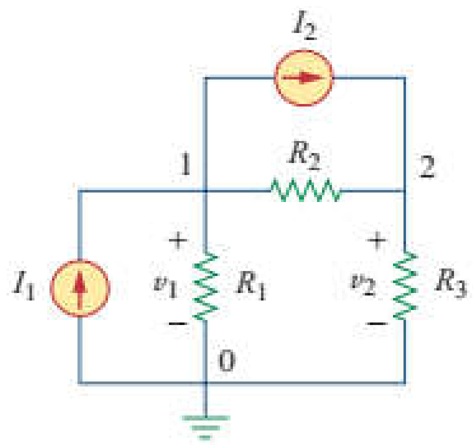
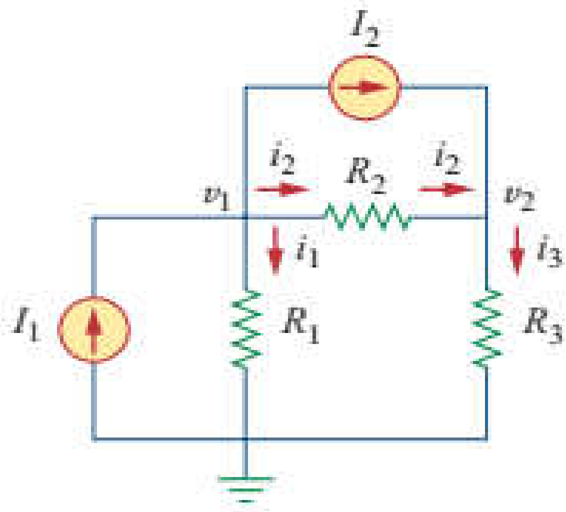
Nodal Analysis

1- Determine the number of nodes within the network.
2- Label each remaining node with a subscripted value of voltage: V_1, V_2 , or V_a, V_b, and so on.
3- Define one of the nodes as a reference node (that is, a point of zero potential or ground).
4- Apply Kirchhoff's current law at each node except the reference node.
5- Solve the resulting equations for the nodal voltages

Now,consider for example the circuit of figure shown below



Steps: (1-3)



V_3=0

<u>At node (1),</u> Applying KCL:

$$\mathrm{I}_1 - i_1 - i_2 - I_2 = 0 \quad \ldots (1)$$

$$i_1 = \frac{v_1 - v_3}{R_1}, \qquad i_2 = \frac{v_1 - v_2}{R_2},$$

v_3=0;

$$i_1 = \frac{v_1}{R_1}$$

$$\mathrm{I}_1 - \left(\frac{v_1}{R_1}\right) - \left(\frac{v_1 - v_2}{R_2}\right) - \mathrm{I}_2 = 0 \quad \ldots (2)$$

At node (2),

$$\mathrm{I}_2 + i_2 - i_3 = 0$$

$$I_2 + \left(\frac{v_1 - v_2}{R_2}\right) - \left(\frac{v_2 - v_3}{R_3}\right) = 0 \quad \dots (3)$$

Simplifying the two equations (2 and 3) then:

$$I_1 - I_2 = v_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - \frac{1}{R_2}v_2 \quad \dots (4)$$

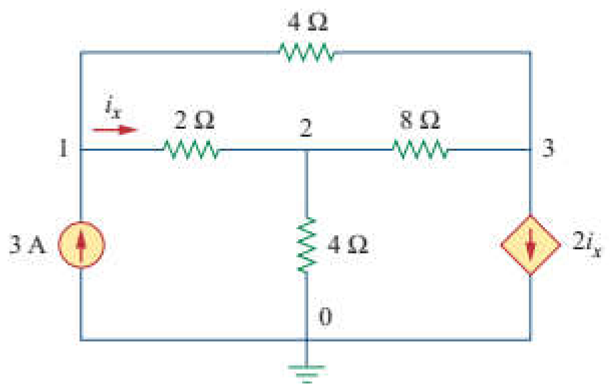
$$I_2 = v_2\left(\frac{1}{R_2} + \frac{1}{R_3}\right) - \frac{1}{R_2} \quad \dots (5)$$

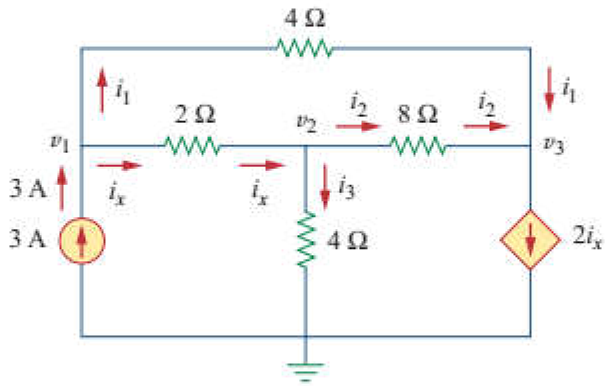
Note: the variables are v_1, v_2.

