

## Lecture Thirteen: Relative Permeability

### 13.1. Definition

The absolute permeability is a property of the porous medium and is a measure of the capacity of the medium to transmit fluids. When two or more fluids flow at the same time, the relative permeability of each phase at a specific saturation is the ratio of the effective permeability of the phase to the absolute permeability, or:

$$K_{ro} = \frac{K_o}{K} \quad \dots\dots (13-1)$$

$$K_{rw} = \frac{K_w}{K} \quad \dots\dots (13-2)$$

$$K_{rg} = \frac{K_g}{K} \quad \dots\dots (13-3)$$

where:

$k_{ro}$  = relative permeability to oil.

$k_{rg}$  = relative permeability to gas.

$k_{rw}$  = relative permeability to water.

$k$  = absolute permeability.

$k_o$  = effective permeability to oil for a given oil saturation.

$k_g$  = effective permeability to gas for a given gas saturation.

$k_w$  = effective permeability to water at some given water saturation.

For example, if the absolute permeability  $k$  of a rock is 200 md and the effective permeability  $k_o$  of the rock at an oil saturation of 80% is 60 md, the relative permeability  $k_{ro}$  is 0.30 at  $S_o = 0.80$ .

Since the effective permeabilities may range from zero to  $k$ , the relative permeabilities may have any value between zero and one, or:

$$0 \leq k_r, k_{ro}, k_{rg} \leq 1.0$$

## 13.2. Two-Phase Relative Permeability

When a wetting (water for example) and a nonwetting phase (as oil) flow together in a reservoir rock, each phase follows separate and distinct paths. The distribution of the two phases according to their wetting characteristics results in characteristic wetting and nonwetting phase relative permeabilities.

The wetting phase occupies the smaller pore openings at small saturations, and these pore openings do not contribute materially to flow. The nonwetting phase occupies the central or larger pore openings that contribute materially to fluid flow through the reservoir.

Figure (13-1) presents a typical set of relative permeability curves for a water-oil system with the water being considered the wetting phase. Figure (13-1) shows the following four distinct and significant observations:

### Observation 1

The wetting phase (water in this case) relative permeability shows that a small saturation of the nonwetting phase (oil in this case) will drastically reduce the relative permeability of the wetting phase. For example see in the figure above at the water saturation of 70%, relative permeability for the water will be reduced from (1 to 0.37). The reason for this is that the nonwetting phase occupies the larger pore spaces, and it is in these large pore spaces that flow occurs with the least difficulty.

### Observation 2

The nonwetting phase relative permeability curve shows that the nonwetting phase begins to flow at the relatively low saturation of the nonwetting phase.

The saturation of the oil at this point is called *critical oil saturation*  $S_{oc}$ , (see in the figure above at oil saturation of 16%).

