Lecture Six: Permeability

6.1. Definition

Permeability is a property of the porous medium to transmit fluids through it. The rock permeability, *k*, is a very important rock property because it controls the directional movement and the flow rate of the reservoir fluids in the formation. This rock characterization was first defined mathematically by (Henry Darcy in 1856). In fact, the equation that defines permeability in terms of measurable quantities is called Darcy's Law.

Darcy's law states that the volumetric flow rate of a homogeneous fluid through a porous medium is proportional to the pressure gradient and to the cross-sectional area normal to the direction of flow, and inversely proportional to the length of the porous medium and the viscosity of the fluid.

Darcy developed a fluid flow equation that has since become one of the standard mathematical tools of the petroleum engineer. If a horizontal linear flow of an incompressible fluid is established through a core sample of length L and a cross-section of area A, then the governing fluid flow equation is defined as:

$$q = -\frac{K\ A}{\mu}\frac{dp}{dx} \qquad \text{……. (6-1)}$$

where:

q = volumetric flowrate, cm3/sec.

k = absolute permeability of the rock, darcy.

A = cross sectional area in the flow direction, cm2.

μ = fluid viscosity, cp. (*μ=mass/length.time =m/L.t =gm/cm.sec (poise)*).

$\frac{dp}{dx}$ = fluid pressure gradient, atm/cm.

One Darcy is a relatively high permeability, as the permeabilities of most reservoir rocks are less than 1 Darcy. In order to avoid the use of fractions in describing permeabilities, the term millidarcy is used.

$1\ darcy = 1000\ md$

The negative sign in equation (6-1) is necessary, as the pressure increases in one direction while the length increases in the opposite direction.

Equation (6-1) can be integrated when the geometry of the system through which fluid flows is known. For the simple linear system shown in Figure (6-1), the integration is performed as follows:

$$q \int_0^L dL = -\frac{KA}{\mu} \int_{P1}^{P2} dP$$

Integrating the above expression yields:

$$q = -\frac{KA}{\mu L}\ (P2 - P1)$$

$$q = \frac{KA}{\mu L}\ (P1 - P2) \qquad \text{……. (6-2)}$$

Example 1

A brine is used to measure the absolute permeability of a core plug. The rock sample is (4 cm) long and (3 cm^2) in cross section. The brine has a viscosity of (1.0 cp) and is flowing at a constant rate of (0.5 cm^3/sec) under a (2.0 atm) pressure differential. Calculate the absolute permeability.

Solution:

$$q = \frac{KA}{\mu L}(P1 - P2)$$

$$0.5 = \frac{K \times 3}{4 \times 1} \times 2$$

k = 0.333 $darcy$

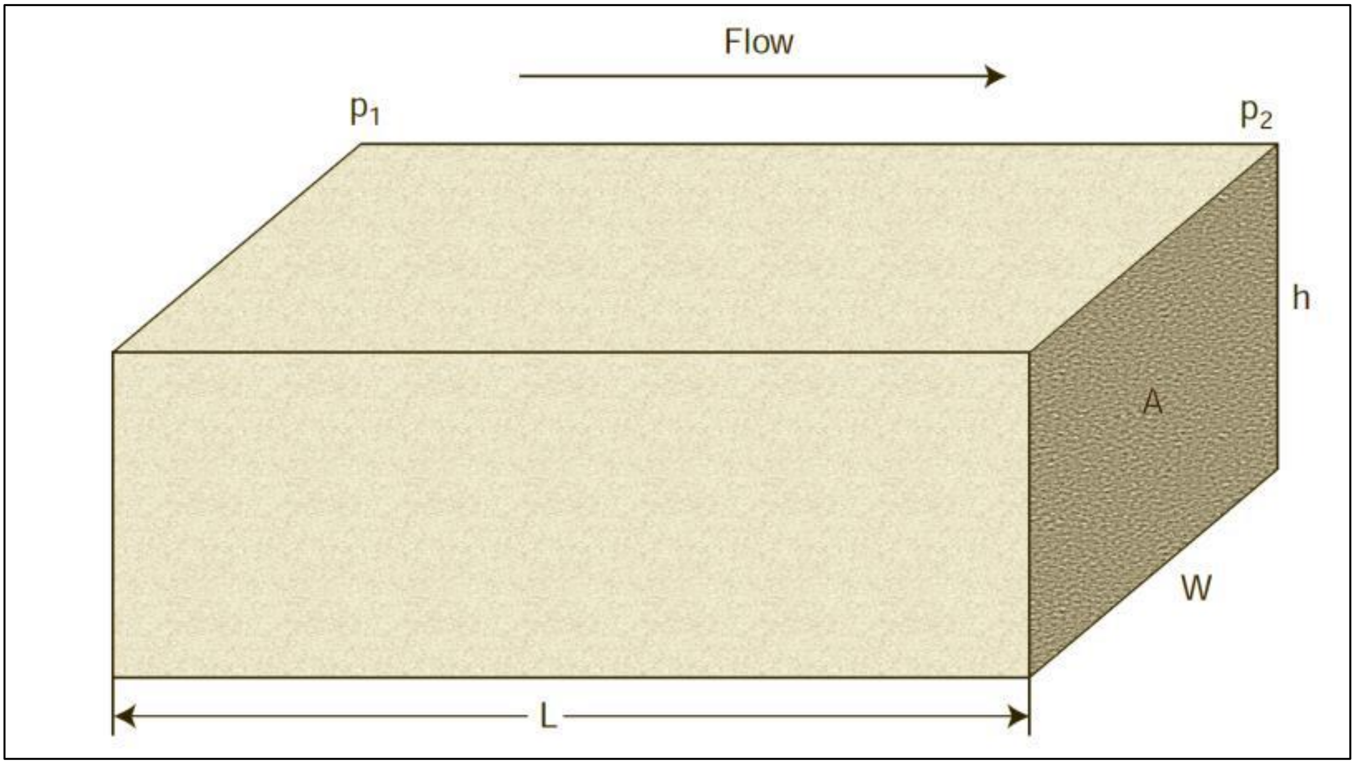

Fig. 6-1 linear flow model.

6.2. Dimensions of Permeability

Darcy's equation for linear flow is

$$q = \frac{KA}{\mu L}(P1 - P2) \qquad \text{……. (6-2)}$$

We can solve this equation for k value

$$K = \frac{q \, \mu \, L}{A(P1 - P2)}$$

- $q = L^3/T$,
- $A= L^2$,
- $\mu= m/L*T$,
- $P= \frac{\text{Force}}{\text{Area}} = \frac{m\,L\,/\,T^2}{L^2} = \frac{m}{L\,T^2}$

$$K = \frac{\frac{L^3}{T} \times \frac{m}{L \times T} \times L}{L^2 \times \frac{m}{L \times T^2}}$$

$K= L^2$

Thus the unit of permeability should be the cm2 in cgs units, or the metre2 in Sl units.

1 darcy is equivalent to 9.869233×10^{-13} m^2 or 0.9869233 μm^2.

The area unit (L2) is physically related to the cross-sectional area of pore-throats in a rock.Relatively large pore-throats of a rock imply relatively large values of L2 and thus correspond to relatively high values of permeability. That is to say, the permeability of a rock represents the average cross-sectional area of pores in the rock. Thereby, the greater the average cross-sectional area of rock pores, the higher the rock permeability, the better the permeable property of the rock, and the easier fluids flow through the rock.

6.3. Units of Permeability

Field units and Lab. units of Darcy law are shown in table below:

For steady-state linear flow in oilfield units equation (6-2) become:

$$q = \frac{0.001127 \times KA}{\mu L}\ (P1 - P2) \qquad \text{……. (6-3)}$$

q = volumetric flowrate, bbl/Day.

Table (6-1): field and Lab units of Darcy's law.

Parameter	Lab units	Field units	Conversion factor
q	cm^3/sec	bbl/day	$\frac{(30.48)^3 ft^3 . 5.615\, bbl/ft^3}{60 \times 60 \times 24\, day}$
k	$darcy$	md	10^{-3}
A	cm^2	ft^2	$(30.48)^2$
Δp	atm	psi	$1/14.7\, psi$
L	cm	ft	30.48
μ	cp	cp	

k = absolute permeability of the rock, millidarcy (md).

A = cross sectional area in the flow direction, ft^2.

P1 = inlet pressure, psi.

P2 = outlet pressure, psi.

μ = fluid viscosity, cp.

L = length of the rock, ft.

6.4. Types of permeability

1. **Absolute permeability**: is the permeability of a porous medium saturated with a single fluid.

2. **Effective permeability**: is a measure of the conductance of a porous medium for one fluid phase when the medium is saturated with more than one fluid.

$$K_e = K_a \quad When\ S_f = 100\%$$

3. **Relative permeability**: is the ratio of the effective permeability of a fluid at a given saturation to the absolute permeability.