هذه الصيغة تسمى (Format Approach)

So, solving equations 4 and 5 simultaneously to find the values of v_1, v_2 .

Example: Calculate the node voltages of figure shown below:





Solution:$[12 = 3v_1 - 3v_3 + 6v_1 - 6v_2] * \frac{1}{3}$

At node 1,

$$3 = i_1 + i_x \quad \Rightarrow \quad 3 = \frac{v_1 - v_3}{4} + \frac{v_1 - v_2}{2}$$

Multiplying by 4 and rearranging terms, we get

$$3v_1 - 2v_2 - v_3 = 12$$

At node 2,

$$i_x = i_2 + i_3 \quad \Rightarrow \quad \frac{v_1 - v_2}{2} = \frac{v_2 - v_3}{8} + \frac{v_2 - 0}{4}$$

Multiplying by 8 and rearranging terms, we get

$$-4v_1 + 7v_2 - v_3 = 0$$

At node 3,

$$i_1 + i_2 = 2i_x \quad \Rightarrow \quad \frac{v_1 - v_3}{4} + \frac{v_2 - v_3}{8} = \frac{2(v_1 - v_2)}{2}$$

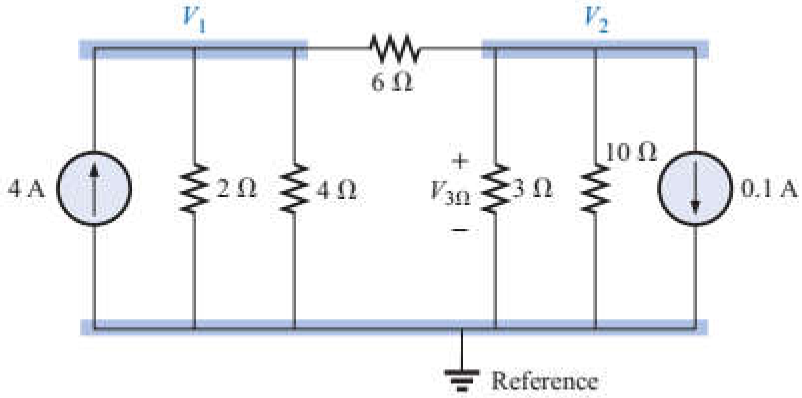
Multiplying by 8, rearranging terms, and dividing by 3, we get

$$2v_1 - 3v_2 + v_3 = 0$$

Solving the three equations, then

$v_1 = 4.8$ V, $v_2 = 2.4$ V, $v_3 = -2.4$ V

Example:- Find the voltage across the 3-Ω resistor of figure below, Using nodal analysis.



Solution:

$$\left(\frac{1}{2\,\Omega}+\frac{1}{4\,\Omega}+\frac{1}{6\,\Omega}\right)V_1-\left(\frac{1}{6\,\Omega}\right)V_2=+4\text{ A}$$

$$\left(\frac{1}{10\,\Omega}+\frac{1}{3\,\Omega}+\frac{1}{6\,\Omega}\right)V_2-\left(\frac{1}{6\,\Omega}\right)V_1=-0.1\text{ A}$$

$$\frac{11}{12}V_1-\frac{1}{6}V_2=4$$

$$-\frac{1}{6}V_1+\frac{3}{5}V_2=-0.1$$

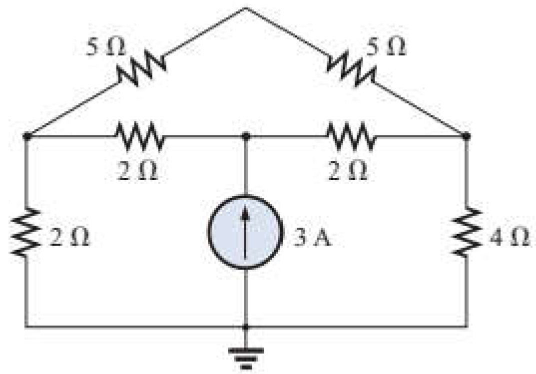
$$11V_1-2V_2=+48$$

$$-5V_1+18V_2=-3$$

Solving these equations:

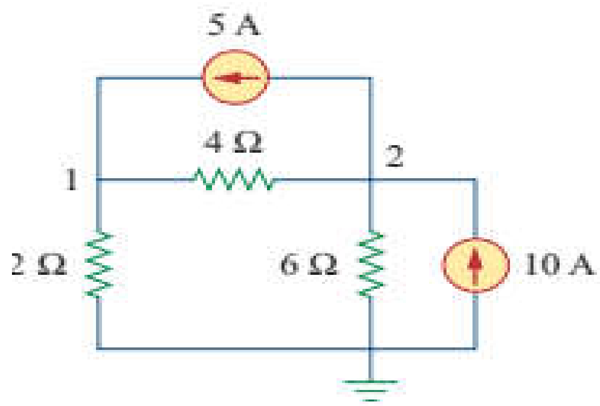
V_2=1.101volt.

