Chapter Three
Network Theorem

-Superposition.

-Thevenen's Theorem

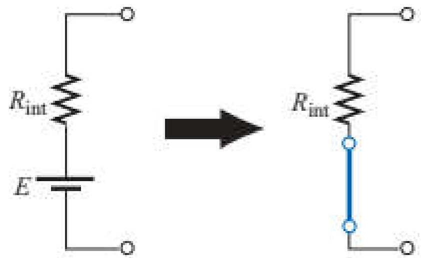
- Norton's Theorem

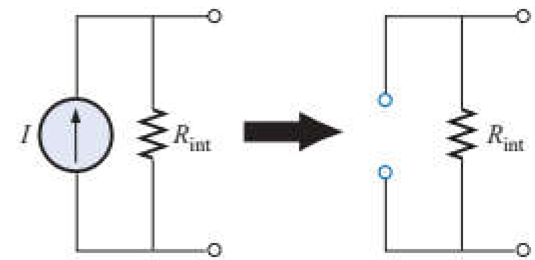
- Maximum Power Transfer

Superposition Theorem

The **superposition** principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

1. We consider one independent source at a time while all other independent sources are *turned off*. This implies that we replace every voltage source by 0 V (or a short circuit), and every current source by 0 A (or an open circuit). This way we obtain a simpler and more manageable circuit.
2. Dependent sources are left intact because they are controlled by circuit variables.





Steps to apply superposition principle:

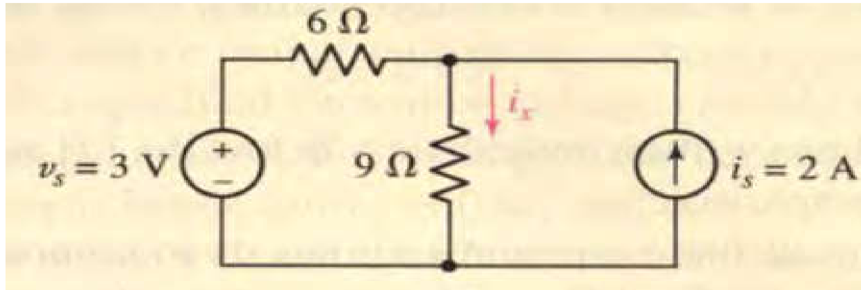
1- Turn off all independent sources except one source. Find the output(voltage or current) due to that active source using the - techniques covered in chapter 1 and 2.

2-Repeat step 1 for each of the other independent sources.

3- Find the total contribution by adding algebraically all the contributions due to the independent sources.

Number of networks to be analyzed = Number of independent sources

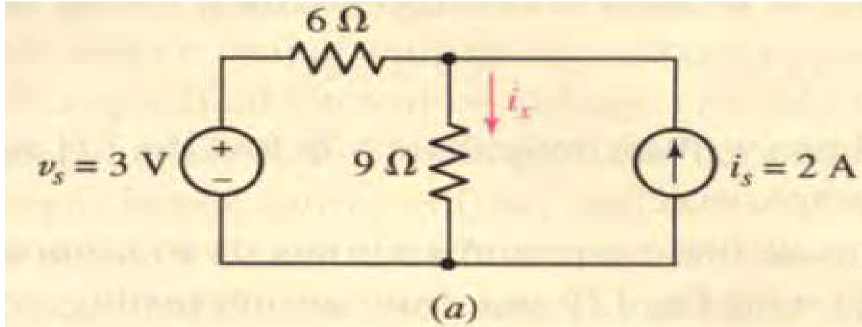
Example: For the circuit of figure below, use superposition to write an expression for the unknown branch current i_x.

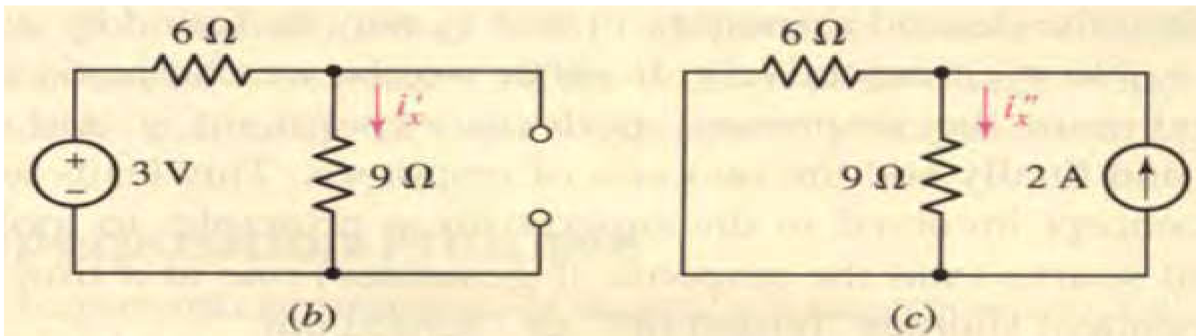


(a)

