Superposition

If a D-hr unit hydrograph is available, and it is desired to develop a unitgraph of nD-hr, where n is an integer, it is easily accomplished by superposing n unit hydrographs with each graph separated from the previous on by D-hr. Figure 17 shows three 4-hr unitgraphs A, B, and C. curve B begins 4 hr after A and C begins 4 hr after B. Thus the combination of these three curves is a DRH of 3 cm due to an ER of 12-hr duration. If the ordinates of this DRH are now divided by 3, one obtains a 12-hr unit hydrograph. The calculations are easy if performed in a tabular form.



Construction a 12-hr unitgraph from a 4-hr unitgraph

Figure No. 17

Example:

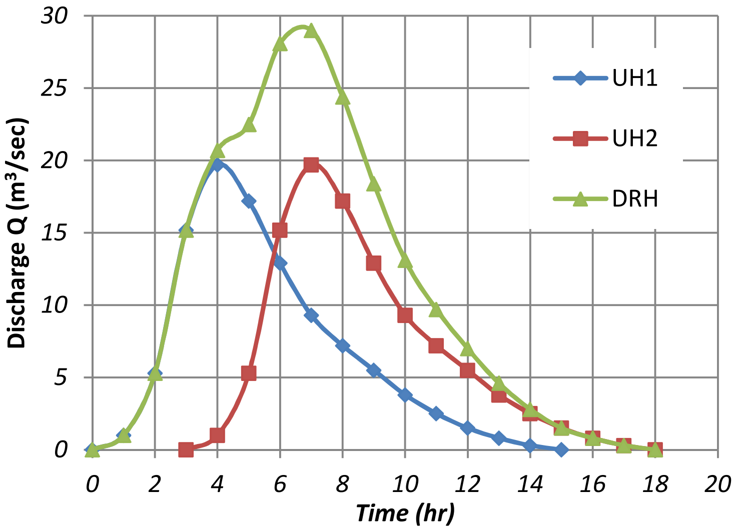
Table below is a 3-hr unitgraph, derive a 6-hr unitgraph by superposition method.

Time, hr	0	1	2	3	4	5	6	7
Flow m^3/s	0	1	5.3	15.2	19.7	17.2	12.9	9.3
Time, hr	8	9	10	11	12	13	14	15
Flow m^3/s	7.2	5.5	3.5	2.5	1.5	0.8	0.3	0

Solution:

نبدأ برسم الهيدروغراف القياسي المعلوم ذو الاستدامة 3-hr من الزمن (0) ثم نرسم نفس الهيدروغراف القياسي ولكن من الزمن (3-hr) بحيث يكون مقدار التزحيف مساوياً الى الاستدامة العلومة. ثم يتم جمع الاحداثيات العمودية للهيروغرافين المرسومين لنحصل على هيدروغراف السيح المباشر ذو الاستدامة (6-hr) وشدة مطرية مقدارها (2×1cm=2cm).

وبقسمة احداثيات هيروغراف السيح المباشر على (2) نحصل على احداثيات الهيدروغراف القياسي المطلوب (6-hr Unit Hydrograph).



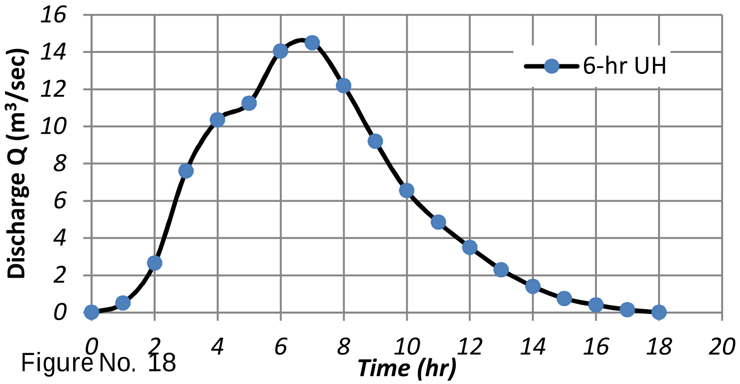


Figure No. 18

Or, it can be solved in table as in below.

1	2	3	4=2+3	5=4/2
Time hr	Q m³/s	lagged UH	DRH	6hr UH
0	0	--	0	0
1	1	--	1	0.5
2	5.3	--	5.3	2.65
3	15.2	0	15.2	7.6
4	19.7	1	20.7	10.35
5	17.2	5.3	22.5	11.25
6	12.9	15.2	28.1	14.05
7	9.3	19.7	29	14.5
8	7.2	17.2	24.4	12.2
9	5.5	12.9	18.4	9.2
10	3.8	9.3	13.1	6.55
11	2.5	7.2	9.7	4.85
12	1.5	5.5	7	3.5
13	0.8	3.8	4.6	2.3
14	0.3	2.5	2.8	1.4
15	0	1.5	1.5	0.75
16	--	0.8	0.8	0.4
17	--	0.3	0.3	0.15
18	--	0	0	0

The S-Curve:

If it is desired to develop a unit hydrograph of duration mD, where m is a fraction, the method of superposition cannot be used. A different technique known as the S-curve method is adopted in such cases.

The S-curve, also called S-hydrograph is a hydrograph produced by a continuous effective rainfall at a constant rate for an infinite period. It is a curve obtained by summation of an infinite series of D-hr unit hydrographs spaced D-hr apart. Figure 19 shows the construction and the use of S-curve in developing unit hydrograph.

A check for the S-curve must be is:

$$Q_e = \frac{2.78\,A}{D}$$

Where: Q_e is the equilibrium (constant) discharge (m^3/sec)

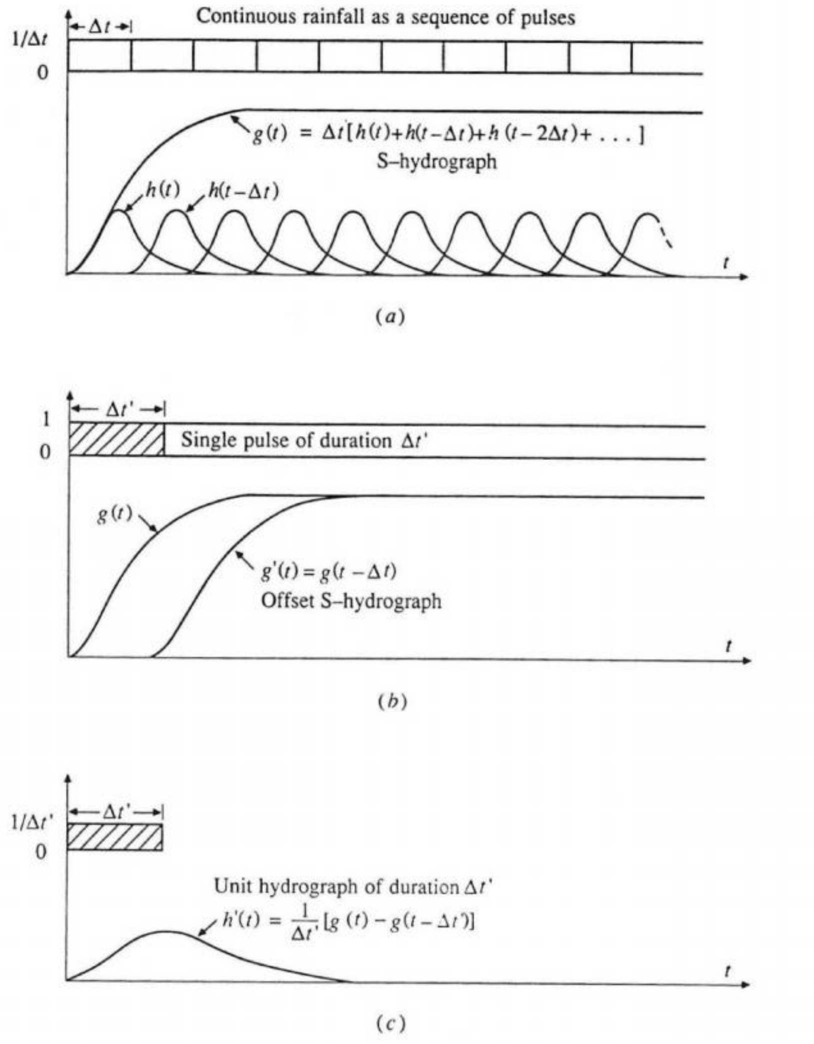
A is the catchment area (km)

D is the duration (hr)

Example:

Given the 4-hr unit hydrograph listed in table. Derive the 3-hr unit hydrograph. The catchment area is 300 sq. km.

Time, hr	0	1	2	3	4	5	6	7
Flow m^3/s	0	6	36	66	91	106	93	79
Time, hr	8	9	10	11	12	13	14	15
Flow m^3/s	68	58	49	41	34	27	23	17
Time, hr	16	17	18	19	20	21		
Flow m^3/s	13	9	6	3	1.5	0		



Using the S-hydrograph to find a unit hydrograph of duration Δt′ from a unit hydrograph Δt.

Figure No. 19

Solution:

The solution is tabulated in table below.

1	2	3	4	5	6	7	8	9	10	11
T hr	U-H	Lag UH	Lag UH	Lag UH	Lag UH	Lag UH	S - curve	Lag S-c	10= 8-9	3-hr UH
0	0						0		0	0
1	6						6		6	8
2	36						36		36	48
3	66						66	0	66	88
4	91	0					91	6	85	113
5	106	6					112	36	76	101
6	93	36					129	66	63	84
7	79	66					145	91	54	72
8	68	91	0				159	112	47	63
9	58	106	6				170	129	41	55
10	49	93	36				178	145	33	44
11	41	79	66				186	159	27	36
12	34	68	91	0			193	170	23	31
13	27	58	106	6			197	178	19	25
14	23	49	93	36			201	186	15	20
15	17	41	79	66			203	193	10	13
16	13	34	68	91	0		206	197	9	12
17	9	27	58	106	6		206	201	5	7
18	6	23	49	93	36		207	203	4	5
19	3	17	41	79	66		206	206	0	0
20	1.5	13	34	68	91	0	207	206	1	-
21	0	9	27	58	106	6	206	207	-1	-

Figure 20 represents the 4-hr S-curve and the 3-hr unit hydrograph.