HW/using Nodal analysis , determine the potential across the 4Ω resistor in figure below.



2-

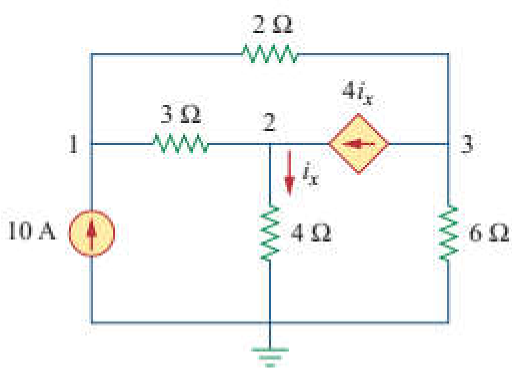
Example 3.1 Calculate the node voltages in the circuit shown in Fig. 3.3(a).



Find the voltages at the three nonreference nodes in the circuit of Fig. 3.6.

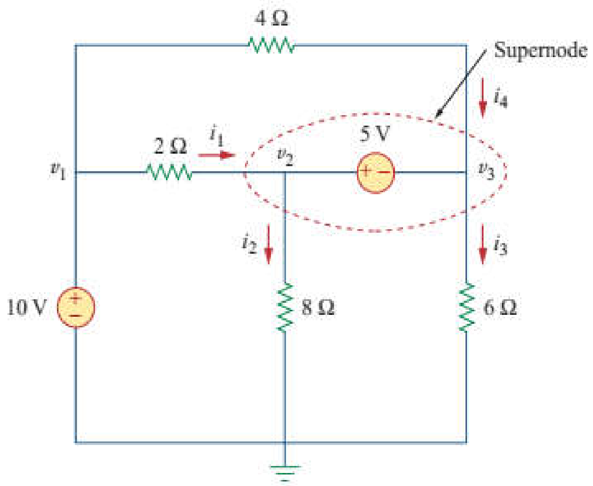
3-

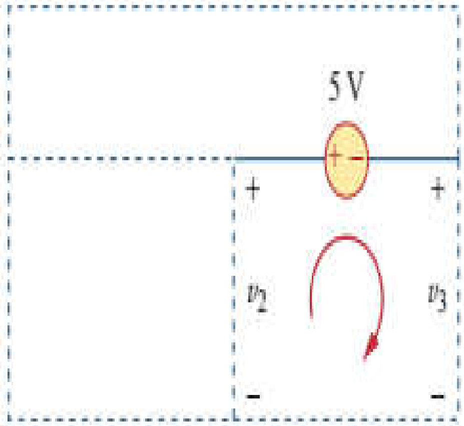
Answer: $v_1 = 80$ V, $v_2 = -64$ V, $v_3 = 156$ V.



Nodal Analysis with Voltage Sources

Example: for the circuit shown below, find the values of v_1, v_2, and v_3.





CASE 1 If a voltage source is connected between the reference node and a nonreference node, we simply set the voltage at the nonreference node equal to the voltage of the voltage source. In Fig. 3.7, for example,

$$v_1 = 10 \text{ V} \tag{3.10}$$

CASE 2 If the voltage source (dependent or independent) is connected between two nonreference nodes, the two nonreference nodes

form a *generalized node* or *supernode*;

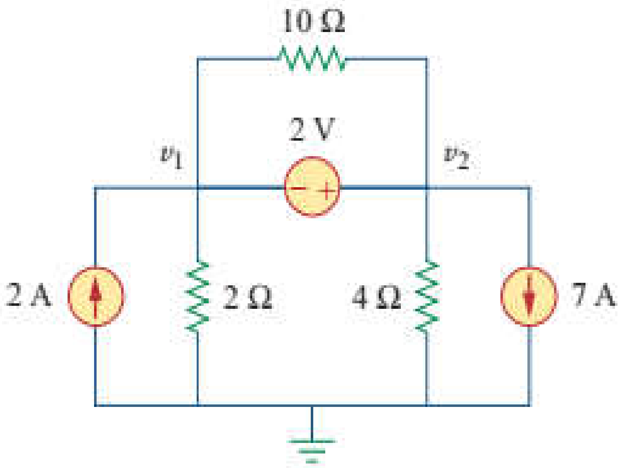
A **supernode** is formed by enclosing a (dependent or independent) voltage source connected between two nonreference nodes and any elements connected in parallel with it.

