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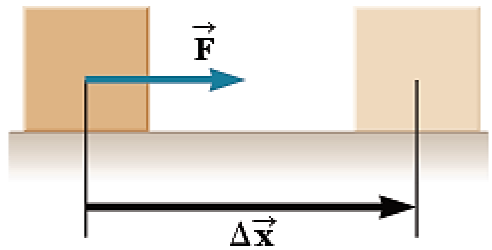
Work and Energy

Work (W) is the energy transferred to or from an object via the application of force ($\vec{F}$) along a displacement ($\overrightarrow{\Delta x}$).

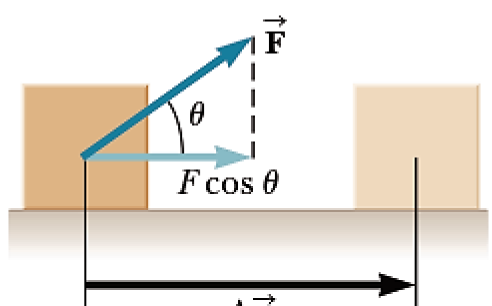
In its simplest form, it is often represented as the product of force and displacement.

$$\boldsymbol{W = \vec{F} \cdot \overrightarrow{\Delta x}}$$



- A force is said to do **positive work** ($\boldsymbol{W > 0}$) if (when applied) it has a component *in the direction of the displacement* of the point of application.
- A force does **negative work** ($\boldsymbol{W < 0}$) if it has a component *opposite to the direction of the displacement* at the point of application of the force.
- For example, when a ball is held above the ground and then dropped, the work done by the gravitational force on the ball as it falls is equal to the weight of the ball (a force) multiplied by the distance to the ground (a displacement).
- When the force **F** is constant and the angle between the force and the displacement $\Delta\boldsymbol{x}$ is θ, then the work done is given by:

$$\boldsymbol{W = F \cos\theta \; \Delta x}$$

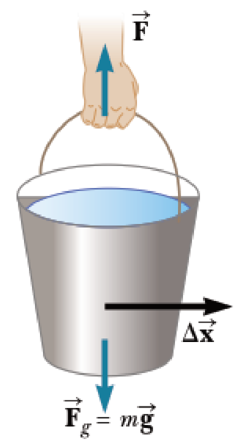


- This equation in last figure is shown; the components of the vector F can be written as $\boldsymbol{F_x = F\cos\theta}$ and $\boldsymbol{F_y = F\sin\theta}$. However, only the *x-component*, which is parallel to the direction of motion, makes a nonzero contribution to the work done on the object.
    - Work negative: W < 0 if 180°> $\boldsymbol{\theta}$ > 90°
    - Work zero: W = 0 if $\boldsymbol{\theta}$ = 90°
    - Work maximum if $\boldsymbol{\theta}$ = 0°
    - Work minimum if $\boldsymbol{\theta}$ = 180°

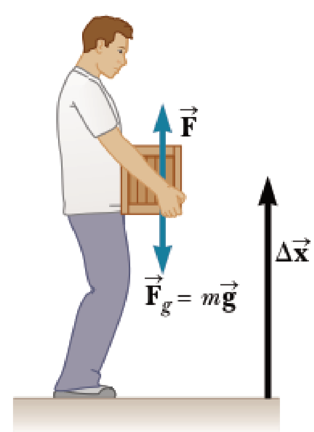
$$\boldsymbol{W = F\cos\theta\ \Delta x}$$

For example:

- A man carries a bucket of water and moves horizontally at constant velocity.
- The force does no work on the bucket
- Displacement is horizontal
- Force is vertical
- cos 90° = 0



- When a student lifts a box as in the figure, the work he does on the box is positive because the force he exerts on the box is upward, in the same direction as the displacement.
- In lowering the box slowly back down, how- ever, the student still exerts an upward force on the box, but the motion of the box is downwards.



- Work is a scalar quantity the **W** unit in SI is:
    - Newton • meter = Joule or
    - N • m = J = (kg • m / s^2) • m = $kg.\frac{m^2}{s^2}$

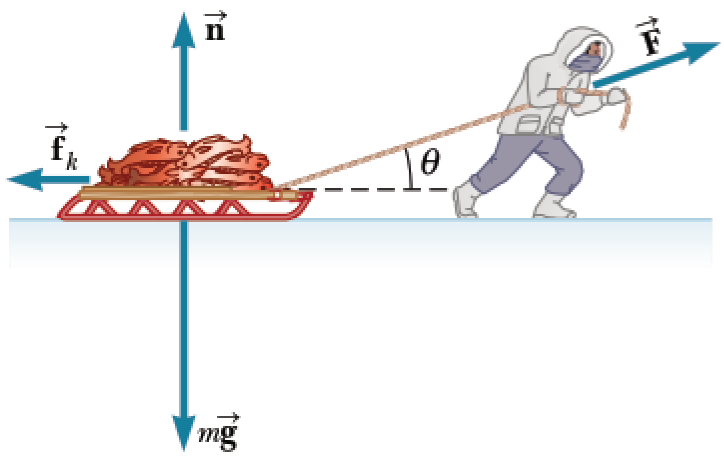
Example 1: An Eskimo returning from a successful fishing trip pulls a sled loaded with salmon. The total mass of the sled and salmon is 50 kg, and the

Eskimo exerts a force of magnitude 1.2×10^2 N on the sled by pulling on the rope.

(a) How much work does he do on the sled if the rope is horizontal to the ground (θ = 0°) and he pulls the sled 5 m?

(b) How much work does he do on the sled if (θ = 30°) and he pulls the sled the same distance?