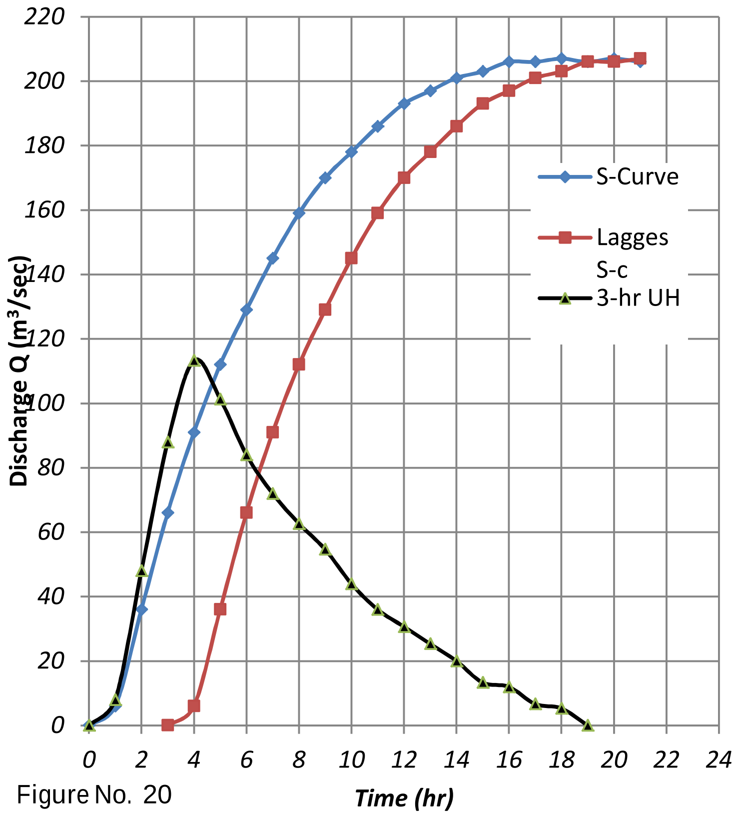


Figure No. 20

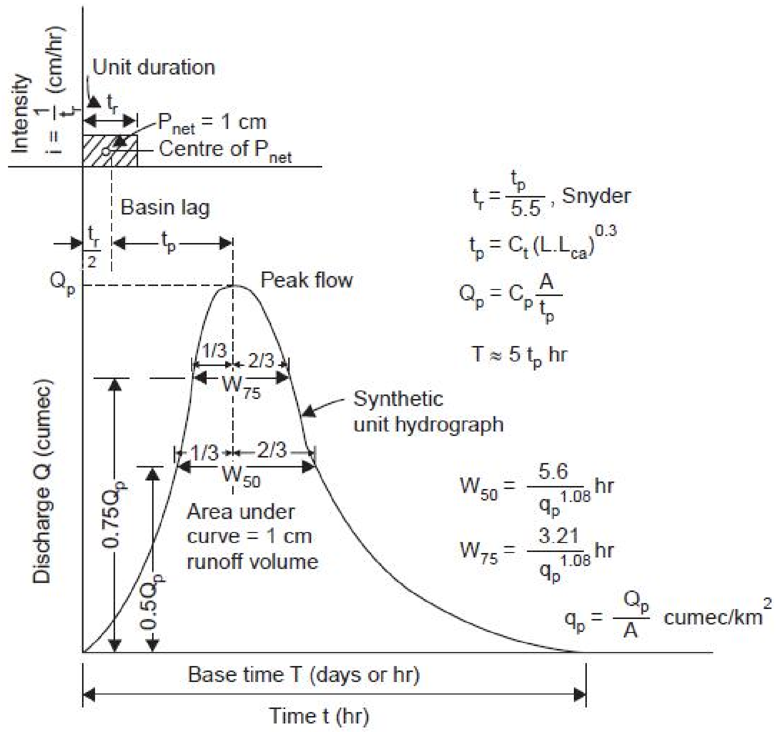
Another method for determining the S-curve is shown in table below.

1	2	3	4	5	6	7
Time hr	U-H	S-c addition	S-curve 4=2+3	Lagged S-c	(4-5)	3-hr UH
0	0		0		0	0
1	6		6		6	8
2	36		36		36	48
3	66		66	0	66	88
4	91	0	91	6	85	113
5	106	6	112	36	76	101
6	93	36	129	66	63	84
7	79	66	145	91	54	72
8	68	91	159	112	47	63
9	58	112	170	129	41	55
10	49	129	178	145	33	44
11	41	145	186	159	27	36
12	34	159	193	170	23	31
13	27	170	197	178	19	25
14	23	178	201	186	15	20
15	17	186	203	193	10	13
16	13	193	206	197	9	12
17	9	197	206	201	5	7
18	6	201	207	203	4	5
19	3	203	206	206	0	0
20	1.5	206	207	206	1	-
21	0	206	206	207	-1	-

Synthetic Hydrograph:

There are many catchments for which there are no runoff records at all and for which unitgraphs may be required. In these cases hydrographs may be synthesized on the basis of past experience in other areas and applied as first approximations to the unrecorded catchment. Such devices are called Synthetic Unitgraph.

The best known approach is due to Snyder who selected the three parameters of hydrograph base width (time), peak flow and basin lag as being sufficient to define the unitgraph. These are shown in figure 21.



Synthetic Unit Hydrograph Parameters

Figure No. 21

Snyder proposed the following empirical formulae for the three parameters.

Lag time, $t_p = C_t (L \times L_{ca})^{0.3}$

standard duration of net rain, $t_r = \frac{t_p}{5.5}$

peak flow, $Q_p = C_p \frac{A}{t_p}$

time base in days, $T = 3 + 3\frac{t_p}{24}$ (days)

peak flow per km² of basin, $q_p = \frac{Q_p}{A}$

Snyder proposed subsequently an expression to allow for some variation in the basin lag with variation in the net rain duration, i.e., if the actual duration of the storm is not equal to t_r given by Eq. above but is t_r', then

$$t_{pr} = t_p + \frac{t_R - t_r}{4}$$

where t_{pr} = basin lag for a storm duration of t_R , and t_{pr} is used instead of t_p in above mentioned Eqs.

In the above equations,

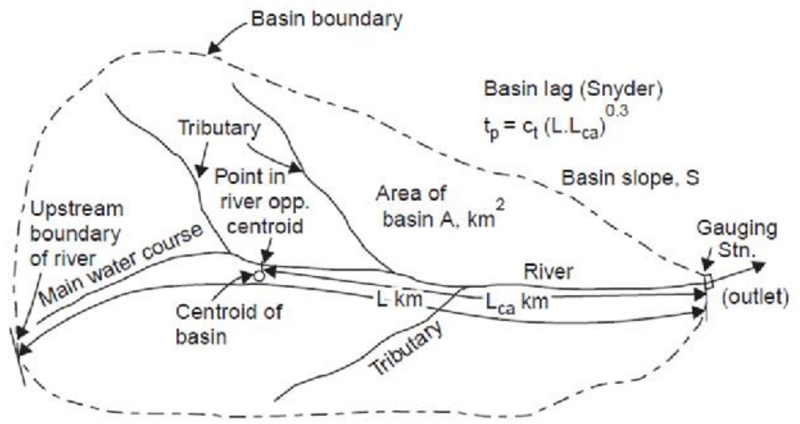
t_p = lag time (basin lag), hr

C_t , C_p = empirical constants ($C_t \approx 0.2$ to 2.2, $C_p \approx 2$ to 6.5, the values depending on the basin characteristics and units)

A = area of the catchment (km²)

L = length of the longest water course, i.e., of the mainstream from the gauging station (outlet or measuring point) to its upstream boundary limit of the basin, (km) (Fig. 22)

L_{ca} = length along the main stream from the gauging station (outlet) to a point on the stream opposite the areal center of gravity (centroid) of the basin



Basin characteristics (Snyder) Figure No. 22

Empirical formulae have been developed by the US Army Corps of Engineers (1959) for the widths of W_{50} and W_{75} of the hydrograph in hours at 50% and 75% height of the peak flow ordinate, respectively, (see Fig. 23) as

$$W_{50} = \frac{5.6}{q_p^{1.08}} \quad , \quad W_{75} = \frac{3.2}{q_p^{1.08}} = \frac{W_{50}}{1.75}$$

Example:

A basin has an area of 360 km² , L=25 km, L_{ca} = 10 km, C_t =1.5, C_p = 2.5. Derive a 3 hr synthetic unit hydrograph by Snyder method.

Solution:

$t_p = C_t (L \times L_{ca})^{0.3}$ = 1.5 $(25*10)^{0.3}$ = 7.86 hr

$t_r = \frac{t_p}{5.5} = \frac{7.86}{5.5} = 1.43\ hr,$ $\therefore$ is differ from required duration (3 hr), *then calculate* t_{pr}

$t_{pr} = t_p + \frac{t_R - t_r}{4} = t_{pr} = 7.86 + \frac{3-1.43}{4} = 8.25\ hr$

$Q_p = C_p \frac{A}{t_p} = 2.5 \frac{360}{8.25} = 109\ \text{m}^3/\text{sec}$

$T = 3 + 3\frac{t_p}{24} = 3 + 3\frac{8.25}{24} = 4 days = 96\ hr$

$q_p = \frac{Q_p}{A} = \frac{109}{360} = 0.303\ \text{m}^3/\text{km}^2$

$W_{50} = \frac{5.6}{q_p^{1.08}} = \frac{5.6}{0.303^{1.08}} = 20.3$ hr □ $W_{75} = 11.6$ hr

Now we can plot the 3-hr unitgraph as in figure 23.

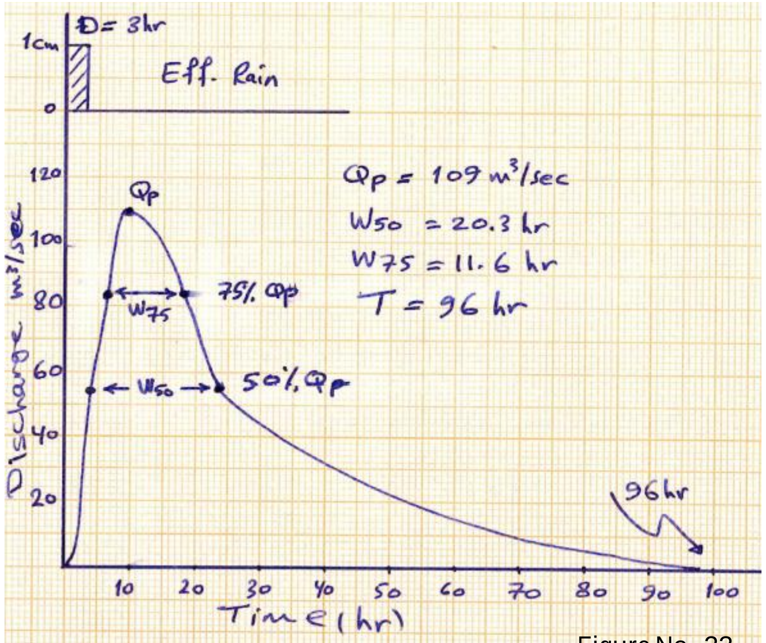


Figure No. 23

Example:

Two catchments B and C are considered similar from metrological aspect and having the following characteristics

Catchment	L (km)	L_{ca} (km)	A (km^2)
B	60	20	400
C	80	30	750

If the peak of 2-hr U-H for catch. B is 100 m3/sec occurs at the 10th hour from the beginning of the U-H. Derive the 2-hr S.U.H for catch C, use Snyder method.

Solution:

- For Catch B:

Time from the beginning of effective rain to peak flow = 10 hr

⇒ $10\text{hr} = t_{pr} + \frac{t_R}{2} = t_{pr} + (2/2)$ ⇒ $t_{pr} = 9$ hr

$t_{pr} = t_p + \frac{t_R - t_r}{4}$ ⇒ $9 = t_p + \frac{2 - t_p/5.5}{4}$ ⇒ $t_p = 8.9$ hr

$t_p = C_t (L \times L_{ca})^{0.3}$ ⇒ $8.9 = C_t (60 \times 20)^{0.3}$ ⇒ $C_t = 1.06$

$Q_p = C_p \frac{A}{t_p}$ ⇒ $100 = C_p \frac{400}{9}$ ⇒ $C_p = 2.25$

Now using C_t and C_p of Catchment B, we can develop a 2-hr S.U.H for catchment C.

HYDROGRAPHS

A *hydrograph* is a graph or table showing flow rate (stream discharge) as a function of time at a given location on the stream (concentration point). Figure No.1 shows an annual hydrograph for Hit station.

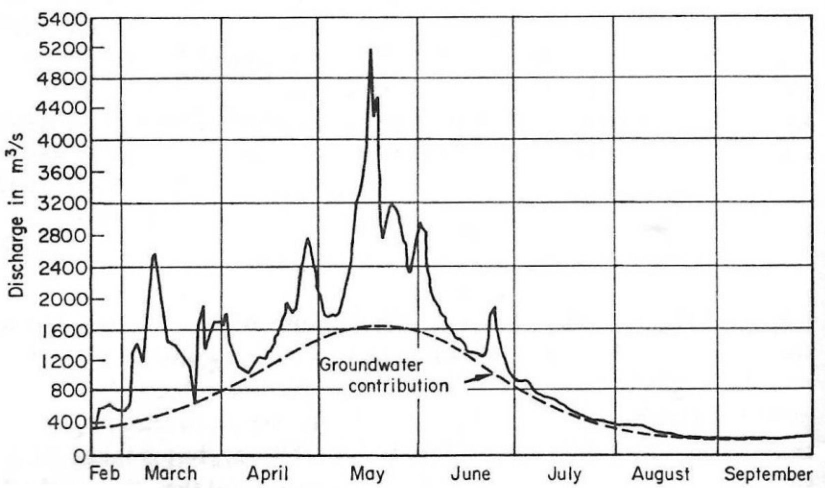


Figure No. 1 *Hydrograph of R. Euphrates at Hit. Feb.–Sept. 1957 (after Directorate of Irrigation. Iraq)*

HYDROGRAPH COMPONENTS

The various components of a natural hydrograph (*storm hydrograph*) are shown in Fig. No.2. At the beginning, there is only base flow (the ground water contribution to the stream) gradually depleting in an exponential form. After the storm starts, the initial losses like interception and infiltration are met and then the surface flow begins. The hydrograph gradually rises (*rising limb*) and reaches its peak value after a time t_p (called *lag time* or *basin lag*) measured from the centroid of the hyetograph of net rain. Thereafter it declines and there is a change of slope at the inflection point, i.e., there has been, inflow of the rain up to this point and after this there is gradual withdrawal of catchment storage.

