PDF-XChange Viewer Click to buy NOW! www.docu-track.com

Two-dimensional motion

First: Concept Explanation

We have learned how to use kinematic equations to describe the motion of an object moving horizontally, like a car , as well as objects moving vertically, like objects falling straight down to the ground,

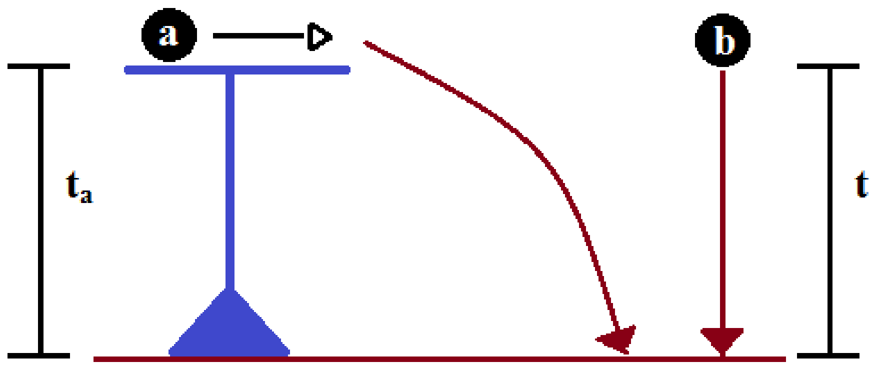
1. $v = v_0 + at$
2. $\Delta x = (\frac{v + v_0}{2})t$
3. $\Delta x = v_0 t + \frac{1}{2}at^2$
4. $v^2 = v_0^2 + 2a\Delta x$

However, what about the object that moves in **both of these directions**? This type of motion, if it involves an object that is thrown (or launched) into the air, can be referred as a **projectile motion** (حركة القذيفة).

Projectile motion: an object moves along a **curved path** under the influence of gravity.

Imagine a cannonball being fired at some angle (Θ) from the horizontal. It will travel some distance up into the air before eventually falling back down and hitting the ground, some distance away from cannon, and we can a use a **parabola** to represent the path of this object.

The important thing to understand about these kinds of examples is that horizontal motion and vertical motion of cannonball are completely independent of one another. This means we can use separate equations to discuss the motion in each directions, one equation that exclusively corresponds of the X-coordinates of the object and another that exclusively corresponds to the Y-coordinates of the object. To drive this idea home consider two situations, one dropped from a particular height and another that rolls off of a surface at that same height with some horizontal velocity.

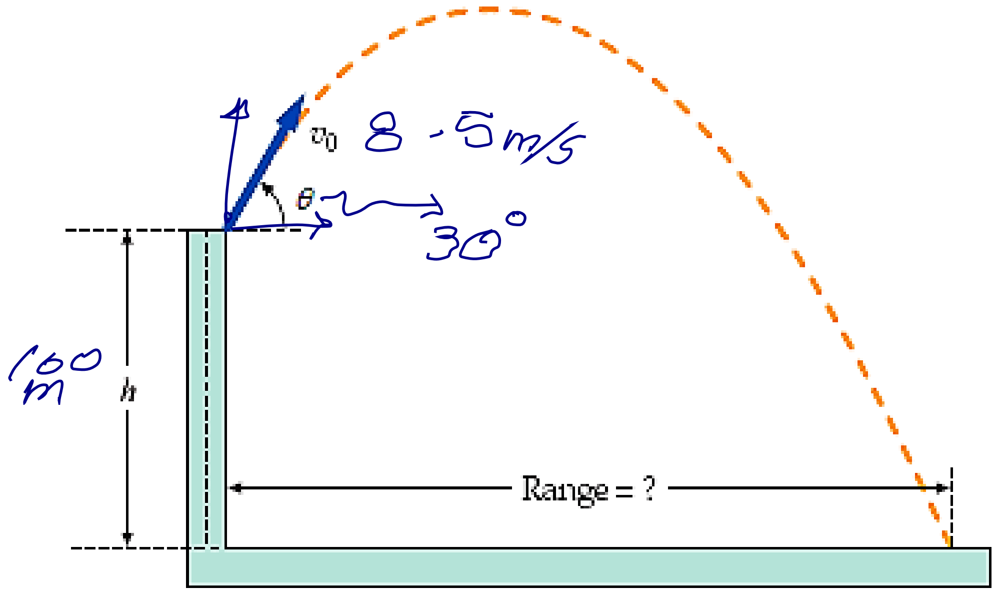


If these begin falling at the same time, they will strike the ground at the same instant (t_a=t_b) because their vertical motion is independent of any horizontal motion. The one with horizontal velocity will cover some distance in the X- direction but it will fall downward at the same rate as the one that falls straight down and so the will have identical airtimes. How can we apply this to other real-world examples?

