Lecture Seven: Average Permeability

7.1. Radial flow system

For radial flow of fluids into a wellbore Darcy's law may be expressed in radial coordinates

$$q = \frac{K\ A}{\mu}\ \frac{dp}{dr} \qquad \text{....... (7-1)}$$

$$q \int_{r_w}^{r_e} dr = \frac{kA}{\mu} \int_{p_w}^{p_e} dp$$

$$q \int_{r_w}^{r_e} \frac{dr}{A} = \frac{k}{\mu} \int_{p_w}^{p_e} dp$$

$$q \int_{r_w}^{r_e} \frac{dr}{2\pi r h} = \frac{k}{\mu} \int_{p_w}^{p_e} dp$$

$$\frac{q}{2\pi h} \int_{r_w}^{r_e} \frac{dr}{r} = \frac{k}{\mu} \int_{p_w}^{p_e} dp$$

$$\frac{q}{2\pi h} [\ln(r_e) - \ln(r_w)] = \frac{k}{\mu}(p_e - p_w)$$

$$\frac{q}{2\pi h} \left[\ln \frac{r_e}{r_w}\right] = \frac{k}{\mu}(p_e - p_w)$$

$$q = \frac{2\pi k h (p_e - p_w)}{\mu \left[\ln \frac{r_e}{r_w}\right]}$$

Darcy's law may beexpressed in radial coordinates in field units as:

$$q = \frac{0.00708\ Kh\ (Pe - Pw)}{\mu\ \ln\left(\frac{re}{rw}\right)} \qquad \text{....... (7-2)}$$

where:

q = volumetric flowrate, bbl/Day.

k = absolute permeability of the rock, millidarcy (md).

h = pay thickness, ft.

pe= pressure at external radius, psi.

pw = pressure at wellbore, psi.

μ = fluid viscosity, cp.

L = length of the rock, ft.

re = external drainage area, ft.

rw = wellbore radius, ft.

ln = natural logarithm.

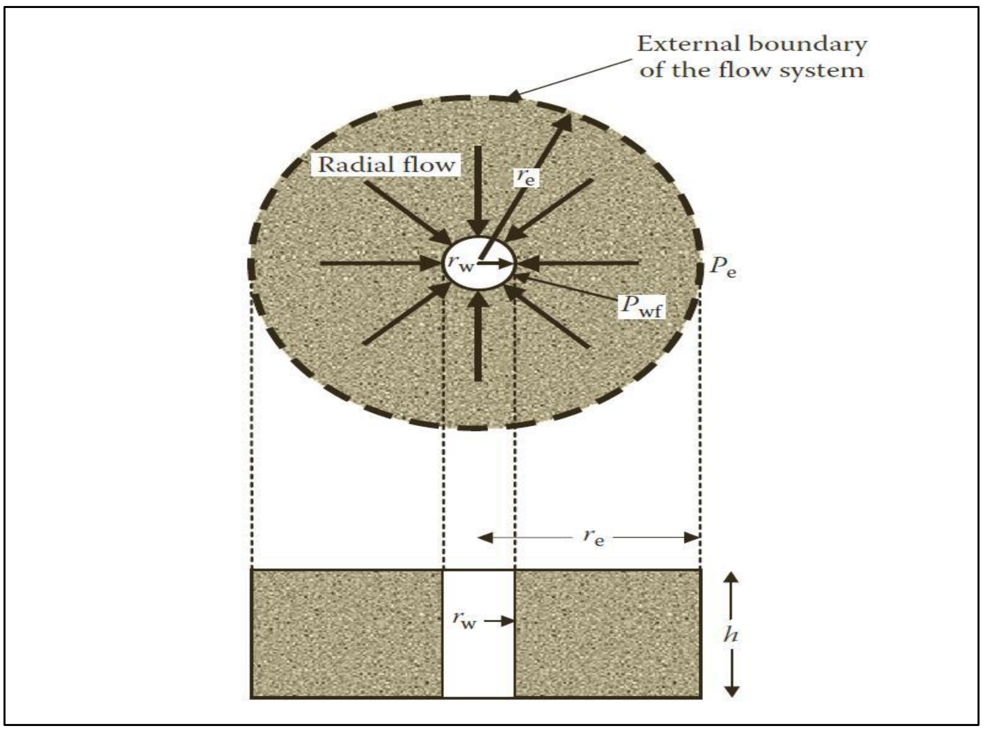

Fig. 7-1 Radial flow model.

