PDF-XChange Viewer
Click to buy NOW!
www.docu-track.com

Chapter 2

Motion in One Dimension

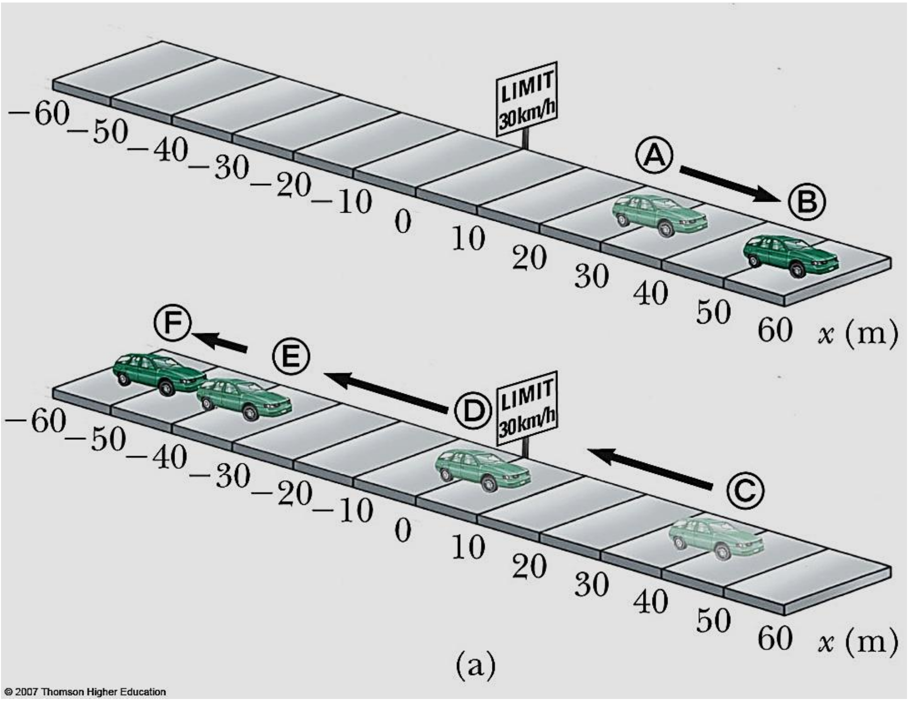
Kinematics (Motion)

PDF-XChange Viewer
Click to buy NOW!
www.docu-track.com

- Describes motion while ignoring the operators that caused the motion
- For now, will consider motion in one dimension
  - Along a straight line
- Will use the particle model
  - A particle is a point-like object, has mass but tiny size

Position

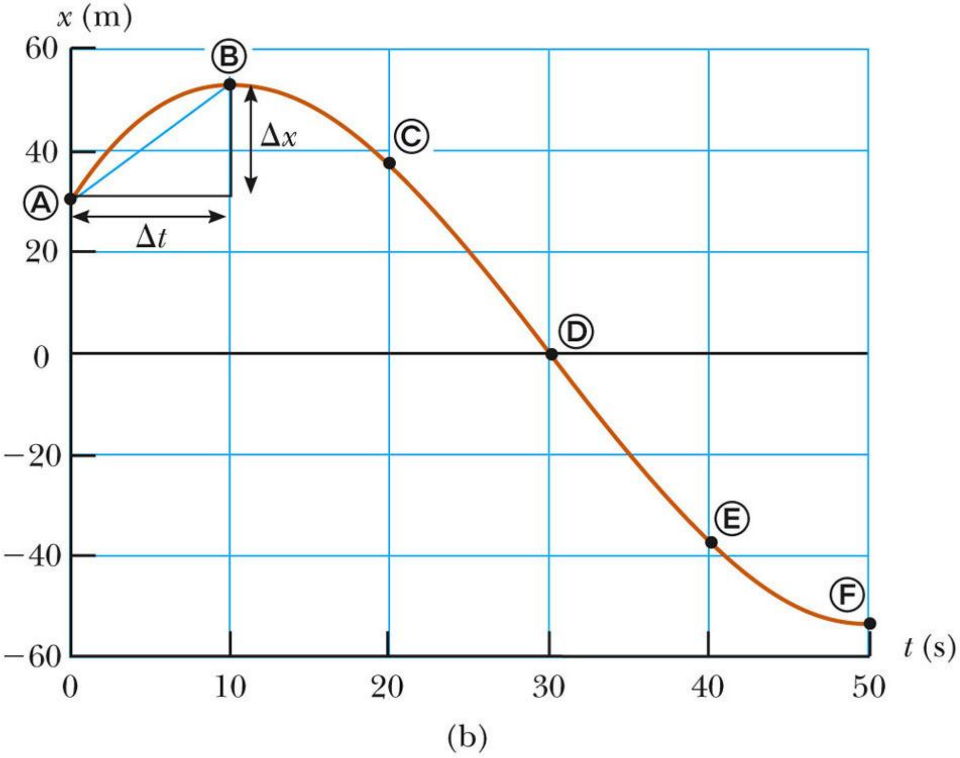
- The object's position is its location with respect to a chosen reference point
  - Consider the point to be the origin of a coordinate system
- In the diagram, allow the road sign to be the reference point



PDF-XChange Viewer Click to buy NOW! www.docu-track.com

Position-Time Graph

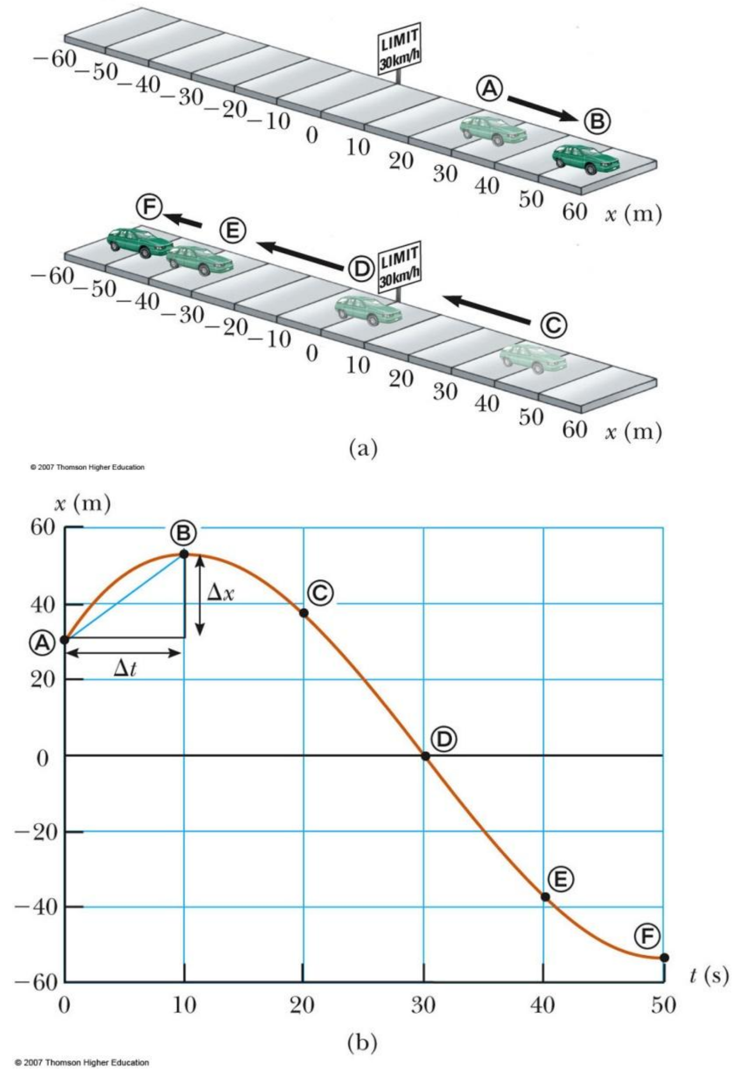
- The position-time graph shows the motion of the particle (car)
- The smooth curve is a guess as to what happened between the data points



PDF-XChange Viewer Click to buy NOW! www.docu-track.com

Motion of Car

- Note the relationship between the position of the car and the points on the graph
- Compare the different representations of the motion



PDF-XChange Viewer Click to buy NOW! www.docu-track.com

Data Table

- The table gives the actual data collected during the motion of the object (car)
- Positive is defined as being to the right

