Introduction:

- The goal of physics is to provide an understanding of the physical world by developing **theories** based on **EXPERIMENTS**.
- A **physical theory**, usually expressed **mathematically**, describes how a given physical system works.
- The theory makes certain **predictions** about the physical system which can then be checked (التحقق منه) by **observations and experiments**.
- If the predictions turn out to (تبيَّن أنها) correspond closely to what is actually observed, then, the theory stands (تبقى قائمة), although it remains provisional (مؤقتة).
- **No theory to date** has given a complete description of all physical **phenomena**, even within a given sub-discipline (مجال فرعي) of physics.
- The basic laws of physics involve such physical quantities as **force, velocity, volume, and acceleration,** all of which can be described in terms of more fundamental quantities.
- **In mechanics**, it is conventional to use the quantities of **length (L), mass (M),** and **time (T)**; all other physical quantities can be constructed from these three.
- Physics is also known as **the science of measurement**.
- **Kelvin** in his own words: When you can measure what you are speaking about, and express it in numbers, you know something about it, when you cannot express it in numbers, your knowledge is of a meager (ضئيل) and unsatisfactory kind; it may be the beginning of knowledge, but you have scarcely (بالكاد), in your thoughts advanced to the stage of science.

Physical Quantity:

- We must first know a **measurement method** to define a physical quantity, or a method for mathematically calculating it from other quantities. For example, **distance** and **time** can be defined by describing the method in which we can measure both of them, and thus the **velocity** of a moving object can be defined by calculating the result of division of **distance** over **time**; ($v=l/t$).
- **Distance** and **Time** FUNDAMENTAL quantity whereas **velocity** DERIVED quantity.
- This method of definition is called an **Operational Definition.**
- To communicate the result of a measurement of a certain physical quantity, a ***unit*** for quantity must be defined.

Unit Systems

- Two systems of units are widely used in the world: the **metric system** (*International System of Units* **(SI)**), used widely in Europe and most of the rest of the world, and the **Imperial or British system**, a form of which is now mainly used in the USA.

Units		Symbol	Measuring
British system	SI		
foot (ft)	**Meter**	**m**	**Distance**
Pound (Ib)	**Kilogram**	**kg**	**Mass**
Second	**Second**	**s**	**Time**
Gallon (gal)	Litre/liter	l	Volume
Pound-force Ibf	Newton	N	Weight/Force
Fahrenheit ^{o}F	Kelvin	K	Temperature
	ampere	A	Electric current
	mole	mol	Number of particles
	candela	cd	Luminous intensity

- **Mass:**
  The SI unit of mass, the **kilogram**, is defined as the mass of a specific platinum–iridium alloy cylinder kept at the International Bureau of Weights and Measures at France.
- **Length:**
  The SI unit of length is measured by **Meter**, the meter was redefined as the distance traveled by light in vacuum during a time interval of 1/299 792 458 second
- **Time:**
  The SI unit of time is the Second, which is the time required for a cesium-133 atom to undergo 9 192 631 770 vibrations (the period of oscillation).

Prefixes for Powers of Ten

Table Some Prefixes for Powers of Ten Used with "Metric" (SI and cgs) Units

Power	Prefix	Abbreviation
10^{-18}	atto-	a
10^{-15}	femto-	f
10^{-12}	pico-	p
10^{-9}	nano-	n
10^{-6}	micro-	μ
10^{-3}	milli-	m
10^{-2}	centi-	c
10^{-1}	deci-	d
10^{1}	deka-	da
10^{3}	kilo-	k
10^{6}	mega-	M
10^{9}	giga-	G
10^{12}	tera-	T
10^{15}	peta-	P
10^{18}	exa-	E

Units Driven of Length Smallest to Largest

1	kilometer	(km)	$=10^{3}$m
1	decimeter	(dm)	$=10^{-1}$m
1	centimeter	(cm)	$=10^{-2}$m
1	millimeter	(mm)	$=10^{-3}$m
1	micrometer	(µm)	$=10^{-6}$m
1	nanometer	(nm)	$=10^{-9}$m
1	angstrom	(Å)	$=10^{-10}$m
1	picometer	(pm)	$=10^{-12}$m
1	femtometer	(fm)	$=10^{-15}$m

Derived quantities

All physical quantities measured by physicists can be expressed in terms of the three basic unit of **length, mass,** and **time**. Speed is simply length divided by time, and the force is actually mass multiplied by length divided by time squared or ($F=m\ l/s^2$).

[Speed] = $L/T = L\cdot T^{-1}$
[Force] = $M\cdot L/T^2 = M\cdot LT^{-2}$

Where [Speed] is meant to indicate the unit of speed, and M, L, and T represents mass, length, and time units.

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Dimensional Analysis:

- In physics the word ***dimension*** denotes the physical nature of a quantity.
- For example, the distance between two points can be measured in feet, or meters, which are different ways of expressing the dimension of *length*.
- One way to analyze such expressions, called dimensional analysis, makes use of the fact that dimensions can be treated as algebraic quantities.
- Adding masses to lengths, for example, makes no sense, so it follows that quantities can be added or subtracted only if they have the same dimensions.
- The procedure can be illustrated by using it to develop some relationships between acceleration, velocity, time, and distance.
- Distance x has the dimension of length: $[x] = \text{L}$. Time t has dimension $[t] = \text{T}$. Velocity v has the dimensions length over time: $[v] = \text{L/T}$, and acceleration the dimensions length divided by time squared: $[a] = \text{L/T}^2$.
- Notice that velocity and acceleration have similar dimensions, except for an extra dimension of time in the denominator of acceleration. It follows that

$$[v] = \frac{\text{L}}{\text{T}} = \frac{\text{L}}{\text{T}^2}\text{T} = [a][t]$$

- From this it might be guessed that velocity equals acceleration multiplied by time, $v = at$, and that is true for the special case of motion with constant acceleration starting at rest.
- Noticing that velocity has dimensions of length divided by time and distance has dimensions of length, it's reasonable to guess that

$$[x] = \text{L} = \text{L}\frac{\text{T}}{\text{T}} = \frac{\text{L}}{\text{T}}\text{T} = [v][t] = [a][t]^2$$

- Dimensional analysis that is used for finding the **Units of Quantity** and the **Mistakes of Derived Quantities** or laws.

