Differentiation Rules

1. **If** $y = k$, **where** k **is constant**, $y' = 0$
2. **If** $y = k.f(x)$, **then** $y' = k.f'(x)$.
3. **If** $f(x) = x^n$, (n **positive integer**) **then for every real value of** x, $f'(x) = n\,x^{n-1}$.
4. **If** $y = f(x) \mp g(x)$, **then** $y' = f'(x) \mp g'(x)$.
5. **If** $y = f(x) \cdot g(x)$, **then** $y' = f(x) * g'(x) + g(x) * f'(x)$.
6. **If** $y = \dfrac{f(x)}{g(x)}$, **where** $(x) \neq 0$ **for every** x , **then** $y' = \dfrac{g(x)*f'(x) - f(x)*g'(x)}{(g(x))^2}$.
7. **If** $y = (f(x))^n$, (**Any real number** n) **then** $y' = n\,(f(x))^{n-1} * f'(x)$.

<u>*Derivative of higher orders:-*</u>

Frist order derivative of $y = f(x) \rightarrow y'$, $\dfrac{dy}{dx}, f'(x)$.

Second order derivative of $y = f(x) \rightarrow y''$, $\dfrac{d^2y}{dx^2}$, $f''(x)$.

Third order derivative of $y = f(x) \rightarrow y'''$, $\dfrac{d^3y}{dx^3}$, $f'''(x)$.

$\vdots$
$\vdots$

The n_{th} *order derivative of* $y = f(x) \rightarrow y^{(n)}, \dfrac{d^n y}{dx^n}, f^{(n)}(x)$

Examples:- Find derivative of following functions:

1. $y = x + \dfrac{1}{x^2} + 3 \longrightarrow y' = 1 - \dfrac{2}{x^3}$

2. $y = \sqrt{x^3 - 2} + \dfrac{1}{\sqrt{x+1}} \longrightarrow y' = \dfrac{1}{2}(x^3 - 2)^{-\frac{1}{2}}.3x^2 - \dfrac{1}{2}(x+1)^{-\frac{3}{2}}.1$

$$= \frac{3}{2}\frac{x^2}{\sqrt{x^3-2}} - \frac{1}{2}\frac{1}{\sqrt{(x+1)^3}}$$

3. $y = (x^2 + x + 1)^3$

$\longrightarrow y' = 3(x^2 + x + 1)^2 * (2x + 1)$

4. $y = \left(\frac{x+1}{x-1}\right)^{2/3}$

$\longrightarrow y' = \frac{2}{3} \cdot \left(\frac{x+1}{x-1}\right)^{-1/3} \cdot \frac{(x-1)-(x+1).(1)}{(x-1)^2} = \frac{4(x+1)^{-1/3}}{3(x-1)^{5/3}}$

5. $y = \frac{x^2}{2x-1}$

$\longrightarrow y' = \frac{(2x-1)*(2x)-x^2*2}{(2x-1)^2}$

$= \frac{4x^2-2x-2x^2}{(2x-1)^2} = \frac{2x^2-2x}{(2x-1)^2} = \frac{2x(x-1)}{(2x-1)^2}$

6. $y = \sqrt{x+3} \to y = (x+3)^{\frac{1}{2}}$

$\longrightarrow y' = \frac{1}{2}(x+3)^{\frac{-1}{2}} * 1 = \frac{1}{2\sqrt{x+3}}$

7. $y = \frac{1}{(x^2+3)^2} \to y = (x^2+3)^{-2}$

$\longrightarrow y' = -2(x^2+3)^{-3} * (2x)$

$= \frac{-4x}{(x^2+3)^3} .$

8. Find y', y'', y''' of $y = 6\sqrt[3]{x^2}$

$y = 6(x)^{\frac{2}{3}} \to y = 6(x)^{\frac{2}{3}}$

$\longrightarrow y' = 6 * \frac{2}{3}(x)^{\frac{-1}{3}} = \frac{4}{\sqrt[3]{x}}$

$\longrightarrow y'' = 4 * \left(\frac{-1}{3}\right) * (x)^{\frac{-4}{3}} = \frac{-4}{3\sqrt[3]{x^4}}$

$y''' = \frac{-4}{3} * \left(\frac{-4}{3}\right) * (x)^{\frac{-7}{3}} = \frac{16}{9\sqrt[3]{x^7}} .$

9. let $f(x) = x^2(x-4)^3$, find $f'(x)$?