TABLE 2.1

Position of the Car at Various Times

Position	t (s)	x (m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53

Alternative Representations

PDF-XChange Viewer Click to buy NOW! www.docu-track.com

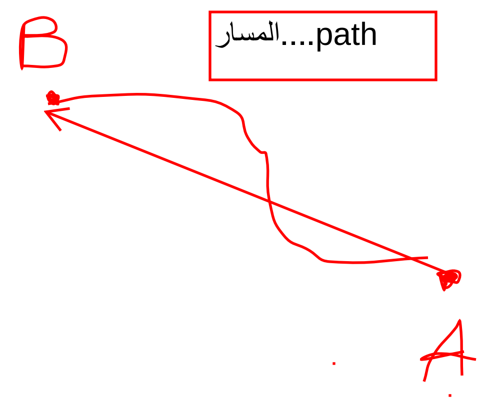
- Using alternative representations is often an excellent strategy for understanding a problem
  - For example, the car problem used multiple representations
    - Pictorial representation
    - Graphical representation
    - Tabular representation
- Goal is often a mathematical representation

PDF-XChange Viewer
Click to buy NOW!
www.docu-track.com

Displacement

- Defined as the change in position during some time interval
  - Represented as Δx

    $\Delta x \equiv x_f - x_i$.
  - SI units are meters (m)
  - Δx can be positive or negative
- Different than distance – the length of a path followed by a particle



Distance vs. Displacement – An Example

PDF-XChange Viewer Click to buy NOW! www.docu-track.com

- Assume a player moves from one end of the court to the other and back
- Distance is twice the length of the court
  - Distance is always positive
- Displacement is zero
  - $\Delta x = x_f - x_i = 0$ since

    $x_f = x_i$

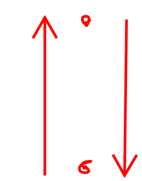


TABLE 2.2

Kinematic Equations for Motion of a Particle Under Constant Acceleration

H.W

Equation Number	Equation	Information Given by Equation
2.13	$v_{xf} = v_{xi} + a_x t$	Velocity as a function of time
2.15	$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$	Position as a function of velocity and time
2.16	$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$	Position as a function of time
2.17	$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$	Velocity as a function of position

Note: Motion is along the x axis.

Vectors and Scalars

- Vector quantities need both magnitude (size or numerical value) and direction to completely describe them
  - Will use + and – signs to indicate vector directions
- Scalar quantities are completely described by magnitude only

PDF-XChange Viewer
Click to buy NOW!
www.docu-track.com

Average Velocity

- The **average velocity** is rate at which the displacement occurs

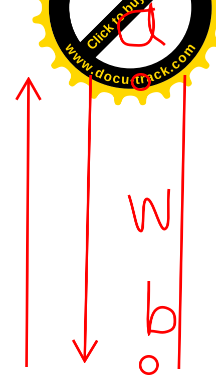
$$v_{x,avg} \equiv \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$$

  - The x indicates motion along the x-axis
- The dimensions are length / time [L/T]
- The SI units are m/s
- Is also the slope of the line in the position – time graph

PDF-XChange Viewer Click to buy NOW! www.docu-track.com

انطلاق

Average Speed



- Speed is a scalar quantity
  - same units as velocity
  - total distance / total time: $$v_{avg} \equiv \frac{d}{t}$$
- The speed has no direction and is always expressed as a positive number
- Neither average velocity nor average speed gives details about the trip described

PDF-XChange Viewer
Click to buy NOW!
www.docu-track.com

Instantaneous Velocity

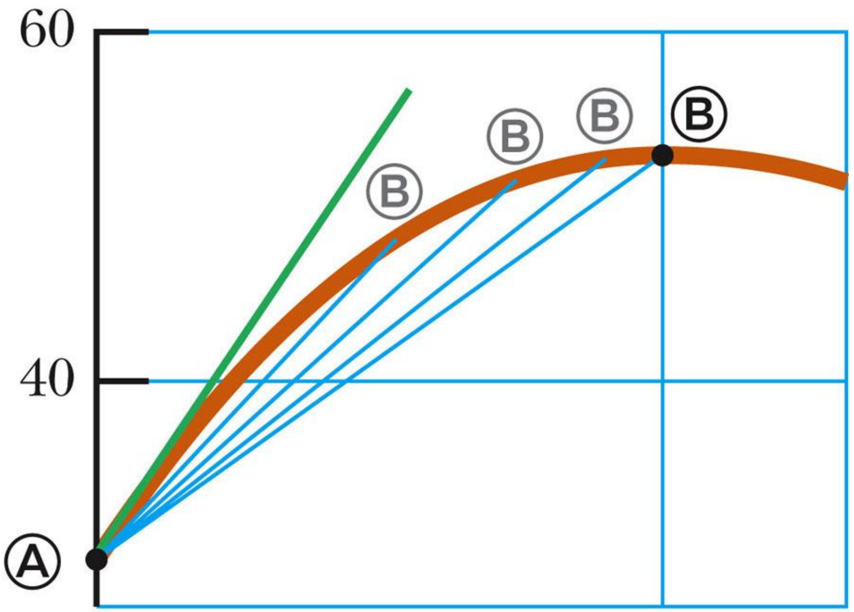
الآني.... اللحظي

- The limit of the average velocity as the time interval becomes infinitesimally short, or as the time interval approaches zero
- The instantaneous velocity indicates what is happening at every point of time

PDF-XChange Viewer Click to buy NOW! www.docu-track.com

Instantaneous Velocity, graph

- The instantaneous velocity is the slope of the line tangent to the *x* vs. *t* curve
- This would be the green line
- The light blue lines show that as Δt gets smaller, they approach the green line



(b)

PDF-XChange Viewer Click to buy NOW! www.docu-track.com

Instantaneous Velocity, equations

PDF-XChange Viewer Click to buy NOW! www.docu-track.com

- The general equation for instantaneous velocity is

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- The instantaneous velocity can be positive, negative, or zero

Instantaneous Speed

- The instantaneous speed is the magnitude of the instantaneous velocity
- The instantaneous speed has no direction associated with it

PDF-XChange Viewer Click to buy NOW! www.docu-track.com

Vocabulary Note

- “Velocity” and “speed” will indicate *instantaneous* values
- *Average* will be used when the average velocity or average speed is indicated

Analysis Models

- Analysis models are an important technique in the solution to problems
- An analysis model is a previously solved problem
  - It describes
    - The behavior of some physical entity
    - The interaction between the entity and the environment
  - Try to identify the fundamental details of the problem and attempt to recognize which of the types of problems you have already solved could be used as a model for the new problem

PDF-XChange Viewer Click to buy NOW! www.docu-track.com

Analysis Models, cont

- Based on four *simplification models*
  - Particle model
  - System model
  - Rigid object
  - Wave

Particle Under Constant Velocity

PDF-XChange Viewer Click to buy NOW! www.docu-track.com

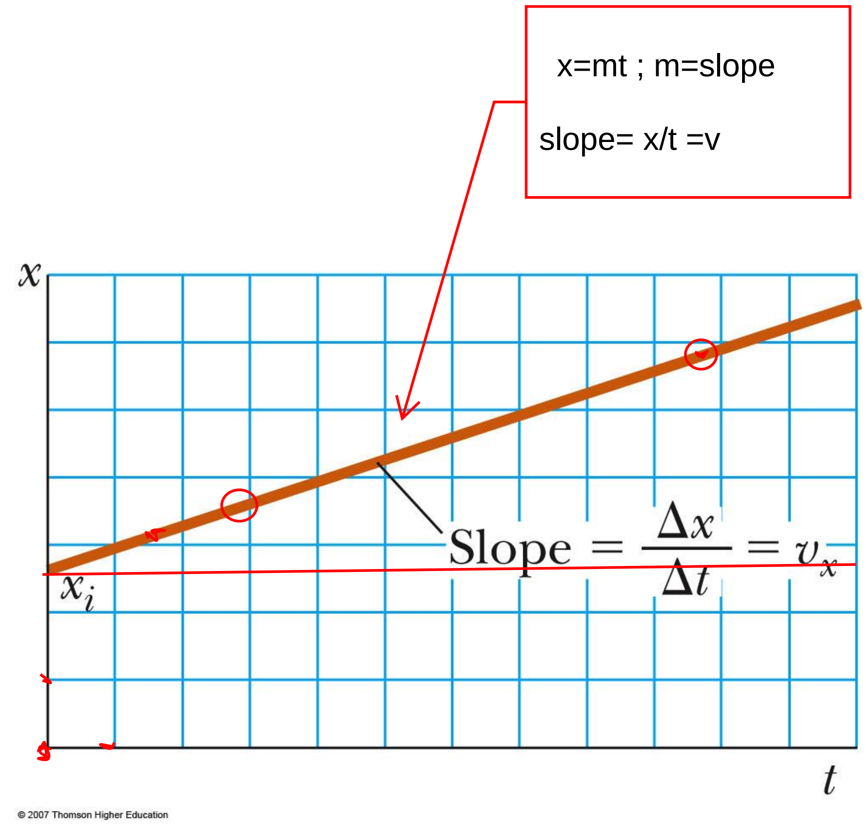
- Constant velocity indicates the instantaneous velocity at any instant during a time interval is the same as the average velocity during that time interval
  - $v_x = v_{x,\ avg}$
  - The mathematical representation of this situation is the equation

$$v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t} \quad or \quad x_f = x_i + v_x \Delta t$$