By this time the ground water table has been built up by the infiltrating and percolating water, and now the ground water contributes more into the stream flow than at the beginning of storm, but thereafter the GWT declines and the hydrograph again goes on depleting in the exponential form called the *ground water depletion curve* or the *recession curve*. If a second storm occurs now, again the hydrograph starts rising till it reaches the new peak and then falls and the ground water recession begins, Fig.3.

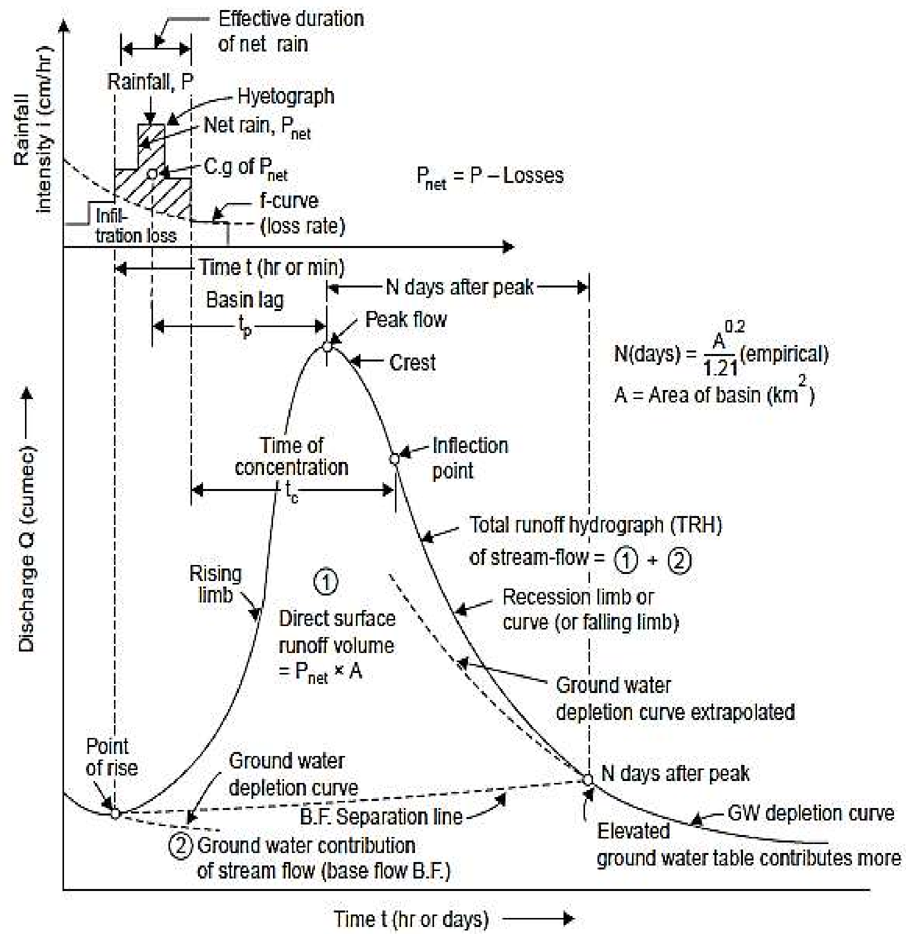


Figure No. 2 Components of streamflow hydrograph

Thus, in actual streams gauged, the hydrograph may have a single peak or multiple peaks according to the complexity of storms. For flood analysis and derivation of unit hydrograph, a single peaked hydrograph is preferred. A complex hydrograph, however, can be resolved into simple hydrographs by drawing hypothetical recession lines as shown in Fig.3.

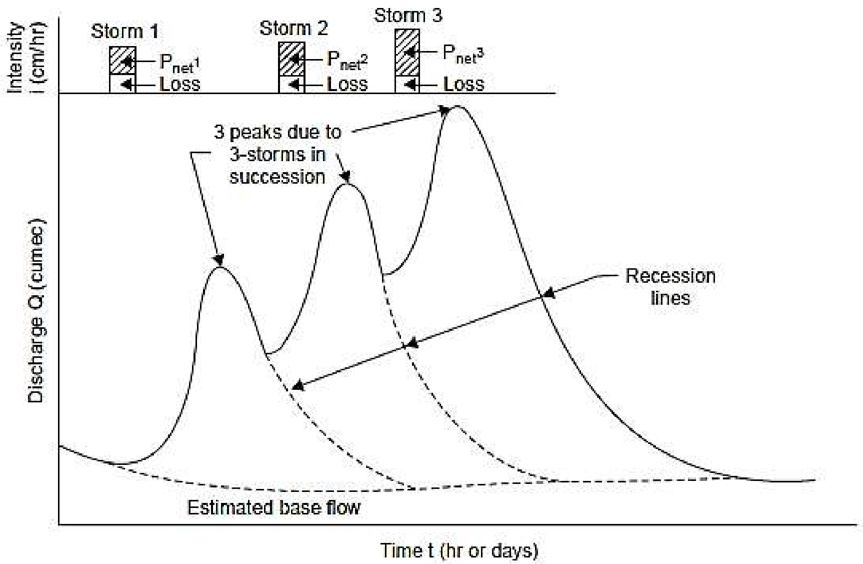


Figure No. 3 Hydrograph with multiple peaks

Factors Affecting Hydrograph:

Factors mainly affecting hydrograph are:

Storm and Metrological characteristics which affects the rising limb of hydrograph.

Basin characteristics which affects recession limb of hydrograph.

(i) Shape of basin, it affects the time of concentration. Thus, it affects the shape of the hydrograph and the location of peak point. Figure No.4.

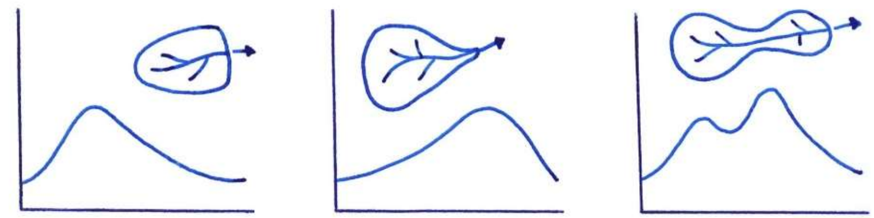
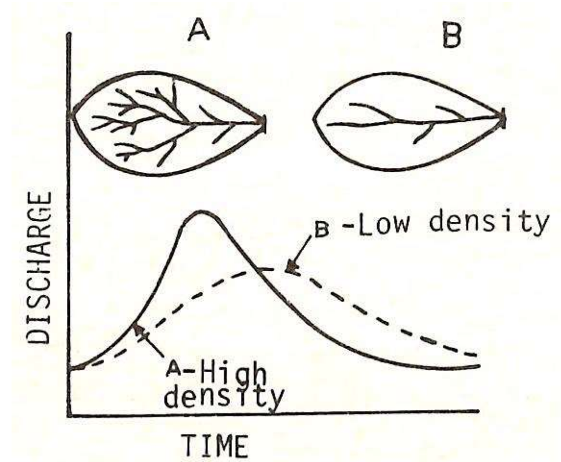


Figure No. 4 تأثير شكل الحابيه على الهيدروغراف

(ii) Slope of basin, it affects the Slope of the recession curve. Thus, it affects the time of base of hydrograph.

(iii) Drainage density, it affects the peak of the hydrograph. Figure No.5.



تأثير كثافة البزل على الهيدروغراف Figure No. 5

HYDROGRAPH SEPARATION

For the derivation of unit hydrograph, the base flow has to be separated from the total runoff hydrograph (i.e., from the hydrograph of the gauged stream flow). Some of the well-known base flow separation procedures are given below, Fig.6.

(i) Simply by drawing a line *AC* tangential to both the limbs at their lower portion.

This method is very simple but is approximate and can be used only for preliminary estimates.

(ii) Extending the recession curve existing prior to the occurrence of the storm up to the point *D* directly under the peak of the hydrograph and then drawing a straight line *DE*, where E is a point on the hydrograph *N* days after the peak, and *N* (in days) is given by

$N = 0.83\,A^{0.2}$

(iii) Simply by drawing a straight line *AE*, from the point of rise to the point *E*, on the hydrograph, *N* days after the peak.

(iv) Construct a line *AFG* by projecting backwards the ground water recession curve after the storm, to a point *F* directly under the inflection point of the falling limb and sketch an arbitrary rising line from the point of rise of the hydrograph to connect with the projected base flow recession. This type of separation is preferred where the ground water storage is relatively large and reaches the stream fairly rapidly, as in lime-stone terrains.

Many times a straight line *AE* meets the requirements for practical purposes. Location of the point *E* is where the slope of the recession curve changes abruptly, and as a rough guide *E* is *N* days after the peak.

.

In all the above four separation procedures, the area below the line constructed represents the base flow, i.e., the ground water contribution to stream flow. Any further refinement in the base flow separation procedure may not be needed, since the base flow forms a very insignificant part of high floods. In fact, very often, a constant value of base flow is assumed

After separation of base flow, the result hydrograph is the *direct runoff hydrograph* or direct flow hydrograph (DRH or DFH). The area under DRH represents the direct surface runoff, and by dividing it by the catchment area we get the *excess rain* or *effective rain*.

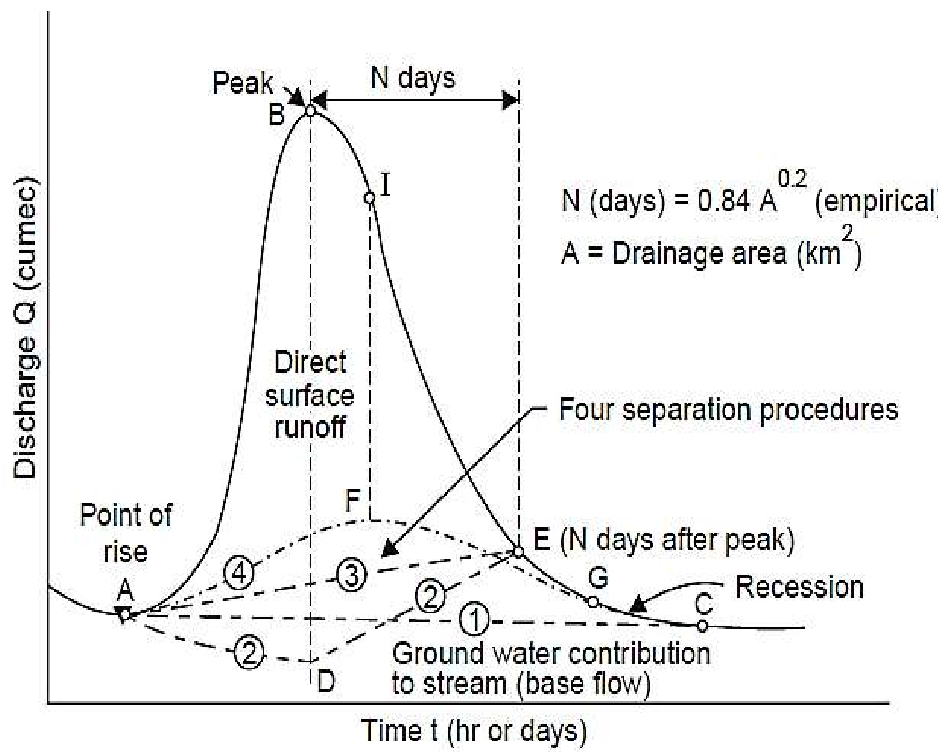


Figure No. 6 Hydrograph separation

Example:

The hydrograph tabulated below was observed for a river draining a 103.6km^2 catchment, following a storm lasting 3hr. Separate base flow from runoff and calculate total runoff volume. What was the net rainfall in mm/hr?. If the total rainfall was 20cm, find the ϕ index for the storm.