Example 1.1

Using the dimensional analysis check that this equation $x = \frac{1}{2} at^2$ is correct, where x: is the distance, a: is the acceleration and t: is the time.

Solution:

$$x = \tfrac{1}{2} at^2 \Rightarrow [m] = \frac{1}{2} \times \frac{v}{t} \times t^2 = \frac{1}{2} \times v \times t = \frac{1}{2} \times \frac{x}{t} \times t = \frac{[m]}{[s]} \times \frac{[s]}{1} = [m]$$

∴ left side=right side

Example 1.2

Determine the dimensions for the following equations.

Area= length × width, Density=mass/volume, Force=mass × acceleration

Solution:

Area = [m] × [m]
= $[m]^2$

Density=[kg]/[m][m][m]
=$[kg]/[m]^3$

F=[kg] × [v]/[t]
=[kg] × [x]/[t]/[t]
=$[kg] \times [m]/[s]^2$
=[N]

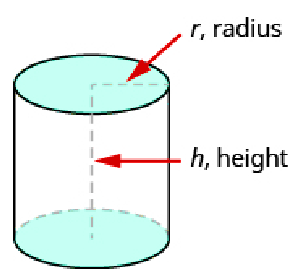
Example 1.3

Use the dimensional analysis to find the mistakes in the given equation that is considered as surface area of cylinder: ➔ $2\pi r^2 + 2\pi r^2 \times h$

Solution:

Left side Surface Area = $[m]^2$,

Right side= $[m]^2 + [m]^2 \times [m] = [m]^2 + [m]^3$



The mistakes is ➔ $2\pi r^2 \times h$

***Exercise*:**

Suppose that the acceleration of a particle moving in circle of radius r with uniform velocity v is proportional to the r^n and v^m. Use the dimensional analysis to determine the power n and m.

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Conversion of Units

- Sometimes it's necessary to convert units from one system to another. Conversion factors between the SI and BS for units of length are as follows:

$$1 \text{ mi} = 1\,609 \text{ m} = 1.609 \text{ km}, \qquad 1 \text{ ft} = 0.304\,8 \text{ m} = 30.48 \text{ cm}$$
$$1 \text{ m} = 39.37 \text{ in.} = 3.281 \text{ ft}, \qquad 1 \text{ in.} = 0.025\,4 \text{ m} = 2.54 \text{ cm}$$

Exercise: If a car is traveling at a speed of **28.0 m/s**, is the driver exceeding the speed limit of **55.0 mi/h**?

CONVERSION FACTORS

Length
1 m = 39.37 in. = 3.281 ft
1 in. = 2.54 cm (exact)
1 km = 0.621 mi
1 mi = 5 280 ft = 1.609 km
1 lightyear (ly) = 9.461×10^{15} m
1 angstrom (Å) = 10^{-10} m

Mass
1 kg = 10^3 g = 6.85×10^{-2} slug
1 slug = 14.59 kg
1 u = 1.66×10^{-27} kg = 931.5 MeV/c^2

Time
1 min = 60 s
1 h = 3 600 s
1 day = 24 h = 1.44×10^3 min = 8.64×10^4 s
1 yr = 365.242 days = 3.156×10^7 s

Volume
1 L = 1 000 cm^3 = 0.035 3 ft^3
1 ft^3 = 2.832×10^{-2} m^3
1 gal = 3.786 L = 231 $in.^3$

Angle
180° = π rad
1 rad = 57.30°
1° = 60 min = 1.745×10^{-2} rad

Speed
1 km/h = 0.278 m/s = 0.621 mi/h
1 m/s = 2.237 mi/h = 3.281 ft/s
1 mi/h = 1.61 km/h = 0.447 m/s = 1.47 ft/s

Force
1 N = 0.224 8 lb = 10^5 dynes
1 lb = 4.448 N
1 dyne = 10^{-5} N = 2.248×10^{-6} lb

Work and energy
1 J = 10^7 erg = 0.738 ft · lb = 0.239 cal
1 cal = 4.186 J
1 ft · lb = 1.356 J
1 Btu = 1.054×10^3 J = 252 cal
1 J = 6.24×10^{18} eV
1 eV = 1.602×10^{-19} J
1 kWh = 3.60×10^6 J

Pressure
1 atm = 1.013×10^5 N/m^2 (or Pa) = 14.70 lb/$in.^2$
1 Pa = 1 N/m^2 = 1.45×10^{-4} lb/$in.^2$
1 lb/$in.^2$ = 6.895×10^3 N/m^2

Power
1 hp = 550 ft · lb/s = 0.746 kW
1 W = 1 J/s = 0.738 ft · lb/s
1 Btu/h = 0.293 W

Vector and Scalar quantities:

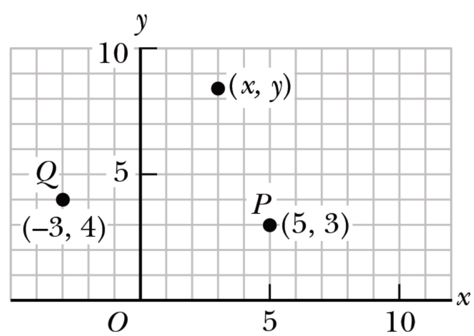
- Many familiar physical quantities can be specified completely by giving a **single number and the appropriate unit**. For example, "a class period lasts **50 min**" or "the gas tank in my car holds **65 L**" or "the distance between two posts is **100 m**."
- A physical quantity that can be specified completely in this manner is called a **Scalar Quantity**. **Scalar** is a synonym of "**number**." **Time, mass, distance, length, volume, temperature, and energy** are examples of scalar quantities.

- Physical quantity specified completely by giving a number of units (magnitude) and a direction are called **Vector Quantities**. Examples of vector quantities include displacement, velocity, position, force, and torque.