(c) At a coordinate position of 12.4 m, the Eskimo lets up on the applied force. A friction force of 45 N between the ice and the sled brings the sled to rest at a coordinate position of 18.2 m. How much work does friction do on the sled?



SOLUTION

(a) Find the work done when the force is horizontal.

600J

$$W = F_x \Delta x = (1.20 \times 10^2 \text{ N})(5.00 \text{ m}) = 6.00 \times 10^2 \text{ J}$$

(b) Find the work done when the force is exerted at a 30° angle.

$$W = (F\cos\theta)\Delta x = (1.20 \times 10^2 \text{ N})(\cos 30°)(5.00 \text{ m})$$

$$= 5.20 \times 10^2 \text{ J}$$

519.615 J

520 J

(c) How much work does a friction force of 45.0 N do on the sled as it travels from a coordinate position of 12.4 m to 18.2 m?

F_x replaced by f_k:

$$W_{\text{fric}} = F_x \Delta x = f_k(x_f - x_i)$$

Substitute $f_k = -45.0$ N and the initial and final coordinate positions into x_i and x_f:

$$W_{\text{fric}} = (-45.0 \text{ N})(18.2 \text{ m} - 12.4 \text{ m}) = -261 \text{ N.m}$$

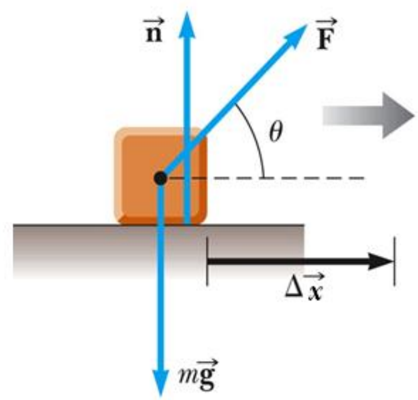
Work Done by Multiple Forces

- If more than one force acts on an object, then the total work is equal to the algebraic sum of the work done by the individual forces

$$\mathbf{W}_{\mathbf{net}} = \sum \mathbf{W}_{individual\ forces}$$

- Remember *Work* is a *Scalar*, so this is the algebraic sum.

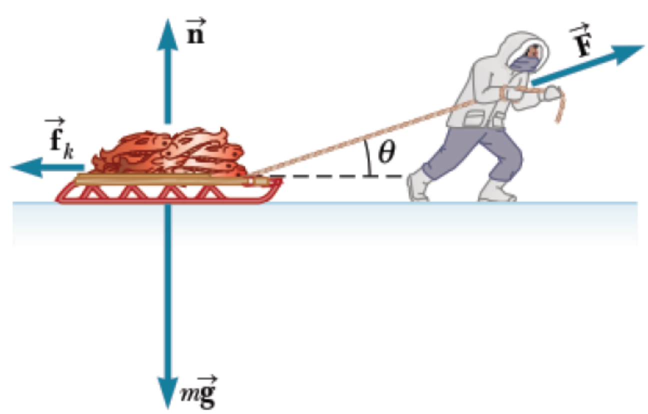
$$\mathbf{W}_{\mathbf{net}} = \mathbf{W}_{g} + \mathbf{W}_{N} + \mathbf{W}_{F} = F\ \cos\theta\ \Delta x$$



Return to Eskimo example: Suppose $\mu_k = 0.200$, How much work done on the sled by friction, and the net work if $\theta = 30°$ and he pulls the sled 5m.

$$F_{net,y} = N - mg + F\sin\theta = 0 \rightarrow N = mg - F\sin\theta$$

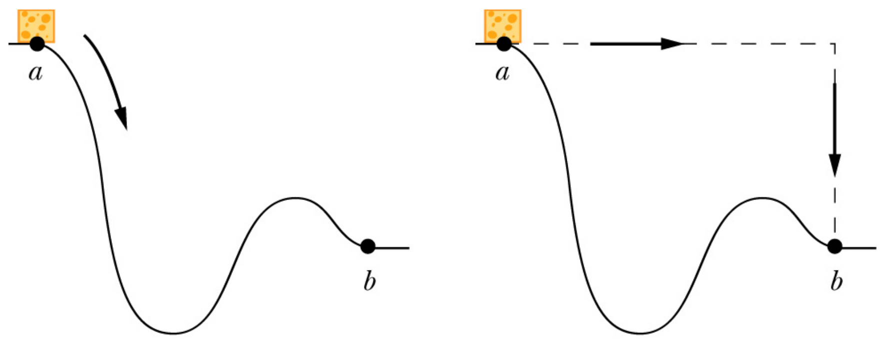
$$W_{fric} = (f_k \cos 180^{\circ})\Delta x = -f_k \Delta x$$
$$= -\mu_k N \Delta x = -\mu_k (mg - F\sin\theta)\Delta x$$
$$= -(0.2)(50kg) \bullet 9.8m/s^2$$
$$-1.2\times10^2 N \sin 30^{\circ} \bullet 5.m)$$
$$= -4.3\times10^2 J$$



$$W_{net} = W_F + W_{fric} + W_N + W_g$$
$$= 5.2\times10^2 J - 4.3\times10^2 J + 0 + 0$$
$$= 90J$$