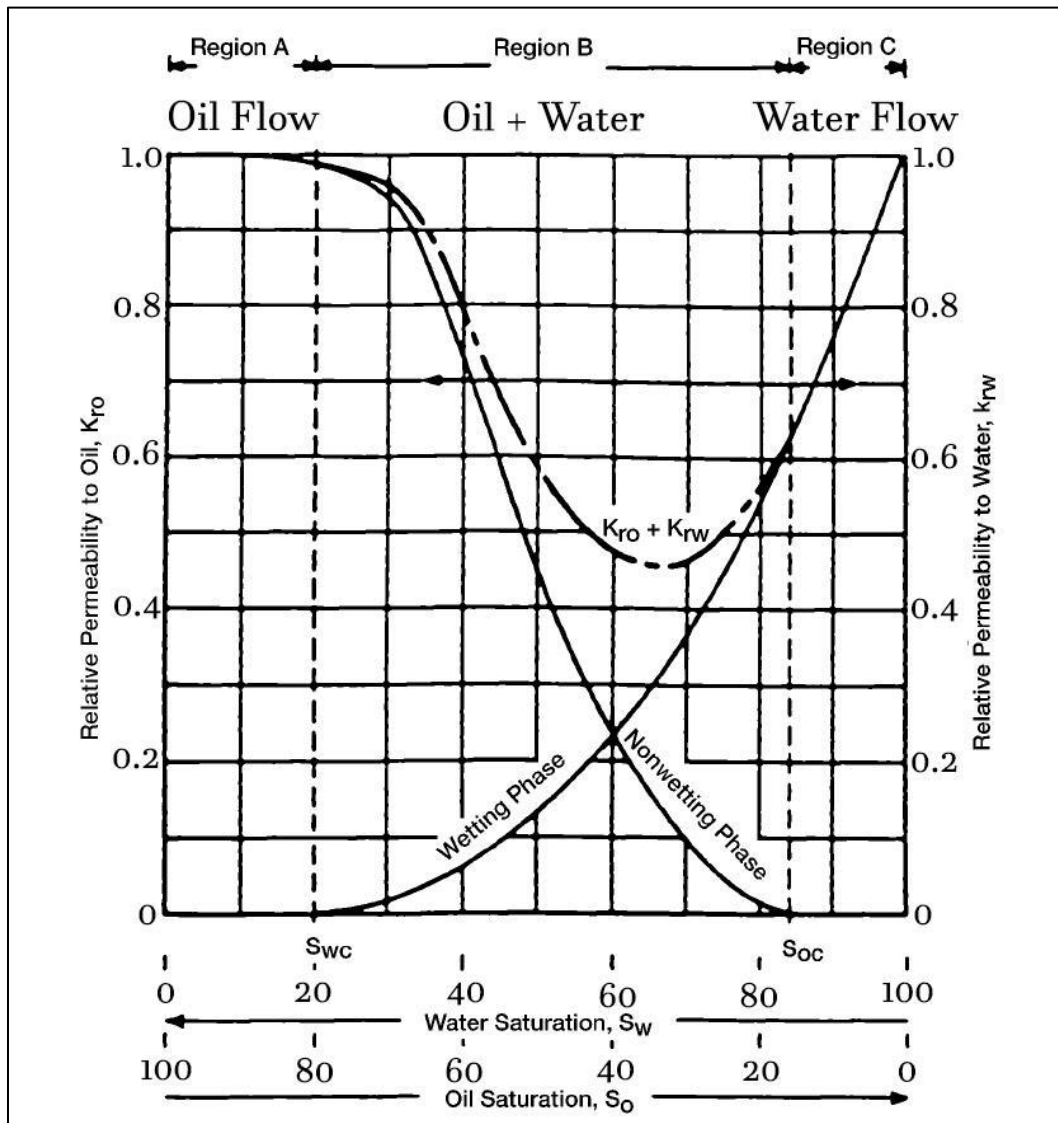


Fig. 13-1 Typical two-phase flow behavior.

### Observation 3

The wetting phase relative permeability curve shows that the wetting phase will cease to flow at a relatively large saturation, (see in the figure above at water saturation of 20%). This is because the wetting phase preferentially occupies the smaller pore spaces, where capillary forces are the greatest. The

saturation of the water at this point is referred to as *the irreducible water saturation*  $S_{wir}$  or *connate-water saturation*  $S_{wc}$ —both terms are used interchangeably.

#### Observation 4

The nonwetting phase relative permeability curve shows that, at the lower saturations of the wetting phase, changes in the wetting phase saturation have only a small effect on the magnitude of the nonwetting phase relative permeability curve, (for example in the figure above at 30% and 40% of water saturation,  $k_{ro}$  will be 0.94 and 0.72 respectively). The reason for this phenomenon is that at the low saturations, the wetting phase fluid occupies the small pore spaces that do not contribute materially to flow, and therefore changing the saturation, in these small pore spaces has a relatively small effect on the flow of the nonwetting phase.

In addition to the observations above note that the total permeability to both phases,  $k_{rw} + k_{ro}$ , is less than 1, in regions *B* and *C*.

