Interval Notation

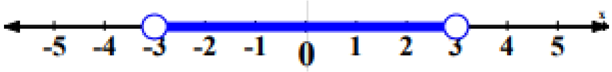
Interval Notation for Linear Inequalities

A set of numbers may be described in many ways; by using rosters, tables, number lines, and other methods. A useful way of describing a set of numbers is by using interval notation.

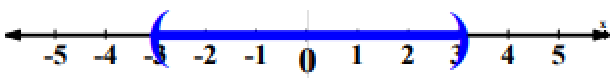
Interval notation is a frequent option to express a set of numbers between two values, a and b. We basically use two symbols: parentheses ( ) and brackets [ ]:

1. ( ) is used for less than, $<$, or greater than, $>$. This means that specified values for a or b are not included.
2. [ ] is used for less than or equal to, $\leq$, or greater than or equal to, $\geq$. This means that specified values for a or b are included.

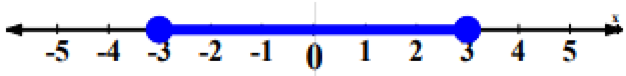
Example 1:- The inequality $-3 < x < 3$ reflects all the real numbers between -3 and 3, without including -3 nor 3. The corresponding graph is:



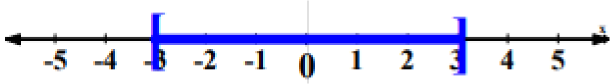
In interval notation, parentheses (and) are equivalent to the open circle on the number line. Since we do not want to include the endpoints, using interval notation we write this inequality as (-3, 3). We could have also used parentheses to graph the solution set:



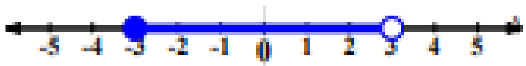
Example 2:- The inequality $-3 \leq x \leq 3$ reflects all the real numbers between -3 and 3, including -3 and 3. The corresponding graph is



In interval notation brackets [and] are equivalent to the closed circle (solid dot) on the number line. Since we now want to include the endpoints, using interval notation, we write this inequality as [-3, 3]. We could have also used brackets to graph the solution set:

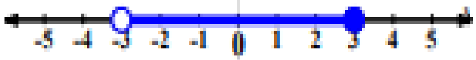


Example 3:- The inequality $-3 \leq x < 3$ reflects all the real numbers between -3 and 3, including -3 but not 3.



In interval notation, we write this inequality as [-3, 3).

Example 4 : The inequality **$-3 < x \leq 3$** reflects all the real numbers between -3 and 3, including 3 but not -3.



In interval notation, we write this inequality as (-3, 3].

Summary of Interval Notation
A closed interval [a, b] describes all real numbers x where $a \leq x \leq b$
An open interval (a, b) describes all real numbers x where $a < x < b$
A half-open interval (or half-closed) describes one of the following:
[a, b) describes all real numbers x where $a \leq x < b$
(a, b] describes all real numbers x where $a < x \leq b$

Caution: When writing the interval notation, make sure you always write the smaller value to the left and the greater value to the right.

Infinity Symbol

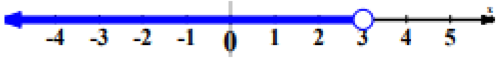
The symbol **"∞"** is called the infinity symbol and we use it when there is no lower or upper bound on the number line. For example, we know that the inequality $x \geq 3$ includes all real numbers greater than or equal to 3, without limit. The corresponding graph is:

Since there is no upper bound, we say that the interval is unlimited, and the interval notation is [3, ∞).

Infinite Intervals
We use ∞ to signify that the values continue getting larger without end (unbounded to the right on the number line).
We use -∞ to signify that the values continue getting smaller without end (unbounded to the left on the number line).
[a, ∞) describes all real numbers x where $x \geq a$
(a, ∞) describes all real numbers x where $x > a$
(-∞, a] describes all real numbers x where $x \leq a$
(-∞, a) describes all real numbers x where $x < a$

Note: Since the use of the infinity symbol implies that the interval is unlimited, we never close it with a bracket. Always use a parenthesis next to the ∞ symbol.

Example 5 :- The inequality x < 3 reflects all the real number less than



In interval notation, we write this inequality as (-∞, 3).

Example 6 :- Use inequality, graphical, and interval notation to write the set of numbers that are:

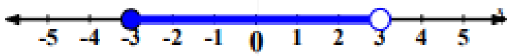
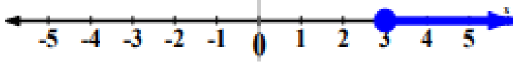
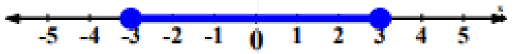
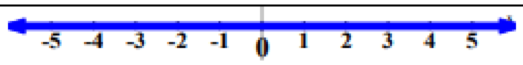
a. between -3 and 3, not including the endpoints.

b. including -3, but excluding 3.

c. greater than or equal to 3.

d. less than 3. e. between -3 and 3, including the endpoints.

f. all the real numbers.