7.2. Horizontal and vertical permeabilities

Permeability is reduced by overburden pressure, and this factor should be considered in estimating permeability of the reservoir rock in deep wells because permeability is an isotropic property of porous rock in some defined regions of the system, that is, in other words, it is directional. Routine core analysis is generally concerned with plug samples drilled parallel to bedding planes and, hence, parallel to direction of flow in the reservoir. These yield horizontal permeabilities (kh). The measured permeability on plugs that are drilled perpendicular to bedding planes is referred to as vertical permeability (kv).

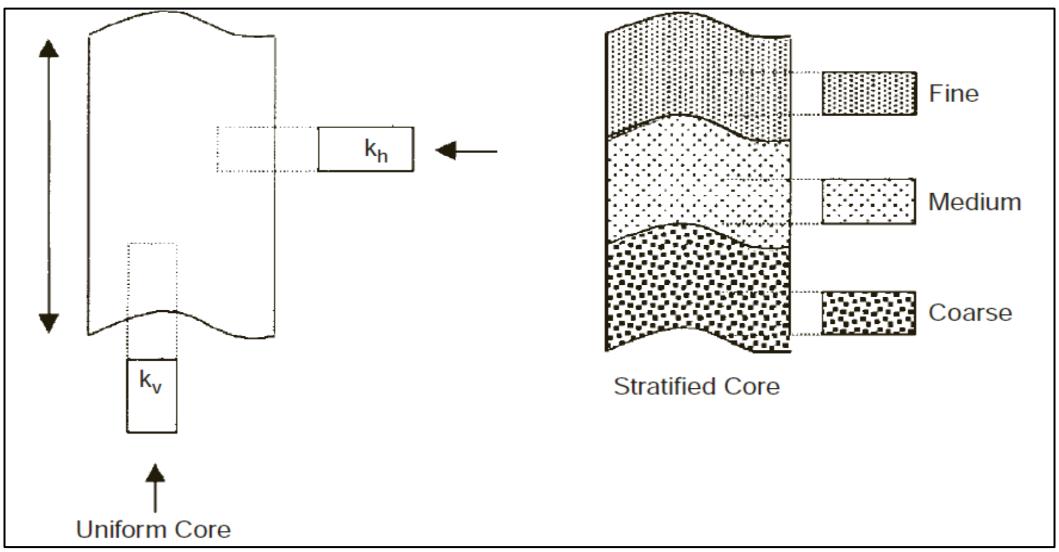

Fig. 7-2 Representative samples of porous media.

7.3. Averaging Absolute Permeabilities

The absolute permeability expression is derived based on a fairly uniform or continuous value of permeability between the inflow and outflow faces. However, such uniformity and consistency is rarely seen in reservoir rocks. Most reservoir rocks have space variations of permeability. For example, reservoir rocks may contain distinct layers, blocks, or concentric rings of fixed permeability. In such cases, the permeability values are averaged according to the particular type of flow: parallel or series. The mathematical expressions for averaging permeability for these cases are developed in the following text.

1- Parallel Flow

Consider the case of fluid flow taking place in parallel through different layers of vertically stacked porous media. These individual layers of porous media that have varying permeability and thickness are separated from one another by infinitely thin impermeable barriers that preclude the possibility of cross flow or vertical flow. The average permeability for such a combination can be easily developed by applying Darcy's law to the individual layers.