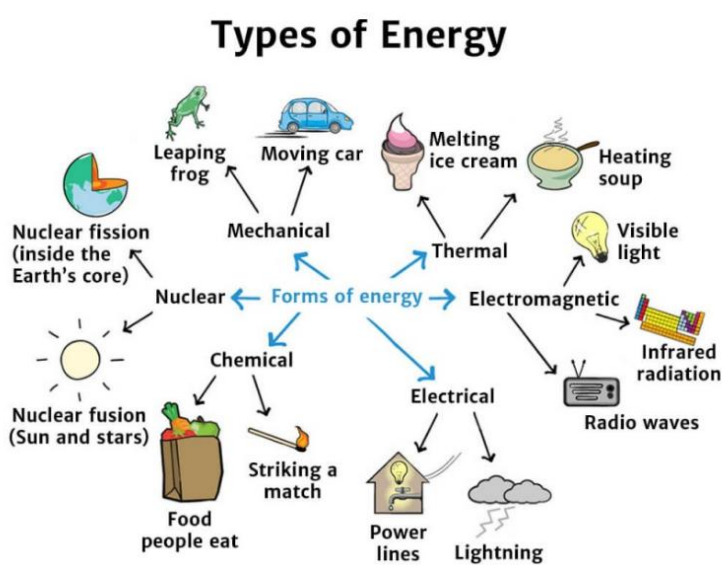
Energy

- The energy approach to describing motion is particularly useful when Newton's Laws are difficult or impossible to use.
- Energy is a *scalar quantity*. It does not have a direction associated with it.



- Energy is a property of the state of a system, not a property of individual objects: we have to expand our view.
- Some forms of energy:
  (a) Mechanical:
    - Kinetic energy (associated with ***motion***, within system)
    - Potential energy (associated with ***position***, within system)

  (b) Chemical
  (c) Electromagnetic
  (d) Nuclear



- Energy is conserved (محفوظة). It can be transferred from one object to another or change in form, but cannot be ***created*** or ***destroyed***

Kinetic Energy (K.E) is energy associated with the state of motion of an object. For an object moving with a speed of v

$$K.E = \frac{1}{2}mv^2$$

- **SI unit**: joule (J)

  1 J = 1 joule = 1 kg m^2/s^2

For constant acceleration

Recall the equation: $$v_f^2 = v_i^2 + 2a\Delta x$$

Or $$v_f^2 - v_i^2 = 2a\Delta x$$

$$[v_f^2 - v_i^2 = 2a\Delta x] \times \frac{1}{2}m$$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = ma\,\Delta x \quad ; but, ma = F$$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = F\,\Delta x = W$$

- Work provides a link between force and energy
- Work done on an object is transferred to/from it
  - If W > 0, energy added: "transferred to the object"
  - If W < 0, energy taken away: "transferred from the object"

Work-Kinetic Energy Theorem

- When work is done by a net force on an object and the only change in the object is its speed, the work done is equal to the change in the object's kinetic energy
  - Speed will increase if work is positive
  - Speed will decrease if work is negative

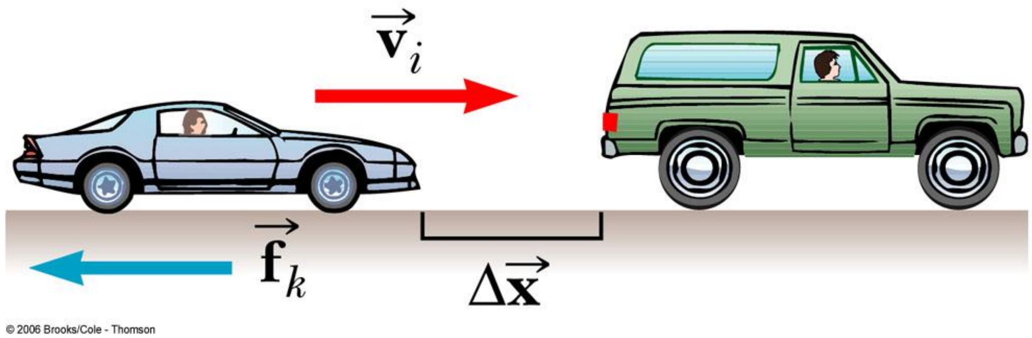
$$W_{net} = K.E_f - K.E_i = \Delta K.E$$

$$W_{net} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Example 2:

The driver of a car ($1\text{x}10^3$ kg) traveling on 35 m/s slam (كبس) on his brakes to avoid hitting a second vehicle in front of him, which had come to rest because of congestion (زحام) ahead. After the breaks are applied, a constant friction force of 8×10^3 N acts on the car. *Ignore air resistance*.

(a) At what minimum distance should the brakes be applied to avoid a collision with the other vehicle?
(b) If the distance between the vehicles were initially only 30m, at what speed would the collisions occur?



SOLUTION:

(a) We know

$$v_i = 35\frac{m}{s}, v_f = 0, m = 10^3 kg, f_k = 8\times10^3 N$$

Find the minimum necessary stopping distance

$$W_{net} = W_{fric} + W_g + W_N = W_{fric} = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$-f_k\Delta x = 0 - \frac{1}{2}mv_0^2$$

$$-(8.00\times10^3 N)\Delta x = -\frac{1}{2}(1.00\times10^3 kg)(35.0m/s)^2$$

$$\Delta x = 76.6m$$

$35\frac{m}{s} = 35 * \frac{km}{1000} * \frac{3600}{hr} = 126\frac{km}{hr}$

$km = 1000m \Rightarrow m = \frac{km}{1000}$

$hr = 3600s \Rightarrow s = \frac{hr}{3600}$

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(b) We know $\Delta x = 30m, v_i = 35\frac{m}{s}, v_f = 0, m = 10^3 kg, f_k = 8 \times 10^3 N$