  - Common practice is to let $t_i = 0$ and the equation becomes: $x_f = x_i + v_x t$ (for constant v_x)

Particle Under Constant Velocity, Graph

- The graph represents the motion of a particle under constant velocity
- The slope of the graph is the value of the constant velocity
- The y-intercept is x_i



متوسط التسارع = معدل التعجيل

التعجيل

v1=30
v2=35
v3=32

v=(30+35+32)/3

Average Acceleration

- Acceleration is the rate of change of the velocity

$$a_{x,avg} \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$

- Dimensions are L/T^2
- SI units are m/s^2
- In one dimension, positive and negative can be used to indicate direction

PDF-XChange Viewer
Click to buy NOW!
www.docu-track.com

Instantaneous Acceleration

- The instantaneous acceleration is the limit of the average acceleration as Δt approaches 0

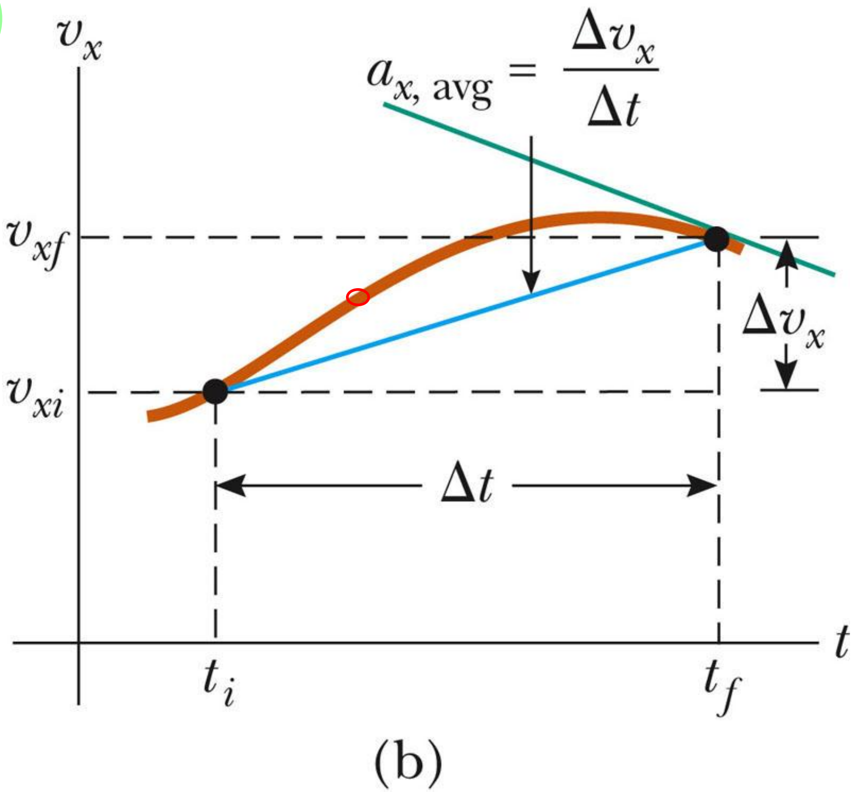
$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$$

- The term acceleration will mean instantaneous acceleration
  - If average acceleration is wanted, the word average will be included

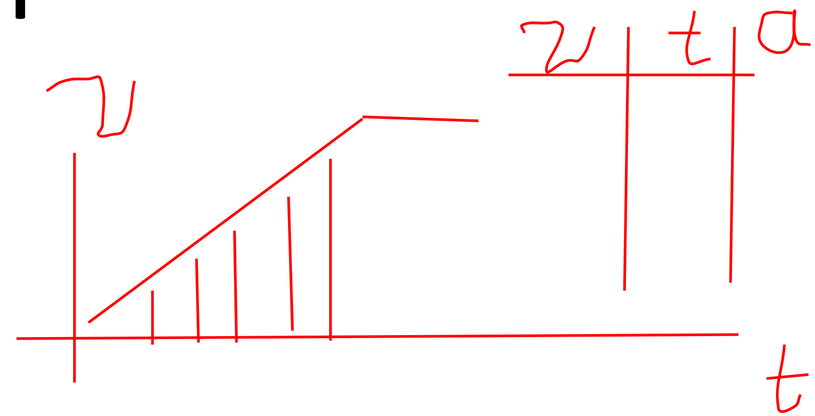
PDF-XChange Viewer Click to buy NOW! www.docu-track.com

Instantaneous Acceleration -- graph

- The slope of the velocity-time graph is the acceleration
- The green line represents the instantaneous acceleration
- The blue line is the average acceleration

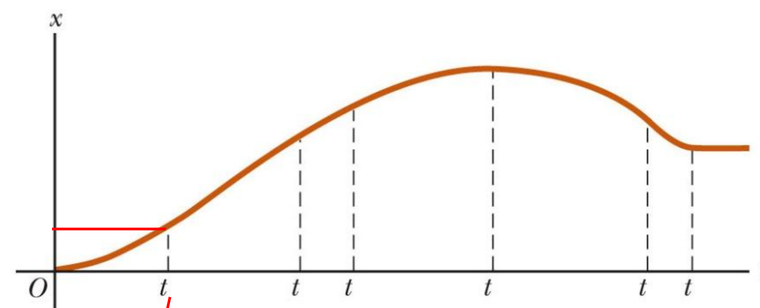


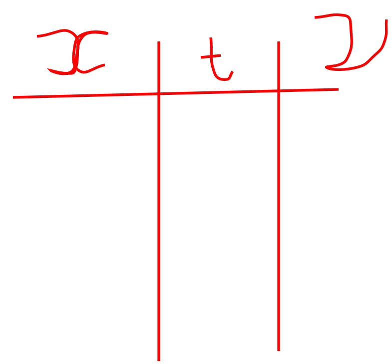
Graphical Comparison



- Given the displacement-time graph (a)
- The velocity-time graph is found by measuring the slope of the position-time graph at every instant
- The acceleration-time graph is found by measuring the slope of the velocity-time graph at every instant

(a)





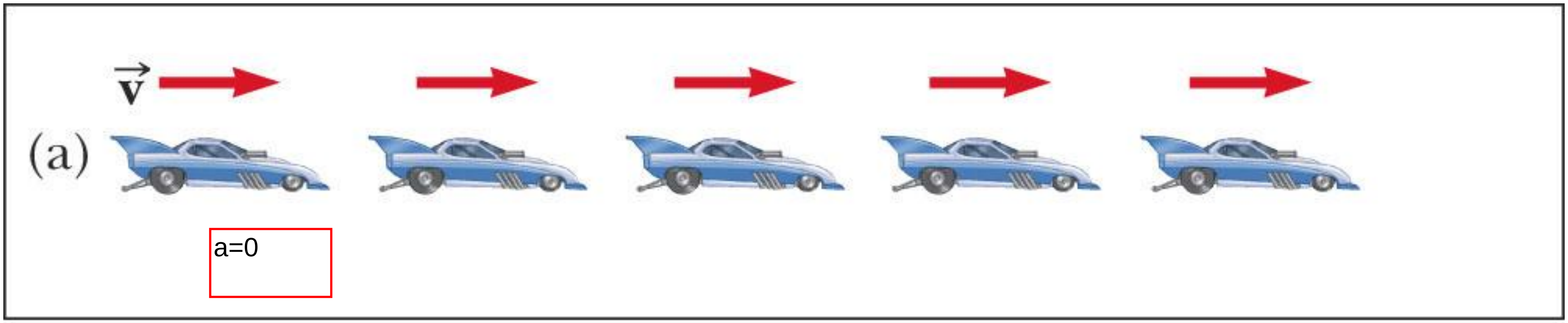
PDF-XChange Viewer Click to buy NOW! www.docu-track.com

