Norton’s Theorem

Norton's Theorem

- The theorem states the following:
- Any two terminal linear dc network can be replaced by an equivalent circuit consisting of a current source and a parallel resistor as shown in figure below:

1. *Remove that portion of the network across which the Norton equivalent circuit is found.*
2. *Mark the terminals of the remaining two-terminal network.*

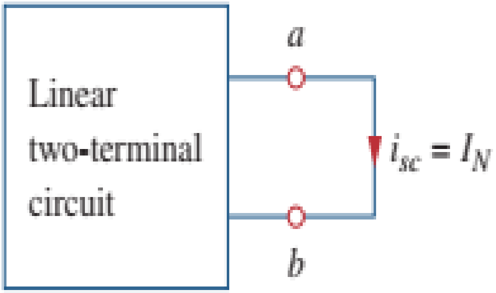
R_N:

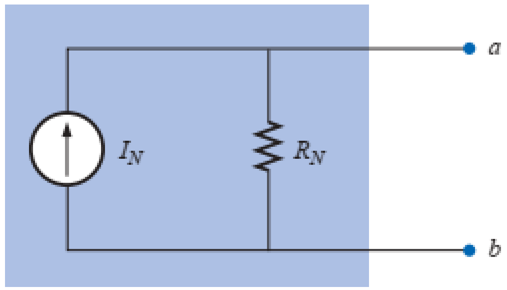
3. *Calculate R_N by first setting all sources to zero (voltage sources are replaced with short circuits, and current sources with open circuits) and then finding the resultant resistance between the two marked terminals. (If the internal resistance of the voltage and/or current sources is included in the original network, it must remain when the sources are set to zero.) Since $R_N = R_{Th}$, the procedure and value obtained using the approach described for Thévenin's theorem will determine the proper value of R_N.*

I_N:

4. Calculate I_N by first returning all sources to their original position and then finding the short-circuit current between the marked terminals. It is the same current that would be measured by an ammeter placed between the marked terminals.

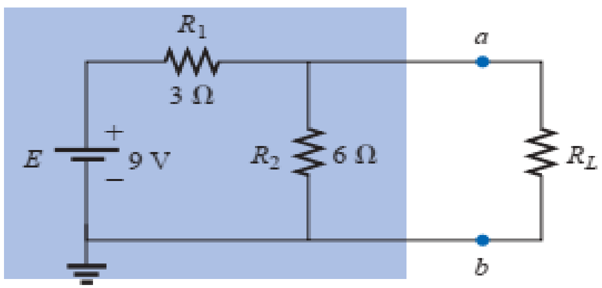
$$I_N = i_{sc}$$



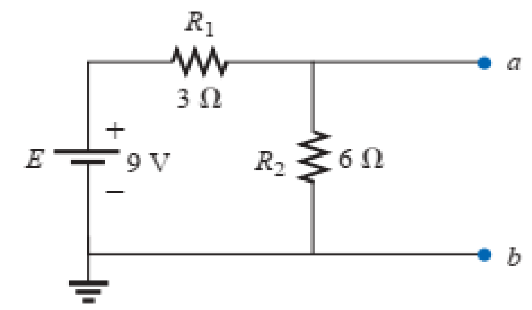


Norton equivalent circuit.

Example (1): Find the Norton equivalent circuit for the network in the shaded area, then calculate the current passing through the R_L .

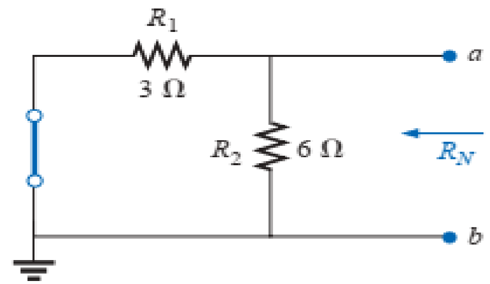


- Steps 1 and 2 is shown in the figure below:

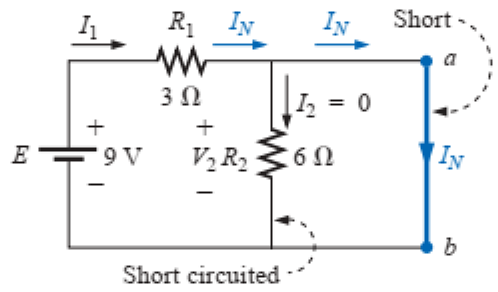


- Step 3:

$$R_N = R_1 \parallel R_2 = 3\,\Omega \parallel 6\,\Omega = \frac{(3\,\Omega)(6\,\Omega)}{3\,\Omega + 6\,\Omega} = \frac{18\,\Omega}{9} = 2\,\Omega$$