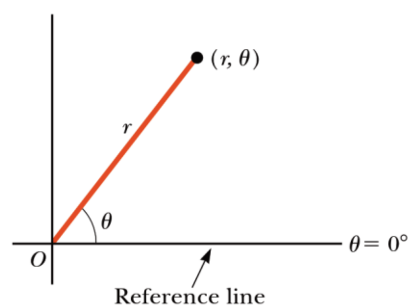
Coordinate Systems

- Many aspects of physics deal with locations in space, which require the definition of a coordinate system.
- A point on a **line** can be located with **one coordinate**.
- A point in a **plane** with **two coordinates**.
- A point in **space** with **three coordinates**.
- A coordinate system used to specify locations in space consists of the following:
    - A fixed reference point *O*, called the *origin*
    - A set of specified axes, or directions, with an appropriate scale and labels on the axes
    - Instructions on labeling a point in space relative to the origin and axes

One convenient and commonly used coordinate system is the **Cartesian Coordinate System (**rectangular coordinate system**).** Such a system in two dimensions is illustrated in Fig. ➔ An arbitrary point in this system is labeled with the coordinates (*x*, *y*). For example, the point *P* in the figure has coordinates (5, 3).



Sometimes it's more convenient to locate a point in space by its **Plane Polar Coordinates** (*r*,Ɵ), as in Fig ➔ the point is represented by *r* distance between origin point,Ɵ angle between the reference line and a line drawn from the origin to the point.



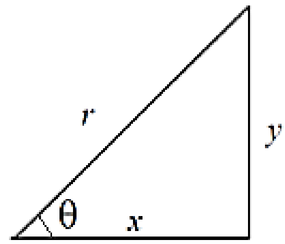
The relation between coordinates

The relation between the rectangular coordinates (x,y) and the polar coordinates (r,θ) is shown in Figure 1.3, where,

$$x = r\cos\theta \qquad (1.1)$$

And

$$y = r\sin\theta \qquad (1.2)$$



Squaring and adding equations (1.1) and (1.2) we get

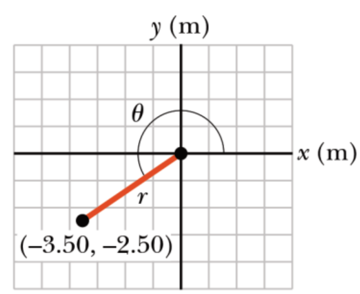
$$r = \sqrt{x^2 + y^2} \qquad (1.3)$$

Dividing equation (1.1) and (1.2) we get

$$\tan\theta = \frac{y}{x} \qquad (1.4)$$

Example 1.4

(a) The Cartesian coordinates of a point in the xy-plane are (x, y) = (-3.50, -2.50), as shown in next Fig. Find the polar coordinates of this point. (b) Convert (r, Θ) = (5, 37°) to rectangular coordinates.



Solution:

(a)

$r = \sqrt{x^2+y^2} = \sqrt{(-3.50)^2+(-2.50)^2}$ = 4.30 m, $\tan\theta = \frac{y}{x} = \frac{-2.50}{-3.50}$ =0.714

$\theta = \tan^{-1}(0.714) =$ 35.5° + 180° = 216°

(b) Polar to Cartesian conversion:

x = *r* cos Θ = (5 m) cos 37° = 3.99 m, *y* = *r* sin Θ = (5 m) sin 37° = 3.009 m ➔ P (3.99, 3.01).

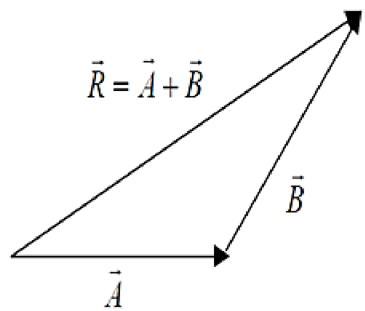
Properties of Vectors

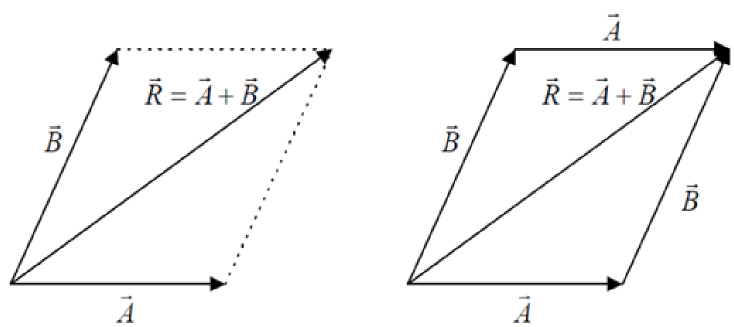
- Vector denote by an arrow marked over the variable. Like $\vec{A}$, $\vec{B}$.
- The magnitude of vector is used $|\vec{A}|$, $|\vec{B}|$, respectively.
- Commutative law: $\vec{A} + \vec{B} = \vec{B} + \vec{A}$
- Associative law: A+(B+C)=(A+B)+C

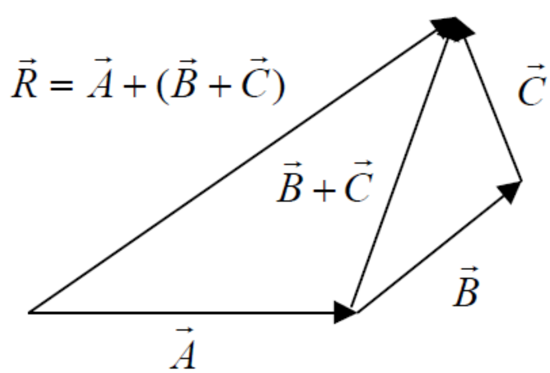
Vector Addition:

Only vectors representing the same physical quantities can be added. To add vector $\vec{A}$ to vector $\vec{B}$ as shown in Fig ⇨, the resultant vector $\vec{R}$ is

$$\vec{R} = \vec{A} + \vec{B}$$

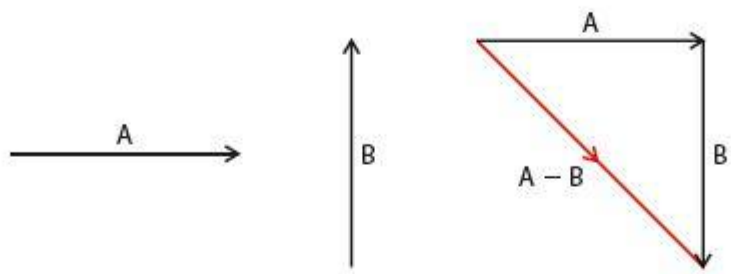






Vector Subtraction:

A - B = A + (-B)



Note that: A+ (-A) = 0

Unit Vector:

A **unit vector** is something that we use to have both magnitude and direction.

$$\text{Unit vector} = \frac{vector}{magnitude of the vector} \quad \text{or} \quad \hat{v} = \frac{\vec{v}}{|\vec{v}|}$$

Example 1.5

There is a vector $\vec{r} = 12\hat{i} - 3\hat{j} - 4\hat{k}.$ Calculate the unit vector $\vec{r}$. Express it in unit vector component formats.

Solution:

$$|\vec{r}| = \sqrt{x^2 + y^2 + z^2} = \sqrt{12^2 + (-3)^2 + (-4)^2} = \sqrt{144 + 9 + 16} = \sqrt{169} = 13$$

So,

$$\hat{r} = \frac{\vec{r}}{|\vec{r}|} = \frac{(x\hat{i}, y\hat{j}, z\hat{k})}{\sqrt{x^2+y^2+z^2}}$$

$$\hat{r} = \frac{12\hat{i} - 3\hat{j} - 4\hat{k}}{13}$$

$$\hat{r} = \frac{12}{13}\hat{i} - \frac{3}{13}\hat{j} - \frac{4}{13}\hat{k}$$

Components of a Vector

In a two-dimensional coordinate system, any **vector** can be broken into x-component and y-component.

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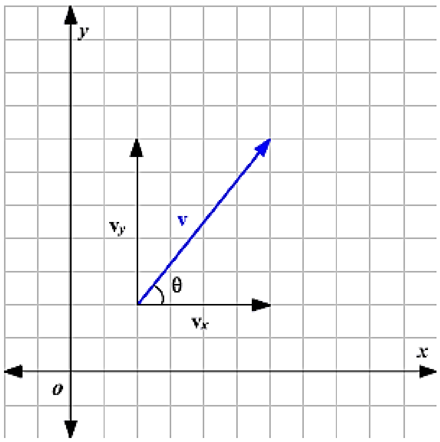
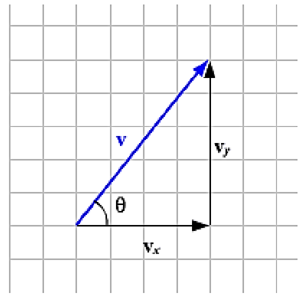
For example, in the figure shown below, the vector $\vec{v}$ is broken into two components, v_x and v_y .
Let the angle between the vector and its x - component be θ .



The vector and its components form a right angled triangle as shown below.



The trigonometric ratios give the relation between magnitude of the vector and the components of the vector.

$\cos\theta = \frac{v_x}{v}$ ⇨ $v_x = v\cos\theta$

$\sin\theta = \frac{v_y}{v}$ ⇨ $v_y = v\sin\theta$

Using the Pythagorean Theorem in the right triangle with lengths v_x and v_y :

$$|v| = \sqrt{{v_x}^2 + {v_y}^2}$$

Here, the numbers shown are the magnitudes of the vectors.

Case 1: Given components of a vector, find the magnitude and direction of the vector.

Use the following formulas in this case.

Magnitude of the vector is $|v| = \sqrt{{v_x}^2 + {v_y}^2}$.

To find direction of the vector, solve $\tan\theta = \frac{v_y}{v_x}$ for θ ⇨ $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)$

Case 2: Given the magnitude and direction of a vector, find the components of the vector.

Use the following formulas in this case.

$$v_x = v\cos\theta$$

$$v_y = v\sin\theta$$

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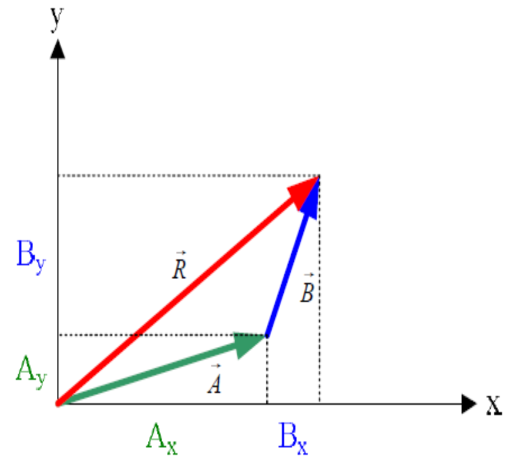
A vector $\vec{A}$ lying in the xy plane, having rectangular components and A_y can be expressed in a unit vector notation

$$\vec{A} = A_x\mathrm{i} + A_y\mathrm{j}$$

ملاحظة: يمكن استخدام طريقة تحليل المتجهات في جمع متجهين $\vec{A}$ و $\vec{B}$ كما في الشكل التالي:



$$\vec{A} = A_x i + A_y j$$

$$\vec{B} = B_x i + B_y j$$

$$\vec{R} = \vec{A} + \vec{B} = (A_x + B_x)i + (A_y + B_y)j$$

Example 1.6

Find the sum of two vectors $\vec{A}$ and $\vec{B}$ given by

$$\vec{A} = 3i + 4j \qquad \text{and} \qquad \vec{B} = 2i - 5j$$

Solution

Note that A_x=3, A_y=4, B_x=2, and B_y=-5

$$\vec{R} = \vec{A} + \vec{B} = (3+2)i + (4-5)j = 5i - j$$

The magnitude of vector $\vec{R}$ is

$$R = \sqrt{R_x^{\,2} + R_y^{\,2}} = \sqrt{25+1} = \sqrt{26} = 5.1$$

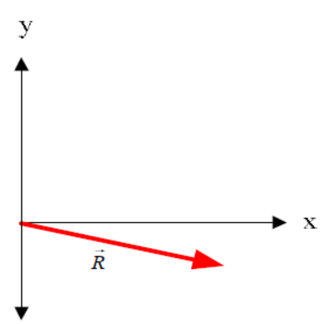


Figure 1.13

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The direction of $\vec{R}$ with respect to x-axis is

$$\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{-1}{5} = -11^o$$

Example 1.6

The polar coordinates of a point are r=5.5m and θ=240°. What are the rectangular coordinates of this point?