Time (hr)	Flow (m^3/sec)	Time (hr)	Flow (m^3/sec)	Time (hr)	Flow (m^3/sec)
0	12.7	24	99.1	48	30.3
3	155.7	27	85.0	51	26.9
6	254.9	30	73.6	54	23.8
9	212.4	33	62.6	57	21.2
12	184.1	36	53.6	60	18.7
15	158.6	39	45.9	63	16.7
18	135.9	42	39.6	66	15.3
21	116.1	45	34.5	---	---

<u>Solution</u>:

By plotting time versus discharge on natural paper we get the total hydrograph, from which we found the base flow as shown.

N days =0.83 A$^{0.2}$ =0.83 (103.6)$^{0.2}$= 2.1 days ≃ 51 hr.

Base Flow Increment = 3((21.2-12.7)/57) = 0.45 m^3/sec each 3hrs.

DRH ordinates = TH ordinates – Base Flow

The result tabulated in the following Table.

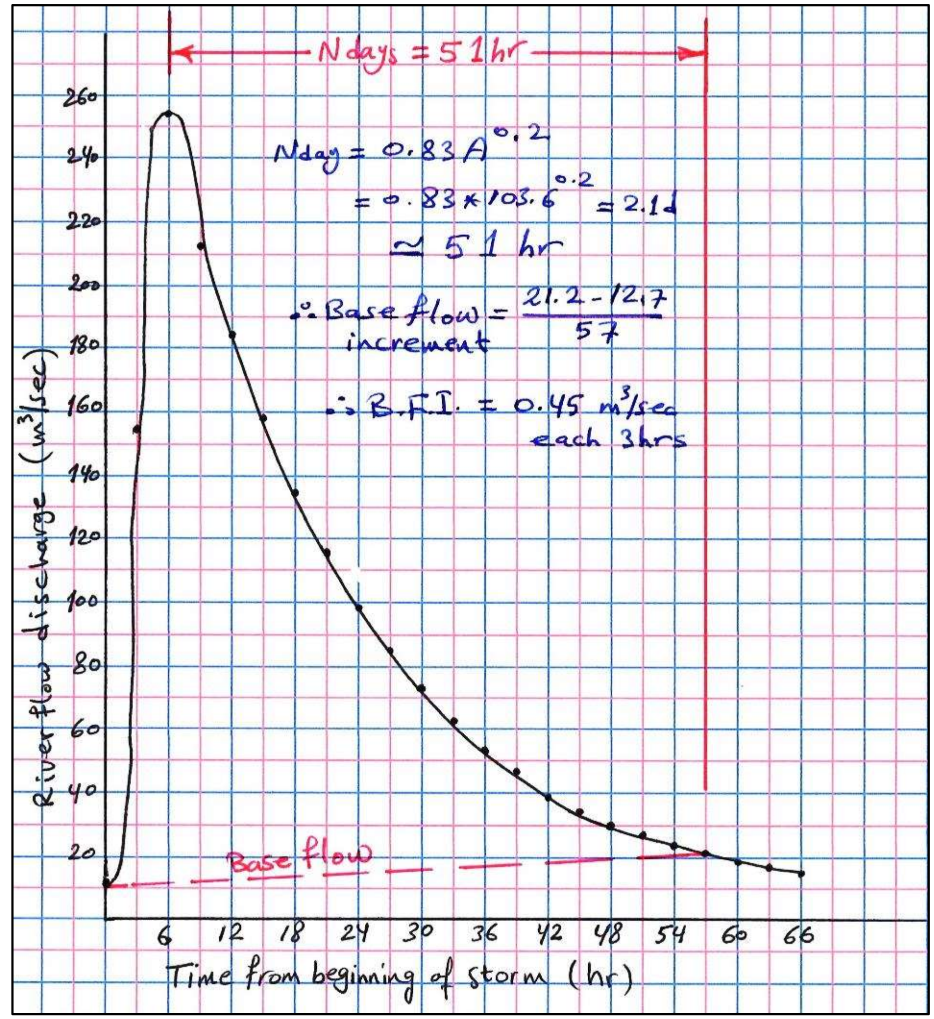


Figure No. 7

Hr	Q	BF	DRH	Hr	Q	BF	DRH
0	12.7	12.7	0	36	53.6	18.10	35.5
3	155.7	13.15	142.55	39	45.9	18.55	27.35
6	254.9	13.60	241.3	42	39.6	19.00	20.6
9	212.4	14.05	198.35	45	34.5	19.45	15.05
12	184.1	14.50	169.6	48	30.3	19.90	10.4
15	158.6	14.95	143.65	51	26.9	20.35	6.55
18	135.9	15.40	120.5	54	23.8	20.80	3
21	116.1	15.85	100.25	57	21.2	21.2	0
24	99.1	16.30	82.8	60	18.7	--	--
27	85.0	16.75	68.25	63	16.7	--	--
30	73.6	17.20	56.4	66	15.3	--	--
33	62.6	17.65	44.95	--	--	--	--

Now we can plot the direct runoff hydrograph as in figure No.8.

Total Surface Runoff = Area Under D.R.H.

$$TSR = \left(\frac{3\times 60\times 60}{2}\right)(0 + 2(142.55 + 241.3 + 198.35 + 169.6 + 143.65 + 120.5 + 100.25 + 82.8 + 68.25 + 56.4 + 44.95 + 35.5 + 27.35 + 20.6 + 15.05 + 10.4 + 6.55 + 3) + 0) = \frac{2\times 1487.05\times 3\times 3600}{2} = 16059600\ m^3 = 16.06 Mm^3$$

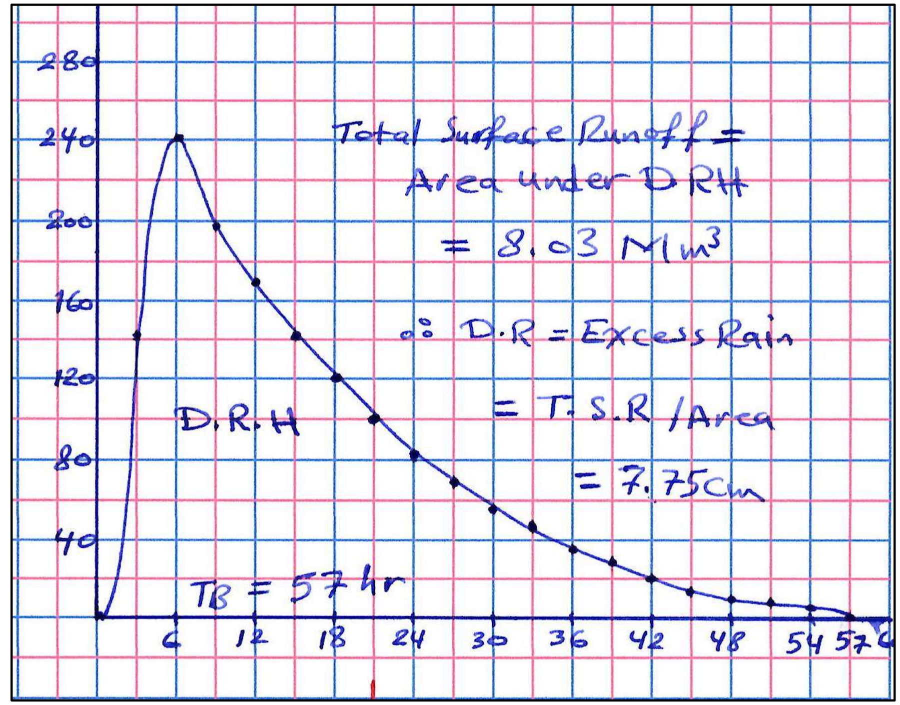


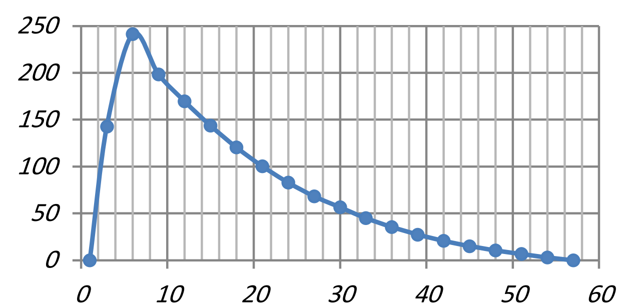
Figure No. 8

∴ Net Rainfall (Excess Rain) = T.S.R. / Area of Catchment

∴Net Rain= 16059600 / (103.6×1000×1000)= 15.5 cm

Or; Net Rain= 16.06/103.6 = 15.5 cm = 15.5/3= 52 mm/hr

⇨ ϕindex = $(P - R)/t_R$ = (20-15.5)/3 = 1.5 cm/hr



UNIT HYDROGRAPH

The unit hydrograph (unit-graph) is defined as the hydrograph of storm runoff resulting from an isolated rainfall of some unit duration occurring uniformly over the entire area of the catchment, produces a unit volume (i.e., 1 cm) of runoff.

Derivation of the unit hydrograph: The following steps are adopted to derive a unit hydrograph from an observed flood hydrograph (Fig. No.9).

(i) Select from the records isolated (single-peaked) intense storms, which occurring uniformly over the catchment have produced flood hydrographs with appreciable runoff.

(iii) Separate the base flow from the total runoff.

(iv) From the ordinates of the total runoff hydrograph (at regular time intervals) deduct the corresponding ordinates of base flow, to obtain the ordinates of direct runoff.

(iv) Divide the volume of direct runoff by the area of the drainage basin to obtain the net precipitation depth over the basin.

(v) Divide each of the ordinates of direct runoff by the net precipitation depth to obtain the ordinates of the unit hydrograph.

(vi) Plot the ordinates of the unit hydrograph against time since the beginning of direct runoff. This will give the unit hydrograph for the basin, for the duration of the unit storm (producing the flood hydrograph) selected in item (i) above.

In unit hydrograph derivation, such storms should be selected for which reliable rainfall and runoff data are available. The net rain graph (hyetograph of excess rain) should be determined by deducting the storm loss and adjusting such that the total volume of net storm rain is equal to the total volume of direct surface runoff. The unit hydrograph derived, which, when applied to the known net rain data, should yield the known direct runoff hydrograph.