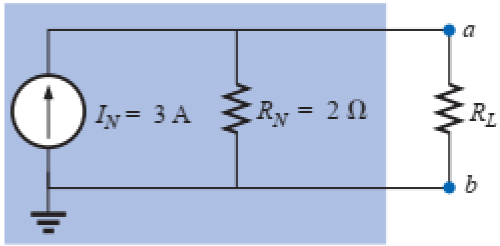
Step 4 is shown in figure below, clearly indicating that the short – circuit connection between terminals a and b is in parallel with R_2 and eliminates its effect I_N id therefore the same as throughR_1, and the full battery voltage appears across R_1 since:



$$V_2 = I_2 R_2 = (0)6\ \Omega = 0\ \text{V}$$

Therefore,

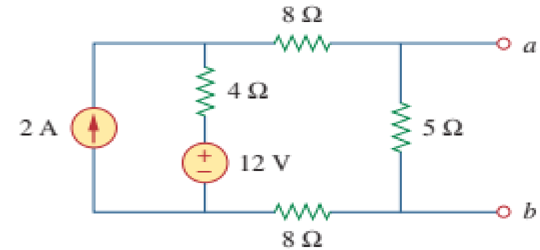
$$I_N = \frac{E}{R_1} = \frac{9\ \text{V}}{3\ \Omega} = \mathbf{3}\text{A}$$



When R_L=4Ω,

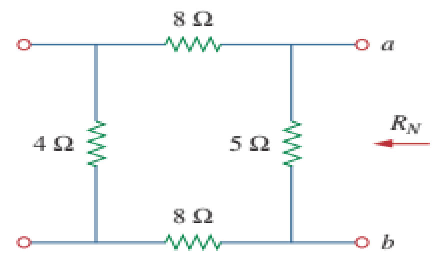
$$I_{R_L} = I_N * \frac{R_N}{R_N + R_L} = 3 * \frac{2}{2+4} = 1A$$

Example (2): For the circuit shown below, Find the Norton's equivalent circuit at terminals (a-b).



- To find R_N, see figure below

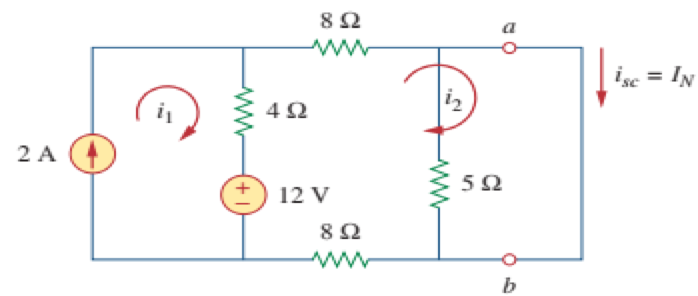
$$R_N = 5 \parallel (8 + 4 + 8) = 5 \parallel 20 = \frac{20 \times 5}{25} = 4\ \Omega$$



To find I_N, we short-circuit terminals a and b, as shown in Fig.

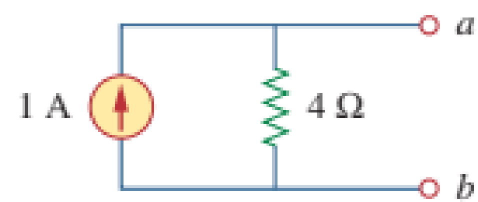
We ignore the 5-Ω resistor because it has been short-circuited. Applying mesh analysis, we obtain

$$i_1 = 2\ \text{A}, \qquad 20i_2 - 4i_1 - 12 = 0$$

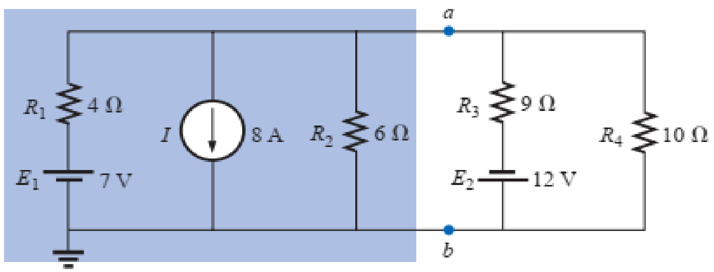


From these equations, we obtain

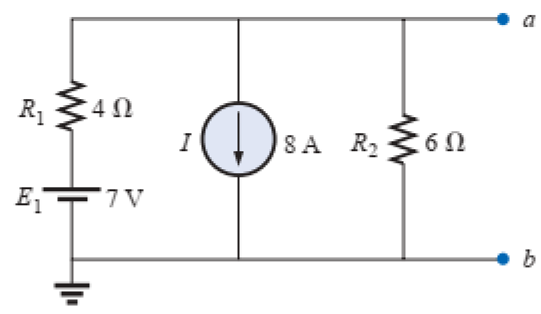
$$i_2 = 1\ \text{A} = i_{sc} = I_N$$



Example(3): Find the Norton's equivalent circuit for the portion of the network to the left of (a-b)

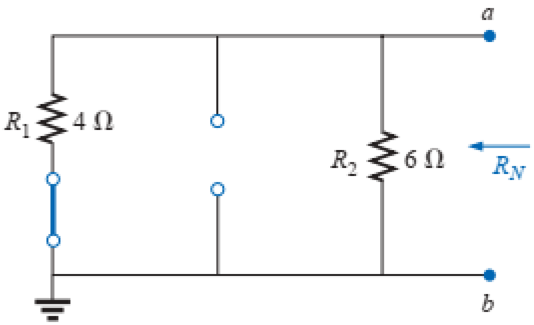


- Step1 : remove the portion of the circuit to the right of a-b as shown in figure below:



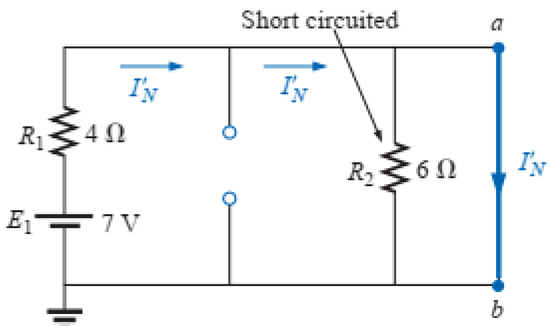
- Step 2: to calculate the Norton's resistance

$$R_N = R_1 \parallel R_2 = 4\,\Omega \parallel 6\,\Omega = \frac{(4\,\Omega)(6\,\Omega)}{4\,\Omega + 6\,\Omega} = \frac{24\,\Omega}{10} = \mathbf{2.4\,\Omega}$$

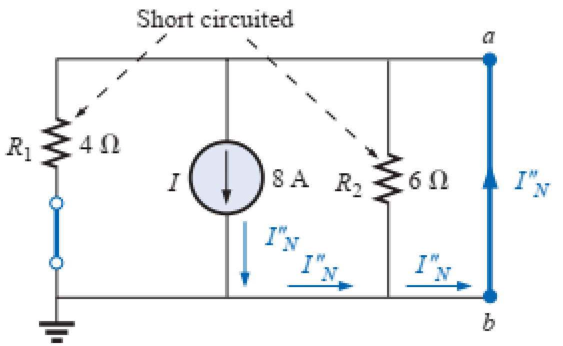


- Step 3: calculate the Norton's current (short circuit current)
- Using superposition
- For the 7v voltage source:

$$I'_N = \frac{E_1}{R_1} = \frac{7\,\text{V}}{4\,\Omega} = 1.75\,\text{A}$$



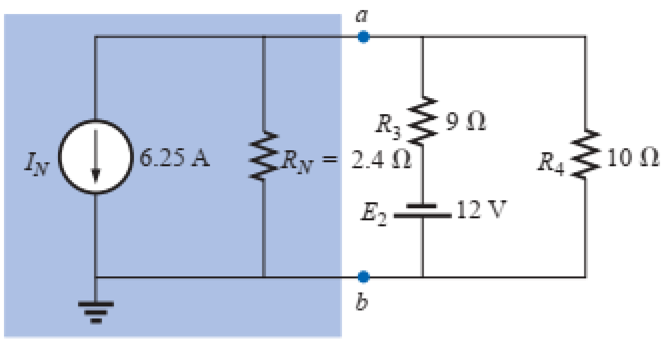
- For the current source:



$$I''_N = I = 8\text{ A}$$

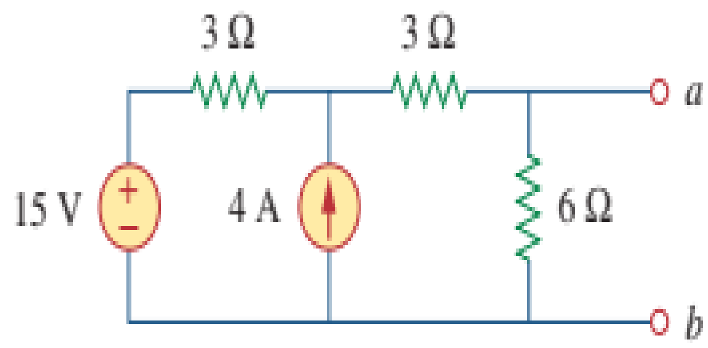
The result is

$$I_N = I''_N - I'_N = 8\text{ A} - 1.75\text{ A} = \mathbf{6.25\text{ A}}$$

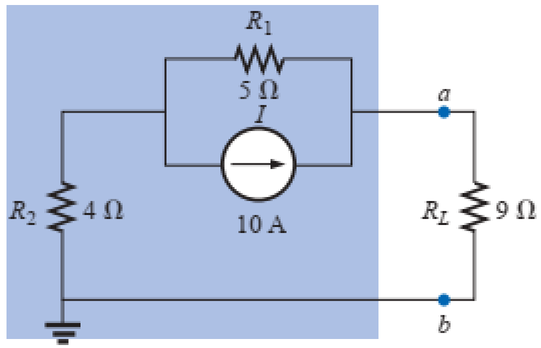


HW

- 1- For the circuit shown below, Find the Norton's equivalent circuit at terminals (a-b).



2- For the circuit shown below, Find the Norton's equivalent circuit for the network external to the 9Ω resistor.



3- For the circuit shown below, Find the Norton’s equivalent circuit at terminals (a-b).