PDF-XChange Viewer
Click to buy NOW!
www.docu-track.com

Acceleration and Velocity, 1

- When an object's velocity and acceleration are in the same direction, the object is speeding up تزايد
- When an object's velocity and acceleration are in the opposite direction, the object is slowing down تناقص

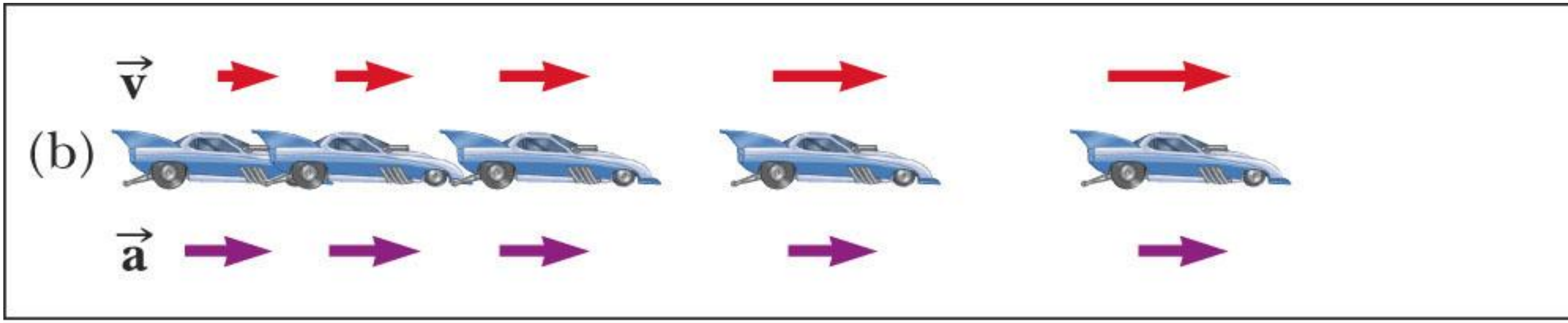
Acceleration and Velocity, 2



- Images are equally spaced. The car is moving with constant positive velocity (shown by red arrows maintaining the same size)
- Acceleration equals zero

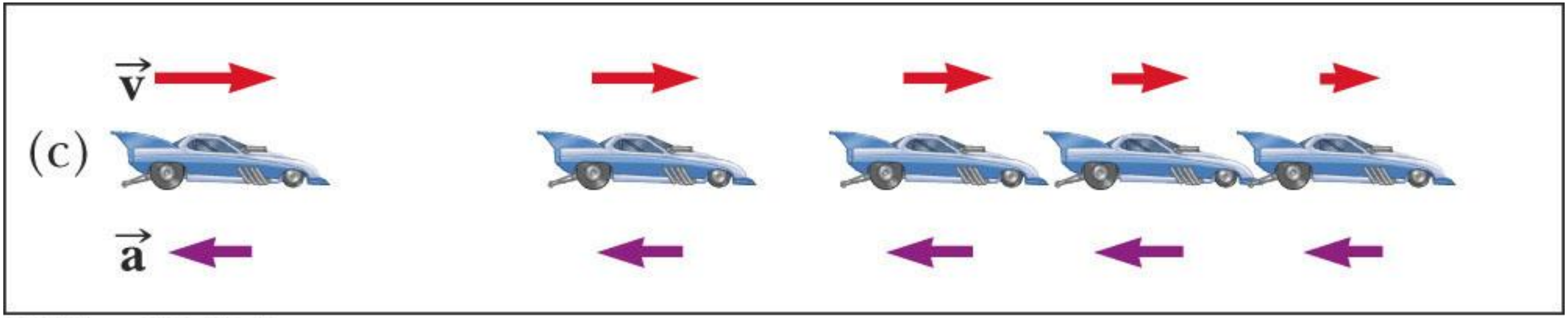
PDF-XChange Viewer
Click to buy NOW!
www.docu-track.com

Acceleration and Velocity, 3



- Images become farther apart as time increases
- Velocity and acceleration are in the same direction
- Acceleration is uniform (violet arrows maintain the same length)
- Velocity is increasing (red arrows are getting longer)
- This shows positive acceleration and positive velocity

Acceleration and Velocity, 4



- Images become closer together as time increases
- Acceleration and velocity are in opposite directions
- Acceleration is uniform (violet arrows maintain the same length)
- Velocity is decreasing (red arrows are getting shorter)
- Positive velocity and negative acceleration

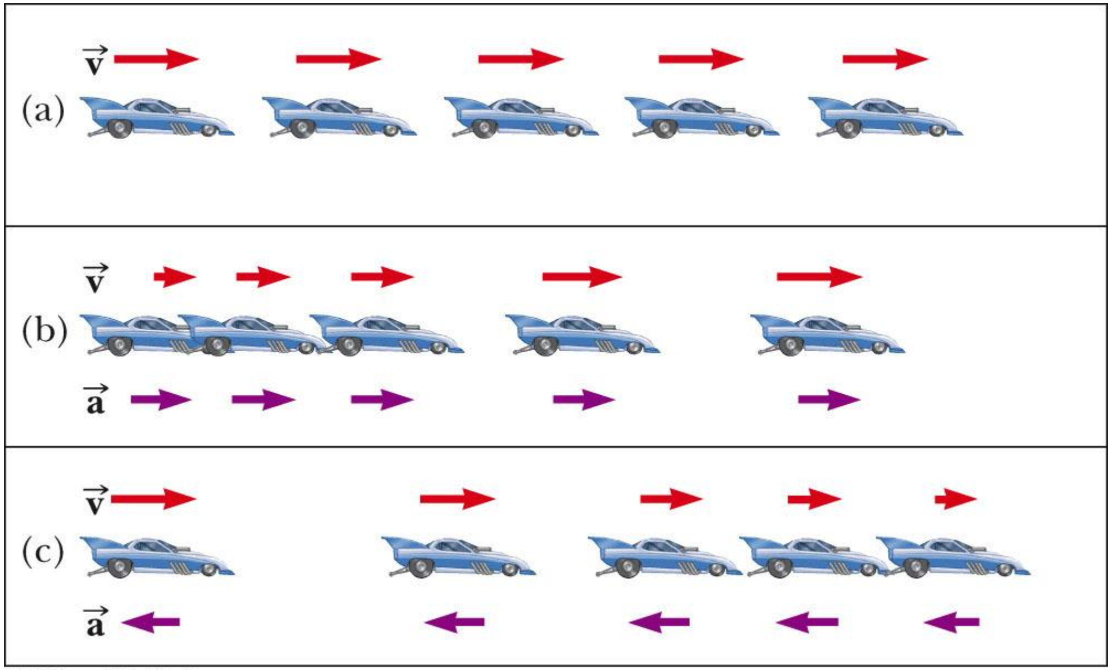
Acceleration and Velocity, final

PDF-XChange Viewer
Click to buy NOW!
www.docu-track.com

- In all the previous cases, the acceleration was constant
  - Shown by the violet arrows all maintaining the same length
- The diagrams represent motion of a particle under constant acceleration
- A particle under constant acceleration is another useful analysis model

Graphical Representations of Motion

- Observe the graphs of the car under various conditions
- Note the relationships among the graphs
  - Set various initial velocities, positions and accelerations



PDF-XChange Viewer Click to buy NOW! www.docu-track.com

Kinematic Equations – summary

TABLE 2.2

Kinematic Equations for Motion of a Particle Under Constant Acceleration

Equation Number	Equation	Information Given by Equation
2.13	$v_{xf} = v_{xi} + a_x t$	Velocity as a function of time
2.15	$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$	Position as a function of velocity and time
2.16	$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$	Position as a function of time
2.17	$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$	Velocity as a function of position

Note: Motion is along the x axis.