### 13.3. Effect of saturation history

There are two types of relative permeability curves:

- 1- Drainage curve - wetting phase is displaced by non-wetting phase, i.e., wetting phase saturation is decreasing
- 2- Imbibition curve - non-wetting phase is displaced by wetting phase, i.e., wetting phase saturation is increasing

The typical relative permeability curve shown below in figure (13-2) represents a process in which:

The process begins with porous rock 100% saturated with wetting phase ( $S_w = 100\%$ ); wetting phase is displaced with non-wetting phase (drainage) until wetting phase ceases to flow ( $S_w =$  critical water saturation).

Then non-wetting phase is displaced with wetting phase (imbibition) until non-wetting phase ceases to flow ( $S_o$  =residual non-wetting phase saturation)

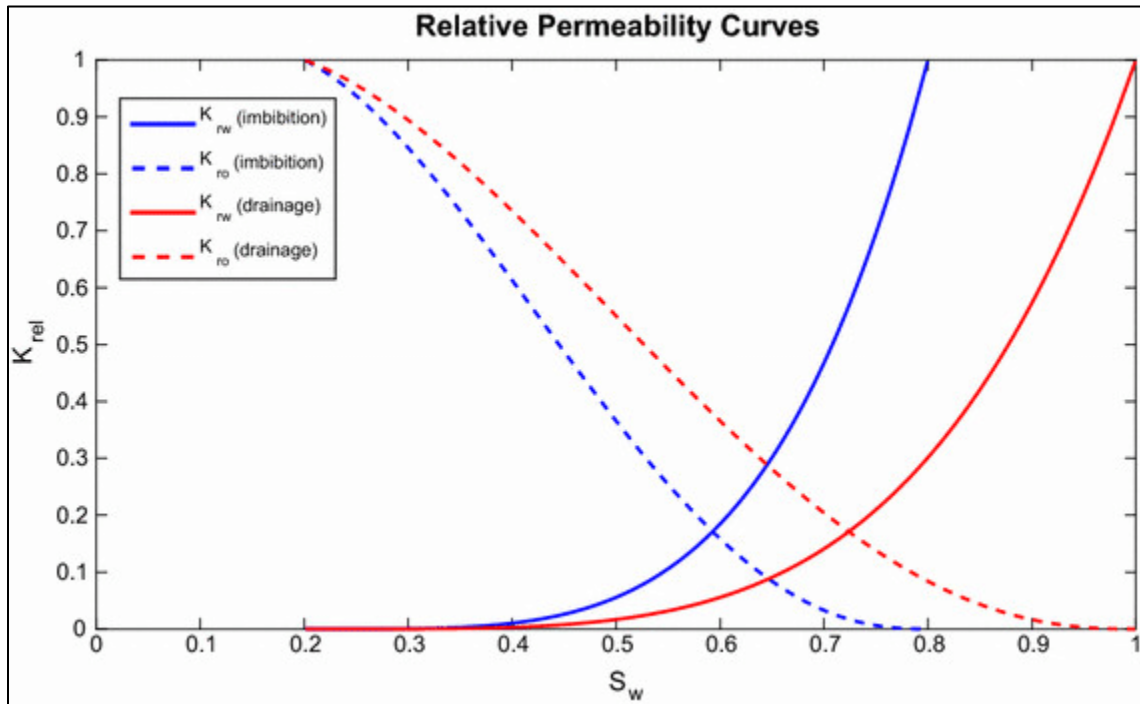


Fig. 13-2 types of relative permeability curves.

At some small saturation, which is presumed to be the saturation at which the displaced phase ceases to be continuous, flow of the displaced phase will cease. This saturation is often referred to as the **residual saturation**. This is an important concept as it determines the maximum recovery from the reservoir. Conversely, a fluid must develop a certain minimum saturation before the phase will begin to flow. This is evident from an examination of the relative permeability curves this saturation at which a fluid will just begin to flow is called the **critical saturation**.