Inequality Notation	*Graphical Notation*	*Interval Notation*	*Type of Interval*
a. $-3 < x < 3$		(-3, 3)	Open
b. $-3 \leq x < 3$		[-3, 3)	Half-open or Half-closed
c. $x \geq 3$		[3, ∞)	Infinite
d. $x < 3$		(-∞, 3)	Infinite
e. $-3 \leq x \leq 3$		[-3, 3]	Closed
f. $-\infty < x < \infty$		(-∞,∞)	Infinite

Exercise 2:-

1. Use inequality, graphical, and interval notation on the table that follows to write the set of numbers that are:

a. between -5 and 6, not including the endpoints.

b. less than 1.5.

c. greater than or equal to -5.

d. between -4 and 0, including the endpoints.

e. including -3.5, but excluding 2.

Absolute Value

The absolute value of a real number a is denoted by **|a|** and it is the distance from a to the origin 0 on the number line. The absolute value is always positive. We can give a formula for the absolute value of the number, which depends on whether a is positive or negative. Because of this we have to make two statements to describe the formula.

Definition:- If a is a real number, the absolute value of a is

$$|a| = \begin{cases} a & \text{if } a \geq 0 \\ -a & \text{if } a < 0 \end{cases}$$

Example 7:- Evaluate **|2|, |-5| , |5-9| , |9-5|**

<u>Distance Between Two Points on The Real Line.</u>

If a and b are real numbers, then the distance between the points a and b on the real line is

d (a, b) = | b -a |

Example 8:- Find the distance between the numbers (-2 , 10) , (-7,0) , (5 ,9)

Example 9:- Suppose we wish to solve the inequality

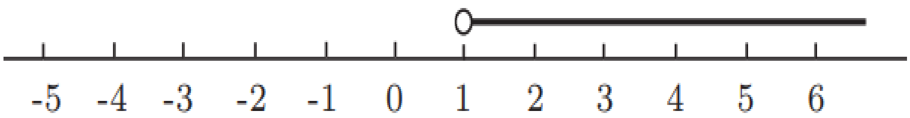
$$4x + 6 > 3x + 7.$$

First we subtract 6 from both sides to give

$$4x > 3x + 1$$

Now we subtract 3x from both sides: $x > 1$

This is the solution. It can be represented on the number line as shown



A number line showing x > 1.

Example 10:- Suppose we wish to solve $3x - 5 \le 3 - x$.

We start by adding 5 to both sides:

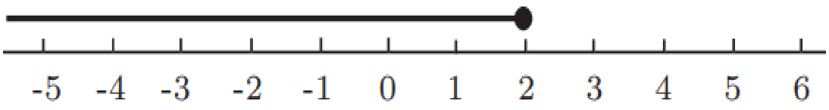
$$3x \le 8 - x$$

Then add x to both sides to give $4x \le 8$

Finally dividing both sides by 4 gives $x \le 2$

This is shown on the number line

The closed circle denotes that x can actually equal 2.



A number line showing $x \le 2$.

Example 11:- Suppose we wish to solve the inequality $-2x > 4$.

In order to solve this we are going to divide both sides by −2, and we need to remember that because we are dividing by a negative number we must reverse the inequality. $\mathbf{x < -2}$

There is often more than one way to solve an inequality.

Example 12 :- Suppose we wish to solve $|5x - 8| \le 12$.

This means $-12 \le 5x - 8 \le 12$

and again we have a double inequality.

On the left: $-12 \le 5x - 8$.

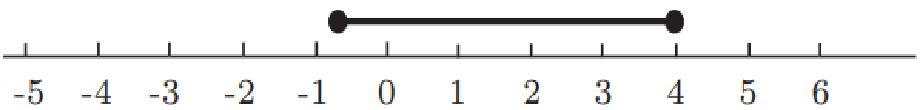
Adding 8 to both sides: $\mathbf{-4 \le 5x}$, and dividing by 5 gives, $\mathbf{-4\ 5 \le x}$.

On the right: $\mathbf{5x - 8 \le 12}$.

Adding 8 to both sides: $\mathbf{5x \le 20}$.

Dividing by 5 gives $\mathbf{x \le 4}$.

Putting these results together gives the solution $\mathbf{-4\ 5 \le x \le 4}$ This range of values is shown on the number line



Example 13-a :- Suppose we wish to solve |3x + 1| - 4 < 7

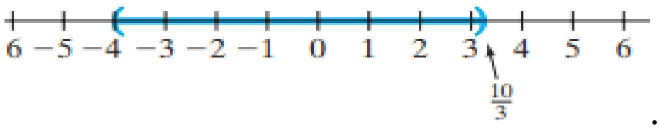
Solution :- $|3x + 1| - 4 < 7$

$\rightarrow |3x + 1| < 11$

$\rightarrow -11 < 3x + 1 < 11$

$\rightarrow -12 < 3x < 10$

$-4 < x < \frac{10}{3}$



Example 13-b :- Suppose we wish to solve $3 \leq 1 + |\frac{t}{2} - 5|$

Solution :- $3 \leq 1 + \left|\frac{t}{2} - 5\right| \rightarrow 1 + \left|\frac{t}{2} - 5\right| \geq 3$

$\rightarrow \left|\frac{t}{2} - \mathbf{5}\right| \geq 2$

*	1)	$\|x\| > a \rightarrow x < -a$ or $x > a$
	2)	$\|x\| \geq a \rightarrow x \leq -a$ or $x \geq a$

Exercises 3 :- Solve the following inequalities

a) $|x| \leq 3$ b) $|x| > 6$ c) $|x - 4| \leq 3$ d) $|x + 4| \geq 2$ g) $|3 - x| - 4 > 1$