Example: Let us throw a rock an upward angle of 30 degrees (30^o) of the horizontal from the very edge of our favorite 100 m cliff and with an initial velocity of 8.5 m/s.

H.W

PDF-XChange Viewer Click to buy NOW! www.docu-track.com

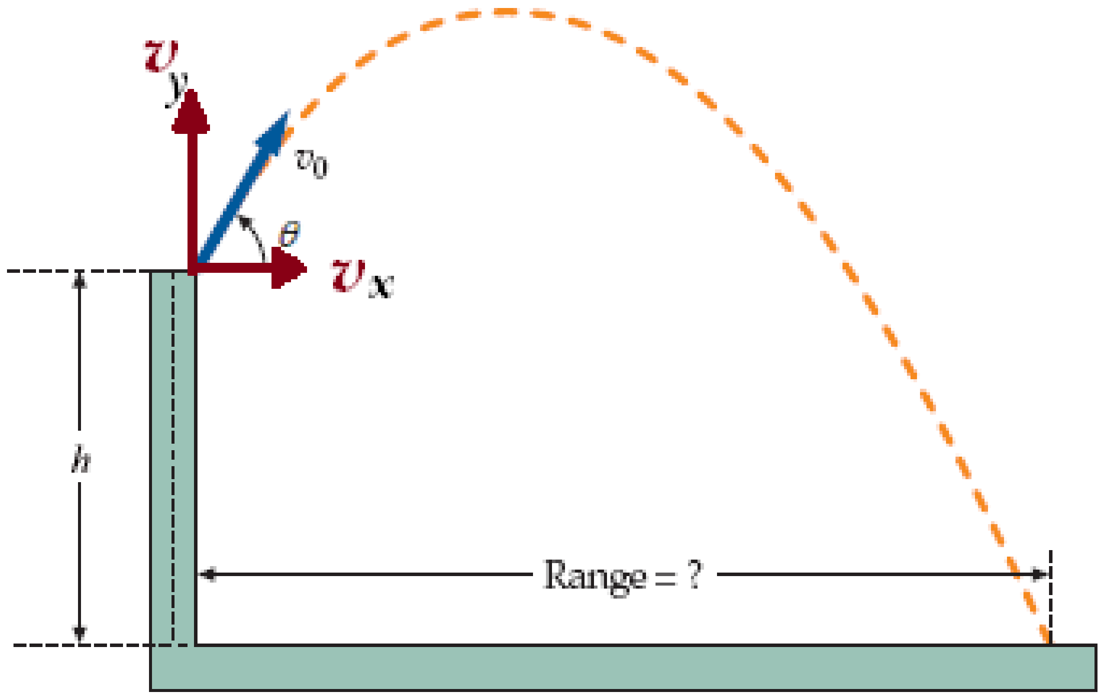


Again, we will ask, how long before the rock hits the ground? And now additionally we want to know how far away from the edge of cliff it will land.