## 13.4. Measurement of Two-Phase Relative Permeability

### 13.4.1. Two-Phase Relative Permeability Correlations

In many cases, relative permeability data on actual samples from the reservoir under study may not be available, in which case it is necessary to obtain the

desired relative permeability data in some other manner. Several methods have been developed for calculating relative permeability relationships.

The effective phase saturation used in the following correlations which is defined by the following set of relationships:

$$S_o^* = \frac{S_o}{1-S_{wc}} \quad \dots\dots (13-4)$$

$$S_g^* = \frac{S_g}{1-S_{wc}} \quad \dots\dots (13-4)$$

$$S_w^* = \frac{S_w - S_{wc}}{1-S_{wc}} \quad \dots\dots (13-4)$$

where

$S_o^*$ ,  $S_w^*$ ,  $S_g^*$  = effective oil, water, and gas saturation, respectively

$S_o$ ,  $S_w$ ,  $S_g$  = oil, water, and gas saturation, respectively

$S_{wc}$  = connate (irreducible) water saturation

## 1. Wyllie and Gardner Correlation

Wyllie and Gardner have suggested the following two expressions that can be used when one relative permeability is available:

- **Oil-water system**

$$k_{rw} = (S_w^*)^2 - k_{ro} \left[ \frac{S_w^*}{1-S_w^*} \right]$$

- **Gas-oil system**

$$k_{ro} = (S_o^*) - k_{rg} \left[ \frac{S_o^*}{1-S_o^*} \right]$$

Conveniently tabulated Wyllie and Gardner correlations shown below:

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**Drainage Oil-Water Relative Permeabilities**

Type of formation	$k_{ro}$	$k_{rw}$
Unconsolidated sand, well sorted	$(1 - S_w^*)$	$(S_w^*)^3$
Unconsolidated sand, poorly sorted	$(1 - S_w^*)^2 (1 - S_w^{*1.5})$	$(S_o^*)^{3.5}$
Cemented sandstone, oolitic limestone	$(1 - S_w^*)^2 (1 - S_w^{*2})$	$(S_o^*)^4$

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**Drainage Gas-Oil Relative Permeabilities**

Type of formation	$k_{ro}$	$k_{rg}$
Unconsolidated sand, well sorted	$(S_o^*)^3$	$(1 - S_o^*)^3$
Unconsolidated sand, poorly sorted	$(S_o^*)^{3.5}$	$(1 - S_o^*)^2 (1 - S_o^{*1.5})$
Cemented sandstone, oolitic limestone, rocks with vugular porosity	$(S_o^*)^4$	$(1 - S_o^*)^2 (1 - S_o^{*2})$

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***Example 1:***

Generate the drainage relative permeability data for an unconsolidated well sorted sand by using the Wyllie and Gardner method. Assume the following critical saturation values:

$$S_{oc} = 0.3, \quad w_c = 0.25, \quad S_{gc} = 0.05$$

***Solution:***

Generate the oil-water relative permeability data by applying Wyllie and Gardner correlations for unconsolidated well sorted sand

$S_w$	$S_w^* = \frac{S_w - S_{wc}}{1 - S_{wc}}$	$k_{ro} = (1 - S_w^*)^3$	$k_{rw} = (S_w^*)$
0.25	0.0000	1.000	0.0000
0.30	0.0667	0.813	0.0003
0.35	0.1333	0.651	0.0024
0.40	0.2000	0.512	0.0080
0.45	0.2667	0.394	0.0190
0.50	0.3333	0.296	0.0370
0.60	0.4667	0.152	0.1017
0.70	0.6000	0.064	0.2160

Figure (13-3) shows the calculated data above.