Solution: The portion of i_x due to voltage source has been designate i'_x (fig.(b)) to avoid the confusion. And due to current source is i''_x fig.(c).

$$i_x = i_x|_{3\text{ V}} + i_x|_{2\text{ A}} = i'_x + i''_x$$



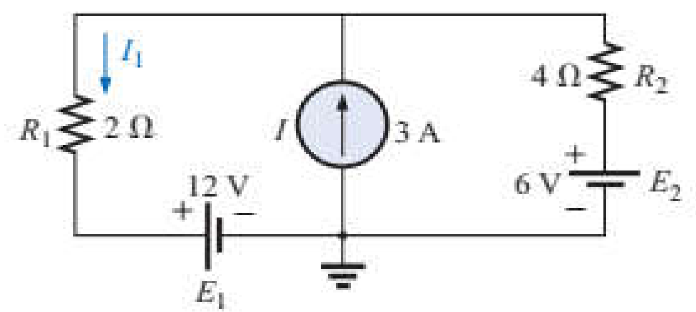


$$i_x' = \frac{3}{6+9} = 0.2A$$

$$i_x'' = 2 * \frac{6}{6+9} = 0.8A$$

$$i_x = 0.2 + 0.8 = 1A$$

Example: Find the current through the 2 Ω resistor of the network of figure below, using superposition theorem.



The presence of three sources will result in three different networks to be analyzed.

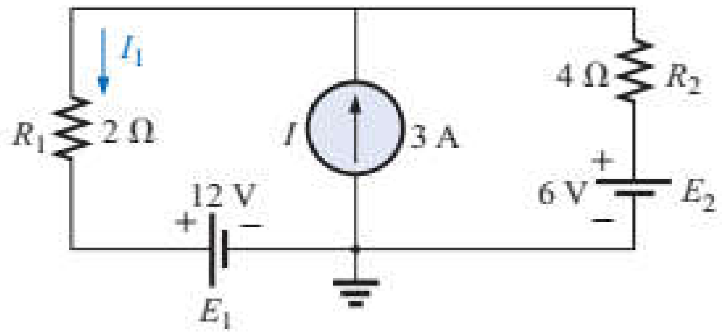
$I_1 = I_1' + I_1'' + I_1'''$

I_1'=due to 12v voltage source.

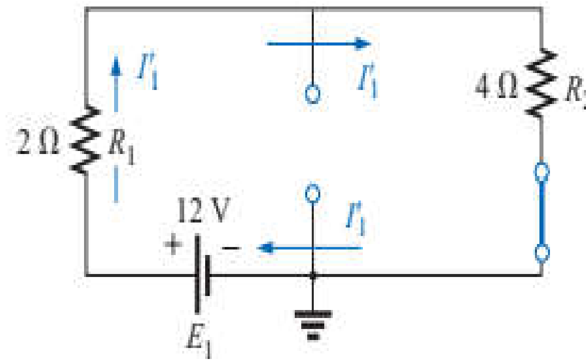
I_1''=due to 6v voltage source.

I_1'''=due to 3A current source.

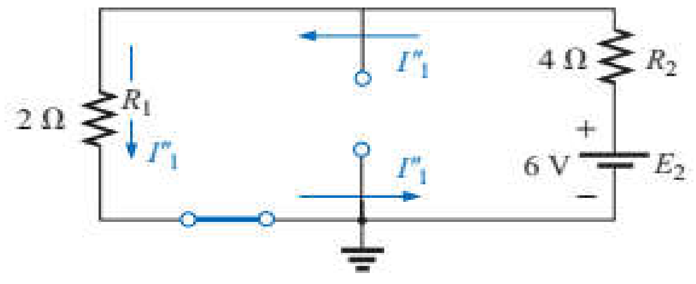
1- For the effect of the 12v voltage source.



$$I'_1 = \frac{E_1}{R_1 + R_2} = \frac{12\text{ V}}{2\,\Omega + 4\,\Omega} = \frac{12\text{ V}}{6\,\Omega} = 2\text{ A}$$