Solution

$x = r\cos\theta = 5.5 \times \cos 240 = -2.75$ m

$y = r\sin\theta = 5.5 \times \sin 240 = -4.76$ m

Example 1.7

Vector $\vec{A}$ is 3 units in length and points along the positive x axis. Vector $\vec{B}$ is 4 units in length and points along the negative y axis. Use graphical methods to find the magnitude and direction of the vector (a) $\vec{A}+\vec{B}$, (b) $\vec{A}-\vec{B}$

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Solution

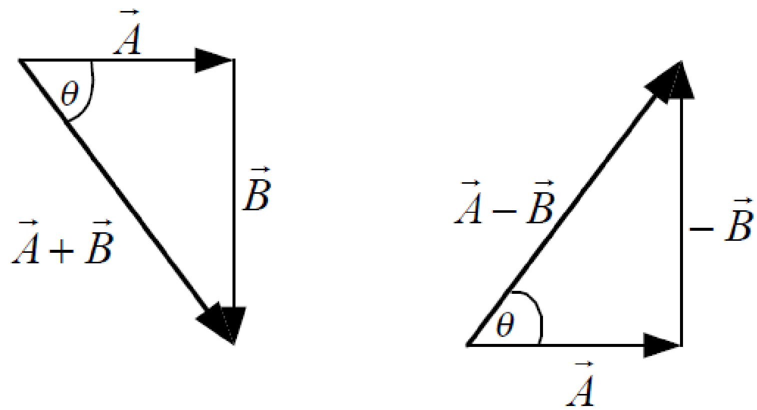

Figure 1.14

Example 1.8

Two vectors are given by $\vec{A} = 3i - 2j$ and $\vec{B} = -i - 4j$. Calculate (a) $\vec{A}+\vec{B}$, (b) $\vec{A}$ - $\vec{B}$, (c) $\left|\vec{A}+\vec{B}\right|$, (d) $\left|\vec{A}-\vec{B}\right|$, and (e) the direction of $\vec{A}+\vec{B}$ and $\left|\vec{A}-\vec{B}\right|$.

$\left|\vec{A}+\vec{B}\right| = 5$ $\qquad$ $\left|\vec{A}-\vec{B}\right| = 5$

$\theta = -53°$ $\qquad$ $\theta = 53°$

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Solution (a) $\vec{A}+\vec{B} = (3i-2j)+(-i-4j) = 2i-6j$

(b) $\vec{A}-\vec{B} = (3i-2j)-(-i-4j) = 4i+2j$

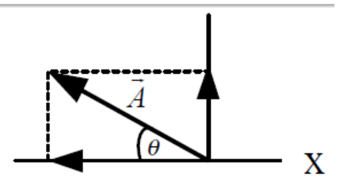
(c) $\left|\vec{A}+\vec{B}\right| = \sqrt{2^2+(-6)^2} = 6.32$

(d) $\left|\vec{A}-\vec{B}\right| = \sqrt{4^2+2^2} = 4.47$

(e) For $\vec{A}+\vec{B}$, $\theta = \tan^{-1}(-6/2) = -71.6^\circ = 288^\circ$

For $\vec{A}-\vec{B}$, $\theta = \tan^{-1}(2/4) = 26.6^\circ$

Solution



$A_x = -3$ units & $A_y = 2$ units

(a) $\vec{A} = A_x i + A_y j = -3i+2j$ units

(b) $\left|\vec{A}\right| = \sqrt{A_x^{\,2}+A_y^{\,2}} = \sqrt{(-3)^2+(2)^2} = 3.61\ units$

$\theta = \tan^{-1}(2/-3) = 33.7^\circ$ (relative to the –x axis)

(c) $R_x = 0$ & $R_y = -4$; since $\vec{R} = \vec{A}+\vec{B}$, $\vec{B} = \vec{R}-\vec{A}$

Example 1.9

A vector $\vec{A}$ has a negative x component 3 units in length and positive y component 2 units in length. (a) Determine an expression for $\vec{A}$ in unit vector notation. (b) Determine the magnitude and direction of $\vec{A}$. (c) What vector $\vec{B}$ when added to $\vec{A}$ gives a resultant vector with no x component and negative y component 4 units in length?

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Solution

(a) $\vec{R}_1 = x_1 i + y_1 j = (-3i - 5j)m$

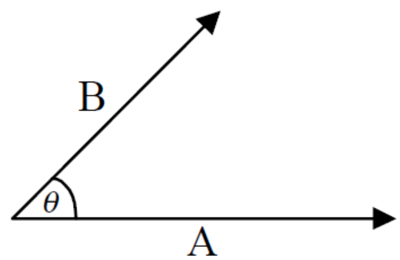
$\vec{R}_2 = x_2 i + y_2 j = (-i + 8j)m$

(b) Displacement $= \Delta\vec{R} = \vec{R}_2 - \vec{R}_1$

$$\Delta\vec{R} = (x_2 - x_1)i + (y_2 - y_1)j = -i - (-3i) + 8j - (-5j) = (2i + 13j)m$$

1.11 Product of a vector

There are two kinds of vector product the first one is called scalar product or dot product because the result of the product is a scalar quantity. The second is called vector product or cross product because the result is a vector perpendicular to the plane of the two vectors.