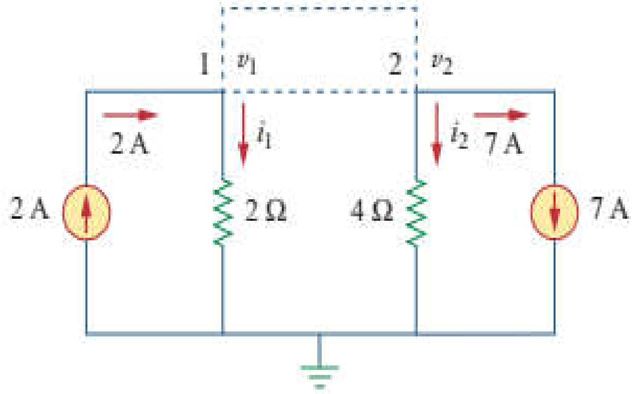
$$i_1 + i_4 = i_2 + i_3$$

$$\frac{v_1 - v_2}{2} + \frac{v_1 - v_3}{4} = \frac{v_2 - 0}{8} + \frac{v_3 - 0}{6}$$

$$-v_2 + 5 + v_3 = 0 \qquad \Rightarrow \qquad v_2 - v_3 = 5$$

Example:- for the circuit shown below, determine the node voltages.





Solutions:-

The supernode contains the 2-V source, nodes 1 and 2, and the 10-Ω resistor. Applying KCL to the supernode as shown in Fig. 3.10(a) gives

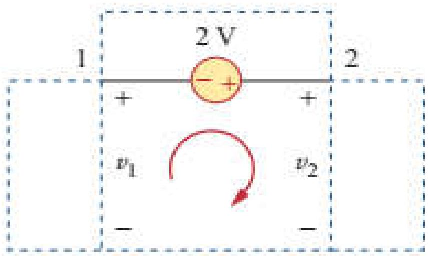
$$2 = i_1 + i_2 + 7$$

Expressing i_1 and i_2 in terms of the node voltages

$$2 = \frac{v_1 - 0}{2} + \frac{v_2 - 0}{4} + 7 \quad \Rightarrow \quad 8 = 2v_1 + v_2 + 28$$

$$v_2 = -20 - 2v_1$$

To get the relation between v_1 and v_2,apply KVL to the circuit in figure below.



$$-v_1 - 2 + v_2 = 0 \quad \Rightarrow \quad v_2 = v_1 + 2$$

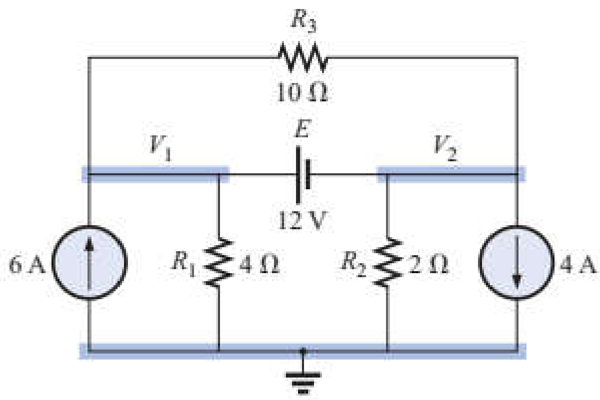
$$v_2 = v_1 + 2 = -20 - 2v_1$$

or

$$3v_1 = -22 \quad \Rightarrow \quad v_1 = -7.333 \text{ V}$$

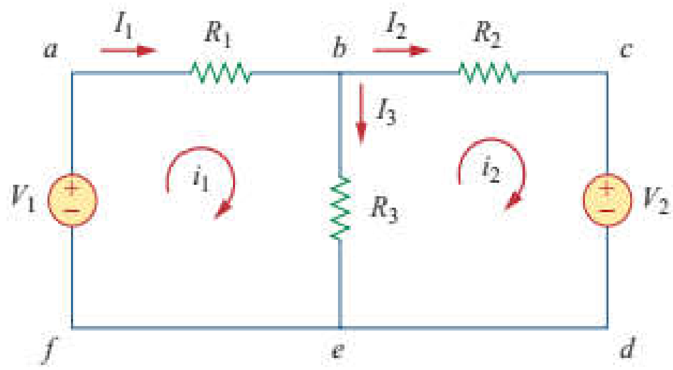
and $v_2 = v_1 + 2 = -5.333$ V. Note that the 10-Ω resistor does not make any difference because it is connected across the supernode.

HW//For the circuit shown below, determine the node voltages.



Mesh Analysis

A **mesh** is a loop which does not contain any other loops within it.



- In the figure shown below, for example, paths(abefa) and (bcdeb) are meshes, but path (abcdefa) is not a mesh.
- The current through a mesh is known as mesh current.
- Applying KVL to find the mesh currents in a given circuit.