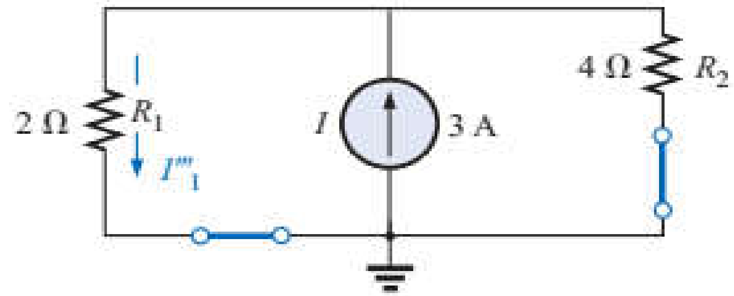


2- For the effect of the 6v voltage source.



$$I''_1 = \frac{E_2}{R_1 + R_2} = \frac{6\text{ V}}{2\,\Omega + 4\,\Omega} = \frac{6\text{ V}}{6\,\Omega} = 1\text{ A}$$

3- For the effect of the 3A current source.



$$I'''_1 = \frac{R_2 I}{R_1 + R_2} = \frac{(4\,\Omega)(3\,\text{A})}{2\,\Omega + 4\,\Omega} = \frac{12\,\text{A}}{6} = 2\,\text{A}$$

The total current through the 2Ω resistor appears in figure below:

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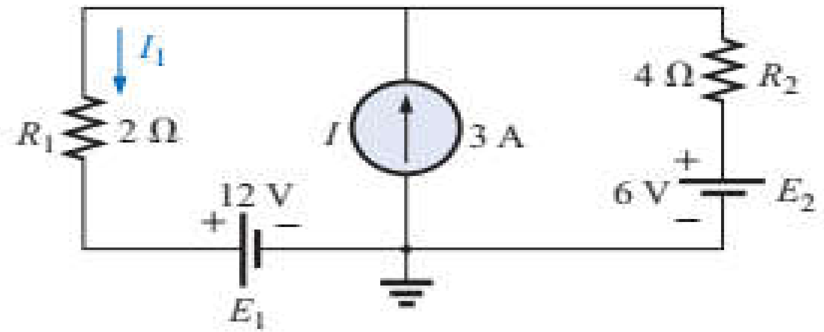
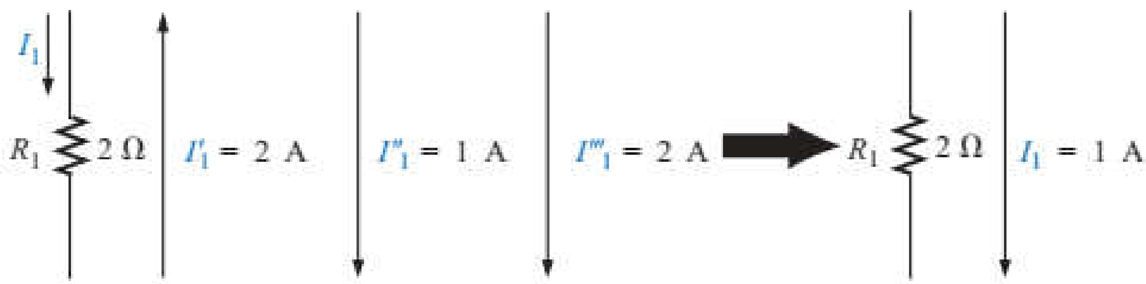
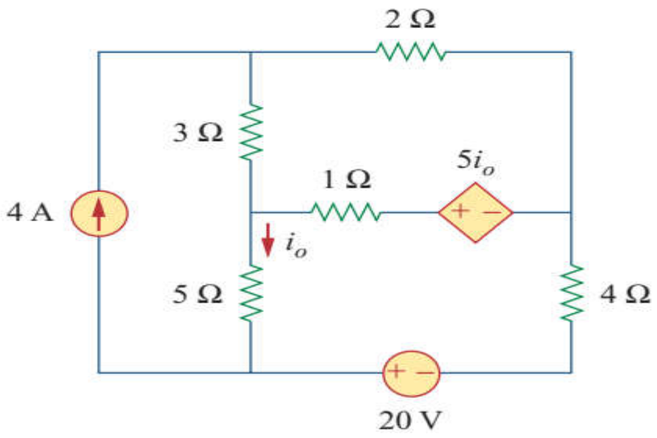




$$I_1 = \overbrace{I''_1 + I'''_1}^{\text{Same direction as } I_1 \text{ in Fig. 9.19}} - \underbrace{I'_1}_{\text{Opposite direction to } I_1 \text{ in Fig. 9.19}}$$

$$= 1\text{A} + 2\text{A} - 2\text{A} = 1\text{A}$$

Example: Find the value of I_o in the circuit shown below, using superposition.