Find the speed at impact. Write down the work-energy theorem:

$$W_{net} = W_{fric} = -f_k \Delta x = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$v_f^2 = v_i^2 - \frac{2}{m} f_k \Delta x$$

$$v_f^2 = (35m/s)^2 - \left(\frac{2}{1\times 10^3 kg}\right)(8\times 10^3 N)(30m)$$

$$= 745 m^2/s^2 \Rightarrow v_f = 27.3 m/s$$

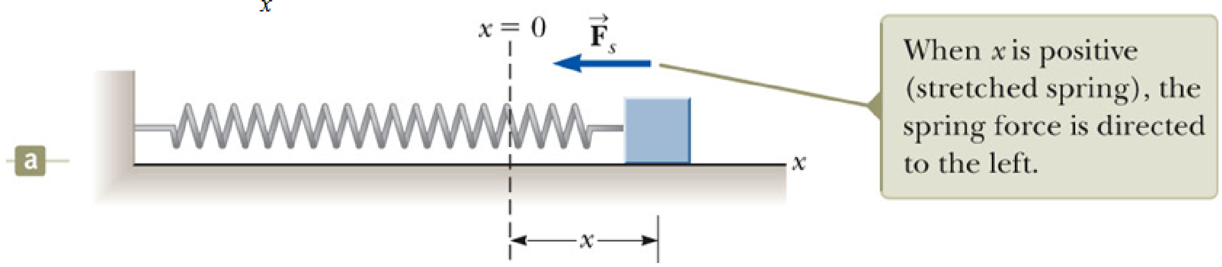
$v_f = 27.3 \times \frac{km}{1000} \times \frac{3600}{hr}$