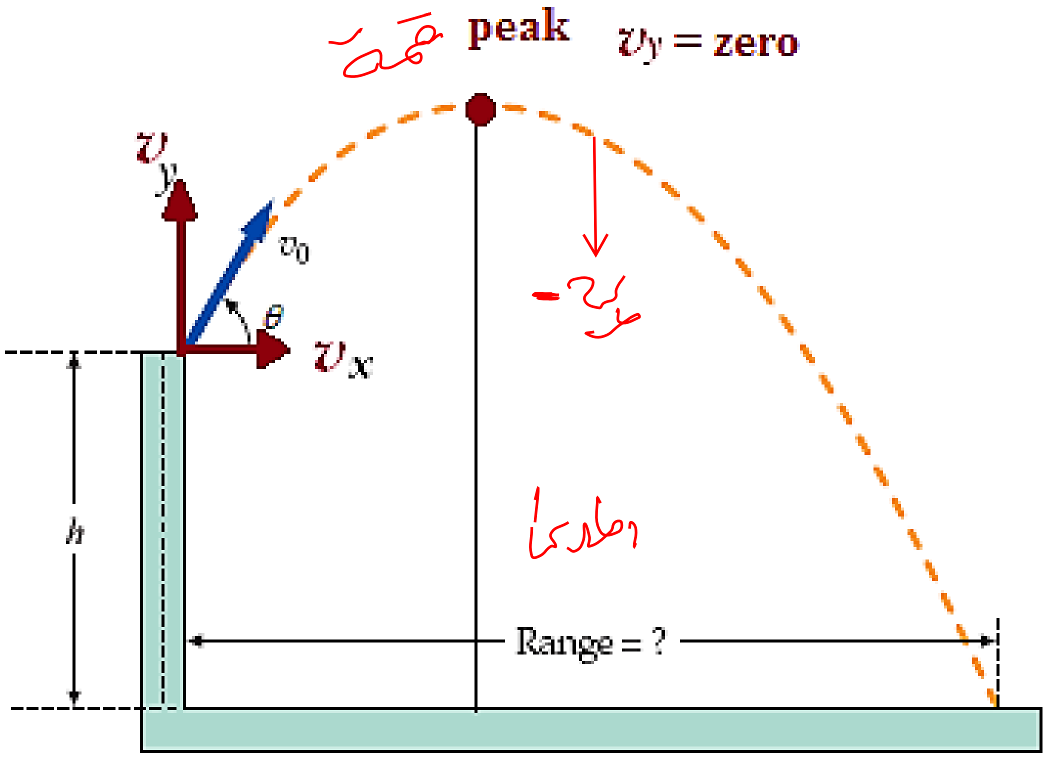
First things first

Let us make sure we understand how these questions relate to motion in both the X and Y direction (**Two-Dimensional motion**). The time is spends in the air only relates to Y direction behavior because it will stop being in the air when it hits the ground, no matter what the horizontal velocity is, from 0 to some huge number. The vertical motion will be independent of that.

The distance (**Rang المدى**) it travels from the edge of the cliff **depends on the horizontal velocity but also the time spends in the air** because once it hits the ground it cannot travel any further. Moreover, the velocity any moment can be split into components.



The horizontal velocity will be the same at every moment (v_x=constant) in this path as long as we disregard wind resistance, but the vertical velocity (v_x) will be the greatest at the moment the rock is thrown and decrease until it reach zero at zenith (**Peak اقصى ارتفاع**) and then increasingly negative until it hits the ground.



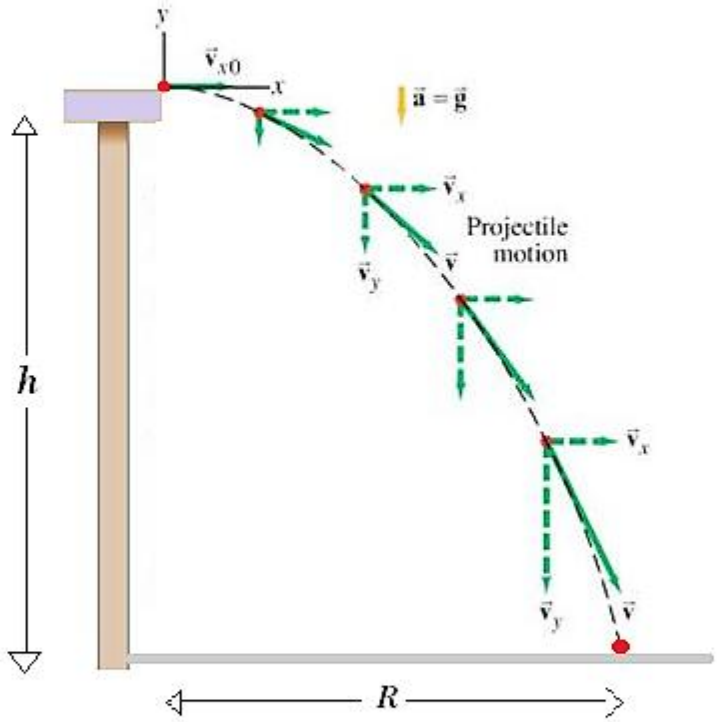
This is because there is a **constant acceleration** in the negative direction due the gravity.

$$v_x = v\cos\theta \qquad \textit{and} \qquad v_y = v\,\sin\theta$$

القذف بقمة

Second: Deriving the Projectile Equations

Case_1:



From equation

$$x_f = x_i + v_i t + \frac{1}{2}\,at^2 \quad \text{Or,} \quad \Delta x = v_i t + \frac{1}{2}\,at^2$$