Steps to Determine Mesh Currents:

1. Assign mesh currents $i_1, i_2, \ldots, i_n$ to the n meshes.
2. Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.
3. Solve the resulting n simultaneous equations to get the mesh currents.

$$-V_1 + R_1 i_1 + R_3(i_1 - i_2) = 0$$

or

$$(R_1 + R_3)i_1 - R_3 i_2 = V_1$$

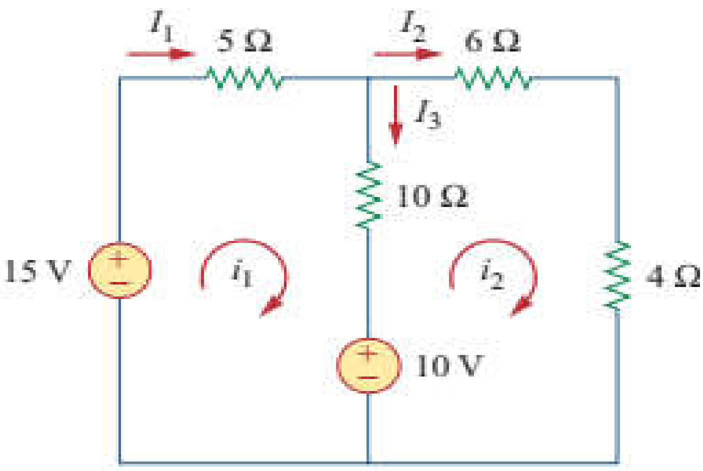
For mesh 2, applying KVL gives

$$R_2 i_2 + V_2 + R_3(i_2 - i_1) = 0$$

or

$$-R_3 i_1 + (R_2 + R_3)i_2 = -V_2$$

Example: For the circuit shown below, find the branch currents I_1, I_2, and I_3 using mesh analysis



Solution:

We first obtain the mesh currents using KVL. For mesh 1,

$$-15 + 5i_1 + 10(i_1 - i_2) + 10 = 0$$

or

$$3i_1 - 2i_2 = 1 \qquad \text{...(1)}$$

For mesh 2,

$$6i_2 + 4i_2 + 10(i_2 - i_1) - 10 = 0$$

or

$$i_1 = 2i_2 - 1 \qquad \text{...(2)}$$

Simplifying the two equations 1, and 2:

$$3i_1 - 2i_2 = 1 \quad \text{...}(3)$$

$$i_1 - 2i_2 = -1 \quad \text{...}(4)$$

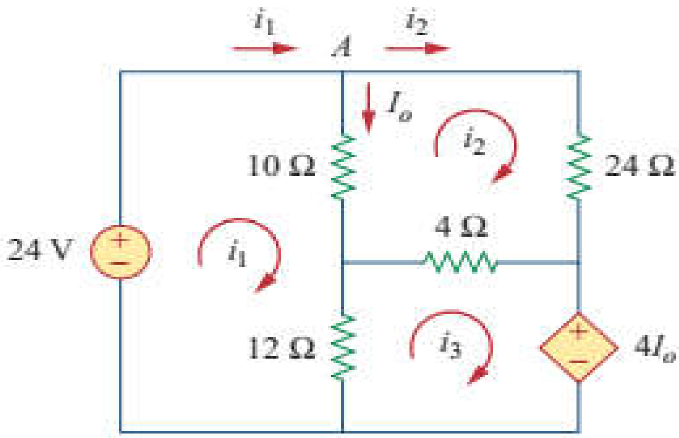
....بالطرح

$$2i_1 = 2$$

i_1=1A

i_2=1A

Example: use mesh analysis to find the current I_o for the circuit of figure shown below.



Solution:

We apply KVL to the three meshes in turn. For mesh 1,

$$-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0$$

or

$$11i_1 - 5i_2 - 6i_3 = 12 \qquad \textbf{...(1)}$$

For mesh 2,

$$24i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0$$

or

$$-5i_1 + 19i_2 - 2i_3 = 0 \qquad \textbf{...(2)}$$

For mesh 3,

$$4I_o + 12(i_3 - i_1) + 4(i_3 - i_2) = 0 \qquad \textbf{...(3)}$$

But at node A, $I_o = i_1 - i_2$, so that Sub into eq. 3

$$4(i_1 - i_2) + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$$

or

$$-i_1 - i_2 + 2i_3 = 0 \qquad \textbf{...(4)}$$

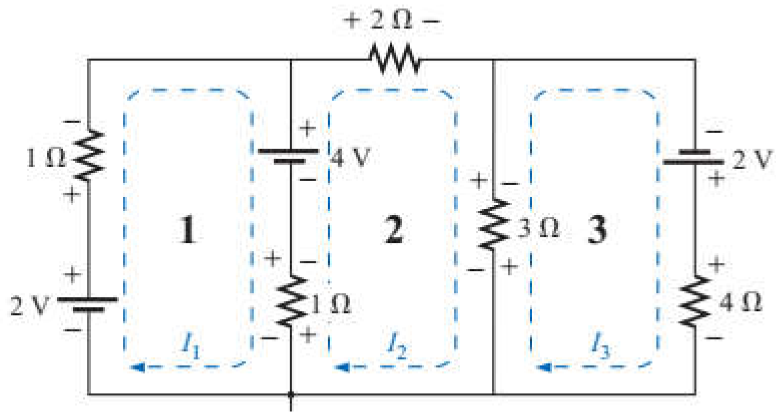
Solving equations 1, 2, and 4;

