Lecture Four: Rock Compressibility

4.1. Introduction

Rock compressibility is required for different engineering applications in petroleum engineering including material balance calculations, reservoir simulation, and well test analysis. The isothermal compressibility is defined as the reduction in volume per unit of volume with a unit change in reservoir pressure, expressed as C. Mathematically it is given by:

$$C = -\frac{1}{V}\left(\frac{dV}{dP}\right)_T \qquad \text{....... (4-1)}$$

4.2. Reservoir Pressures

Rocks when buried at reservoir depths are subject to both internal and external stresses. The internal stress results from the pressure of fluids in rock pores. The external stresses are exerted by the weight of the overburden (overlying formations) and any accompanying tectonic stresses.

Overburden pressures vary from area to area depending on factors such as depth, nature of the structure, consolidation of the formation, and possibly the geologic age and history of the rocks.

Depth of the formation is the most important consideration, and a typical value of overburden pressure gradient is approximately 1 psi/ft.

Before production, there is a balance in a reservoir system between the pressure gradients representing rock overburden pressure (p_{ob}), pore fluids pressure (p_f) and sediment grain pressure (p_g):

$$p_{0b} = p_g + p_f \qquad \text{....... (4-2)}$$

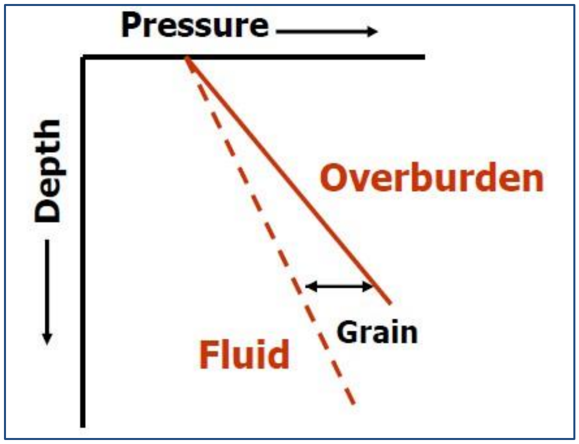


Fig. 4-1 Schematic diagram of Pressure gradient in the reservoir

Once the reservoir is disturbed by drilling and consequently production, the pressure balance is broken.

The pressure difference between overburden and internal pore pressure is referred to as the effective overburden pressure. During pressure depletion (oil reservoir production), the initial pore pressure decreases and the effective overburden pressure increases since the overburden load remains constant. This increase in pressure difference causes the following effects:

(a) The bulk volume of the reservoir rock is reduced;

(b) Sand grains constituting the rock expand;

(c) Fluids within the pore spaces expand.

The first two volume changes tend to reduce the pore space of the rock. Obviously, both the decrease in pore volume and the expansion of fluids will continuously drive fluids flowing out from pores. Moreover, the output of fluids from the reservoirs will lead to the farther decline in pore pressure and the release of the elastic energies of rock and fluids. The elastic energy of a rock is actually an exhibition of rock compressibility.

4.3. Reservoir Rock Compressibility

Geetsma states that three kinds of compressibilities must be distinguished in reservoir rocks, which are

1. Rock matrix (grains) compressibility is the fractional change in the volume of the solid rock material with a unit change in pressure and is mathematically expressed as

$$C_g = -\frac{1}{V_g}\left(\frac{\partial V_g}{\partial P}\right)_T \quad \text{....... (4-3)}$$

2. Rock bulk compressibility is the fractional change in volume of the bulk of the rock with a unit change in pressure and is mathematically expressed as

$$C_b = -\frac{1}{V_b}\left(\frac{\partial V_b}{\partial P}\right)_T \quad \text{....... (4-4)}$$

3. Pore compressibility is the fractional change in the pore volume of the rock with a unit change in pressure and is mathematically expressed as

$$C_p = -\frac{1}{V_p}\left(\frac{\partial V_p}{\partial P}\right)_T \quad \text{....... (4-5)}$$