Because the motion is vertical;

$\Delta y = v_{iy} t + \frac{1}{2}\,at^2$, $\Delta y = h$ *and* $v_{iy} = 0$; The object moves horizontal and down

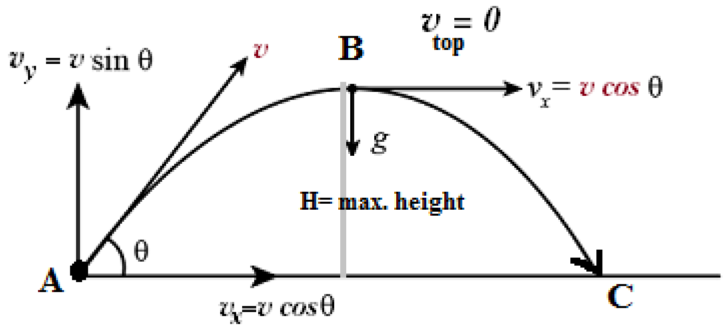
$$h = \frac{1}{2}\,gt^2$$, $$R = \Delta x = v_x t$$, $a_x = 0$ *where* $v_x = constant$

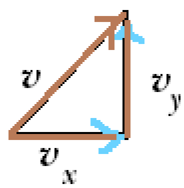
The velocity before hitting the ground

$$v_f = \sqrt{v_x^2 + v_y^2} \qquad and \quad \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$$

PDF-XChange Viewer
Click to buy NOW!
www.docu-track.com

Case_2:





$$v_x = v\cos\theta \quad , \qquad v_y = v\sin\theta \quad and\ \theta = \tan^{-1}\frac{v_y}{v_x}$$

A➔ B

To calculate the Time from A to B; we have,

$$\boldsymbol{v_{fy} = v_{iy} + a_y\, t}$$

$$0 = v\sin\theta + (-g)t$$

$$t_{A\to B} = \frac{v\sin\theta}{g}$$

so,

$$t_{A\to C} = 2\, t_{A\to B}$$; due to symmetric ➔ $$\boxed{t_{A\to C} = 2\frac{v\sin\theta}{g}}$$

How do you find the maximum height (H)?

$$v_f^2 = v_i^2 + 2a\Delta x$$

or

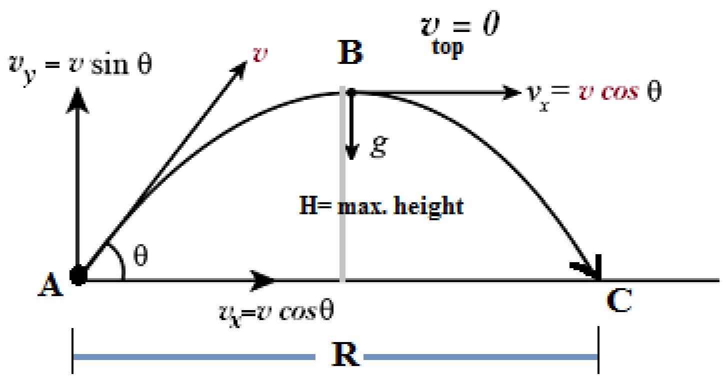
$$v_{fy}^2 = v_{iy}^2 + 2a\Delta y \quad ➔ \quad v_{fy}^2 = v_{iy}^2 + 2a\mathrm{H}\ ;\ \Delta y = \mathrm{H}$$

PDF-XChange Viewer
Click to buy NOW!
www.docu-track.com

$\boldsymbol{v_{fy}}$ **=** 0 due to $\boldsymbol{v_{fy}}$ at the top of path =0

$$0 = v_{iy}^2 - 2g\text{H} \rightarrow 0 = (v \sin\theta)^2 - 2g\text{H} \rightarrow \boxed{\mathbf{H = \frac{v^2 \sin^2 \theta}{2g}}}$$

How do you find the Range (R)?



$$\Delta x = v_x t = \text{R} = v \cos\theta \ t$$

We have,

$$t_{A \to C} = 2\frac{v \sin\theta}{g}$$

Therefore:

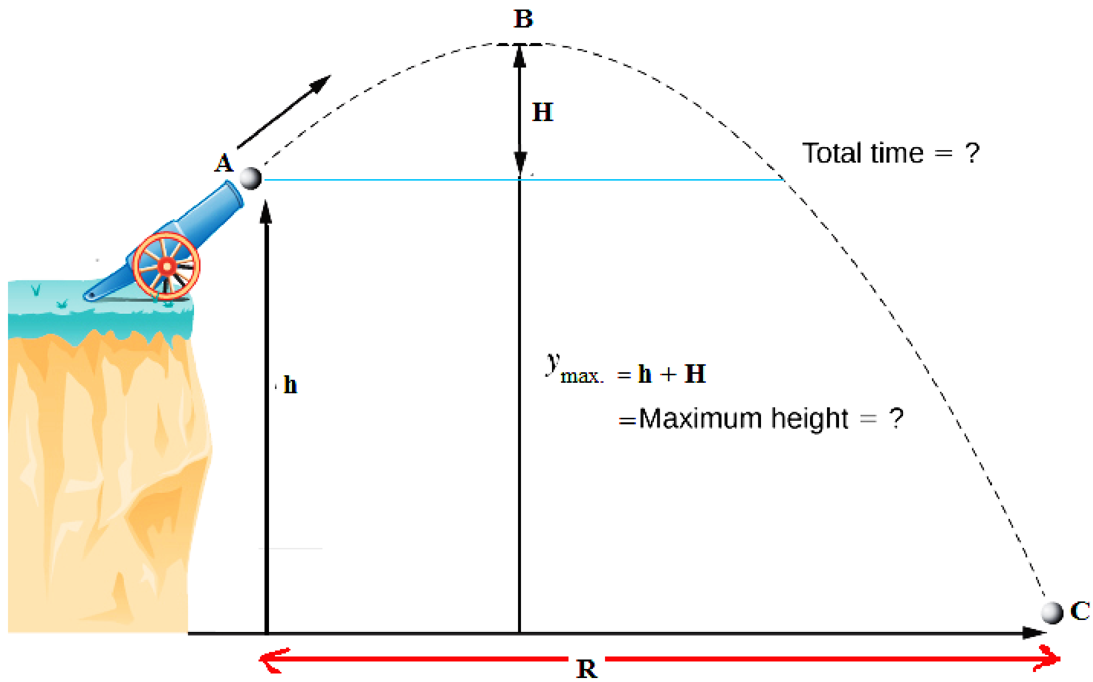
$$\text{R} = v \cos\theta \ t = v \cos\theta \cdot 2\frac{v \sin\theta}{g}$$

$$\text{R} = v \cos\theta \ t = v^2 \cdot \frac{2\cos\theta \sin\theta}{g}$$

From trigonometry the double angle formula

$$\sin 2\theta = 2\cos\theta \sin\theta \rightarrow \boxed{\text{R} = \frac{v^2}{\text{g}} \sin 2\theta}$$

Case_3:



How long bomb is taken (the total time) to strike the ground?

We have, $x_f = x_i + v_i t + \frac{1}{2}\,at^2$ or $y_f = y_i + v_{iy} t + \frac{1}{2}\,at^2$

$y_f = 0$; due to at end bomb hit ground, $y_i = h, and\ a = g$

$\frac{1}{2}\,gt^2 + v_{iy}t + h = 0$ ➔ $\frac{1}{2}\,g\,t^2 + v\sin\theta\ t + h = 0$ …… #

Use the quadratic formula to solve (#), you should have position value for the time.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

when $ax^2+bx+c=0$

a, b, c = constants, where a ≠ 0

x = the unknown

There is another way to total time:

Total time = $t_{A\to B} + t_{B\to C}$

We have , $t_{A\to B} = \frac{v\sin\theta}{g}$

$y_{max} = \text{h} + \text{H}$, $\text{H} = \frac{v^2 \sin^2 \theta}{2g}$ and $h = \frac{1}{2} g t^2$

$y_{max} = \frac{1}{2} g\, t_{B \to C}^2$ ➔ $t_{B \to C} = \sqrt{\frac{2 y_{max}}{g}}$

Total time = $t_{A \to B} + t_{B \to C}$

$t_{A \to B} = \frac{v \sin \theta}{g}$

$t_{B \to C} = \sqrt{\frac{2 y_{max}}{g}}$

How do you find the Range (R)?

$R = v_x t$ *and* $v_x = v \cos \theta$ ➔ $R = v \cos\theta\, t$

How do you find velocity before hit ground?

v_x *is constant*, **we have** $v_f = v_i + at$ ➔ $v_{yf} = v \sin \theta + g t_{A \to C}$

$$v = \sqrt{v_x^2 + v_y^2}$$