i_1=2.25A i_2=0.75A i_3=1.5A

$I_o = i_1 - i_2$

=2.25-0.75=1.5A

Example: Write a mesh equations for the network of figure below:(Format approach)



I_1 does not pass through an element mutual with I_3.

$$
\begin{aligned}
I_1 &: \quad (1\,\Omega + 1\,\Omega)I_1 - (1\,\Omega)I_2 + 0 = 2\text{ V} - 4\text{ V} \\
I_2 &: \quad (1\,\Omega + 2\,\Omega + 3\,\Omega)I_2 - (1\,\Omega)I_1 - (3\,\Omega)I_3 = 4\text{ V} \\
I_3 &: \quad (3\,\Omega + 4\,\Omega)I_3 - (3\,\Omega)I_2 + 0 = 2\text{ V}
\end{aligned}
$$

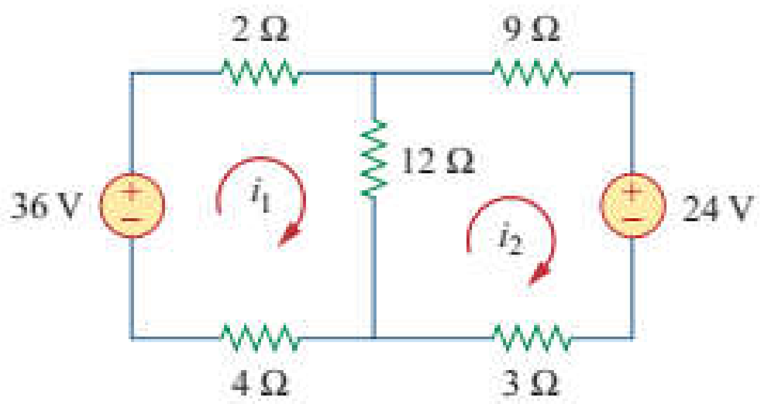
I_3 does not pass through an element mutual with I_1.

Summing terms yields

$$
\begin{aligned}
2I_1 - I_2 + 0 &= -2 \\
6I_2 - I_1 - 3I_3 &= 4 \\
7I_3 - 3I_2 + 0 &= 2
\end{aligned}
$$

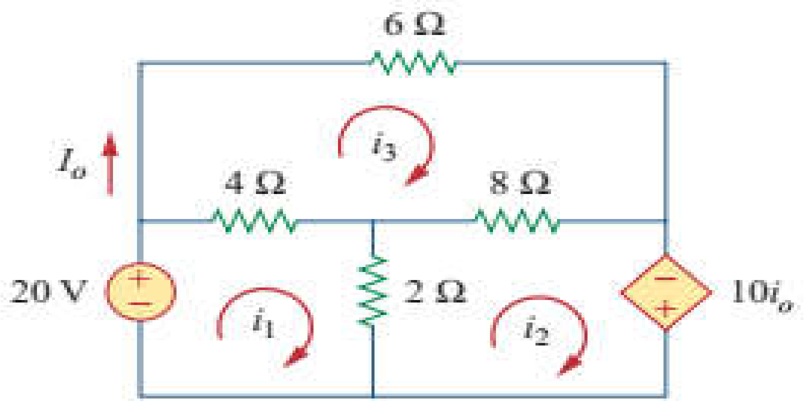
HW//

1- Calculate the mesh currents $i_1$and i_2 of the circuit shown below.



Answer: $i_1 = 2$ A, $i_2 = 0$ A.

2- Using mesh analysis, find the current I_o in the circuit shown below:



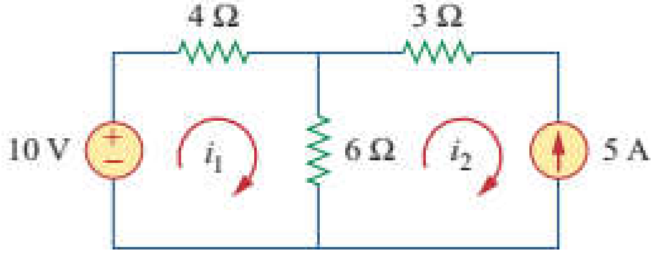
Answer: −5 A.