An alternative expression of Equation (4-5) in terms of porosity was suggested, ϕ, by noting that porosity increases with the increase in the pore pressure:

$$C_p = \frac{1}{\phi}\left(\frac{\partial \phi}{\partial P}\right)_T \quad \text{....... (4-6)}$$

Since the rock and bulk compressibilities are considered small in comparison with the pore compressibility, the formation compressibility C_f is the term commonly used to describe the total compressibility of the formation and is equated to C_p:

$$C_f = C_p = \frac{1}{\phi}\left(\frac{\partial \phi}{\partial P}\right)_T$$

where

C_f = The formation compressibility (psi^{-1})

V_p = The bulk volume of the rock

∂V_p = The change in pore volume

∂P = the change in reservoir pressure (psi)

Of principal interest to the reservoir engineer is the change in the pore volume of the rock or also reffered to as effective rock compressibility or formation compressibility.

4.4. Measurement of compressibility

4.4.1 Rock Compressibility Correlations

1- Newman Correlation

For consolidate sandstones ($0.02<\phi<0.23$):

$$c_f = \frac{97.3 \cdot 10^{-6}}{[1 + 55.9\phi]^{1.429}}$$

For limestones ($0.02<\phi<0.33$):

$$c_f = \frac{0.856}{[1 + 2.48 \cdot 10^{6}\phi]^{0.930}} \quad \text{……. (4-7)}$$

2-Hall (1953) correlation:

$$c_f = \frac{1.782}{\phi^{0.438}} \quad \text{……. (4-8)}$$

4.4.2 Laboratory measurements

Rock compressibility is not measured in a conventional core analysis, but can be determined using hydrostatic or triaxial tests.

1- First the core volumes and porosity are calculated as studied in previous lectures, then the core is saturated 100% with water

2- The core exposes a gradual increase in applied pressure, which leads to a decrease in Pore volume and water expelled.
3- expelled volume increases as pore volume, vp, decreases, thus,
$dV_p = - dV_{expelled}$

$$c_f = + \frac{1}{V_p}\left(\frac{\Delta (V_p)expelled}{\Delta P}\right) \quad \text{....... (4-9)}$$

Example 1

Given the following lab data, calculate the pore volume compressibility for a sandstone sample at 4,000 and 6,000 psi.

pore volume = 50.0 cc

pressure, psi	vol. fluid expelled, cc
1000	0.244
2000	0.324
3000	0.392
4000	0.448
5000	0.500
6000	0.546
7000	0.596
8000	0.630

Solution:

from graph

@ 4,000 psi:

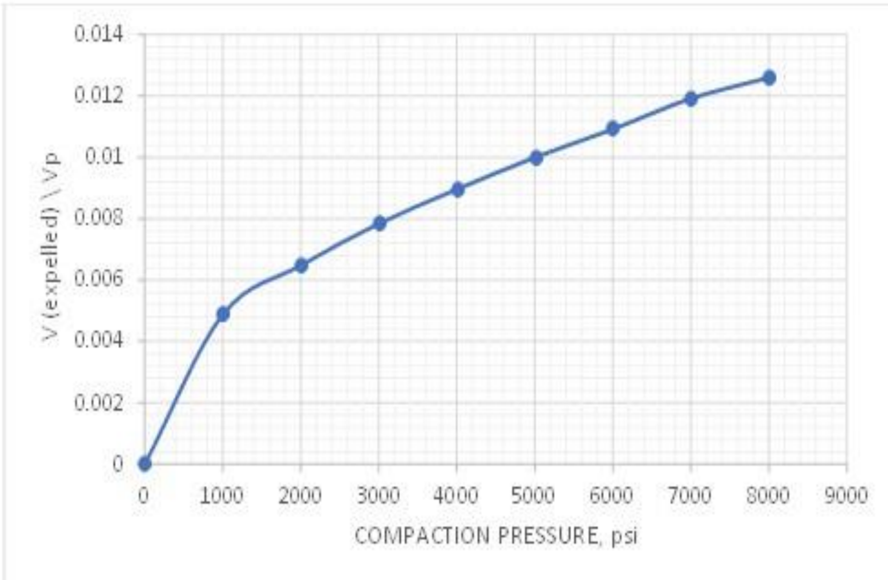
$$\text{Slope} = \left(\frac{0.009}{4000 \text{ psi}}\right)$$