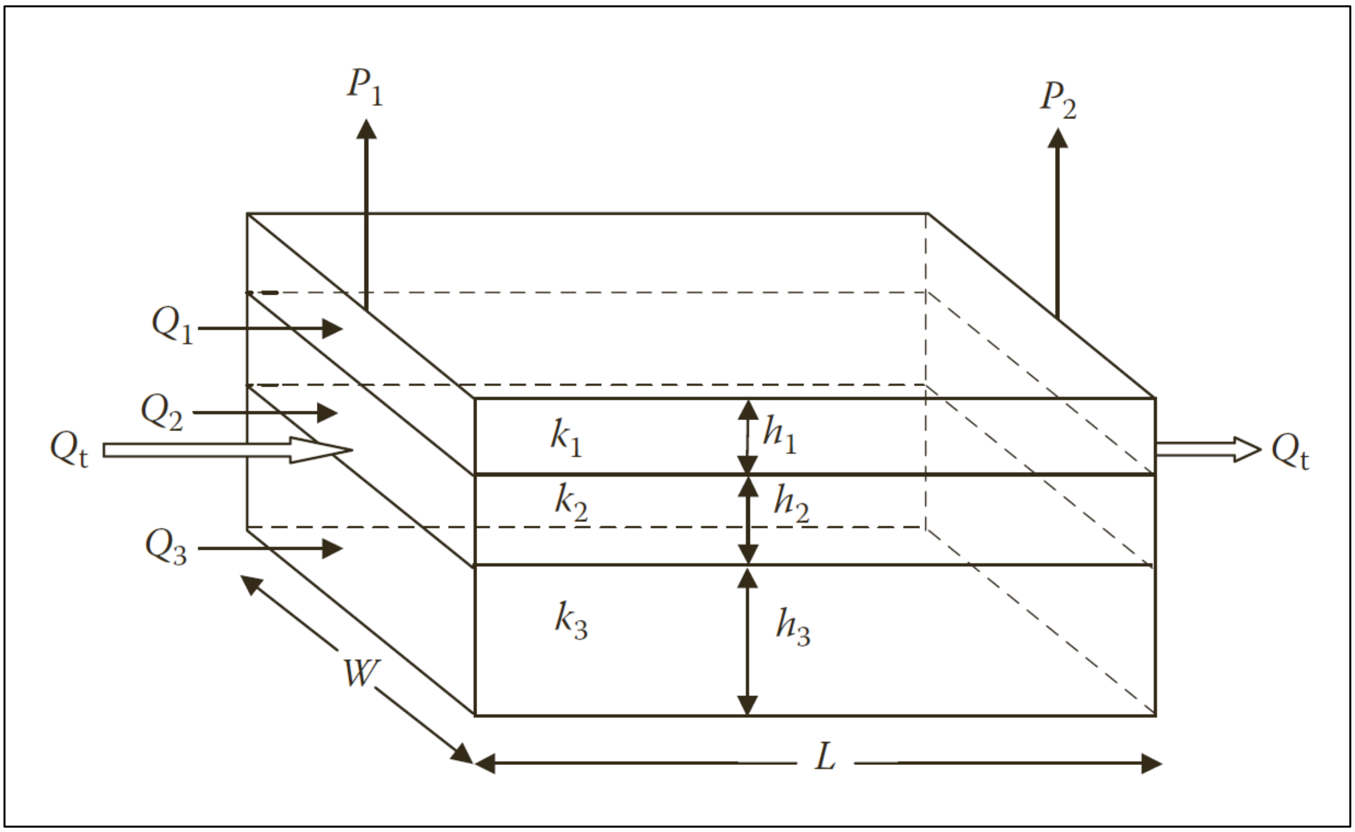

Fig. 7-3 fluid flow through a parallel combination.

For **layer 1**: $q_1 = \frac{K_1\, W\, h_1\, \Delta P}{\mu\, L}$ ……. (7-3)

For **layer 2**: $q_2 = \frac{K_2\, W\, h_2\, \Delta P}{\mu\, L}$ ……. (7-4)

For **layer 3**: $q_3 = \frac{K_3\, W\, h_3\, \Delta P}{\mu\, L}$ ……. (7-5)

However, since flow is taking place in parallel, the total volumetric flow rate can be equated to the summation of the individual flow rates through the three layers:

$$q_t = q_1 + q_2 + q_3 \quad \text{……. (7-6)}$$

Similarly, the total height is given by:

$$h_t = h_1 + h_2 + h_3 \quad \text{……. (7-7)}$$

Based on Equations (7-6) and (7-7), Darcy's law can be written for the total flow rate for the entire systems using k_{avg} as the average absolute permeability:

$$q_t = \frac{K_{avg}\, W\, h_t\, \Delta P}{\mu\, L} = \frac{K_1\, W\, h_1\, \Delta P}{\mu\, L} + \frac{K_2\, W\, h_2\, \Delta P}{\mu\, L} + \frac{K_3\, W\, h_3\, \Delta P}{\mu\, L} \quad \text{……. (7-8)}$$

$$k_{avg} h_t \;=\; k_1 h_1 \;+ k_2 h_2 \;+\; k_3 h_3 \quad \text{……. (7-9)}$$

That subsequently leads to the final generalized expression for calculating the average absolute permeability for a parallel system of *n* layers:

$$k_{avg} = \frac{\sum_{i=1}^{n} k_i h_i}{\sum_{i=1}^{n} h_i} \quad \text{……. (7-10)}$$