Kinematic Equations

- The kinematic equations can be used with any particle under uniform acceleration.
- The kinematic equations may be used to solve any problem involving one-dimensional motion with a constant acceleration
- You may need to use two of the equations to solve one problem
- Many times there is more than one way to solve a problem

Kinematic Equations, specific

PDF-XChange Viewer
Click to buy NOW!
www.docu-track.com

- For constant *a*, $v_{xf} = v_{xi} + a_x t$
- Can determine an object's velocity at any time *t* when we know its initial velocity and its acceleration
  - Assumes $t_i = 0$ and $t_f = t$
- Does not give any information about displacement

Kinematic Equations, specific

PDF-XChange Viewer Click to buy NOW! www.docu-track.com

- For constant acceleration,

$$v_{x,avg} = \frac{v_{xi} + v_{xf}}{2}$$

Average

- The average velocity can be expressed as the arithmetic mean of the initial and final velocities

Kinematic Equations, specific

v=(x2 -x1)/t

- For constant acceleration,

$$x_f = x_i + v_{x,avg}\, t = x_i + \frac{1}{2}\left(v_{xi} + v_{fx}\right)t$$

- This gives you the position of the particle in terms of time and velocities
- Doesn't give you the acceleration

PDF-XChange Viewer Click to buy NOW! www.docu-track.com

Kinematic Equations, specific

PDF-XChange Viewer
Click to buy NOW!
www.docu-track.com

- For constant acceleration,

$$x_f = x_i + v_{xi} t + \frac{1}{2} a_x t^2$$

- Gives final position in terms of velocity and acceleration
- Doesn't tell you about final velocity

Kinematic Equations, specific

PDF-XChange Viewer Click to buy NOW! www.docu-track.com

- For constant *a*,

$$v_{xf}^2 = v_{xi}^2 + 2a_x\left(x_f - x_i\right)$$

- Gives final velocity in terms of acceleration and displacement
- Does not give any information about the time

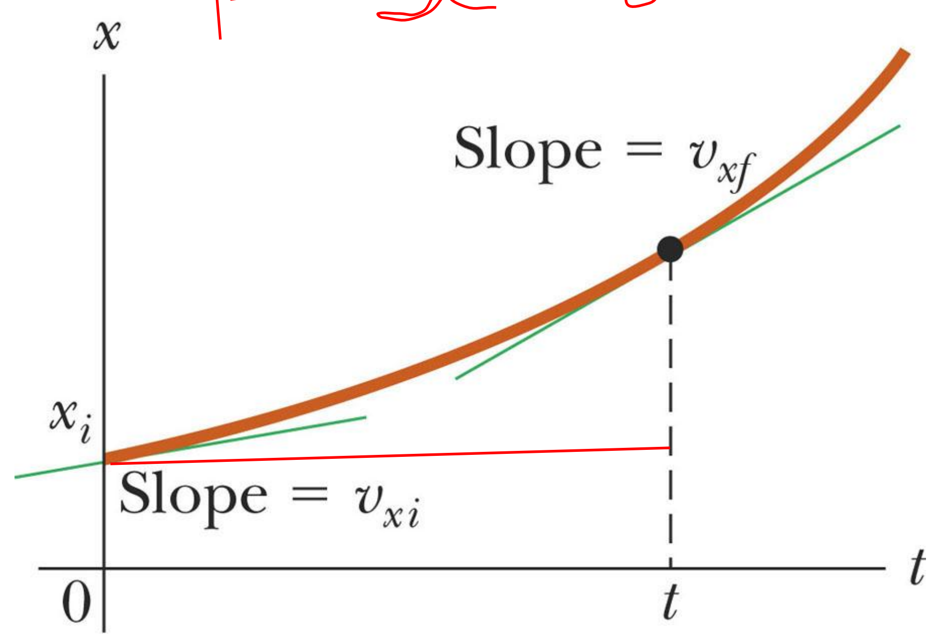
When a = 0

- When the acceleration is zero,
  - $v_{xf} = v_{xi} = v_x$
  - $x_f = x_i + v_x t$
- The constant acceleration model reduces to the constant velocity model

PDF-XChange Viewer
Click to buy NOW!
www.docu-track.com

Graphical Look at Motion: displacement – time curve

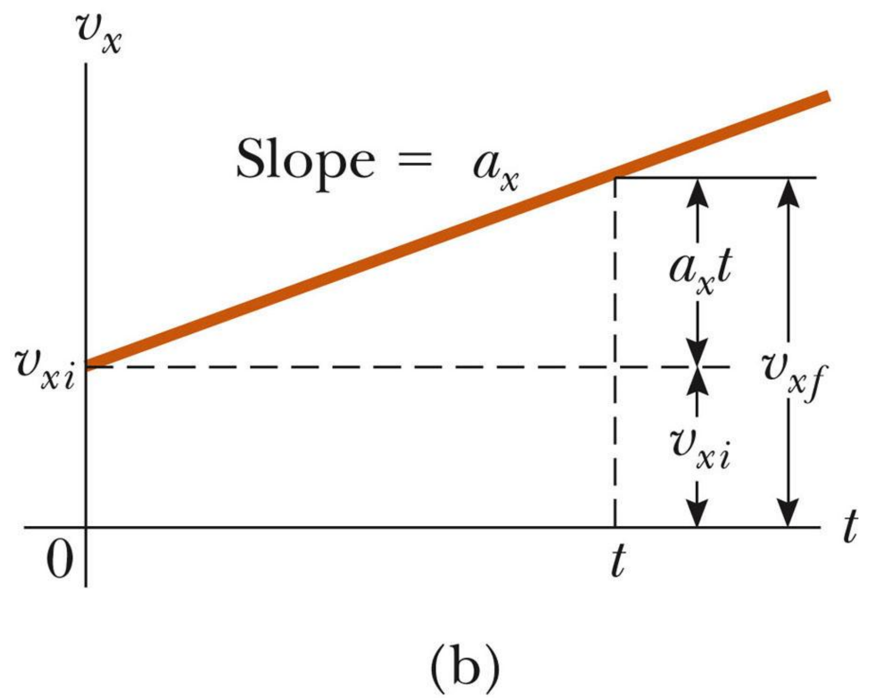
- The slope of the curve is the velocity
- The curved line indicates the velocity is changing
  - Therefore, there is an acceleration



(a)

Graphical Look at Motion: velocity – time curve

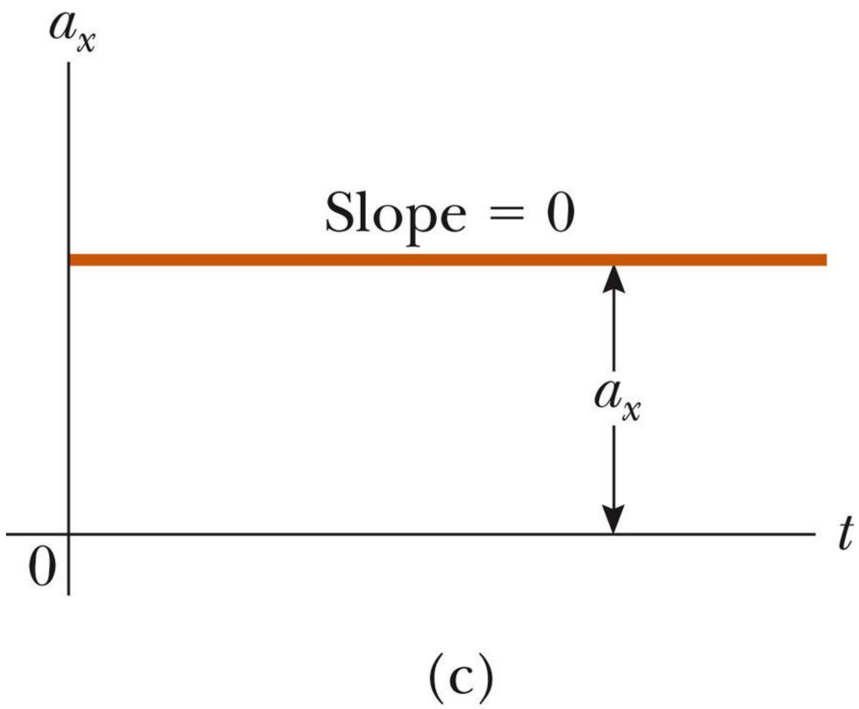
- The slope gives the acceleration
- The straight line indicates a constant acceleration



PDF-XChange Viewer Click to buy NOW! www.docu-track.com

Graphical Look at Motion: acceleration – time curve

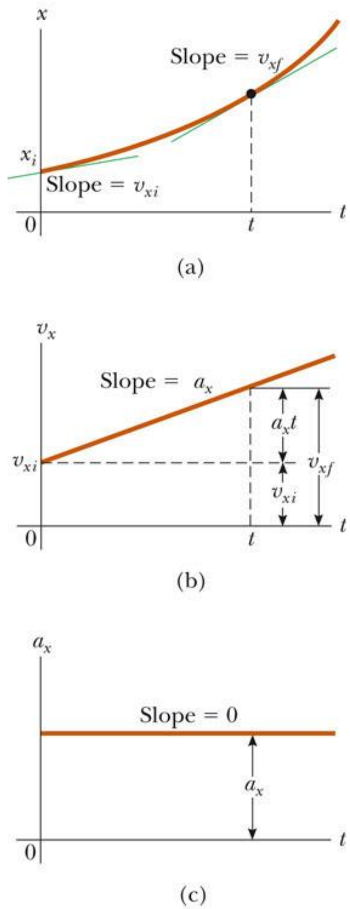
- The zero slope indicates a constant acceleration