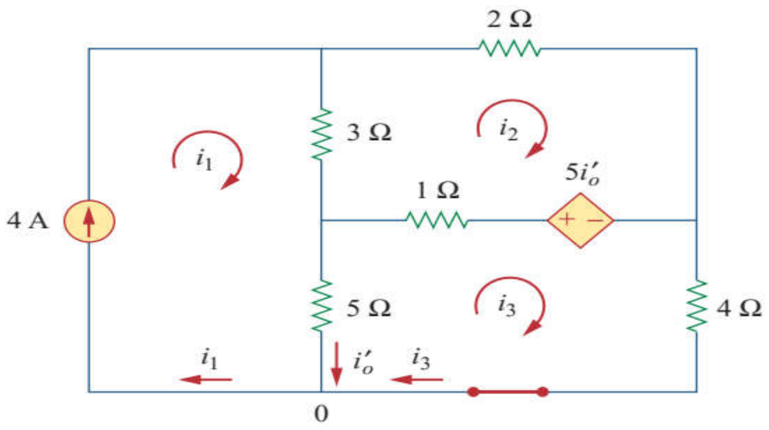
$$i_o = i'_o + i''_o$$

Where

i_o' is due to the 4A current sourc

i_o'' is due to the 20v voltage source.

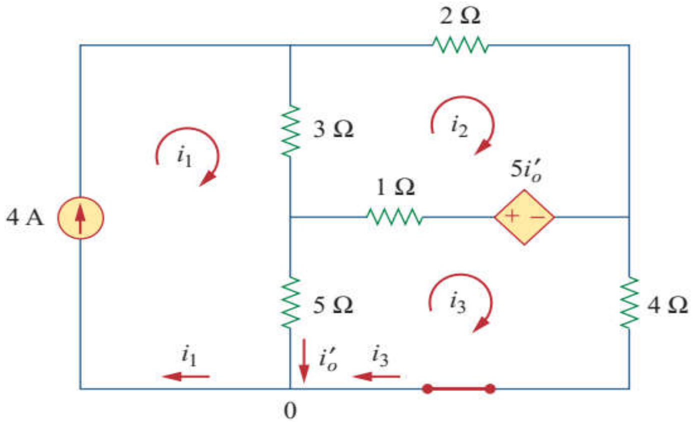
To obtain i_o', the 20v voltage source is turned off.



Now, apply mesh analysis in order to obtain i_o'.

For mesh 1:-

$$i_1 = 4\text{ A} \qquad ...(1)$$



For mesh 2:-

$$-3i_1 + 6i_2 - 1i_3 - 5i_o' = 0 \qquad ...(2)$$

For mesh 3:-

$$-5i_1 - 1i_2 + 10i_3 + 5i_o' = 0 \qquad ...(3)$$

At node o,

$$i_3 = i_1 - i_o' = 4 - i_o' \qquad ...(4)$$

Sub. 1, and 4 into 2

$$3i_2 - 2i'_o = 8$$

$$i_2 + 5i'_o = 20$$

which can be solved to get

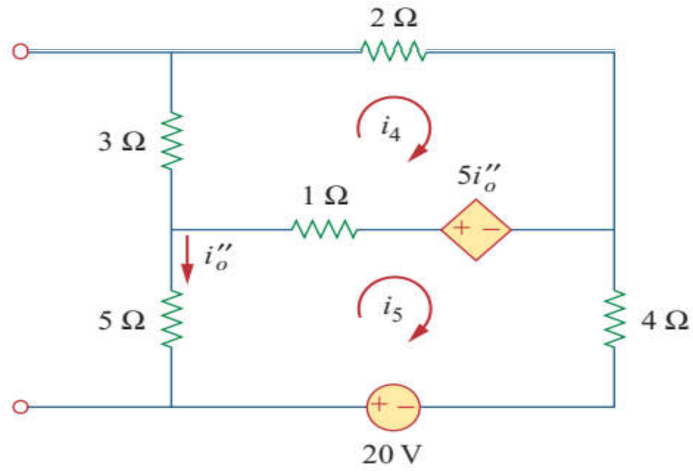
$$i'_o = \frac{52}{17} \text{ A}$$

To obtain i_o'', the 4A current source is turned off.

$$6i_4 - i_5 - 5i''_o = 0$$

$$-i_4 + 10i_5 - 20 + 5i''_o = 0$$

But $i_5 = -i''_o$.