The average absolute permeability expressions for a more generalized case of the parallel flow can also be developed using an approach similar to the one described earlier. In such a case, the width of the layers is varied rather than being kept constant. For example, Figure (7-3) can be modified such that layer 1 has shortest width $\mathbf{W_1}$, layer 2 has medium width $\mathbf{W_2}$, and layer 3 has the largest width $\mathbf{W_3}$ as explained in figure (7-4) below:

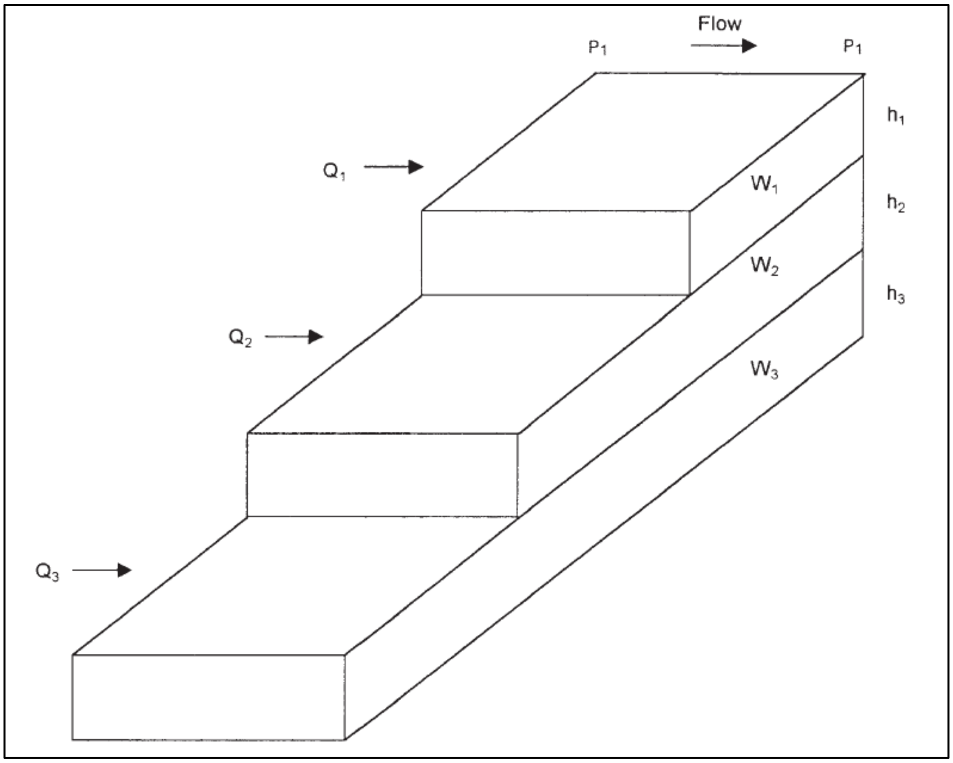

Fig. 7-4 fluid flow through a parallel combination.

Given this arrangement, the following equations can be set up for the three individual layers and the average for the entire system:

For **layer 1**: $q_1 = \frac{K_1\, W_1\, h_1\, \Delta P}{\mu\, L}$ ……. (7-11)

For **layer 2**: $q_2 = \frac{K_2\, W_2\, h_2\, \Delta P}{\mu\, L}$ ……. (7-12)

For **layer 3**: $q_3 = \frac{K_3\, W_3\, h_3\, \Delta P}{\mu\, L}$ ……. (7-13)

$$q_t = \frac{K_{avg}\,(W_1\, h_1 + W_2 h_2 + W_3 h_3)\,\Delta P}{\mu\, L} = \frac{K_1\, W_1\, h_1\, \Delta P}{\mu\, L} + \frac{K_2\, W_2\, h_2\, \Delta P}{\mu\, L} + \frac{K_3\, W_{\ 3} h_3\, \Delta P}{\mu\, L} \quad .(7\text{-}14)$$

$$k_{avg} = \frac{k_1 h_1 w_1 + k_2 h_2 w_2 + k_3 h_3 w_3}{W_1\, h_1 + W_2 h_2 + W_3 h_3} \quad …….\ (7\text{-}15)$$

$$k_{avg} = \frac{\sum_{i=1}^{n} k_i h_i W_i}{\sum_{i=1}^{n} h_i\, W_i} = \frac{\sum_{i=1}^{n} k_i A_i}{\sum_{i=1}^{n} A_i} \quad …….\ (7\text{-}16)$$

Example 1

Given the following permeability data from a core analysis report, calculate the average permeability of the reservoir.

