# Momentum and Collisions

What happens when two automobiles collide?

How does the impact affect the motion of each vehicle, and what basic physical principles determine the likelihood of serious injury?

How do rockets work, and what mechanisms can be used to overcome the limitations imposed by exhaust speed?

Why do we have to brace (يحصن) ourselves when firing small projectiles at high velocity?

Finally, how can we use physics to improve our golf game?

## Momentum and Impulse

- Momentum has a precise definition. Momentum will depend on an object's mass and velocity
- The linear momentum $\vec{P}$of an object of mass $m$ moving with velocity $\vec{v}$ is the product of its mass and velocity:

$$\vec{\boldsymbol{P}} = \boldsymbol{m}\vec{\boldsymbol{v}}\,; \quad SI\ unit: (kg \cdot m/s)$$

- Momentum is a vector quantity with the same direction as the object's velocity. Its components are given in two dimensions by

$$p_x = mv_x \qquad p_y = mv_y$$

- The magnitude of the momentum $p$ of an object of mass m can be related to its kinetic energy KE:

$$\boldsymbol{KE} = \frac{\boldsymbol{p}^2}{2\boldsymbol{m}}$$

- Changing the momentum of an object requires the application of a force. This is, in fact, how Newton originally stated his second law of motion. Starting from the more common version of the second law, we have

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$$\vec{\mathbf{F}}_{\text{net}} = m\vec{\mathbf{a}} = m\frac{\Delta\vec{\mathbf{v}}}{\Delta t} = \frac{\Delta(m\vec{\mathbf{v}})}{\Delta t}$$

**The change in an object's momentum $\Delta\vec{p}$ divided by the elapsed time $\Delta t$ equals the constant net force $\vec{F}_{net}$ acting on the object:**

$$\frac{\Delta\vec{P}}{\Delta t} = \frac{\textit{change in momentum}}{\textit{time interval}} = \vec{F}_{net} \; \ldots$$ *Newton's second law and momentum*

- This equation is also valid when the forces are not constant, provided the limit is taken as $\Delta t$ becomes extremely tiny.
- Last equation says that if the net force on an object is zero, the object's momentum does not change.
- In other words, the linear momentum of an object is conserved when $\vec{F}_{net}=0$.
- Same equation also tells us that changing an object's momentum requires the continuous application of a force over a period of time $\Delta t$, leading to the definition of ***impulse***:
- If a constant force $\vec{F}$ acts on an object, the impulse $\vec{I}$ delivered to the object over a time interval $\Delta t$ is given by

$$\vec{\mathbf{I}} \equiv \vec{\mathbf{F}}\,\Delta t \quad \Longrightarrow \quad \textbf{SI unit: } (\mathbf{kg \cdot m/s})$$

- Impulse is a vector quantity with the same direction as the constant force acting on the object. When a single constant force $\vec{F}$ acts on an object, the Impulse equation can be written as

$$\vec{\mathbf{I}} = \vec{\mathbf{F}}\Delta t = \Delta\vec{\mathbf{p}} = m\vec{\mathbf{v}}_f - m\vec{\mathbf{v}}_i$$

- This is a special case of the impulse–momentum theorem. Last equation shows that ***the impulse of the force acting on an object equals the change in momentum.***
- The **average force** ($\vec{F}_{av}$) is the constant force delivering the same impulse to the object in the time interval $\Delta t$ as the actual time-varying force. We can then write the impulse–momentum theorem as

$$\vec{\mathbf{F}}_{\text{av}}\,\Delta t = \Delta\vec{\mathbf{p}}$$

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**<u>Example 8:</u>**

A golf ball with mass $5\times10^{-2}$ kg is struck with a club. The force on the ball varies from zero when contact is made up to some maximum value (when the ball is maximally deformed) and then back to zero. Assume that the ball leaves the club face (وجه المضرب الذي يضرب كرة الكَولف) with a velocity of +44 m/s.

(a) Find the magnitude of the impulse due to the collision.

(b) Estimate the duration of the collision and the average force acting on the ball.