$$6i_4 - 4i_o'' = 0$$

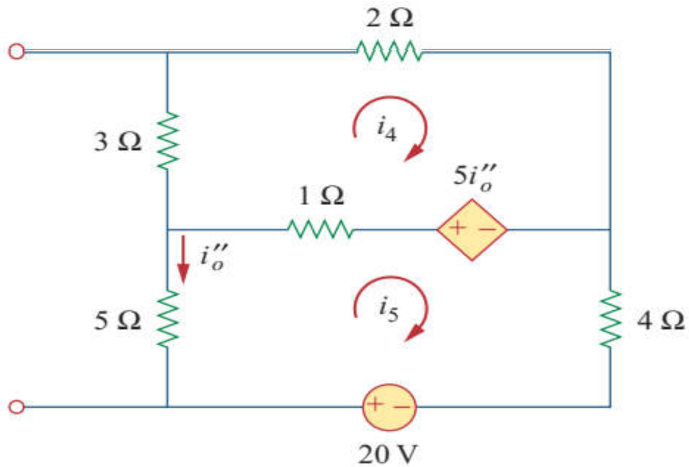
$$i_4 + 5i_o'' = -20$$

which we solve to get

$$i_o'' = -\frac{60}{17}\,\text{A}$$

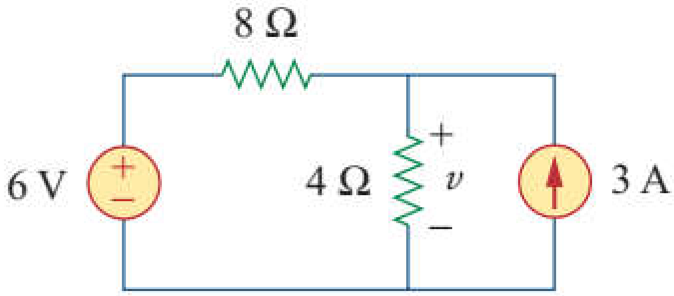
$$i_o = i_o' + i_o''$$

$$i_o = -\frac{8}{17} = -0.4706\ \text{A}$$

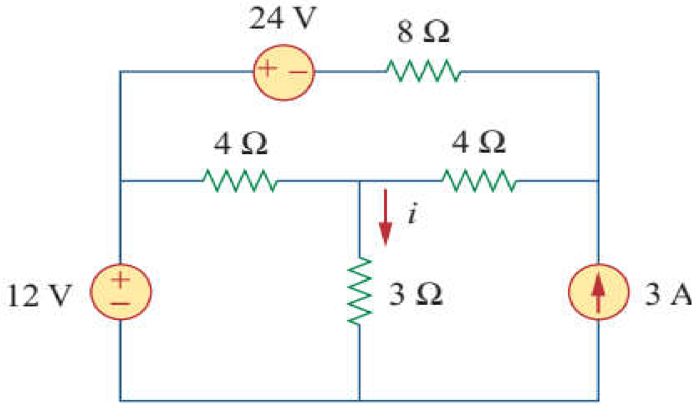


HW

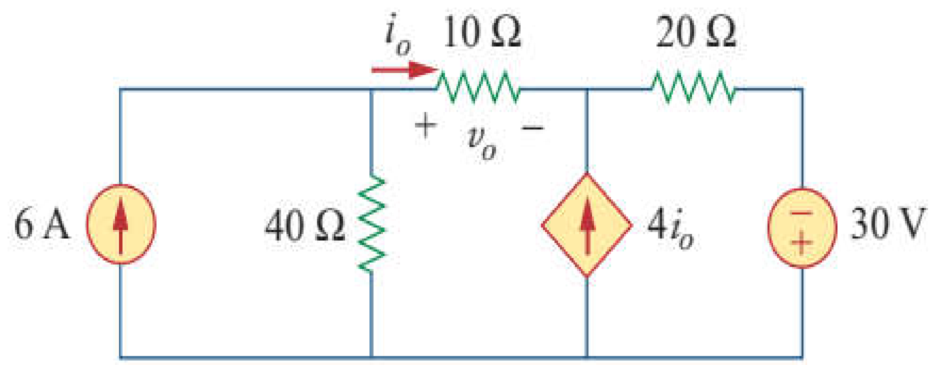
1- Use superposition theorem to find the voltage v in the circuit shown below.



2- For the circuit shown below, find the value of i, using superposition theorem.



3- Use superposition theorem, find the value of v_o and i_o in the circuit shown below.



4- **Referring to the circuit of Fig. 5.5*a*, determine the maximum *positive* current to which the source I_X can be set before any resistor exceeds its power rating and overheats.**