Depth, ft	Permeability, md
3998-4002	200
4002-4004	130
4004-4006	170
4006-4008	180
4008-4010	140

Solution:

h_i, *ft*	k_i	$h_i k_i$
4	200	800
2	130	260
2	170	340
2	180	360
2	140	280
h_t=12		$\sum h_i k_i = 2040$

$$k_{avg} = \frac{2040}{12} = 170\ md$$

2- Series Flow (Harmonic Average Permeability)

Permeability variations can occur laterally in a reservoir as well as in the vicinity of a wellbore. Figure (7-5) illustrates a series flow taking place through a stack of porous media of varying absolute permeabilities and lengths. The mathematical expression for calculating the average absolute permeability for a flow system shown in Figure (7-5) is developed in the following text. Again writing Darcy's law for each of the layers or blocks of porous medium stacked in series,

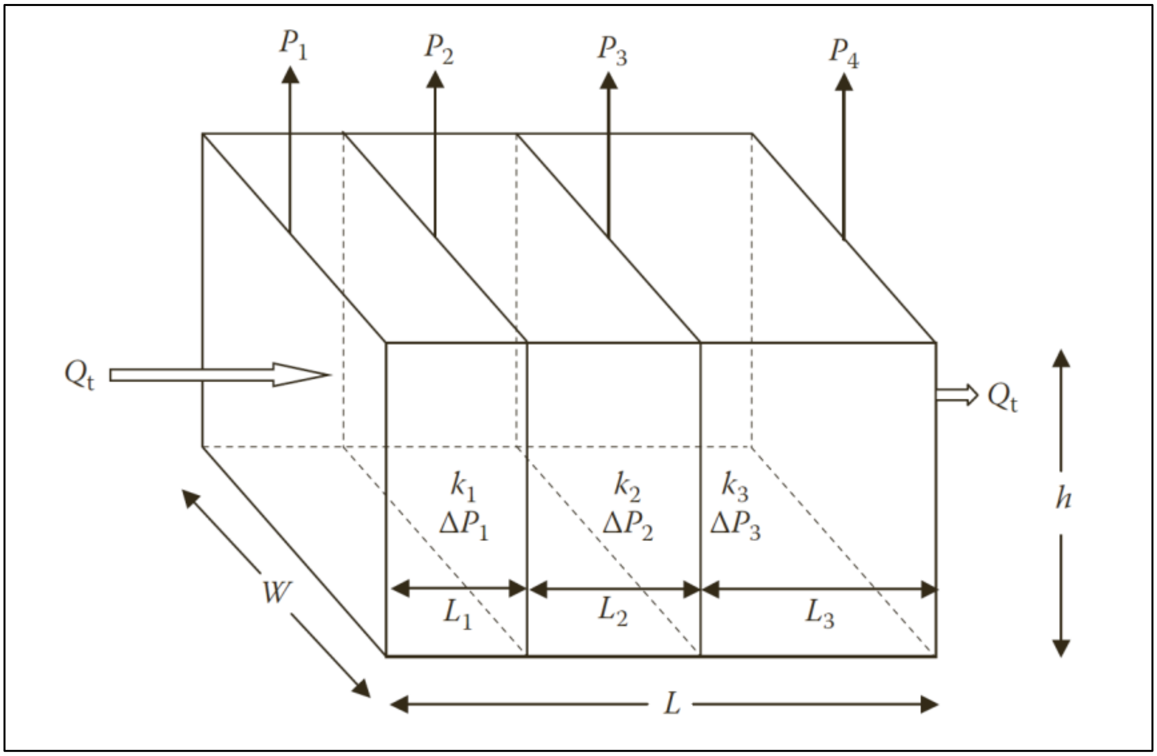

Fig. 7-5 fluid flow through a parallel combination.