$= 98.28 \frac{km}{hr}$

Work Done By a Spring

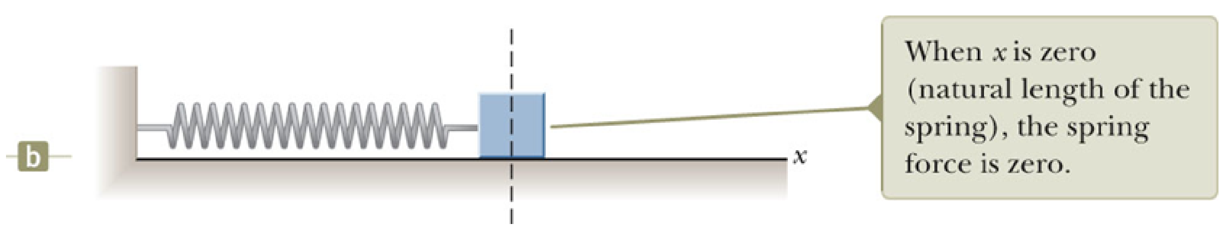
- Spring force

$$F_x = -kx$$

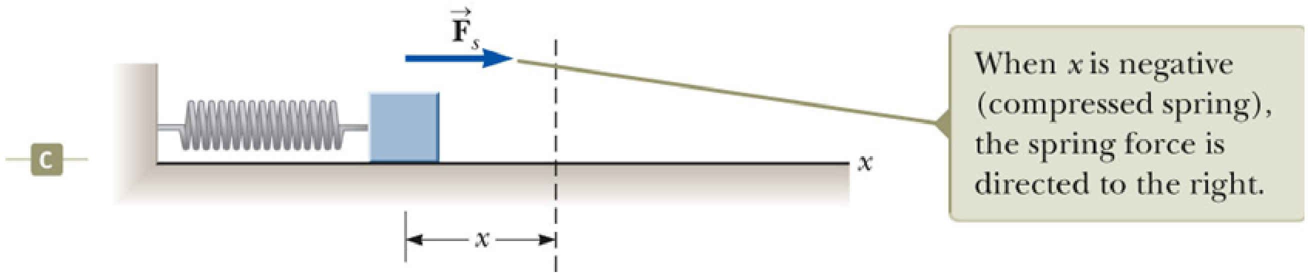


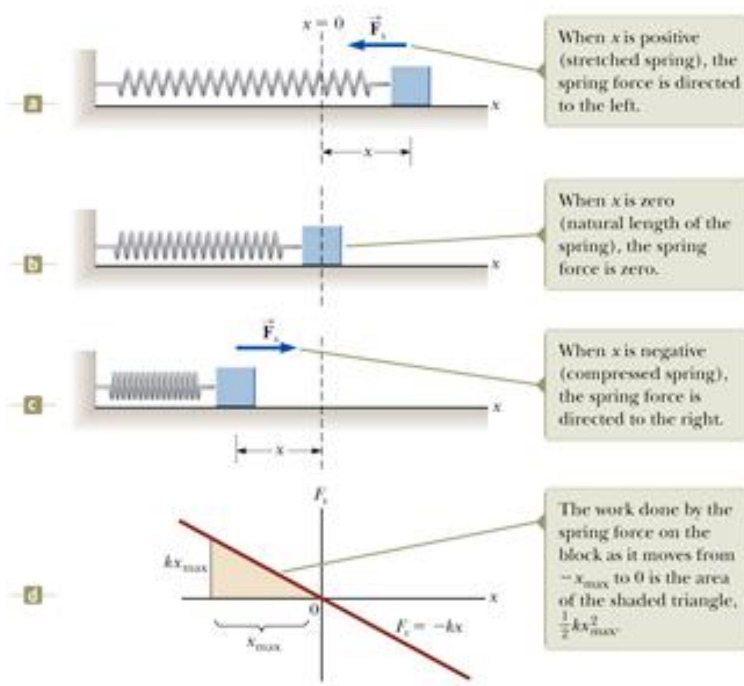
Spring at Equilibrium

- F = 0



Spring Compressed





$$\lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x \, dx$$

$$W = \int_{x_i}^{x_f} F_x \, dx = \int_{x_i}^{x_f} -kx \, dx$$

$$= \int_{-x_{max}}^{0} -kx \, dx = \tfrac{1}{2} kx^2$$

$$W = \int_{x_i}^{x_f} -kx \, dx = \tfrac{1}{2} kx_i^2 - \tfrac{1}{2} kx_f^2$$

Work done by spring on block

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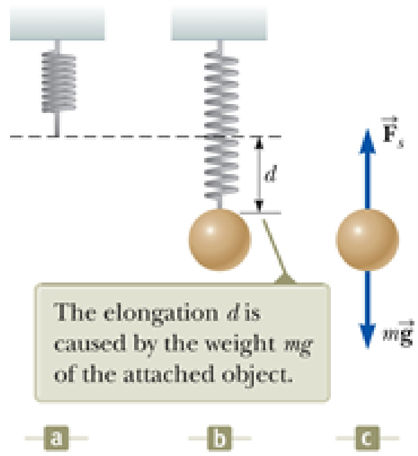
Measuring Spring Constant

- Start with spring at its natural equilibrium length.
- Hang a mass on spring and let it hang to distance d (stationary)
- From $F_x = kx - mg = 0$

$$k = \frac{mg}{d}$$

so can get spring constant.



Potential Energy P.E

- Determined by the configuration of the system
- Gravitational and Elastic

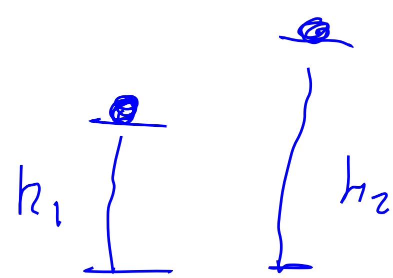
$PE_2 > PE_1$

$mgh_2 > mgh_1$

w = force

mgh

$E_{Total} = K.E + P.E$



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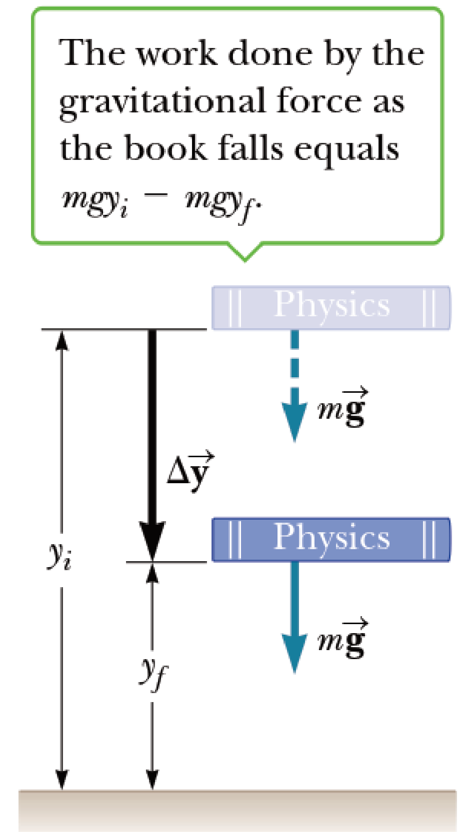
Work and Energy….

Gravitational Work and Potential Energy (P. E)

- Determined by the configuration of the system
- Gravitational and Elastic.
- Using the work–energy theorem in problems involving gravitation requires computing the work done by gravity.
- For a ball traversing a parabolic arc—finding the gravitational work done on the ball requires sophisticated techniques from calculus. Fortunately, for conservative fields there's a simple alternative: ***Potential Energy*.**
- *Potential energy* is a scalar quantity.
- When book mass (m) falls, How much *work (W)* is done?



- The magnitude of the force is mg and that of the displacement is $\Delta y = y_i - y_f$.
- Both $\vec{F}$ *and* $\Delta\vec{y}$ are pointing downwards, so the angle $\rightarrow \theta = 0$, then the work is equal;

$$W = F\cos\theta\,\Delta y = mg\,\cos\theta\,(y_i - y_f) = mg\,(y_i - y_f)$$

- The gravitational ***Potential Energy*** of a system consisting of Earth and an object of mass m near Earth's surface is given by

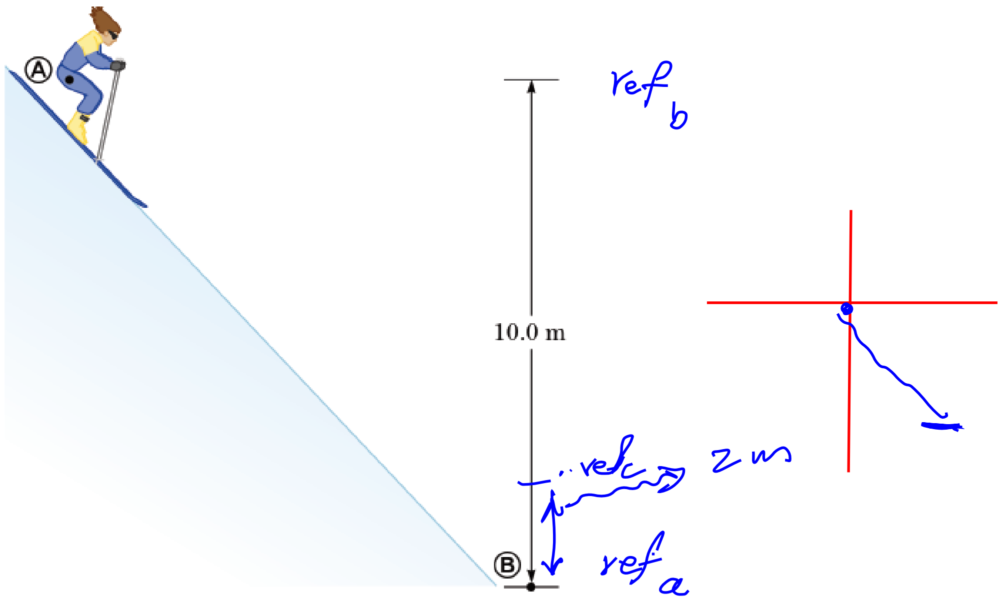
$$\boldsymbol{PE} = mgy \;\dots\dots\dots\dots (\#)$$; **PE** units in SI also joule