ينتج من الضرب القياسي كمية قياسية وينتج من الضرب الإتجاهي كمية متجهة

1.11.1 The scalar product

يعرف الضرب القياسي scalar product بالضرب النقطي dot product وتكون نتيجة الضرب القياسي لمتجهين كمية قياسية، وتكون هذه القيمة موجبة إذا كانت الزاوية المحصورة بين المتجهين بين 0 و 90 درجة وتكون النتيجة سالبة إذا كانت الزاوية المحصورة بين المتجهين بين 90 و 180 درجة وتساوي صفراً إذا كانت الزاوية 90.

$$\vec{A}.\vec{B} = +\text{ve when } 0 \geq \theta > 90^o$$

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$\vec{A}.\vec{B}$ = -ve when $90^o < \theta \leq 180^o$

$\vec{A}.\vec{B}$ = zero when $\theta = 0$

يعرف الضرب القياسي لمتجهين $\vec{A},\vec{B}$ بحاصل ضرب مقدار المتجه الأول $\vec{A}$ في مقدار المتجه الثاني $\vec{B}$ في جيب تمام الزاوية المحصورة بينهما.

$$\vec{A}.\vec{B} = |A||B|\cos\theta \qquad (1.16)$$

يمكن إيجاد قيمة الضرب القياسي لمتجهين باستخدام مركبات كل متجه كما يلي:

$$\vec{A} = A_x i + A_y j + A_z k \qquad (1.17)$$

$$\vec{B} = B_x i + B_y j + B_z k \qquad (1.18)$$

The scalar product is

$$A.B = (A_x i + A_y j + A_z k).(B_x i + B_y j + B_z k) \qquad (1.19)$$

بضرب مركبات المتجه $\vec{A}$ في مركبات المتجه $\vec{B}$ ينتج التالي:

$$\begin{aligned}\vec{A}.\vec{B} = (&A_x i.B_x i + A_x i.B_y j + A_x i.B_z k \\ &+ A_y j.B_x i + A_y j.B_y j + A_y j.B_z k \qquad (1.20)\\ &+ A_z k.B_x i + A_z k.B_y j + A_z k.B_z k)\end{aligned}$$

Therefore

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Therefore

$$\therefore \vec{A}.\vec{B} = A_x B_x + A_y B_y + A_z B_z \qquad (1.21)$$

The angle between the two vectors is

$$\cos\theta = \frac{\vec{A}.\vec{B}}{|A||B|} = \frac{A_x B_x + A_y B_y + A_z B_z}{|A||B|} \qquad (1.22)$$

Example 1.11

Find the angle between the two vectors

$$\vec{A} = 2i + 3j + 4k\,, \quad \vec{B} = i - 2j + 3k$$

Solution $\cos\theta = \dfrac{A_x B_x + A_y B_y + A_z B_z}{|A||B|}$

$$A_x B_x + A_y B_y + A_z B_z = (2)(1)+(3)(\text{-}2)+(4)(3)=8$$

$$|A| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$$

$$|B| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{14}$$

$$\cos\theta = \frac{18}{\sqrt{29}\sqrt{14}} 0.397 \Rightarrow \theta = 66.6^o$$

1.11.2 The vector product

يعرف الضرب الاتجاهي *vector product* بـــ *cross product* وتكون نتيجة الضرب الاتجاهي لمتجهين كمية متجهة. قيمة هذا المتجه $\vec{C} = \vec{A} \times \vec{B}$ واتجاهه عمودي على كل من المتجهين $\vec{A}$ و $\vec{B}$ وفي اتجاه دوران بريمة من المتجه $\vec{A}$ إلى المتجه $\vec{B}$ كما فـــي الشكل التالي:

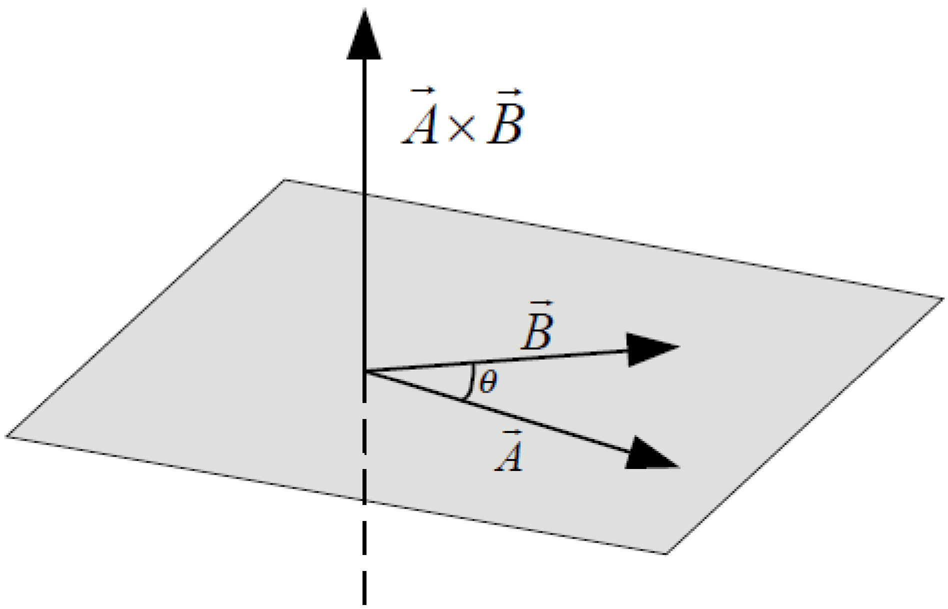


Figure 1.16

$$\vec{A} \times \vec{B} = AB \sin \theta \qquad (1.23)$$

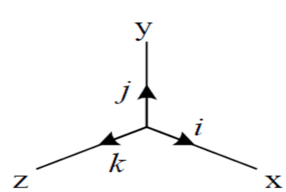
$$\vec{A} \times \vec{B} = \left(A_x i + A_y j + A_z k\right) \times \left(B_x i + B_y j + B_z k\right) \qquad (1.24)$$

To evaluate this product we use the fact that the angle between the unit vectors i, j , k is 90°.