PDF-XChange Viewer Click to buy NOW! www.docu-track.com

Graphical Motion with Constant Acceleration

- A change in the acceleration affects the velocity and position
- Note especially the graphs when a = 0

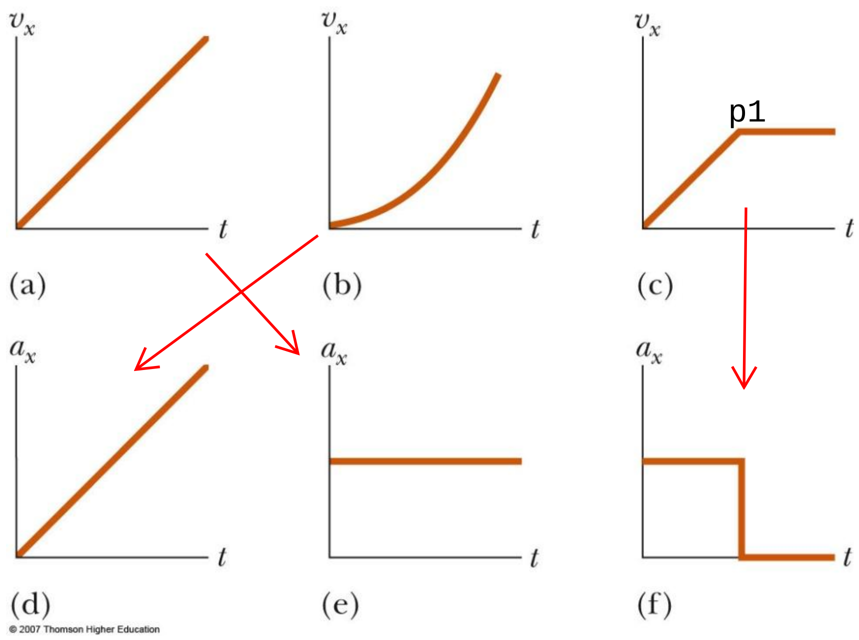


PDF-XChange Viewer Click to buy NOW! www.docu-track.com

Test Graphical Interpretations

- Match a given velocity graph with the corresponding acceleration graph
- Match a given acceleration graph with the corresponding velocity graph(s)

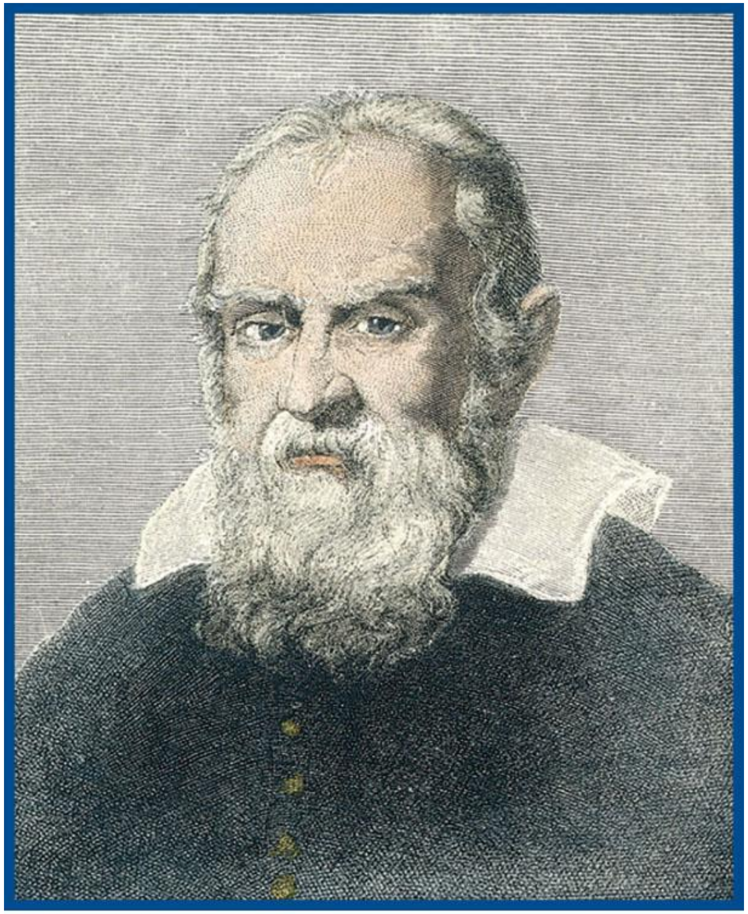
$$a = \frac{\Delta V}{\Delta t}$$



PDF-XChange Viewer
Click to buy NOW!
www.docu-track.com

Galileo Galilei

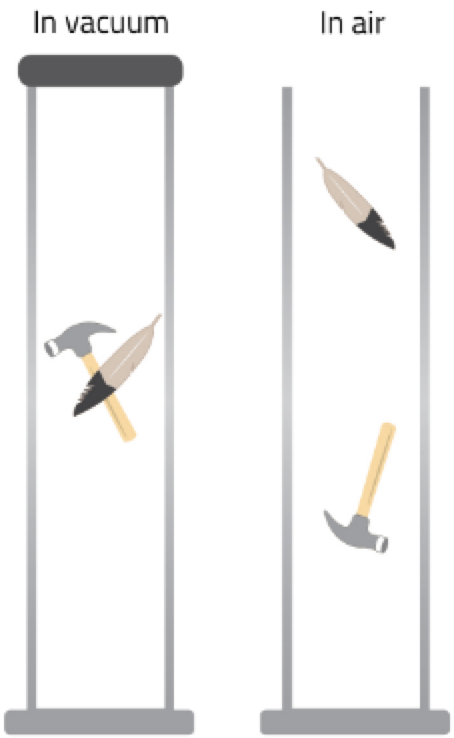
- 1564 – 1642
- Italian physicist and astronomer
- Formulated laws of motion for objects in free fall
- Supported heliocentric universe



PDF-XChange Viewer Click to buy NOW! www.docu-track.com

Freely Falling Objects

- A ***freely falling object*** is any object moving freely under the influence of gravity alone.
- It does not depend upon the initial motion of the object
  - Dropped – released from rest
  - Thrown downward
  - Thrown upward



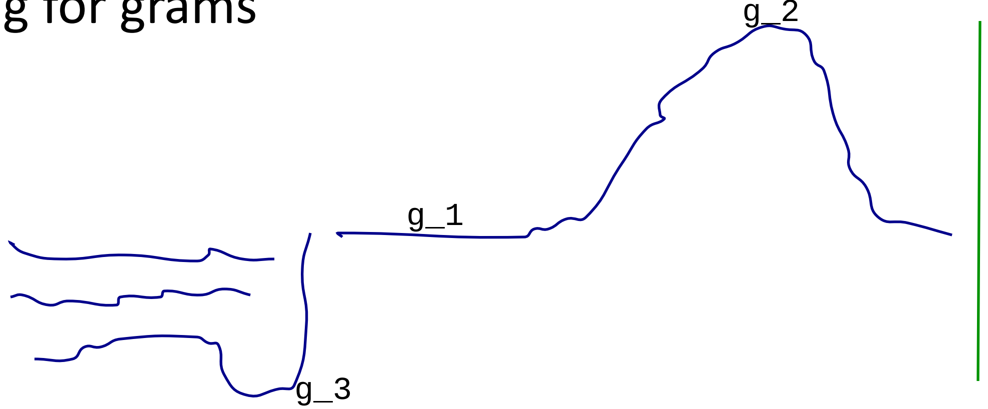
PDF-XChange Viewer Click to buy NOW! www.docu-track.com

Acceleration of Freely Falling Object

PDF-XChange Viewer
Click to buy NOW!
www.docu-track.com

- The acceleration of an object in free fall is directed downward, regardless of the initial motion
- The magnitude of free fall acceleration is g = 9.80 m/s^2
  - *g* decreases with increasing altitude
  - *g* varies with latitude
  - 9.80 m/s^2 is the average at the Earth's surface
  - The italicized *g* will be used for the acceleration due to gravity
    - Not to be confused with g for grams

unit of mass : gm



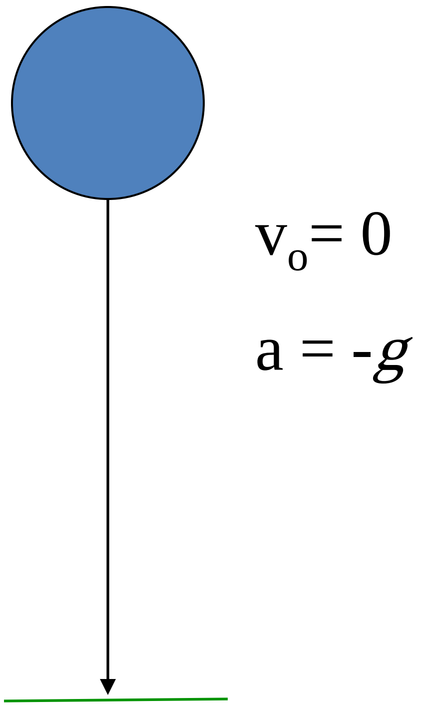
Acceleration of Free Fall, cont.