  Where g is the acceleration of gravity and y is the vertical position of the mass relative the surface of Earth (or some other reference point).
- Finally the work–energy theorem;

$$W = \Delta KE + \Delta PE = (KE_f - KE_i) + (PE_f - PE_i)$$

Example 3:

A 60.0-kg skier is at the top of a slope, as shown in Next Figure At the initial point Ⓐ, she is 10.0 m vertically above point Ⓑ. **(a)** Setting the zero level for gravitational potential energy at Ⓑ, find the gravitational potential energy of this system when the skier is at Ⓐ and then at Ⓑ. Finally, find the change in potential energy of the skier–Earth system as the skier goes from point Ⓐ to point Ⓑ. **(b)** Repeat this problem with the zero level at point Ⓐ. **(c)** Repeat again, with the zero level 2.00 m higher than point Ⓑ.



SOLUTION

(a) Let $y = 0$ at Ⓑ. Calculate the potential energy at Ⓐ and at Ⓑ, and calculate the change in potential energy.

Find PE_i, the potential energy at Ⓐ, from Equation # :

$$PE_i = mgy_i = (60.0\text{ kg})(9.80\text{ m/s}^2)(10.0\text{ m}) = 5.88 \times 10^3\text{ J}$$

$PE_f = 0$ at Ⓑ by choice. Find the difference in potential energy between Ⓐ and Ⓑ:

$$PE_f - PE_i = 0 - 5.88 \times 10^3\text{ J} = \boxed{-5.88 \times 10^3\text{ J}}$$

(b) Repeat the problem if $y = 0$ at Ⓐ, the new reference point, so that PE $= 0$ at Ⓐ.

Find PE_f, noting that point Ⓑ is now at $y = -10.0$ m:

$$PE_f = mgy_f = (60.0\text{ kg})(9.80\text{ m/s}^2)(-10.0\text{ m}) = -5.88 \times 10^3\text{ J}$$

$$PE_f - PE_i = -5.88 \times 10^3\text{ J} - 0 = \boxed{-5.88 \times 10^3\text{ J}}$$

(c) Repeat the problem, if $y = 0$ two meters above Ⓑ.

Find PE_i, the potential energy at Ⓐ:

$$PE_i = mgy_i = (60.0\text{ kg})(9.80\text{ m/s}^2)(8.00\text{ m}) = 4.70 \times 10^3\text{ J}$$

Find PE_f, the potential energy at Ⓑ:

$$PE_f = mgy_f = (60.0\text{ kg})(9.8\text{ m/s}^2)(-2.00\text{ m}) = -1.18 \times 10^3\text{ J}$$

Compute the change in potential energy:

$$PE_f - PE_i = -1.18 \times 10^3\text{ J} - 4.70 \times 10^3\text{ J} = \boxed{-5.88 \times 10^3\text{ J}}$$

REMARKS: These calculations show that the change in the potential energy when the skier goes from the top of the slope to the bottom is -5.88×10^{3} J, regardless of the zero level selected.

Gravity and the Conservation of Mechanical Energy

Conservation principles play a very important role in physics. **When a physical quantity is conserved the numeric value of the quantity remains the same throughout the physical process**. Although the form of the quantity may change in some way, **its final value is the same as its initial value.**

The kinetic energy KE of an object falling only under the influence of gravity is ***constantly*** changing, as is the gravitational potential energy PE.

$$KE_i + PE_i = KE_f + PE_f$$

According to last equation, the sum of the kinetic energy and the potential energy remains constant at all times and hence is a conserved quantity. We denote the total mechanical energy by ***E = KE + PE***, and say that the total mechanical energy is conserved (Conservation Principles).

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$$

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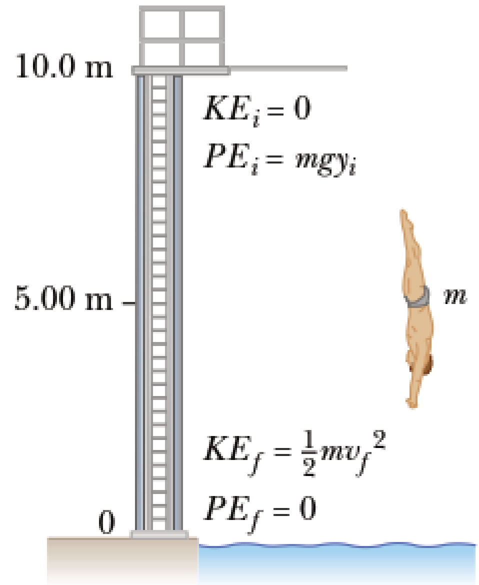
Example 4:

A diver of mass m drops from a board 10 m above the water's surface, as in the fig. Neglect air resistance.