$$i \times i = 0 \qquad i \times j = k \qquad i \times k = -j$$

$$j \times j = 0 \qquad j \times k = i \qquad j \times i = -k$$

$$k \times k = 0 \qquad k \times i = j \qquad k \times j = -i$$



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$$\vec{A}\times\vec{B} = \left(A_yB_z - A_zB_y\right)i + \left(A_zB_x - A_xB_z\right)j + \left(A_xB_y - A_yB_x\right)k \qquad (1.25)$$

If $\vec{C} = \vec{A}\times\vec{B}$, the components of $\vec{C}$ are given by

$$C_x = A_yB_z - A_zB_y$$

$$C_y = A_zB_x - A_xB_z$$

$$C_z = A_xB_y - A_yB_x$$

Example 1.12

If $\vec{C} = \vec{A}\times\vec{B}$, where $\vec{A} = 3i - 4j$, and $\vec{B} = -2i + 3k$, what is $\vec{C}$?

Solution

$$\vec{C} = \vec{A}\times\vec{B} = (3i - 4j)\times(-2i + 3k)$$

which, by distributive law, becomes

$$\vec{C} = -(3i\times 2i) + (3i\times 3k) + (4j\times 2i) - (4j\times 3k)$$

Using equation (123) to evaluate each term in the equation above we get

$$\vec{C} = 0 - 9j - 8k - 12i = -12i - 9j - 8k$$

The vector $\vec{C}$ is perpendicular to both vectors $\vec{A}$ and $\vec{B}$.

Vector Operations

$$\vec{A}+\vec{B}, \qquad \vec{A}-\vec{B}, \qquad C\vec{A}$$

We study the vector addition and subtraction. As for, the multiplication of vectors with scalar $C\vec{A}$.

Example:

A vector represented in orthogonal system as $\vec{A} = 3i + j + k$. What would be the resultant vector if $\vec{A}$ is multiplied by 5?

Solution:

As the vector is to be multiplied by a scalar, the resultant

$$5\vec{A} = 5(3i + j + k)$$

$$\overrightarrow{5A} = (15i + 5j + 5k)$$

Now, we study Vector Multiplication:

Vector multiplication is two type

- Dot product ($\vec{A} \cdot \vec{B}$)
- Cross product ($\vec{A} \times \vec{B}$)

Dot product

If we have two vectors:

$$\vec{A} = a_1 i + a_2 j + a_3 k \quad \text{and} \quad \vec{B} = b_1 i + b_2 j + b_3 k$$

The dot product of these vectors would be

$$\vec{A} \cdot \vec{B} = a_1 b_1 + a_2 b_2 + a_3 b_3 = \text{number (scalar quantity)}$$

Example:

Find the dot product for the following vectors:

$$\vec{A} = 3i + 2j + 4k, \vec{B} = 2i - 3j + 2k$$

Solution:

$$\vec{A} \cdot \vec{B} = (3*2) + (2*-3) + (4*2) = 6 - 6 + 8 = 8$$

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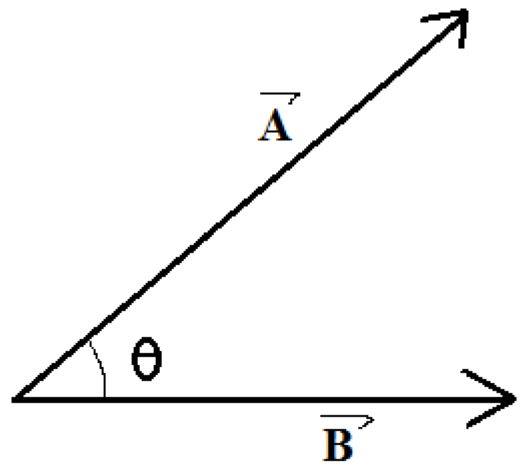
Dot product properties:

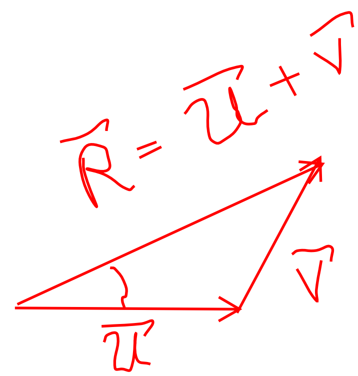
$$\vec{A}\cdot\vec{B}=\vec{B}\cdot\vec{A}$$ ………… Commutative Law

$$\vec{A}\cdot(\vec{B}+\vec{C})=(\vec{A}\cdot\vec{B})+(\vec{A}\cdot\vec{C})$$ … Distributive Law

$$(C\vec{A})\cdot\vec{B}=C(\vec{A}\cdot\vec{B})=\vec{A}\cdot(C\vec{B})$$

$$\vec{A}\cdot\vec{A}=|\vec{A}|^2, \qquad 0\cdot\vec{A}=0$$





$$\vec{A}\cdot\vec{B}=|\vec{A}||\vec{B}|\cos\theta$$

For calculating angle between two vectors, simply we use previous equation:

$$\cos\theta=\frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|}$$

NOTE: the dot product is useful to calculate the angle between two vectors.

Example: what is the angle between following vectors

$\vec{A}=2i+2j-k, \vec{B}=5i-3j+2k$ or you can write

$\vec{A}=<2,2,-1>, \vec{B}=<5,-3,2>$

Solution:

$|\vec{A}| = \sqrt{2^2 + 2^2 + (-1^2)} = \sqrt{9} = \mathbf{3}$, $|\vec{B}| = \sqrt{5^2 + (-3)^2 + (2^2)} = \sqrt{\mathbf{38}}$

$$\vec{A} \cdot \vec{B} = (2 * 5) + (2 * -3) + (-1 * 2) = 10 - 6 - 2 = 2$$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \frac{2}{3\sqrt{38}} = 0.108 \Rightarrow \theta = \cos^{-1}\cos\theta = \cos^{-1} 0.108$$

$\theta = 1.4\,rad = 84°$; degree =radian $\frac{180}{\pi}$, or, radian = degree $\frac{\pi}{180}$

If $\boldsymbol{\Theta = 90^o}$, which means the two vectors **orthogonal** (perpendicular). In another word for any orthogonal vectors:

$$\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} = |\vec{A}||\vec{B}|\cos\theta = \mathbf{0} \text{ ; Because of } \mathbf{\cos\frac{\pi}{2}} = \mathbf{0}$$

Exercise:

$$\vec{A} =< 4, -2{,}5 >, \vec{B} =< -1{,}3, -6 > , \vec{C} =< 7, -5{,}1 >$$

Find $\vec{A} \cdot \vec{B}$, $\vec{B} \cdot \vec{C}$, and find angles vectors.