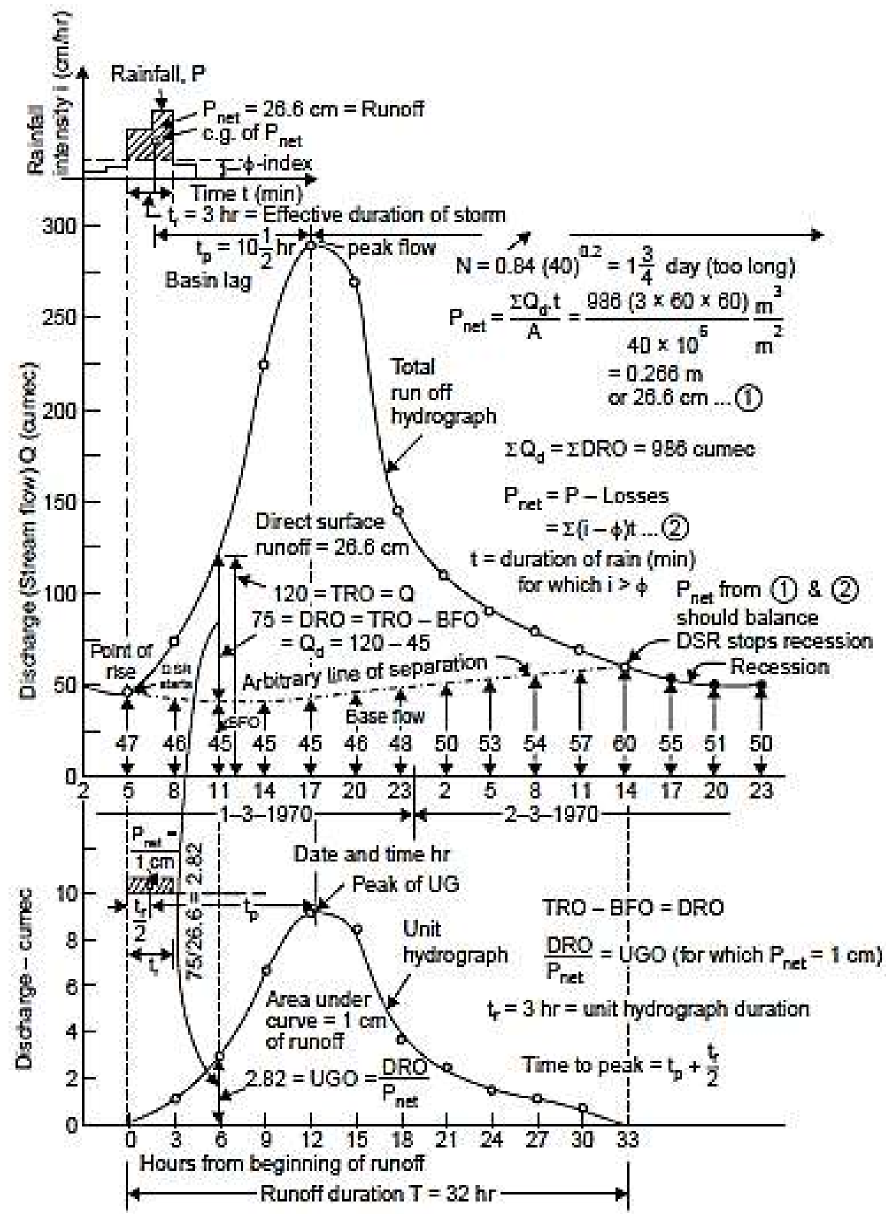


Figure No. 9

Derivation of a unit hydrograph (Example 5.1)

Example:

Data below represent the discharge results from a storm of 6-hr duration falls on a catchment of 500km². Assume base flow equal to zero, drive the 6-hr unit hydrograph.

Time (hr)	Q m³/sec	Time (hr)	Q m³/sec
0	0	42	50
6	100	48	35
12	250	54	25
18	200	60	15
24	150	66	5
30	100	72	0
36	70	--	---

Solution:

للحصول على الهيدروغراف القياسي نتبع الخطوات التالية:

١- نقوم بفصل الجريان القاعدي عن السيح السطحي للحصول على احداثيات هيدروغراف الجريان السطحي DRH(اي طرح الجريان القاعدي من احداثيات هيدروغراف الجريان الكلي) وفي مثالنا هذا لا يوجد جريان قاعدي.

٢- نقوم بحساب السيح السطحي المكافي (المطر الفعال) والذي يمثل المساحة تحت هيدروغراف السيح السطحي DRH.

٣- احداثيات الهيروغراف القياسي UH هي عباره عن احداثيات السيح المباشر مقسومه على كمية المطر الفعال.

الجدول والشكل اللاحقين هما جدول الحسابات للحل وشكل هيدروغراف الجريان المباشر والهيدروغراف القياسي ونلاحظ ما يلي:

- يلاحظ بان الزمن القاعدي (TB) لـ UH هو نفسة زمن القاعده لـ DRH.
- الاستدامة المطرية لـ UH هي نفس الاستدامة للهيدروغراف الكلي TH للعاصفة المطرية التي اشتق منها الهيدروغراف القياسي.

Time (hr)	Q m³/sec	UH Ordinate m³/sec	Time (hr)	Q m³/sec	UH Ordinate m³/sec
0	0	0	42	50	**11.6**
6	100	23.1	48	35	**8.1**
12	250	57.9	54	25	**5.8**
18	200	46.3	60	15	**3.5**
24	150	34.7	66	5	**1.2**
30	100	23.1	72	0	**0**
36	70	16.2	--	---	**---**
			Σ	1000	**----**

Total Runoff=1000*6*60*60 = 21600000 m³ ⇨⇨
Equivalent Depth of Runoff= Excess Rain =
21600000/(500*1000*1000) = 0.0432 m = 4.32 cm

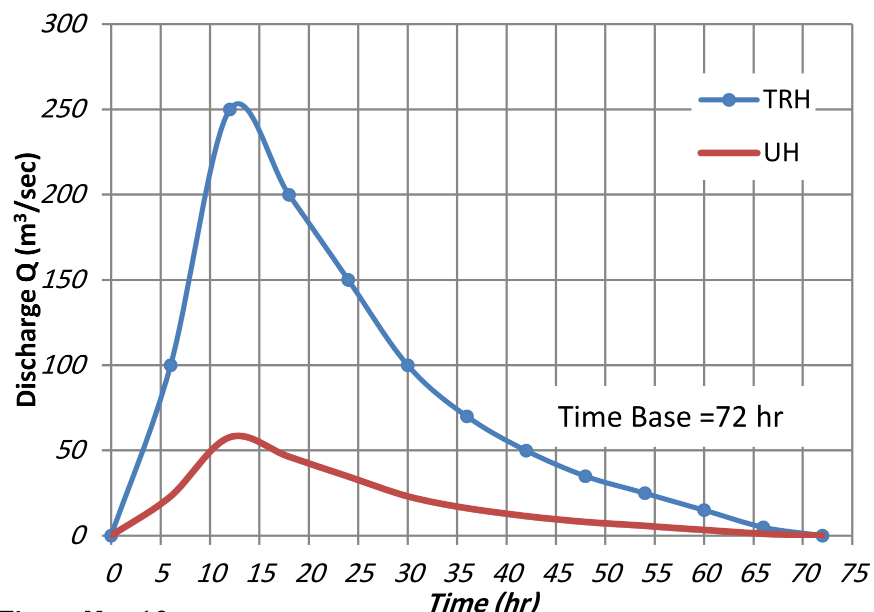


Figure No. 10

هذا عندما يكون المطلوب هو انشاء UH ولكن بعد انشاء الـــ UH للعاصفه الممطره لجابية معينه فاننا يمكن ان نستفيد منه في حساب حجم السيح السطحي للامطار المستقبلية الساقطه على نفس الجابية وذلك بضرب المطر الفعال (المؤثر) باحداثيات الهيدروغراف القياسي للحصول على احداثيات الـــ DRH) والذي تمثل احداثياته تصريف النهر بعد اضافة الجريان القاعدي اليها) والمساحه تحته تمثل حجم السيح السطحي الكلي.

Example:

Peak flow of total hydrograph due to 6-hr storm was 470 m^3/sec and average depth of rain=8cm. Assume average infiltration losses =0.25 cm/hr and base flow is constant and equal to 15 m^3/sec. Estimate the peak flow of a 6-hr unit-graph.

Solution:

Direct flow = total flow – base flow = 470-15 = 455 m^3/sec

Infiltration losses = average losses * duration = 0.25*6 = 1.5 cm

∴ Effective Rain = average depth of rain – infiltration losses

= 8-1.5 = 6.5 cm

⇨ Peak flow of UH = peak of DRH / ER = 455/6.5 = 70 m^3/sec.

Example:

For the following 6-hr unit hydrograph, find the total flow hydrograph due to a 3.5 cm excess rain. Suppose base flow increases linearly from 60 m^3/sec at starting of hydrograph to 126m^3/sec at the end.

Tim110e 60(hr)	UH (Q) m^3/sec	Time (hr)	UH (Q) m^3/sec
0	0	30	110
3	25	36	60
6	50	42	36
9	85	48	25
12	125	54	16
15	160	60	8
18	185	66	0
24	160	---	---

Solution:

Time	UH	BF	DRH	TH
0	0	60	0	60
3	25	63	88	151
6	50	66	175	241
9	85	69	298	367
12	125	72	438	510
15	160	75	560	635
18	185	78	648	726
24	160	84	560	644
30	110	90	385	475
36	60	96	210	306
42	36	102	126	228
48	25	108	88	196
54	16	114	56	170
60	8	120	28	148
66	0	126	0	126

في الجدول السابق تم الحصول على الاحداثيات العمودية لـ DRH بضرب المطر المؤثر والذي مقداره (3.5 cm) باحداثيات الهيدروغراف القياسي. ثم تم الحصول على الاحداثيات العمودية لهيدروغراف النهر او الهيدروغراف الكلي Total Hydrograph باضافة مقدار الجريان القاعدي الى احداثيات هيروغراف الجريان السطحي المباشر Direct Runoff Hydrograph .

والشكل التالي يمثل حل المثال السابق.

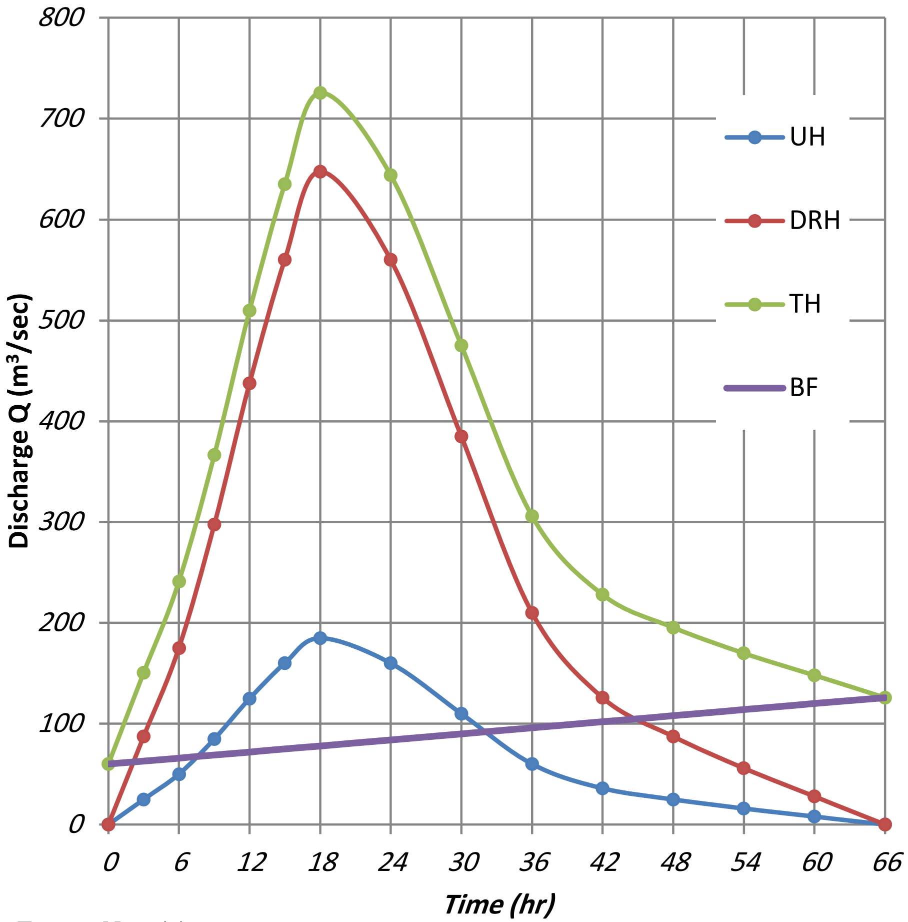


Figure No. 11

Example:

For the same 6-hr unit hydrograph in previous example, if tow storms (6-hr duration) occur successively, the first have an excess rain of 3cm and the second of 2cm. Find and draw the resulted direct runoff hydrograph (direct flow hydrograph, DRH or DFH) from these storms.