(a) Use conservation of mechanical energy to find his speed 5 m above the water's surface.

(b) Find his speed as he hits the water.

SOLUTION

(a) Find the diver's speed halfway down, at $y = 5.00$ m.

We write the energy conservation equation and supply the proper terms:

$$KE_i + PE_i = KE_f + PE_f$$

$$\frac{1}{2}mv_i^2 + mgy_i = \frac{1}{2}mv_f^2 + mgy_f$$

Substitute $v_i = 0$, cancel the mass m, and solve for v_f:

$$0 + gy_i = \frac{1}{2}v_f^2 + gy_f$$

$$v_f = \sqrt{2g(y_i - y_f)} = \sqrt{2(9.80 \text{ m/s}^2)(10.0 \text{ m} - 5.00 \text{ m})}$$

$$v_f = 9.90 \text{ m/s}$$

(b) Find the diver's speed at the water's surface, $y = 0$.

Use the same procedure as in part (a), taking $y_f = 0$:

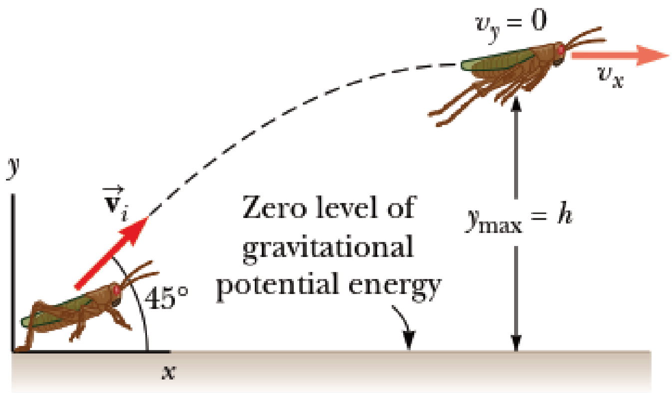
$$0 + mgy_i = \frac{1}{2}mv_f^2 + 0$$

$$v_f = \sqrt{2gy_i} = \sqrt{2(9.80 \text{ m/s}^2)(10.0 \text{ m})} = 14.0 \text{ m/s}$$

REMARKS: Notice that the speed halfway down is not half the final speed. Another interesting point is that the final answer doesn't depend on the ***mass***. That is really a consequence of neglecting the change in kinetic energy of Earth, which is valid when the mass of the object, the diver in this case, is much smaller than the mass of Earth. In reality, Earth also falls towards the diver, reducing the final speed, but the reduction is so ***tiny*** it could never be measured.

Example 5:

A powerful grasshopper launches itself at an angle of 45° above the horizontal and rises to a maximum height of 1 m during the jump. (See Fig.) With what speed v_i did it leave the ground? Neglect air resistance.



SOLUTION

Use energy conservation:

$$\tfrac{1}{2}mv_i^2 + mgy_i = \tfrac{1}{2}mv_f^2 + mgy_f$$

Substitute $y_i = 0$, $v_f = v_x$, and $y_f = h$:

$$\tfrac{1}{2}mv_i^2 = \tfrac{1}{2}mv_x^2 + mgh$$

Multiply each side by $2/m$, obtaining one equation and two unknowns:

$$(1) \quad v_i^2 = v_x^2 + 2gh$$

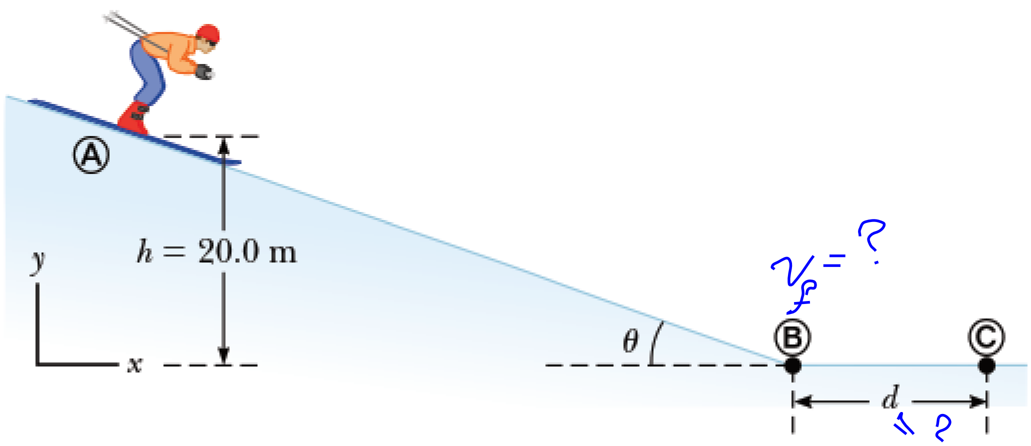
Eliminate v_x by substituting $v_x = v_i \cos 45°$ into Equation (1), solving for v_i, and substituting known values:

$$v_i^2 = (v_i \cos 45°)^2 + 2gh = \tfrac{1}{2}v_i^2 + 2gh$$

$$v_i = 2\sqrt{gh} = 2\sqrt{(9.80 \text{ m/s}^2)(1.00 \text{ m})} = \boxed{6.26 \text{ m/s}}$$

REMARKS: The final answer is a surprisingly high value and illustrates how strong insects are relative to their size.

Example 6:



A skier starts from rest at the top of a frictionless incline of height 20 m, as in the above fig. At the bottom of the incline, the skier encounters a horizontal surface where the coefficient of kinetic friction (μ_k) between skis and snow is 0.21. (a) Find the skier's speed at the bottom. (b) How far does the skier travel on the horizontal surface before coming to rest? Neglect air resistance.