<u>Cross Product:</u>

The result of a dot product is a **number** and the result of a cross product is a **vector**! Be careful not to confuse the two.

So, let us start $\vec{A} = \langle a_1, a_2, a_3 \rangle$ and $\vec{B} = \langle b_1, b_2, b_3 \rangle$

Then the cross product is obtained by the **determinant of a 3x3 matrix**,

$$\vec{A} \times \vec{B} = \begin{vmatrix} i & j & k \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

<====== محدد

$$\vec{A}\times\vec{B}=\begin{vmatrix}a_2 & a_3\\ b_2 & b_3\end{vmatrix} i-\begin{vmatrix}a_1 & a_3\\ b_1 & b_3\end{vmatrix} j+\begin{vmatrix}a_1 & a_2\\ b_1 & b_2\end{vmatrix} k$$

and $\begin{vmatrix}a & b\\ c & d\end{vmatrix}= ad\text{-}bc$

Example:

Find the cross product of the following vectors

$\vec{A}=i+3j+4k, \vec{B}=2i+7j-5k$ or $\vec{A}=<1,3,4>, \vec{B}=<2,7,-5>$

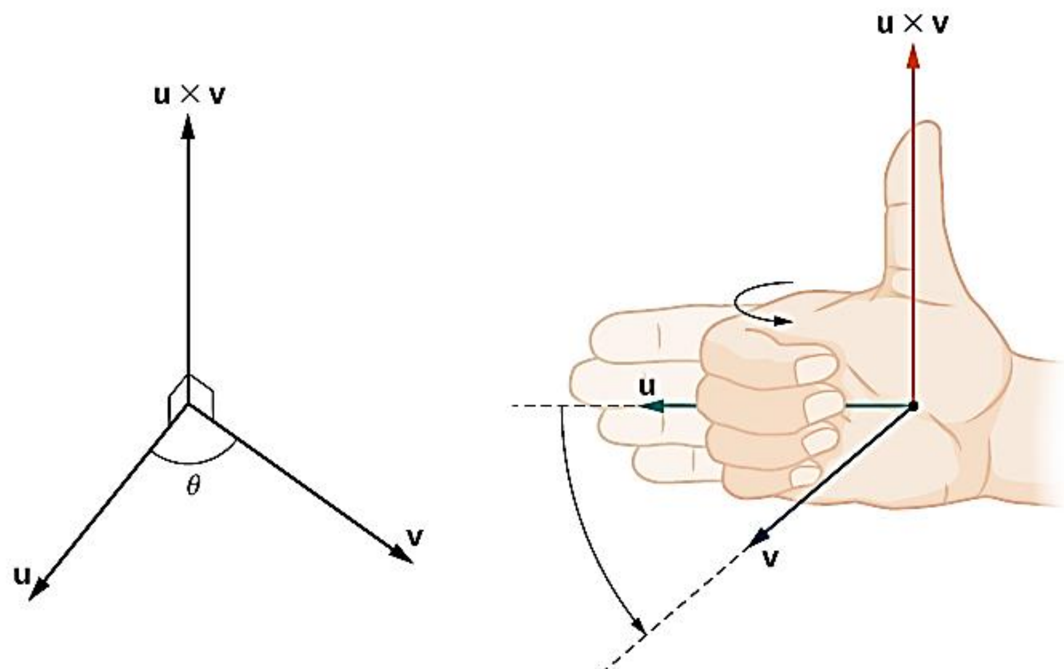
$$\vec{A}\times\vec{B}=\begin{vmatrix}i & j & k\\ 1 & 3 & 4\\ 2 & 7 & -5\end{vmatrix}=\begin{vmatrix}3 & 4\\ 7 & -5\end{vmatrix} i-\begin{vmatrix}1 & 4\\ 2 & -5\end{vmatrix} j+\begin{vmatrix}1 & 3\\ 2 & 7\end{vmatrix} k$$

$$\vec{A}\times\vec{B}=[(3*-5)-(7*4)]i-[(1*-5)-(2*4)]\,j+[(1*7)-(2*3)]k$$

$$=[-15-28]i-[-5-8]\,j+[7-6]k=-43i+13\,j+k$$

$$\boldsymbol{\vec{A}\times\vec{B}=-43i+13\,j+k}$$

- The cross product will always be orthogonal to the plane that containing the original two vectors.
- Its direction is found by **Right-Hand rule**.



Cross Product Properties:

- $\boldsymbol{\vec{A}\times\vec{A}=0}$ … … … . prove that (H. W)

$$|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin\theta$$

We can find the cross product vector magnitude using previous formula.

- $\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$ ....prove that (H.W)
- But, $\vec{A} \times \vec{B} = -(\vec{B} \times \vec{A})$ ....prove that (H.W)
- $(\vec{A} \times \vec{B}) \times \vec{C} \neq \vec{A} \times (\vec{B} \times \vec{C})$ ....prove that (H.W)
- $\vec{A} \times (\vec{B} + \vec{C}) = (\vec{A} \times \vec{B}) + (\vec{A} \times \vec{C})$ ....prove that (H.W)
- What is the cross product of parallel vectors? (H.W)

Exercise:

$\vec{A} =< 4, -2,5 >, \vec{B} =< -1,3, -6 >, \vec{C} =< 7, -5,1 >$

Find:

$\vec{A} \times \vec{B}, \vec{B} \times \vec{C}, \vec{A} \times \vec{C}$