$$c_f = \left(2.25 \text{ X } 10^{-6} \left(\frac{1}{\text{psi}}\right)\right)$$

@ 6000 psi:

$$\text{Slope} = \left(\frac{0.011}{6000 \text{ psi}}\right)$$

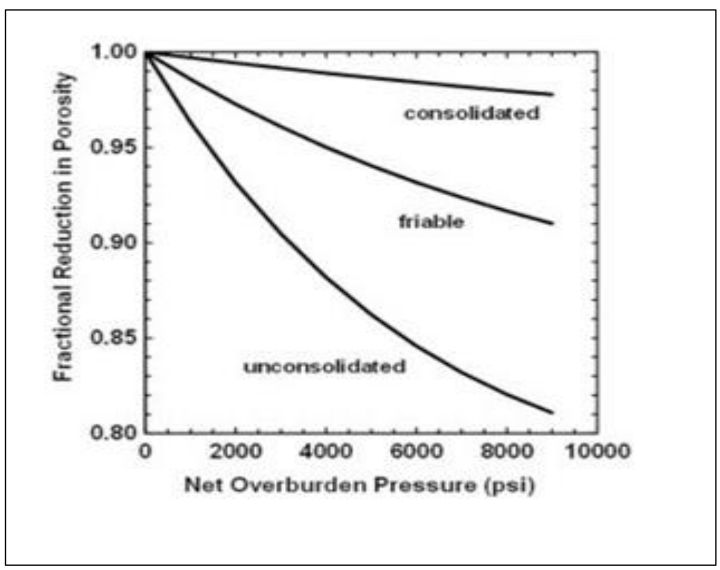
$$c_f = \left(1.83 \text{ X } 10^{-6} \left(\frac{1}{\text{psi}}\right)\right)$$



vol expelled /Vp	vol. fluid expelled, cc	pressure, psi
0	0	0
0.005	0.244	1000
0.006	0.324	2000
0.008	0.392	3000
0.009	0.448	4000
0.01	0.5	5000
0.011	0.546	6000
0.012	0.596	7000
0.013	0.63	8000

4.5. The effect of overburden pressure on porosity

Rock at reservoir conditions is subjected to external stress, while at the surface, recovered core has been relieved of the overburden stresses. It is not usual to perform routine porosity measurements under stress approaching reservoir conditions. Because of this, laboratory-measured porosities are generally expected to be higher than in situ values. Pore volume compressibility can be used to correct Laboratory-measured porosity to an in situ value and for other reservoir engineering calculations.



Usually, the effect of overburden pressure on porosity is negligible for consolidated rock, but can be significant for friable and unconsolidated rock. It is recommended to conduct compressibility tests whenever the reservoir rock is expected to be friable or unconsolidated.

Example 2

A 160-acre and 100 ft thick reservoir has a pore volume of 76,665,600 ft^3. The pore compressibility is 5×10^{-6} (1/psi). If the pressure decreases 3,000 psi, what is the change in porosity?

Solution

A = 160 acre (43,560) = 6,969,600 ft^2

V_b = 100 (6,969,600) = 696,960,000 ft^3

$$\emptyset = \frac{V_p}{V_b} = \frac{76{,}665{,}600}{696{,}960{,}000} = 0.11$$

$$C_p = \frac{1}{\emptyset}\frac{\Delta\emptyset}{\Delta P}$$

$$\Delta\emptyset = 0.11 \times 5 \times 10^{-6} \times 3{,}000 = 0.00165 = 0.165\%$$

Note: The total reservoir compressibility, denoted by C_t, is extensively used in reservoir engineering calculations and reservoir simulations defined by the following expression:

$$C_t = S_g C_g + S_o C_o + S_w C_w + C_f$$

where

Sg, So, and Sw are the gas, oil, and water saturations, respectively

Cg, Co, and Cw are the gas, oil, and water compressibilities, respectively, in psi^{-1}

Ct is the total reservoir compressibility in psi