For **layer 1**: $q_1 = \frac{K_1\ W\ h\ \Delta P_1}{\mu\ L_1}$ ……. (7-17)

For **layer 2**: $q_2 = \frac{K_2\ W\ h\ \Delta P_2}{\mu\ L_2}$ ……. (7-18)

For **layer 3**: $q_3 = \frac{K_3\ W\ h\ \Delta P_3}{\mu\ L_3}$ ……. (7-19)

It should be noted that for series flow, each of these layers or blocks has a different differential pressure and the summation of these is equal to the total or overall differential pressure of the entire flow system. Additionally, the total flow rate is also equal to the individual flow rates:

$\Delta p_1 = p_1 - p_2,\ \Delta p_2 = p_2 - p_3,\ \Delta p_3 = p_3 - p_4$ ……. (7-20)

$\Delta p = p_1 - p_4 = \Delta p_1 + \Delta p_2 + \Delta p_3$ ……. (7-21)

and,

$q_t = q_1 = q_2 = q_3$ ……. (7-22)

Now, Darcy's law can be written for the total flow rate as:

$$q_t = \frac{K_{avg}\, W\, h\, \Delta P}{\mu\, L}$$ ……. (7-23)

Subsequently, the differential pressures can be separated from Equations (7-17) through (7-19) and (7-23) and substituted into Equation (7-21):

$\Delta p = \Delta p_1 + \Delta p_2 + \Delta p_3$

$$\frac{q_t\, \mu\, L}{K_{avg}\, W\, h} = \frac{q_1\, \mu\, L_1}{K_1\, W\, h} + \frac{q_2\, \mu\, L_2}{K_2\, W\, h} + \frac{q_3\, \mu\, L_3}{K_3\, W\, h}$$

$$\frac{L}{K_{avg}} = \frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3}$$

$$K_{avg} = \frac{\sum_{i=1}^{n} L_i}{\sum_{i=1}^{n} L_i / K_i}$$ ……. (7-24)

Equation (7-24) is used for calculating the average absolute permeability for the linear flow system. In the radial system shown in figure below, the preceding averaging methodology can be applied to produce the following generalized expression:

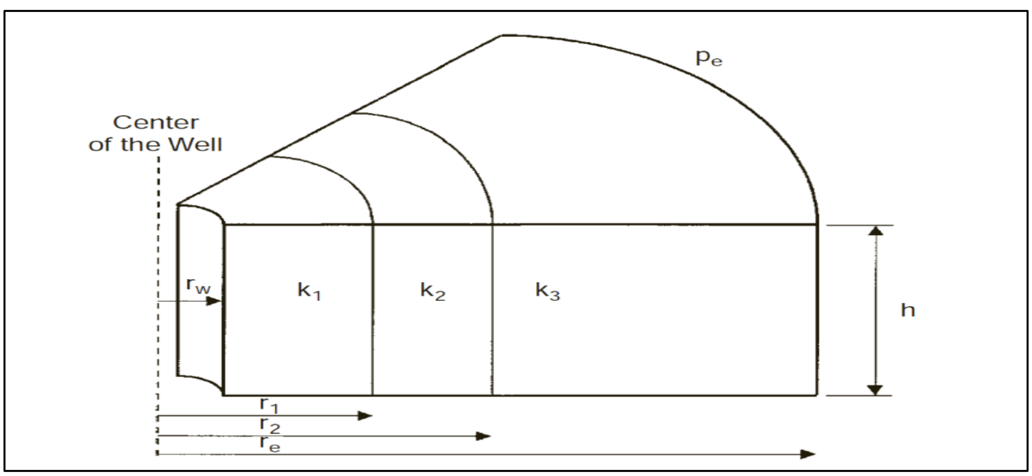

Fig. 7-6 Radial Flow through series beds..

$$K_{avg} = \frac{\ln\left(\frac{re}{rw}\right)}{\sum_{j=1}^{n}\left(\frac{ln\,(r_j/r_{j-1})}{K_i}\right)} \qquad \text{……. (7-25)}$$

Equation (7-25) comes from the following expressions:

$q_t = q_1 = q_2 = q_3$

$\Delta p = \Delta p_1 + \Delta p_2 + \Delta p_3$

$$\frac{q_t\,\mu\;ln\left(\frac{re}{rw}\right)}{K_{avg}\;W\;h} = \frac{q_1\,\mu\;ln\left(\frac{r_1}{rw}\right)}{K_1\;W\;h} + \frac{q_2\,\mu\;ln\left(\frac{r_2}{r_1}\right)}{K_2\;W\;h} + \frac{q_3\,\mu\;ln\left(\frac{r_3}{r_2}\right)}{K_3\;W\;h}$$

$$\frac{ln\left(\frac{re}{rw}\right)}{K_{avg}} = \frac{ln\left(\frac{r_1}{rw}\right)}{K_1} + \frac{ln\left(\frac{r_2}{r_1}\right)}{K_2} + \frac{ln\left(\frac{r_3}{r_2}\right)}{K_3}$$

$$K_{avg} = \frac{ln\left(\frac{re}{rw}\right)}{\frac{ln\left(\frac{r_1}{rw}\right)}{K_1} + \frac{ln\left(\frac{r_2}{r_1}\right)}{K_2} + \frac{ln\left(\frac{r_3}{r_2}\right)}{K_3}}$$

$$K_{avg} = \frac{\ln\left(\frac{re}{rw}\right)}{\sum_{j=1}^{n}\left(\frac{\ln\,(r_j/r_{j-1})}{K_i}\right)} \qquad \text{……. (7-25)}$$

The relationship in Equation (7-25) can be used as a basis for estimating a number of useful quantities in production work. For example, the effects of mud invasion, acidizing, or well shooting can be estimated from it.

Example 2

A hydrocarbon reservoir is characterized by five distinct formation segments that are connected in series. Each segment has the same formation thickness. The length and permeability of each section of the five-bed reservoir follow:

Lenght, ft	Permeability, md
150	80
200	50
300	30
500	20
200	10

Calculate the average permeability of the reservoir by assuming:

a. Linear flow system.

b. Radial flow system.

Solution:

For a ***linear system:***

L_i, ft	k_i, md	L_i/k_i
150	80	1.8750
200	50	4.000
300	30	10.000
500	20	25.000
200	10	20.000
1350		$\sum L_i/k_i = 60.785$

Using Equation (7-24) gives:

$$K_{avg} = \frac{1350}{60.785} = 22.81\ md$$

For a ***radial system:***

The solution of the radial system can be conveniently expressed in the following tabulated form. The solution is based on Equation (7-25) and assumes a wellbore radius of 0.25 ft:

Segment	r_j, ft	$\ln(r_j/r_{j-1})$	k_j	$[\ln(r_j/r_{j-1})]/k_j$
Wellbore	0.25	-	-	-
1	150	ln(150/0.25)=6.397	80	0.080
2	350	ln(350/150)=0.847	50	0.017
3	650	ln(650/350)=0.619	30	0.021
4	1150	ln(1150/650)=0.571	20	0.029
5	1350	ln(1350/1150)=0.160	10	0.016
				0.163

From Equation (7-25),

$$K_{avg} = \frac{\ln(1350/0.25)}{0.163} = 52.72\ md$$

3- Geometric Average Permeability

Warren and Price (1961) illustrated experimentally that the most probable behavior of a heterogeneous formation approaches that of a uniform system having a permeability that is equal to the geometric average. The geometric average is defined mathematically by the following relationship:

$$K_{avg} = \exp\left[\frac{\sum_{i=1}^{n}(h_i\ ln\ K_i)}{\sum_{i=1}^{n} h_i}\right] \qquad \text{……. (7-26)}$$

where:

k_i= permeability of core sample i.

h_i= thickness of core sample i.

n = total numbers of samples.

Example 3

Given the following core data, calculate the geometric average permeability:

Sample	h_i, ft	k_i, md
1	1	10
2	1	30
3	0.5	100
4	1.5	40
5	2	80
6	1.5	70
7	1	15
8	1	50
9	1.5	35
10	0.5	20

Solution:

Sample	h_i, ft	k_i, md	$h_i * \ln(k_i)$
1	1	10	2.303
2	1	30	3.401
3	0.5	100	2.303
4	1.5	40	5.533
5	2	80	8.764
6	1.5	70	6.373
7	1	15	2.708
8	1	50	3.912
9	1.5	35	5.333
10	0.5	20	1.498
	sum = 11.5		sum = 42.128

$$k_{avg} = exp\left[\frac{\sum_{i=1}^{n}(h_i \ln(k_i))}{\sum_{i=1}^{n}(h_i)}\right]$$

$$k_{avg} = exp\left[\frac{42.128}{11.5}\right] = 39\ md$$