SOLUTION

(a) Find the skier's speed at the bottom.

Follow the procedure used in part (a) of the previous example as the skier moves from the top, point Ⓐ, to the bottom, point Ⓑ:

$$v_{Ⓑ} = \sqrt{2gh} = \sqrt{2(9.80\ \text{m/s}^2)(20.0\ \text{m})} = \boxed{19.8\ \text{m/s}}$$

(b) Find the distance traveled on the horizontal, rough surface.

$$W_{net} = \Delta KE + \Delta PE$$

Apply the work–energy theorem as the skier moves from Ⓑ to Ⓒ:

$$W_{\text{net}} = -f_k d = \Delta KE = \tfrac{1}{2}mv_{Ⓒ}^2 - \tfrac{1}{2}mv_{Ⓑ}^2$$

Substitute $v_{Ⓒ} = 0$ and $f_k = \mu_k n = \mu_k mg$:

$$-\mu_k mgd = -\tfrac{1}{2}mv_{Ⓑ}^2$$

Solve for d:

$$d = \frac{v_{Ⓑ}^2}{2\mu_k g} = \frac{(19.8\ \text{m/s})^2}{2(0.210)(9.80\ \text{m/s}^2)} = \boxed{95.2\ \text{m}}$$

REMARKS: Substituting the symbolic expression $v_{Ⓑ} = \sqrt{2gh}$ into the equation for the distance d shows that d is linearly proportional to h: Doubling the height doubles the distance traveled.

EXERCISE: Find the horizontal distance the skier travels before coming to rest if the incline also has a coefficient of kinetic friction equal to 0.21. Assume that $\theta = 20°$.

ANSWER: 40.3 m

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Spring Potential Energy

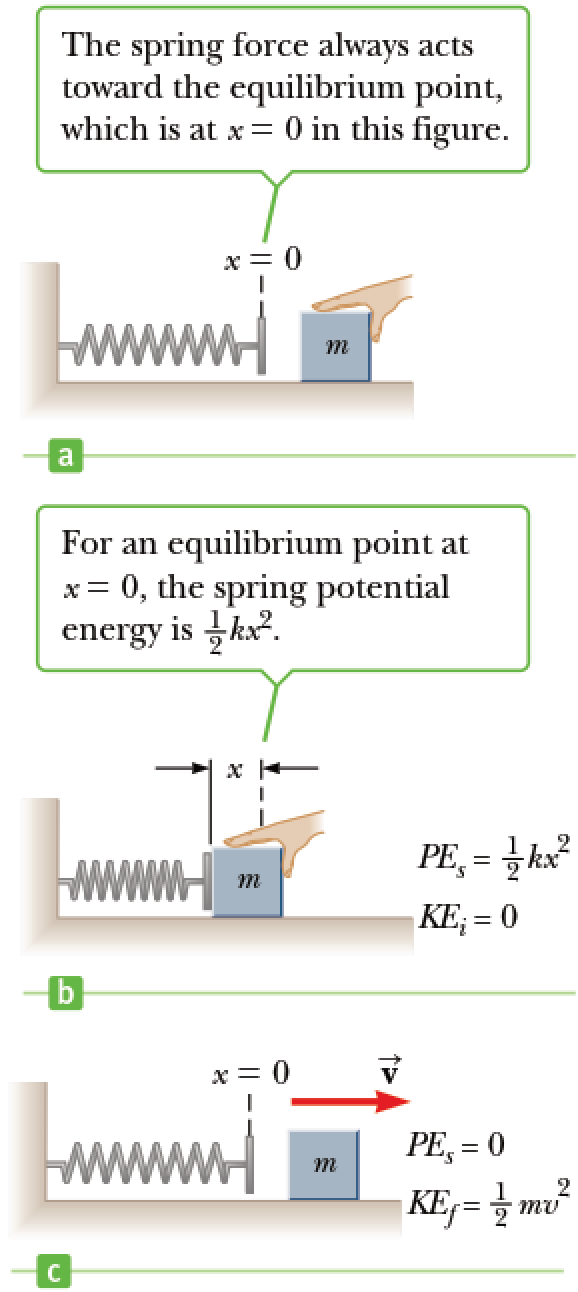
- Work done by an applied force in stretching or compressing a spring can be recovered by removing the applied force, so like gravity, the spring force is conservative, as long as losses through internal friction of the spring can be neglected.
- This means a *Potential Energy* function can be found and used in the work–energy theorem.

  $F_s = -kx$ ... (***Hooke's law***)
- Where k is a constant of proportionality, the *spring constant*, carrying units of (N/m).
- the *Elastic Potential Energy* associated with the spring force, $\boldsymbol{PE_s}$, by

$$\boldsymbol{PE_s} = \frac{1}{2}kx^2$$

- Finally, because $\boldsymbol{PE_s}$ is proportional to x^2, the potential energy is always positive when the spring is not in the equilibrium position.
- The extended form for conservation of mechanical energy results:

$$(KE + PE_g + PE_s)_i = (KE + PE_g + PE_s)_f$$



Example 7: A block with mass of 5 kg is attached to a horizontal spring with spring constant $k = 4\times10^2$ N/m. The surface the block rests upon is frictionless. If the block is pulled out to $x_i = 0.05$ m and released, (a) find the speed of the block when it first reaches the equilibrium point, (b) find the speed when $x_i = 0.025$m, and (c) repeat part (a) if friction acts on the block, with coefficient μ_k =0.15. ➔ *p176 Serway v.9.*