3- Write a mesh equations for the network of figure of the HW 2 above. (Format approach)

Mesh Analysis with current source

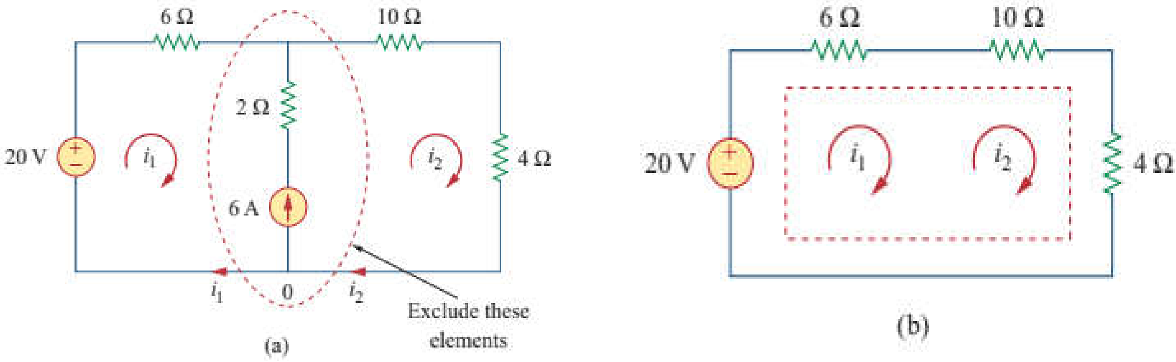
Case : when a current source exists only in one mesh:

For example see figure below. Set i_2=-5A and write a mesh equation for the other mesh.



$$-10 + 4i_1 + 6(i_1 - i_2) = 0 \quad \Rightarrow \quad i_1 = -2\text{ A}$$

Case 2: when a current source exists between two meshes: consider the circuit in figure (a).creating a supermesh by excluding the current source and any elements connected in series with it as shown in figure (b).



A **supermesh** results when two meshes have a (dependent or independent) current source in common.

$$-20 + 6i_1 + 10i_2 + 4i_2 = 0$$

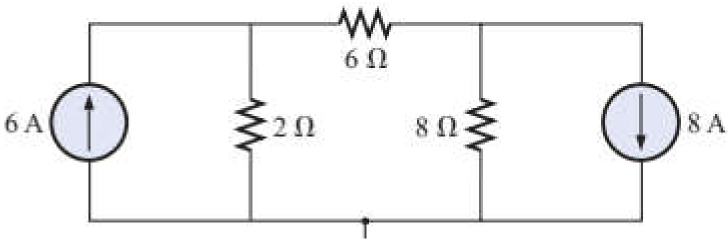
$$6i_1 + 14i_2 = 20$$

$$i_2 - i_1 = 6$$

$$i_2 = i_1 + 6$$

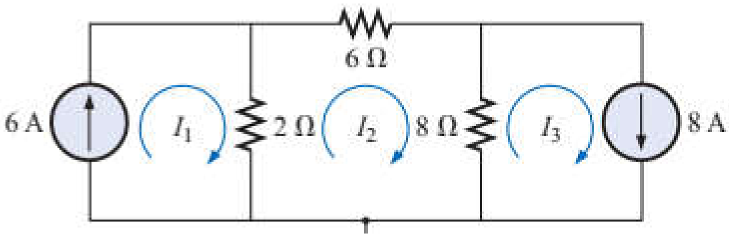
$i_1 = -3.2\ A, \qquad i_2 = 2.8\ A$

Example: using mesh analysis, determine the currents for the network of figure below.

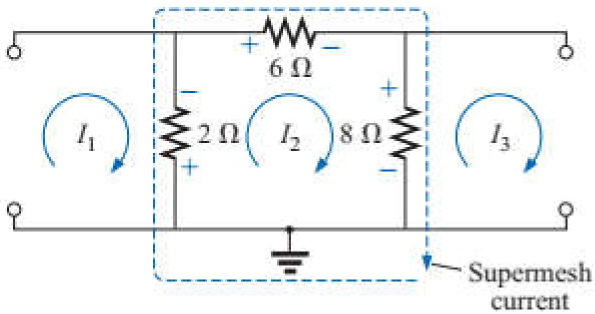


Solution:

The mesh currents are defined in figure (a). The current sources are removed, and the single supermesh path is defined in figure (b)



a



b

$$I_1 = 6A \quad ...(1)$$

$$I_2 = 8A \quad ...(2)$$

$$2(I_2 - I_1) + 6I_2 + 8(I_2 - I_3) = 0 \quad ...(3)$$

results in the following solutions:

$$2I_1 - 16I_2 + 8I_3 = 0$$

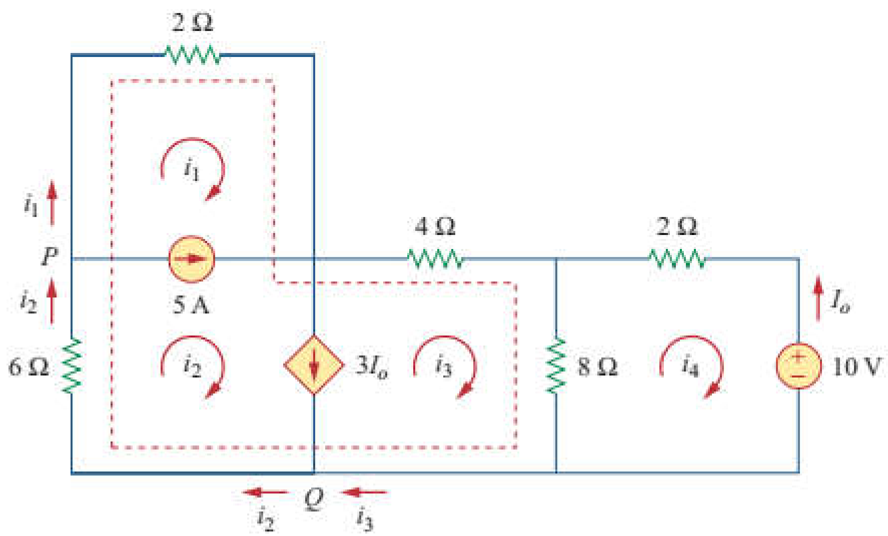
$$2(6\text{ A}) - 16I_2 + 8(8\text{ A}) = 0$$

and $$I_2 = \frac{76\text{ A}}{16} = \mathbf{4.75\text{ A}}$$

Then $$I_{2\Omega}\downarrow = I_1 - I_2 = 6\text{ A} - 4.75\text{ A} = \mathbf{1.25\text{ A}}$$

and $$I_{8\Omega}\uparrow = I_3 - I_2 = 8\text{ A} - 4.75\text{ A} = \mathbf{3.25\text{ A}}$$

Example: for the circuit of figure below, find i_1 to i_4.using mesh analysis.



$$2i_1 + 4i_3 + 8(i_3 - i_4) + 6i_2 = 0$$

or

$$i_1 + 3i_2 + 6i_3 - 4i_4 = 0$$

For the independent current source, we apply KCL to node P:

$$i_2 = i_1 + 5$$

For the dependent current source, we apply KCL to node Q:

$$i_2 = i_3 + 3I_o$$

But $I_o = -i_4$, hence,

$$i_2 = i_3 - 3i_4$$

Applying KVL in mesh 4,

$$2i_4 + 8(i_4 - i_3) + 10 = 0$$

or

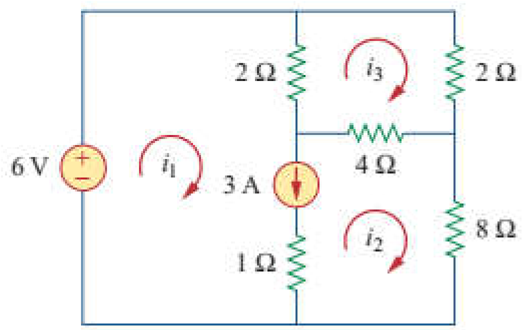
$$5i_4 - 4i_3 = -5$$

$i_1 = -7.5$ A, $i_2 = -2.5$ A, $i_3 = 3.93$ A, $i_4 = 2.143$ A

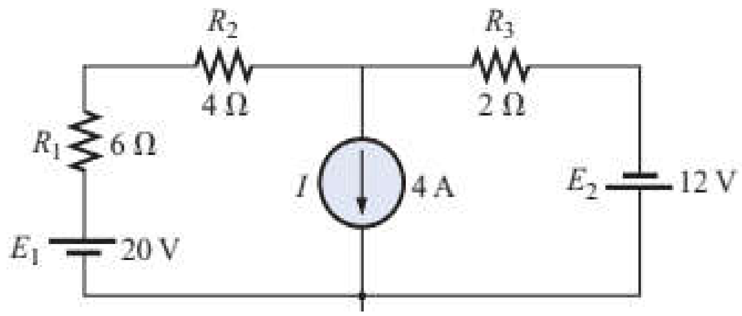
HW//

1- Use mesh analysis to determine i_1, i_2, and i_3 in figure below.

Answer: $i_1 = 3.474$ A, $i_2 = 0.4737$ A, $i_3 = 1.1052$ A.



2- Use mesh analysis, determine the current of figure below.



[Ans. I_1=3.33A, I_2=-0.67A]

47. For the circuit of Fig. 4.76, determine the value of resistor **X** if $i_2 = 2.273$ A.