$\longrightarrow f'(x) = x^2 * 3(x-4)^2 + (x-4)^3 * (2x)$

$= x(x-4)^2(3x + 2(x-4))$
$= x(x-4)^2(3x + 2x - 8).$
$= x(x-4)^2(5x - 8)$

10. $f(x) = x^3 * \sqrt[3]{(7x^2+x-1)^2} = x^3 (7x^2+x-1)^{2/3}$

$$f'(x) = x^3 * \frac{2}{3}(7x^2 + x - 1)^{\frac{-1}{3}} * (14x + 1) + (7x^2 + x - 1)^{2/3} * (3x^2)$$

Trigonomtric Functions

sinx , cosx , tanx, cotx, secx , and cscx.
Important relations:
1. $\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$
2. $\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$
3. $\sin^2 x + \cos^2 x = 1$
4. $\sec^2 x - \tan^2 x = 1$
5. $\csc^2 x - \cot^2 x = 1$
6. $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
7. $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$
8. $\lim_{a \to 0} \frac{\sin a}{a} = 1$
9. $\lim_{a \to 0} \frac{\cos a - 1}{a} = 0$
10. $\tan x = \frac{\sin x}{\cos x}$
11. $\cot x = \frac{\cos x}{\sin x}$
12. $\sec x = \frac{1}{\cos x}$
13. $\csc x = \frac{1}{\sin x}$

Derivatives of Trigonomtric Functions

Let $u = f(x)$ be a differentiable function , then : $-$

1. $\frac{d}{dx}\sin u = \cos u \cdot \frac{du}{dx}$
2. $\frac{d}{dx}\cos u = -\sin u \cdot \frac{du}{dx}$
3. $\frac{d}{dx}\tan u = \sec^2 u \cdot \frac{du}{dx}$
4. $\frac{d}{dx}\cot u = -\csc^2 u \cdot \frac{du}{dx}$
5. $\frac{d}{dx}\sec u = \sec u \cdot \tan u * \frac{du}{dx}$
6. $\frac{d}{dx}\csc u = -\csc u \cdot \cot u * \frac{du}{dx}$

1. $y = sin(2x) + sec(2x)$

$\longrightarrow y' = 2cos(2x) + 3sec(3x).tan(3x)$

2. $y = tan^4 x$

$\longrightarrow y' = 4tan^3 x . sec^2 x$.

3. $y = sec^4(x^2 + 1)$

$\longrightarrow y' = 4 sec^3(x^2 + 1) * sec(x^2 + 1) * tan(x^2 + 1) * (2x)$

$= 8x sec^4(x^2 + 1) * tan(x^2 + 1)$.

4. $y = cos(3x - 2)$

$\longrightarrow y' = - sin(3x - 2) * (3) = -3sin(3x - 2)$.

5. *Find y' and y'' if* $y = \frac{sin x}{1+cos x}$?

$\longrightarrow y' = \frac{(1+cos x)*cos x - sin x *(- sin x)}{(1+cos x)^2}$

$= \frac{cos x + cos^2 x + sin^2 x}{(1 + cos x)^2} = \frac{1 + cos x}{(1 + cos x)^2} = \frac{1}{1 + cos x}$

$\longrightarrow y'' = \frac{(1+cos x)*(0)-(-sin x)}{(1+cos x)^2} = \frac{sin x}{(1+cos x)^2}$.

6. $f(x) = (7sin 5x - cos \sqrt{x})^3$

$\longrightarrow f'(x) = 3(7sin 5x - cos \sqrt{x})^2 * (35 cos 5x + \frac{1}{2\sqrt{x}} sin \sqrt{x})$.

7. $f(x) = sin^3(5x)$

$\longrightarrow f'(x) = 3(sin5x)^2 * cos5x * (5) = 15 sin^2 5x * cos5x$.

8. $y = cot x * csc x$

$\longrightarrow y' = cot x * (-csc x * cot x) + csc x * (- csc^2 x)$

$= - csc x * cot^2 x - csc^3 x$.

9. $y = \sqrt{x^3 - sin x} = (x^3 - sin x)^{\frac{1}{2}}$

$\longrightarrow y' = \frac{1}{2}(x^3 - sin x)^{\frac{-1}{2}} * (3x^2 - cos x)$

$= \frac{3x^2 - cos x}{2\sqrt{x^3 - sin x}}$

10. $y = sin(\sqrt[3]{x})$

$$\longrightarrow y' = \cos \sqrt[3]{x} * \left(\frac{1}{3\sqrt[3]{x^2}}\right)$$

11. $f(x) = \cos^3 7x = (\cos 7x)^3$

$$\longrightarrow f'(x) = 3\,(\cos 7x)^2 * (-\sin 7x) * 7$$
$$= -21\,\cos^2 7x * \sin 7x.$$

12. $f(x) = \cos 3x - \tan 5x + \sec 4x$

$$\longrightarrow f'(x) = -\sin 3x * (3) - \sec^2 5x * (5) + \sec 4x * \tan 4x * 4$$
$$= -3\,\sin 3x - 5\,\sec^2 5x + 4\,\sec 4x * \tan 4x.$$