- We will neglect air resistance
- Free fall motion is constantly accelerated motion in one dimension
- Let upward be positive
- Use the kinematic equations with $a_y = -g = -9.80 \text{ m/s}^2$

PDF-XChange Viewer Click to buy NOW! www.docu-track.com

Free Fall – an object dropped

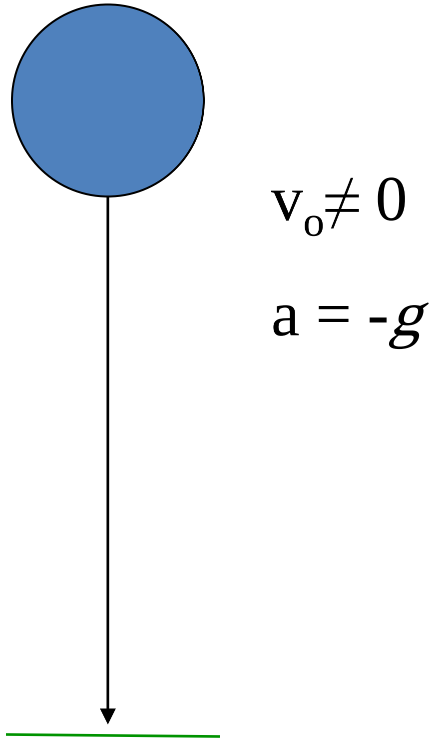
- Initial velocity is zero
- Let up be positive
- Use the kinematic equations
  - Generally use y instead of x since vertical
- Acceleration is
  - $a_y = -g = -9.80\ \text{m/s}^2$



Free Fall – an object thrown downward

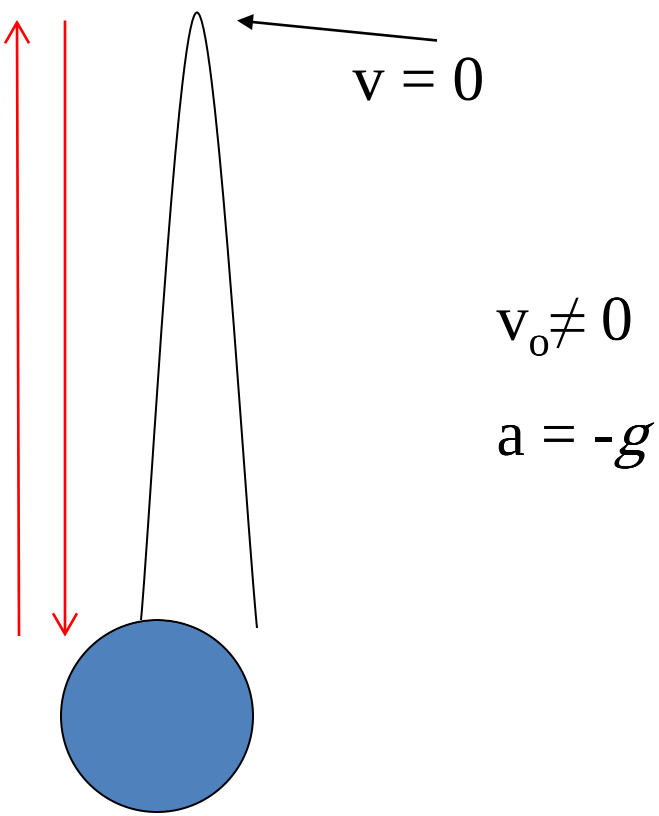
PDF-XChange Viewer
Click to buy NOW!
www.docu-track.com

- $a_y = -g = -9.80\ m/s^2$
- Initial velocity $\neq 0$
  - With upward being positive, initial velocity will be negative

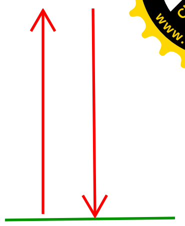


Free Fall -- object thrown upward

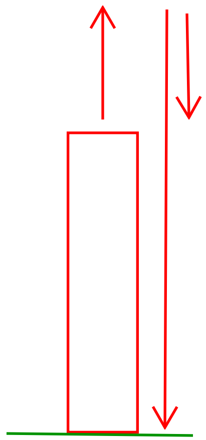
- Initial velocity is upward, so positive
- The instantaneous velocity at the maximum height is zero
- $a_y = -g = -9.80\ m/s^2$ everywhere in the motion



Thrown upward, cont.



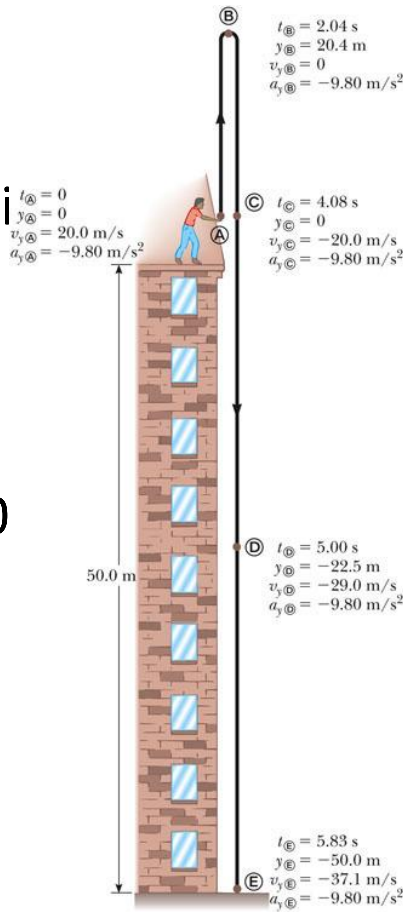
- The motion may be symmetrical
  - Then $t_{up} = t_{down}$
  - Then $v = -v_o$



- The motion may not be symmetrical
  - Break the motion into various parts
    - Generally up and down

Free Fall Example

- Initial velocity at A is upward (+) and acceleration is -*g* (-9.8 m/s^2)
- At B, the velocity is 0 and the acceleration i *g* (-9.8 m/s^2)
- At C, the velocity has the same magnitude at A, but is in the opposite direction
- The displacement is –50.0 m (it ends up 50 m below its starting point)



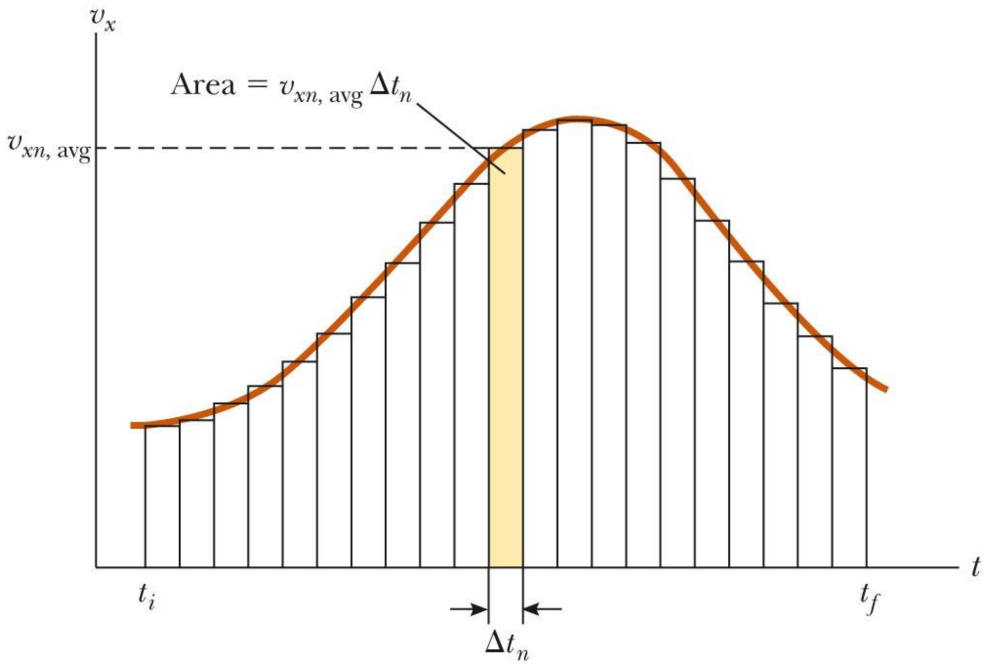
Kinematic Equations from Calculus

$v = \frac{\Delta x}{\Delta t} \rightarrow x = \Delta v \, \Delta t$

- Displacement equals the area under the velocity – time curve

$$\lim_{\Delta t_n \to 0} \sum_n v_{xn} \Delta t_n = \int_{t_i}^{t_f} v_x(t)\,dt$$