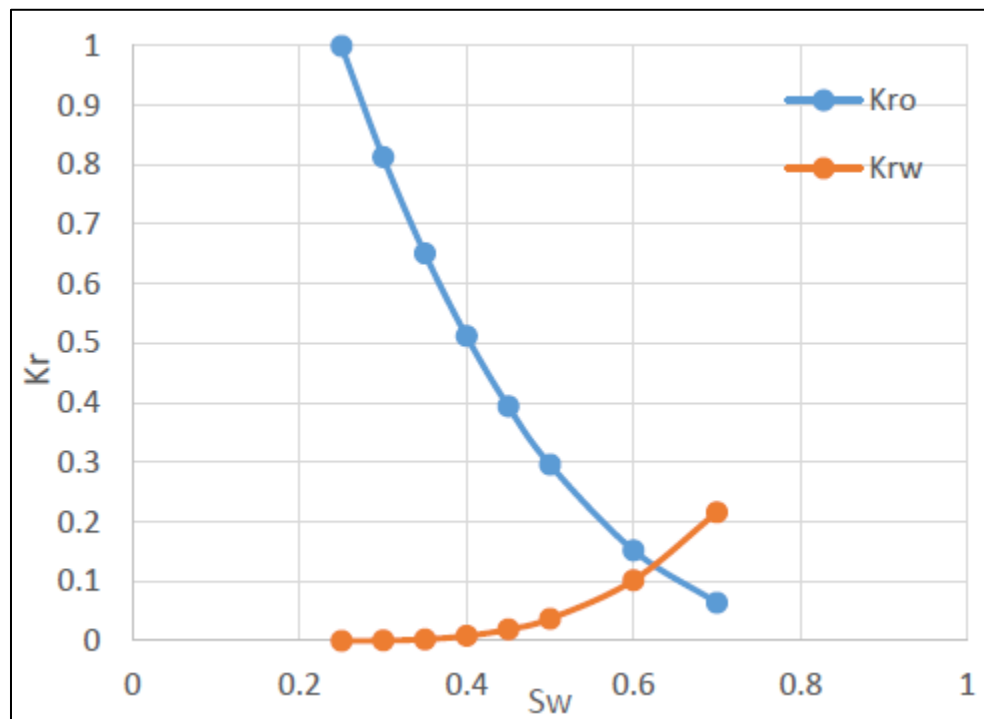


Fig. 13-3 drainage Oil-Water Relative Permeabilities.

### 13.4.2. Two-Phase Relative Permeability Laboratory Methods

Laboratory measurement techniques for obtaining the two-phase relative permeability data based on the flow experiments are fairly well established.



Essentially two different types of flow experiments can be conducted in reservoir rock samples from which relative permeability data are determined. These methods are called steady state (SS) and unsteady state (USS) flows, which are by far the most common.

### **Steady-state flow process:**

- 1- Saturate core with wetting-phase fluid
- 2- Inject wetting-phase fluid through core (this will determine absolute permeability)
- 3- Inject a mix of wetting-phase and non-wetting phase (start with small fraction of non-wetting phase)
- 4- When inflow and outflow rates and portion of non-wetting phase equalize, record inlet pressure, outlet pressure and flow rates of each phase to determine effective permeability

$$K_o = \frac{q_o \mu_o L}{A \Delta P}$$

$$K_w = \frac{q_w \mu_w L}{A \Delta P}$$

- 5- Measure fluid saturation in core

$$S_w = \left( \frac{R_o}{R_t} \right)^{\frac{1}{n}} = \left( \frac{E_o}{E_t} \right)^{\frac{1}{n}}$$

where:

$R_o$  = resistivity of core 100% saturated with wetting-phase, ohm-m

$R_t$  = resistivity of core with saturation of wetting phase less than 100%, ohm-m

$E_o$  = voltage across core 100%, saturated with wetting phase, volts

$E_t$  = voltage across core with saturation of wetting

phase less than 100%, volts

6- Calculate relative permeability

$$K_{r0} = \frac{K_o}{K}$$

$$K_{rw} = \frac{K_w}{K}$$