Solution:

The solution tabulated in table below.

Time	UH	DFH 1	DFH 2	TDFH
0	0	0	---	0
6	50	150	0	150
12	125	375	100	475
18	185	555	250	805
24	160	480	370	850
30	110	330	320	650
36	60	180	220	400
42	36	108	120	228
48	25	75	72	147
54	16	48	50	98
60	8	24	32	56
66	0	0	16	16
72	0	0	0	0

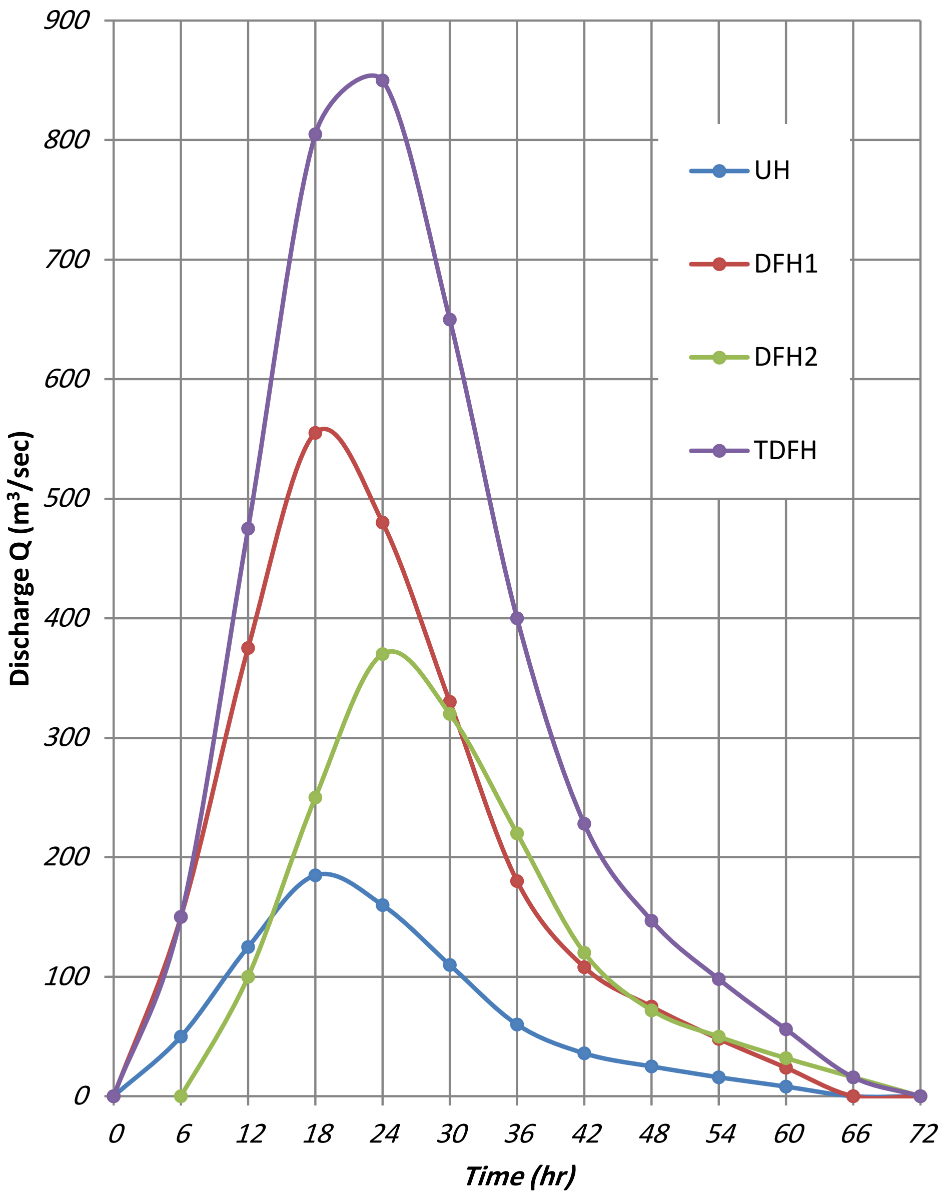


Figure No. 12

Example:

The vertical ordinates of a 6-hr U-H is

Time hr	**0**	**6**	**12**	**18**	**24**	**30**
U-H m³/s	0	250	600	800	700	600
Time hr	36	42	48	54	60	66
U-H m³/s	450	320	200	100	50	0

A storm of three successive intervals of 6-h duration was occurs. Assume it's total rain is 3 , 5 , and 4 cm respectively, and ϕ-index=0.2 cm/hr and base flow is constant and equals 150m³/sec. determine and draw the total hydrograph for this storm.

Solution:

The effective rain for:

1st interval = 3 – (0.2×6) = 1.8 cm

2nd interval = 5 – (0.2×6) = 3.8 cm

3rd interval = 4 – (0.2×6) = 2.8 cm

Ordinates of DRH = Ordinates of UH × Effective rain of interval

Ordinates of TRH = Sum of ordinates of DR-Hydrographs

Ordinates of T-H = Ordinates of TRH + Base flow

The solution was tabulated in table below.

1	2	3	4	5	6	7	8
Time hr	UH m^3/s	DF int1	DF int2	DF int3	TDF	BF	T-H m^3/s
0	0	0	-	-	0	150	250
6	250	450	0	-	450	150	700
12	600	1080	950	0	2030	150	2280
18	800	1440	2280	700	4420	150	4670
24	700	1260	3040	1680	5980	150	6230
30	600	1080	2660	2240	5980	150	6230
36	450	810	2280	1960	5050	150	5300
42	320	576	1710	1680	3966	150	4216
48	200	360	1216	1260	2836	150	3086
54	100	180	760	896	1836	150	2086
60	50	90	380	560	1030	150	1280
66	0	0	190	280	470	150	720
72			0	140	140	150	390
78				0	0	150	250

The following drawing is the total hydrograph.

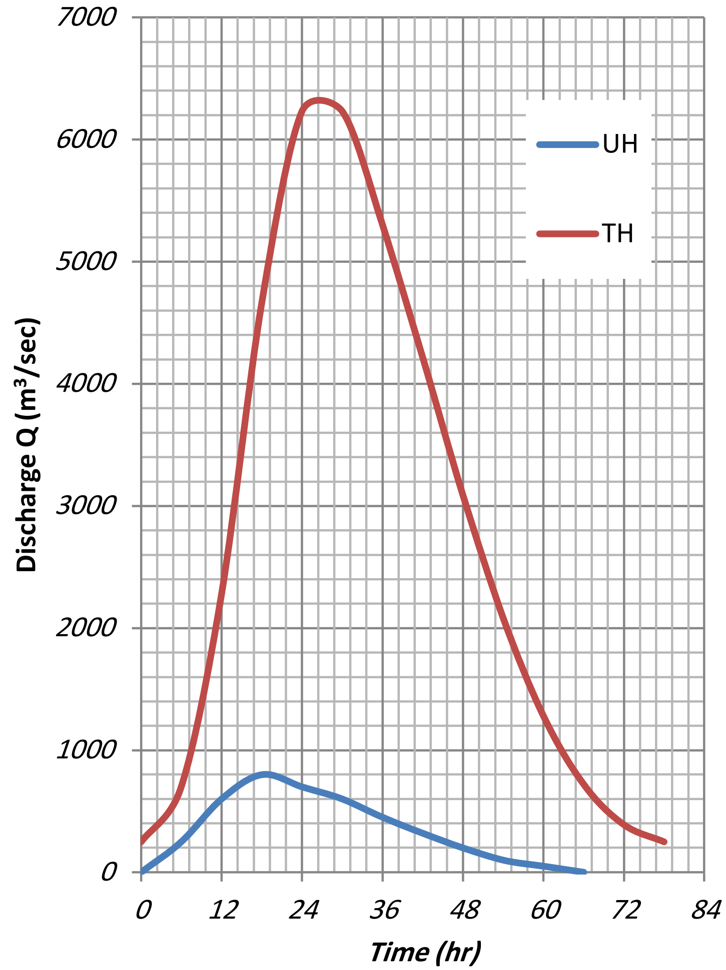


Figure No. 13

The Average Unit-Hydrograph

A unit hydrograph derived from a single storm may not be representative, and it is therefore desirable to average unit hydrographs from several storms of about the same duration. This should not be an arithmetic average of superimposed ordinates. The proper procedure is to compute average peak flow and time to peak. The average unit hydrograph is then sketched to conform to the shape of other graphs, passing through the computed average peak, and having the required unit volume.

Example:

Given below are three unit hydrographs derived from separate storms on a small catchment, all are considered to have resulted from 3hr rains. Derive the average unit hydrograph for this catchment.

Time hr	UH1 ft³/s	UH2 ft³/s	UH3 ft³/s	Time hr	UH1 ft³/s	UH2 ft³/s	UH3 ft³/s
0	0	0	0	8	195	255	322
1	165	37	25	9	143	195	248
2	547	187	87	10	97	135	183
3	750	537	260	11	60	90	135
4	585	697	505	12	33	52	90
5	465	608	660	13	15	30	53
6	352	457	600	14	7	12	24
7	262	330	427	15	0	0	0

Solution:

The peaks of unit hydrographs are 750 , 697, and 660 resp.

∴ Average peak flow = (750+697+660) / 3 = 702 ft^3/s

Time of average peak = (3+4+5) / 3 = 4 hrs

Now we can plot the hydrographs and the average hydrograph.

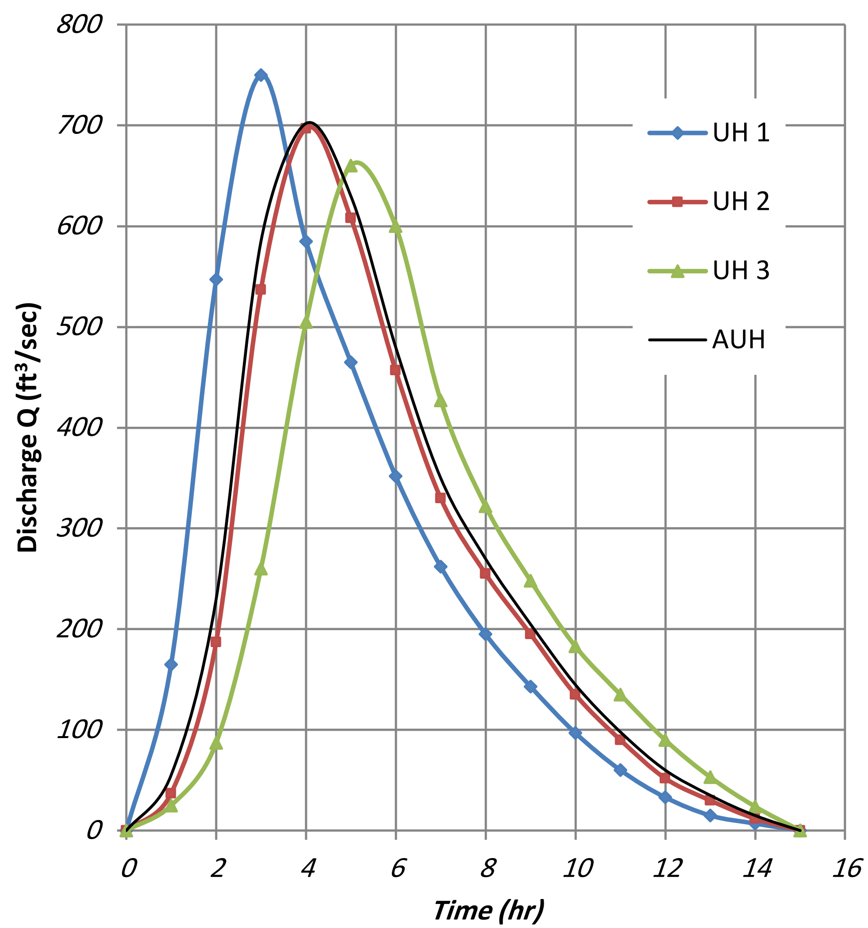


Figure No. 14

The Unit-Hydrograph from Complex or Multi-period Storms:

When suitable simple isolated storms are not available, data from complex storms of long duration will have to be used in unit hydrograph derivation. The problem is to decompose a measured composite flood hydrograph into its component *DRHs* and base flow. A common unit hydrograph of appropriate duration is assumed to exist. Consider a rainfall excess made up of three consecutive durations of D-hr and Equivalent-Rain values *of* i_1, i_2, *and* i_3. Figure 15 shows the *ERH*. By base flow separation of the resulting composite flood hydrograph, a composite DRH is obtained (fig. 15). Let the ordinates of the composite DRH be drawn at a time interval of D-hr. At various time interval *1D, 2D, 3D*,… from the start of the *ERH*, let the ordinates of the unit hydrograph be u_1, u_2, u_3, … and the ordinates of the composite DRH be Q_1, Q_2, Q_3, ….,

Then:

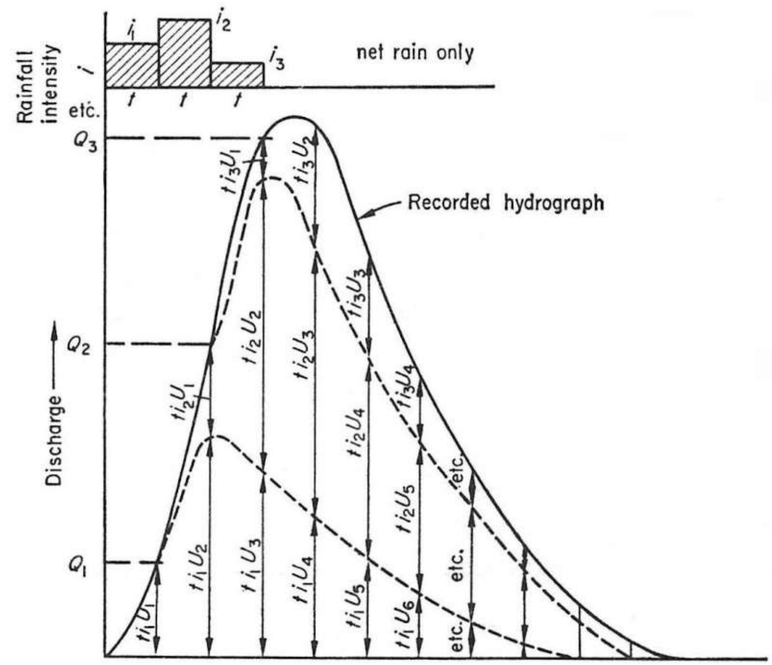
$$Q_1 = i_1 u_1$$

$$Q_2 = i_1 u_2 + i_2 u_1$$

$$Q_3 = i_1 u_3 + i_2 u_2 + i_3 u_1$$

$$Q_4 = i_1 u_4 + i_2 u_3 + i_3 u_2 \qquad \text{……………. and so on}$$

From eq. above the values of u_1, u_2, u_3, … can be determined.



Derivation of unit hydrograph from a multi-period storm hydrograph

Figure No. 15

Example:

The hydrograph tabulated below resulted from three consecutive 6-hr periods of rainfall having runoffs estimated as 0.6, 1.2, 0.9 cm respectively. Find the 6-hr unit hydrograph for the basin. Drainage area was 58 sq. km. Base flow have been subtracted.

Time, hr	0	3	6	9	12	15	18
Flow m³/s	0	75	280	283	662	432	645
Time, hr	21	24	27	30	33	36	
Flow m³/s	314	195	93	31	9	0	

Solution:

Time, hr	Flow m^3/s		
0	0		
3	75		
6	280	←	Q_1
9	283		
12	662	←	Q_2
15	432		
18	645	←	Q_3
21	314		
24	195	←	Q_4
27	93		
30	31	←	Q_5
33	9		
36	0		

Then:

$Q_1 = i_1 u_1 \Rightarrow 280 = 0.6 \times u_1 \Rightarrow u_1 = 467\ m^3/sec.$

$Q_2 = i_1 u_2 + i_2 u_1 \Rightarrow 662 = 0.6 \times u_2 + 1.2 \times u_1$

$\Rightarrow 662 = 0.6\, u_2 + 1.2 \times 467 \Rightarrow u_2 = 170\ m^3/sec.$

$Q_3 = i_1 u_3 + i_2 u_2 + i_3 u_1 \Rightarrow 645 = 0.6\, u_3 + 1.2\, u_2 + 0.9\, u_1$

$\Rightarrow 645 = 0.6\, u_3 + 1.2 \times 170 + 0.9 \times 467 \Rightarrow u_3 = 35\ m^3/sec.$

$Q_4 = i_1 u_4 + i_2 u_3 + i_3 u_2 = 195 \Rightarrow u_4 \simeq 0$

Now, we can establish the table of the unit hydrograph as follows:

Time, hr	0	6	12	18	24
Flow m^3/s	0	467	170	35	0

The Unit-Hydrograph can be plotted as in figure 16

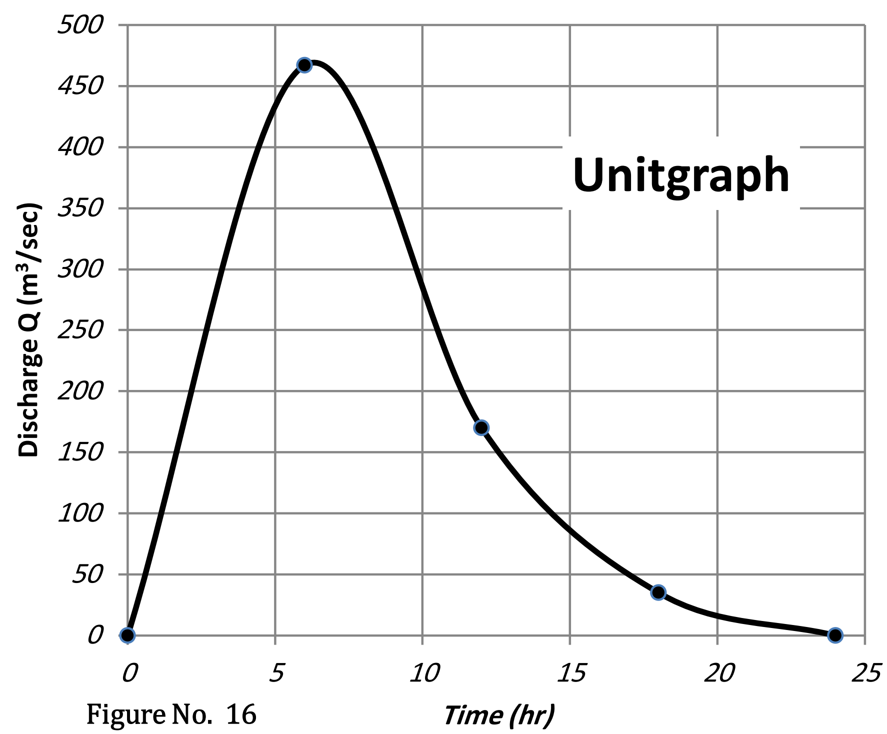


Figure No. 16

The Conversion of U-H Duration:

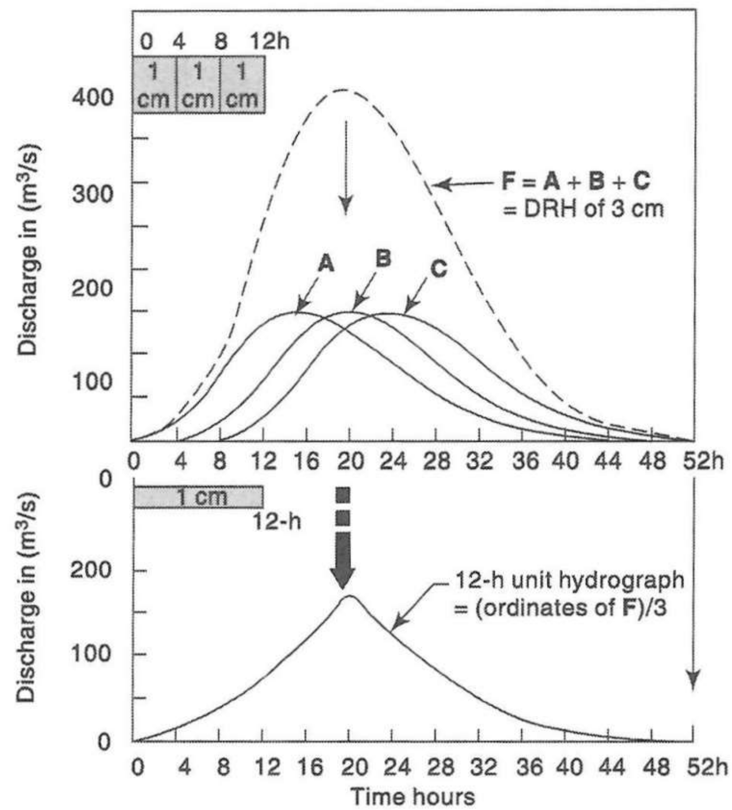
Ideally, unit hydrographs are derived from simple isolated storms and if the duration of the various storms do not differ very much they would all be grouped under one average duration of *D*-hr. If in practical applications unit hydrographs of different durations are needed they are best derived from field data. Lack of adequate data normally prevents development of unitgraphs covering a wide range of durations for a given catchment. Under such conditions a *D*-hr unitgraph is used to develop unitgraphs of differing durations *nD*-hr.

Two methods are available for this purpose.

- Method of superposition
- The S-curve

Method of Superposition

If a *D*-hr unit hydrograph is available, and it is desired to develop a unitgraph of *nD*-hr, where n is an integer, it is easily accomplished by superposing *n* unit hydrographs with each graph separated from the previous on by *D*-hr. Figure 17 shows three 4-hr unitgraphs A, B, and C. curve B begins 4 hr after A and C begins 4 hr after B. Thus the combination of these three curves is a *DRH* of 3 cm due to an *ER* of 12-hr duration. If the ordinates of this *DRH* are now divided by 3, one obtains a 12-hr unit hydrograph. The calculations are easy if performed in a tabular form.



Construction a 12-hr unitgraph from a 4-hr unitgraph

Figure No. 17

Example:

Table below is a 3-hr unitgraph, derive a 6-hr unitgraph by superposition method.

Time, hr	0	1	2	3	4	5	6	7
Flow m^3/s	0	1	5.3	15.2	19.7	17.2	12.9	9.3
Time, hr	8	9	10	11	12	13	14	15
Flow m^3/s	7.2	5.5	3.5	2.5	1.5	0.8	0.3	0

Solution:

نبدأ برسم الهيدروغراف القياسي المعلوم ذو الاستدامة 3-hr من الزمن (0) ثم نرسم نفس الهيدروغراف القياسي ولكن من الزمن (3-hr) بحيث يكون مقدار التزحيف مساوياً الى الاستدامة العلومة. ثم يتم جمع الاحداثيات العمودية للهيروغرافين المرسومين لنحصل على هيدروغراف السيح المباشر ذو الاستدامة (6-hr) وشدة مطرية مقدارها (2×1cm=2cm).

وبقسمة احداثيات هيروغراف السيح المباشر على (2) نحصل على احداثيات الهيدروغراف القياسي المطلوب (6-hr Unit Hydrograph).