**SOLUTION**

**(a)** Find the impulse delivered to the ball.

The problem is essentially one dimensional. Note that $v_i = 0$, and calculate the change in momentum, which equals the impulse:

$$I = \Delta p = p_f - p_i = (5.0 \times 10^{-2}\ \text{kg})(44\ \text{m/s}) - 0 = \boxed{+2.2\ \text{kg}\cdot\text{m/s}}$$

**(b)** Estimate the duration of the collision and the average force acting on the ball.

Estimate the time interval of the collision, $\Delta t$, using the approximate displacement (radius of the ball) and its average speed (half the maximum speed):

$$\Delta t = \frac{\Delta x}{v_{\text{av}}} = \frac{2.0 \times 10^{-2}\ \text{m}}{22\ \text{m/s}} = \boxed{9.1 \times 10^{-4}\ \text{s}}$$

Estimate the average force from Equation 6.6:

$$F_{\text{av}} = \frac{\Delta p}{\Delta t} = \frac{2.2\ \text{kg}\cdot\text{m/s}}{9.1 \times 10^{-4}\ \text{s}} = \boxed{+2.4 \times 10^{3}\ \text{N}}$$

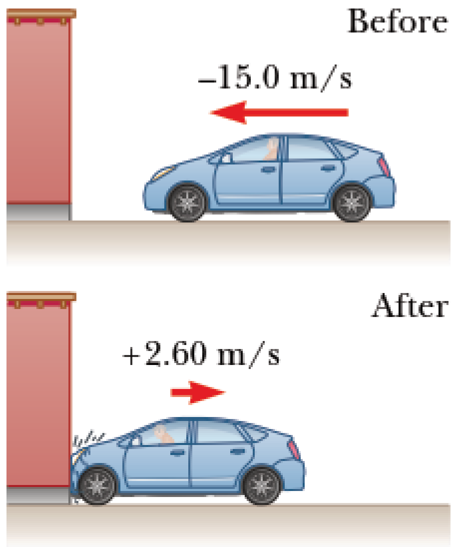
***REMARKS:*** This estimate shows just how large such contact forces can be. A good golfer achieves maximum momentum transfer by shifting weight from the back foot to the front foot, transmitting the body's momentum through the shaft and head of the club. This timing, involving a short movement of the hips, is more effective than a shot powered exclusively by the arms and shoulders. Following through with the swing ensures that the motion is not slowed at the critical instant of impact.

## Example 8:

In a crash test, a car of mass $1.5\times10^{3}$ kg collides with a wall and rebounds (ارتدت) as in the fig. The initial and final velocities of the car are $v_i$ = -15m/s and $v_f$ =2.6 m/s, respectively. If the collision lasts for 0.15 s, find

(a) the impulse delivered to the car due to the collision and

(b) The size and direction of the average force exerted on the car.




**SOLUTION**

**(a)** Find the impulse delivered to the car.

Calculate the initial and final momenta of the car:

$$p_i = mv_i = (1.50 \times 10^3\ \text{kg})(-15.0\ \text{m/s}) = -2.25 \times 10^4\ \text{kg}\cdot\text{m/s}$$

$$p_f = mv_f = (1.50 \times 10^3\ \text{kg})(+2.60\ \text{m/s}) = +0.390 \times 10^4\ \text{kg}\cdot\text{m/s}$$

The impulse is just the difference between the final and initial momenta:

$$I = p_f - p_i = +0.390 \times 10^4\ \text{kg}\cdot\text{m/s} - (-2.25 \times 10^4\ \text{kg}\cdot\text{m/s})$$

$$I = \boxed{2.64 \times 10^4\ \text{kg}\cdot\text{m/s}}$$

**(b)** Find the average force exerted on the car.

Apply Equation 6.6, the impulse–momentum theorem:

$$F_{\text{av}} = \frac{\Delta p}{\Delta t} = \frac{2.64 \times 10^4\ \text{kg}\cdot\text{m/s}}{0.150\ \text{s}} = \boxed{+1.76 \times 10^5\ \text{N}}$$

***REMARKS:*** When the car does not rebound off the wall, the average force exerted on the car is smaller than the value just calculated. With a final momentum of zero, the car undergoes a smaller change in momentum.