- The limit of the sum is a definite integral



Kinematic Equations – General Calculus Form

$$a_x = \frac{dv_x}{dt}$$

$$v_{xf} - v_{xi} = \int_0^t a_x dt$$

$$v_x = \frac{dx}{dt}$$

$$x_f - x_i = \int_0^t v_x dt$$

PDF-XChange Viewer
Click to buy NOW!
www.docu-track.com

Kinematic Equations – Calculus Form with Constant Acceleration

- The integration form of $v_f - v_i$ gives

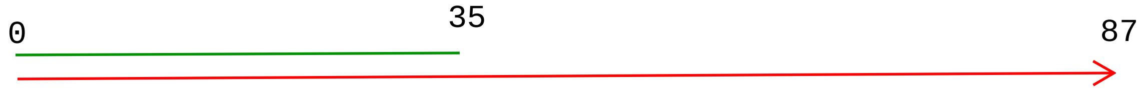
$$v_{xf} - v_{xi} = a_x t \qquad v_{xf} - v_{xi} = \int_0^t a_x dt \quad = a \int_0^t dt$$

$$= a\,[t]_0^t \quad = a[t-0]$$

- The integration form of $x_f - x_i$ gives

$$x_f - x_i = v_{xi} t + \frac{1}{2} a_x t^2 \qquad x_f = x_i + v_i t + \frac{1}{2} a t^2$$

PDF-XChange Viewer Click to buy NOW! www.docu-track.com



Example 2-1

The position of an object is 35 meters when the time was 2 seconds and then the position changes to 87 meters at 15 seconds.

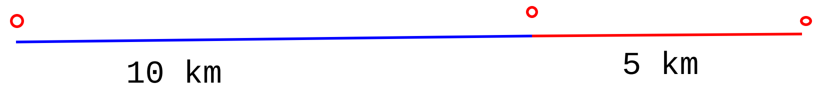
Calculate the average velocity of the object?

Solution

$$v_{Ave} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

$$= \frac{87 - 35}{15 - 2} = \frac{52}{13} = 4\, m/s$$

PDF-XChange Viewer Click to buy NOW! www.docu-track.com



Example 2-2

A Car travels 10 km at velocity of 90 km/h, and then the car stopped because the car run out of the fuel. This is causing the driver to walk a distance of 5 km to reach the nearest petrol station for half an hour.

What is the average velocity for the driver?

Solution

The distance x_1 equal to zero because the driver was still at home, x_2 is the distance covered by car until the fuel run out, and x_3 is the distance that the driver walked to the petrol station.

Convert the distance from kilometers to meters, and convert the units of other velocity from km/h to m/s as follows

$$x_f = x_2 + x_3 = 10\times10^3 + 5\times10^3 = 10{,}000 + 5{,}000 = 15{,}000\ m$$

$$\upsilon = 90\frac{km}{h} = \frac{90\times1000}{60\times60} = 25\ m/s$$

the time taken before the fuel run out

$$t_1 = \frac{x}{\upsilon} = \frac{10000}{25} = 400\,s$$

PDF-XChange Viewer
Click to buy NOW!
www.docu-track.com

the time that the driver takes to reach the petrol station

$t_2 = 30 \times 60 = 1800\ s$

Total time for the car and the driver is

$t_{Total} = t_f = 400 + 1800 = 2200\ s$

The average speed is

$$\bar{\upsilon} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

$$= \frac{x_f - 0}{t_f - 0} = \frac{15000}{2200} = 6.8\, m/s$$

PDF-XChange Viewer
Click to buy NOW!
www.docu-track.com

Example 2-3

Moving object in the positive direction of the x-axis with a relationship, as the following $x(t) = 8 + 2t + 3t^2$ *where the distance is measured in meters and the time in second.*

A) Find the instantaneous velocity of the object after two seconds?

B) *Find the instantaneous acceleration of the object after two seconds?*

C) Find the distance of the object after two seconds?

Solution

A)The instantaneous velocity of the object after two seconds is:

PDF-XChange Viewer
Click to buy NOW!
www.docu-track.com

$$\upsilon = \frac{dx}{dt} = \frac{d}{dt}(8 + 2t + 3t^2)$$

$$= 2 + 3 \times 2t = 2 + 6 \times 2 = 14\, m/s$$

B)*The instantaneous acceleration of the object after two seconds is:*

$$a = \frac{d\upsilon}{dt} = \frac{d}{dt}(2 + 6t)$$

$$= 6\, m/s^2$$

C) *The distance of the object after two seconds is:*

$$x = 8 + 2t + 3t^2$$

$$= 8 + 2 \times 2 + 3 \times (2)^2$$

$$= 8 + 4 + 12 = 24\, m$$

PDF-XChange Viewer
Click to buy NOW!
www.docu-track.com

Example 2-4

A car travels with a velocity of 20 m/s, the driver increased the velocity until it reaches 100 km/h in three seconds. Then the driver decided to stop, the car stopped after four seconds.

Find the average acceleration in both cases?

Solution

First: convert the units from km /h to m /s.

$$\upsilon_f = \frac{100 \times 1000}{60 \times 60} = 27.8\, m/s$$

$$a_{Ave.1} = \frac{\Delta\upsilon}{\Delta t}$$

$$= \frac{27.8 - 20}{3} = 2.6\, m/s^2$$

$$a_{Ave.2} = \frac{\Delta\upsilon}{\Delta t}$$

$$= \frac{0 - 27.8}{4} = -6.95\, m/s^2$$

Example 2-5

An object moves in x-axis according to the following formula: $x = t^3 + 5$

Note that the distance and the time are measured in meters and seconds, respectively.

Find:

A) *The velocity and the acceleration of the object?*

B) *The velocity and the acceleration when t = 3s, t = 2s?*

C) *Average of the velocity and the acceleration at = 3s, t = 2s?*

Solution

A) *Defines the distance equation*

$$x = t^3 + 5 \; \ldots \ldots \ldots (1)$$

The velocity is:

$$\upsilon = \frac{dx}{dt} = \frac{d}{dt}(t^3+5) = 3t^2 \quad \ldots\ldots\ldots\ldots\ldots(2)$$

The acceleration is

$$a = \frac{dv}{dt} = \frac{d}{dt}(3t^2) = 6t \quad \ldots\ldots\ldots\ldots\ldots(3)$$

B) When the time (t = 2 s) the distance, velocity and the acceleration will be

The distance after 2 sec.

$$x_1 = t^3 + 5 = (2)^3 + 5 = 13\,m$$

The velocity after 2 sec.

$$\upsilon_1 = \frac{dx}{dt} = 3t^2 = 3(2)^2 = 12\,m/s$$

The acceleration after 2 sec.

$$a_1 = \frac{d\upsilon}{dt} = 6t = 6(2) = 12\,m/s^2$$

When the time (t=3 s), the distance, velocity and the acceleration, as what we have

PDF-XChange Viewer Click to buy NOW! www.docu-track.com

done in the previous (السابق) answers we get:

$x = 32\ m, \quad v = 27\ m/s \quad a = 18\ m/s^2$

C) The average velocity and acceleration are

$$\upsilon_{Ave.} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

$$= \frac{32 - 13}{3 - 2} = 19\ m/s$$

$$a_{Ave.} = \frac{\Delta \upsilon}{\Delta t} = \frac{\upsilon_2 - \upsilon 1}{t_2 - t_1}$$

$$= \frac{18 - 12}{3 - 2} = 6\ m/s^2$$