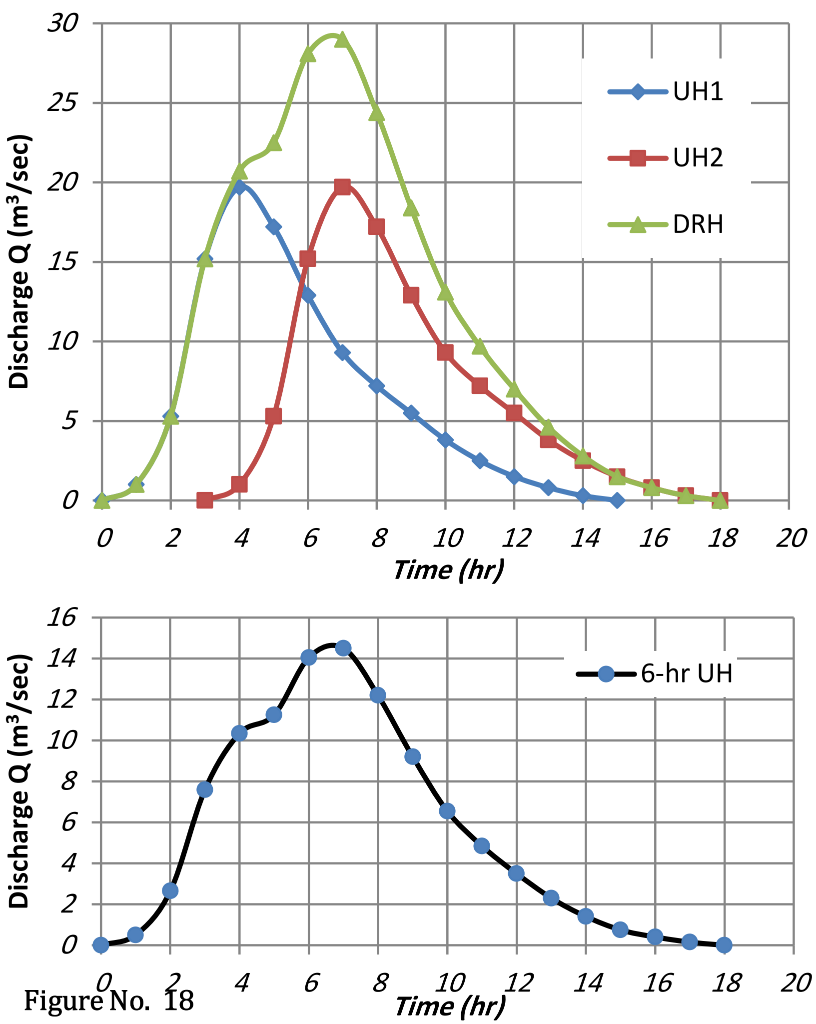


Figure No. 18

Or, it can be solved in table as in below.

1	2	3	4=2+3	5=4/2
Time hr	Q m^3/s	lagged UH	DRH	6hr UH
0	0	--	0	0
1	1	--	1	0.5
2	5.3	--	5.3	2.65
3	15.2	0	15.2	7.6
4	19.7	1	20.7	10.35
5	17.2	5.3	22.5	11.25
6	12.9	15.2	28.1	14.05
7	9.3	19.7	29	14.5
8	7.2	17.2	24.4	12.2
9	5.5	12.9	18.4	9.2
10	3.8	9.3	13.1	6.55
11	2.5	7.2	9.7	4.85
12	1.5	5.5	7	3.5
13	0.8	3.8	4.6	2.3
14	0.3	2.5	2.8	1.4
15	0	1.5	1.5	0.75
16	--	0.8	0.8	0.4
17	--	0.3	0.3	0.15
18	--	0	0	0

The S-Curve:

If it is desired to develop a unit hydrograph of duration *mD*, where *m* is a fraction, the method of superposition cannot be used. A different technique known as the S-curve method is adopted in such cases.

The S-curve, also called S-hydrograph is a hydrograph produced by a continuous effective rainfall at a constant rate for an infinite period. It is a curve obtained by summation of an infinite series of *D*-hr unit hydrographs spaced *D*-hr apart. Figure 19 shows the construction and the use of S-curve in developing unit hydrograph.

A check for the S-curve must be is:

$$Q_e = \frac{2.78\,A}{D}$$

Where: Q_e is the equilibrium (constant) discharge (m^3/sec)

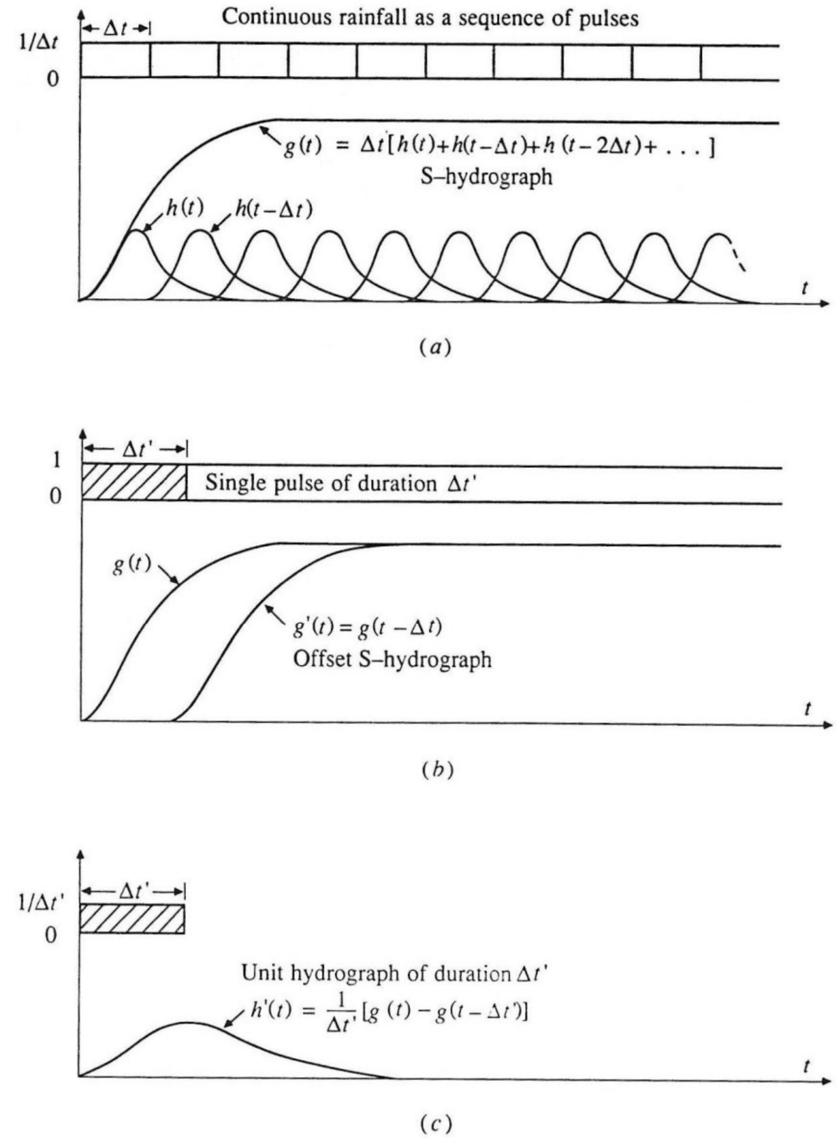
A is the catchment area (km)

D is the duration (hr)

Example:

Given the 4-hr unit hydrograph listed in table. Derive the 3-hr unit hydrograph. The catchment area is 300 sq. km.

Time, hr	0	1	2	3	4	5	6	7
Flow m^3/s	0	6	36	66	91	106	93	79
Time, hr	8	9	10	11	12	13	14	15
Flow m^3/s	68	58	49	41	34	27	23	17
Time, hr	16	17	18	19	20	21		
Flow m^3/s	13	9	6	3	1.5	0		



Using the S-hydrograph to find a unit hydrograph of duration Δt′ from a unit hydrogra Δt.

Figure No. 19

Solution:

The solution is tabulated in table below.

1	2	3	4	5	6	7	8	9	10	11
T hr	U-H	Lag UH	Lag UH	Lag UH	Lag UH	Lag UH	S - curve	Lag S-c	10= 8-9	3-hr UH
0	0						0		0	0
1	6						6		6	8
2	36						36		36	48
3	66						66	0	66	88
4	91	0					91	6	85	113
5	106	6					112	36	76	101
6	93	36					129	66	63	84
7	79	66					145	91	54	72
8	68	91	0				159	112	47	63
9	58	106	6				170	129	41	55
10	49	93	36				178	145	33	44
11	41	79	66				186	159	27	36
12	34	68	91	0			193	170	23	31
13	27	58	106	6			197	178	19	25
14	23	49	93	36			201	186	15	20
15	17	41	79	66			203	193	10	13
16	13	34	68	91	0		206	197	9	12
17	9	27	58	106	6		206	201	5	7
18	6	23	49	93	36		207	203	4	5
19	3	17	41	79	66		206	206	0	0
20	1.5	13	34	68	91	0	207	206	1	-
21	0	9	27	58	106	6	206	207	-1	-

Figure 20 represents the 4-hr S-curve and the 3-hr unit hydrograph.

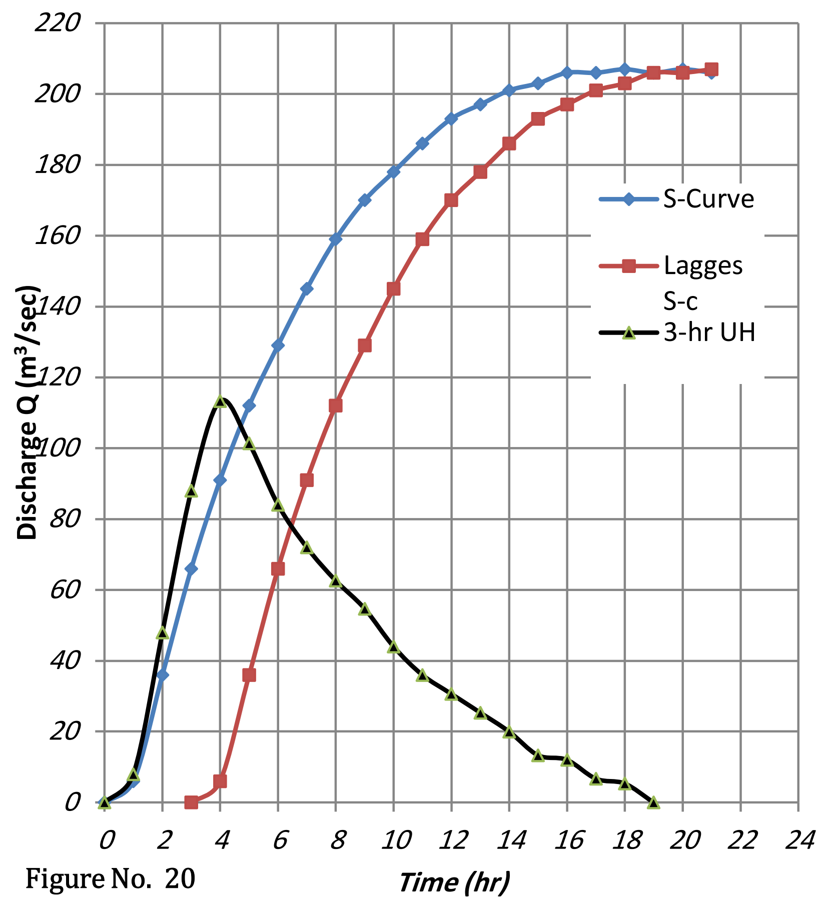


Figure No. 20

Another method for determining the S-curve is shown in table below.

1	2	3	4	5	6	7
Time hr	U-H	S-c addition	S-curve 4=2+3	Lagged S-c	(4-5)	3-hr UH
0	0		0		0	0
1	6		6		6	8
2	36		36		36	48
3	66		66	0	66	88
4	91	0	91	6	85	113
5	106	6	112	36	76	101
6	93	36	129	66	63	84
7	79	66	145	91	54	72
8	68	91	159	112	47	63
9	58	112	170	129	41	55
10	49	129	178	145	33	44
11	41	145	186	159	27	36
12	34	159	193	170	23	31
13	27	170	197	178	19	25
14	23	178	201	186	15	20
15	17	186	203	193	10	13
16	13	193	206	197	9	12
17	9	197	206	201	5	7
18	6	201	207	203	4	5
19	3	203	206	206	0	0
20	1.5	206	207	206	1	-
21	0	206	206	207	-1	-